CORDIC Algorithm – Theory Note

The CORDIC (Coordinate Rotation Digital Computer) algorithm is an efficient iterative method for computing trigonometric, hyperbolic, exponential, logarithmic, and other transcendental functions using only shifts, additions, and table lookups. It avoids hardware multipliers, making it well-suited for FPGA and embedded DSP implementations.

Historical Background

The CORDIC method was introduced by Jack E. Volder in 1959 to simplify real-time navigation computations in airborne systems. Later, it was extended to handle a wide range of functions, and became widely used in calculators, DSPs, and FPGA-based signal processing designs.

Principle of Operation

The algorithm works by rotating a vector through a sequence of predefined angles. At each iteration, the rotation direction is chosen to reduce the remaining angle toward zero. Each rotation uses a shift-and-add operation corresponding to a tangent value of 2^{-i}. The sequence converges to the target angle, and the vector's coordinates approach the sine and cosine values of that angle.

Rotation Mode Equations

Given an initial vector (x■, y■) and angle z■, each iteration i updates the values as follows:

$x_{i+1} = x_i - d_i \times y_i \times 2^{-i}$
$y_{i+1} = y_i + d_i \times x_i \times 2^{-i}$
$z_{i+1} = z_i - d_i \times \mathrm{atan}(2^{-i})$

where d_i = +1 or -1 depending on the sign of the residual angle z_i.

Scaling Factor

Each micro-rotation scales the vector length by $\sqrt{(1 + 2^{-2i})}$. After N iterations, the cumulative gain approaches a constant $K \approx 0.607252935$ for large N. In hardware designs, this scaling can be compensated once at the end or incorporated into the initial vector.

Applications

- Generation of sine and cosine waveforms
- Phase rotation in complex signal processing
- Digital modulation/demodulation
- Polar-to-Cartesian and Cartesian-to-Polar conversions
- Calculators and function generators

The CORDIC algorithm's efficiency in shift-add operations makes it a timeless choice for resource-constrained systems. Modern FPGA-based oscillators, mixers, and vector rotators still rely on CORDIC for precision and speed without sacrificing silicon area.