INFO-6205

Fall 2023 Project

Fibonacci = Pythagoras

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Introduction:

Aim of the project:

- In this project we use the Fibonacci numbers video by mathologer as a base with the following goals in mind.
- We are to build a tree of Pythagorean triples. For example, 3, 4, 5 or 5, 12, 13..
- The next part would include building a (lazy) tree of Rational Number pairs as described in the video. Given a sequence of branches from the root of the tree {3,4,5}, you should be able to return either a Pythagorean triple or a pair of Rationonals.
- Implement rational multiplication by traversing the tree appropriately (provide the formula in terms of tree branching).

Program:

1. Base code:

Here in the fibonacci number we make use of the notation using u and v as mentioned by the professor. Thus in each node of the tree we will have two variables u and v with the Fibonacci box given by: [v-u, u , v , v+u]

The base class for each node looks like this:

```
public class Node {

//    Value of that particular node
    public int v,u;
    public int level;
//    Three nodes representing the three branches of the tree
    public Node left;
    public Node mid;
    public Node right;
// Supplier for lazy printing of node
    private Supplier<String> rSupplier;
    private Supplier<String> pSupplier;
```

```
// Functional interfaces for lazy evaluation
    private Supplier<Double> aSupplier;
    private Supplier<Double> bSupplier;

//    Getter and setter methods

// Functional interfaces for lazy evaluation
    public Node() {
        // Initialize lazy printing of rational and pythagorean
        rSupplier = () -> "("+(this.v - this.u)+"/ "+(this.v + this.u)+","+u +"/"+v+")";
        pSupplier = () -> "{"+((this.v*this.v )- (this.u*this.u))+","+(2*this.u*this.v)+","+(this.u*(this.v+this.u)
+this.v*(this.v-this.u))+"}";

        // Initialize lazy evaluation for a and b of rational
        aSupplier = () -> (v - u) / (double) (v + u);
        bSupplier = () -> u / (double) v;
    }

    // Getter for lazy-printed rational
    public String getRationalLazy() {
        return rSupplier.get();
    }

    // Getter for lazy-evaluated rational
    public double getRationalALazy() {
        return aSupplier.get();
    }

    // Getter for lazy-evaluated rational
    public double getRationalBALazy() {
        return bSupplier.get();
    }

    // Getter for lazy-printed pythagorean
    public String getPythagoreanLazy() {
        return pSupplier.get();
    }
}
```

Here we make use of the supplier class for lazy evaluation of the Rational numbers an Pythagorean Triples.

Tree Traversal:

The goal of this is to traverse a tree given a array of string which represent a path on the tree and output the resultant Pythagorean triple or Rational numbers or Fibonacci box.

Code snippet:

```
public String pythagoreanTraversal(List<String> path){
        if (path == null || path.size() <= 0 || path.size() > height) return "";

        Node curNode = root;
        String res = "";
        for(String i : path){
            if(i == "l") curNode = curNode.left;
            else if (i == "m") curNode = curNode.mid;
            else if (i == "r") curNode = curNode.right;
        }

        res = "["+((curNode.v*curNode.v )- (curNode.u*curNode.u))+","+(2*curNode.u*curNode.v)+","+
(curNode.u*(curNode.v+curNode.u)+curNode.v*(curNode.v-curNode.u))+"]";

        return res;
    }
```

Generate the values of Pythagorean/Rational tree values :

To output the values of the tree we make use of a BFS approach. Printing the values level by level.

Code Snippet:

```
public void pythagoreanBFS(Node tree){

      res = "";
      if(tree == null) return ;
      List<Node> queue = new ArrayList<>();
      queue.add(tree);
      while(!queue.isEmpty()){
         Node temp = queue.remove(0);
         res = res + temp.getPythagoreanLazy()+", ";
         if(temp.getLeft() != null) {
            queue.add(temp.getLeft());
            queue.add(temp.getMid());
            queue.add(temp.getRight());
         }

      }

  }
```

Multiplication as a Traversal of the tree:

```
private boolean searchTree(Node node, int targetU, int targetV, StringBuilder path) {
      if (node == null) {
         return false;
      }

      if (node.u == targetU && node.v == targetV || (((node.v - node.u) == targetU) && ((node.v + node.u)
== targetU))){
         return true; // Found a matching node
      }

      // Check left subtree
      path.append("l");
```

```
        if (searchTree(node.getLeft(), targetU, targetV, path)) {
            return true;
        }
        path.deleteCharAt(path.length() - 1);

        // Check middle subtree
        path.append("m");
        if (searchTree(node.getMid(), targetU, targetV, path)) {
            return true;
        }
        path.deleteCharAt(path.length() - 1);

        // Check right subtree
        path.append("r");
        if (searchTree(node.getRight(), targetU, targetV, path)) {
            return true;
        }
        path.deleteCharAt(path.length() - 1);

        return false;
    }
```

2. Unit test:

We have added unit tests for testing the generation of the Fibonacci tree, Pythagorean Triples and Rational Number. We have also added the testing for Tree traversal given a path.

Fibonacci Test File:

```
@Test
    public void baseCase0(){

        FibonacciSquares test = new FibonacciSquares();
        test.generateSquares(0);
        String exp = "[1,1,2,3], ";
```

```
        assertEquals(exp,test.treeBFS());
        System.out.println("Expected: "+exp+"\nRecieved:"+test.treeBFS());
    }

    @Test
    public void baseCaseN(){

        FibonacciSquares test = new FibonacciSquares();
        test.generateSquares(3);
        String exp = "[1,1,2,3], [3,1,4,5], [3,2,5,7], [1,2,3,5], [5,1,6,7], [5,4,9,13], [3,4,7,11], [7,2,9,11],
[7,5,12,17], [3,5,8,13], [5,2,7,9], [5,3,8,11], [1,3,4,7], [7,1,8,9], [7,6,13,19], [5,6,11,17], [13,4,17,21],
[13,9,22,31], [5,9,14,23], [11,4,15,19], [11,7,18,25], [3,7,10,17], [11,2,13,15], [11,9,20,29], [7,9,16,25],
[17,5,22,27], [17,12,29,41], [7,12,19,31], [13,5,18,23], [13,8,21,29], [3,8,11,19], [9,2,11,13], [9,7,16,23],
[5,7,12,19], [11,3,14,17], [11,8,19,27], [5,8,13,21], [7,3,10,13], [7,4,11,15], [1,4,5,9], ";
        assertEquals(exp,test.treeBFS());
        System.out.println("Expected: "+exp+"\nRecieved:"+test.treeBFS());
    }
```

Pythagorean Triples:

```
@Test
    public void baseCase0(){

        PythagoreanTriples t = new PythagoreanTriples();
        String genRes = t.generateTriples(0);
        String exp = "{3,4,5}, ";
        assertEquals(genRes, exp);

    }

    @Test
    public void baseCaseN(){

        PythagoreanTriples t = new PythagoreanTriples();
        String genRes = t.generateTriples(3);
```

```
        String exp = "{3,4,5}, {15,8,17}, {21,20,29}, {5,12,13}, {35,12,37}, {65,72,97}, {33,56,65}, {77,36,85},
{119,120,169}, {39,80,89}, {45,28,53}, {55,48,73}, {7,24,25}, {63,16,65}, {133,156,205}, {85,132,157},
{273,136,305}, {403,396,565}, {115,252,277}, {209,120,241}, {275,252,373}, {51,140,149}, {165,52,173},
{319,360,481}, {175,288,337}, {459,220,509}, {697,696,985}, {217,456,505}, {299,180,349},
{377,336,505}, {57,176,185}, {117,44,125}, {207,224,305}, {95,168,193}, {187,84,205}, {297,304,425},
{105,208,233}, {91,60,109}, {105,88,137}, {9,40,41}, ";
        assertEquals(genRes, exp);

    }
```

Rational Numbers:

```
    @Test
    public void baseCase0(){

        RationalPairs p = new RationalPairs();
        String genRes = (p.generatePairs(0));
        String exp = "(1/3, 1/2), ";

        assertEquals(genRes, exp);
    }

    @Test
    public void baseCase1(){

        RationalPairs p = new RationalPairs();
        String genRes = (p.generatePairs(3));
        String exp = "(1/3, 1/2), (3/5, 1/4), (3/7, 2/5), (1/5, 2/3), (5/7, 1/6), (5/13, 4/9), (3/11, 4/7), (7/11, 2/9),
(7/17, 5/12), (3/13, 5/8), (5/9, 2/7), (5/11, 3/8), (1/7, 3/4), (7/9, 1/8), (7/19, 6/13), (5/17, 6/11), (13/21, 4/17),
(13/31, 9/22), (5/23, 9/14), (11/19, 4/15), (11/25, 7/18), (3/17, 7/10), (11/15, 2/13), (11/29, 9/20), (7/25,
9/16), (17/27, 5/22), (17/41, 12/29), (7/31, 12/19), (13/23, 5/18), (13/29, 8/21), (3/19, 8/11), (9/13, 2/11),
(9/23, 7/16), (5/19, 7/12), (11/17, 3/14), (11/27, 8/19), (5/21, 8/13), (7/13, 3/10), (7/15, 4/11), (1/9, 4/5), ";

        assertEquals(genRes, exp);
    }
```

Tree Traversal :

```
@Test
    public void baseCase0(){

        Traversal t = new Traversal(4);
        List<String> path = new ArrayList<>();
        path.add("l");
        path.add("m");
        String rec = t.getNode(path);
        String exp = "[13,9,4,5]";

        assertEquals(exp, rec);

    }

    @Test
    public void baseCase1(){

        Traversal t = new Traversal(4);
        List<String> path = new ArrayList<>();
        path.add("l");
        path.add("m");
        path.add("r");
        String rec = t.getNode(path);
        String exp = "[23,14,9,5]";

        assertEquals(exp, rec);

    }

    @Test
    public void errorCase0(){

        // Length of path > Height of gebnerated tree
        Traversal t = new Traversal(4);
```

```
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    path.add("r");
    path.add("l");
    path.add("m");
    path.add("r");
    String rec = t.getNode(path);
    String exp = "";

    assertEquals(exp, rec);

}

@Test
public void pythagoreanTraversalErrorTest(){

    // Length of path > Height of generated tree
    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    path.add("r");
    path.add("l");
    path.add("m");
    path.add("r");
    String rec = t.pythagoreanTraversal(path);
    String exp = "";

    assertEquals(exp, rec);

}

@Test
public void pythagoreanTraversalTest0(){
```

```
    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    String rec = t.pythagoreanTraversal(path);
    String exp = "[65,72,97]";

    assertEquals(exp, rec);

}

@Test
public void rationalTraversalErrorTest(){

    // Length of path > Height of generated tree
    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    path.add("r");
    path.add("l");
    path.add("m");
    path.add("r");
    String rec = t.rationalTraversal(path);
    String exp = "";

    assertEquals(exp, rec);

}

@Test
public void rationalTraversalTest0(){

    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
```

```
        path.add("m");
        String rec = t.rationalTraversal(path);
        String exp = "(5/13, 4/9)";

        assertEquals(exp, rec);

    }
```

Rational Number Multiplication:

```
@Test
    public void baseCase0(){

        FibonacciSquares test = new FibonacciSquares();
        test.generateSquares(6);
        PathSearcher searcher = new PathSearcher();
        String exp = "lllr";
        assertEquals(exp,searcher.searchPath(test.getTreeRoot(), 2, 3, 4, 5));
        System.out.println("Expected: "+exp+"\nRecieved:"+searcher.searchPath(test.getTreeRoot(), 2, 3, 4,
5));
    }

    @Test
    public void baseCaseN(){

        FibonacciSquares test = new FibonacciSquares();
        test.generateSquares(6);
        PathSearcher searcher = new PathSearcher();
        String exp = "lrl";
        assertEquals(exp,searcher.searchPath(test.getTreeRoot(), 1, 3, 4, 5));
        System.out.println("Expected: "+exp+"\nRecieved:"+searcher.searchPath(test.getTreeRoot(), 2, 3, 4,
5));
    }
```

Output and Graphs:

Output generated for 6 levels:

FibonacciSquares:

[1,1,2,3], [3,1,4,5], [3,2,5,7], [1,2,3,5], [5,1,6,7], [5,4,9,13], [3,4,7,11], [7,2,9,11], [7,5,12,17], [3,5,8,13], [5,2,7,9], [5,3,8,11], [1,3,4,7], [7,1,8,9], [7,6,13,19], [5,6,11,17], [13,4,17,21], [13,9,22,31], [5,9,14,23], [11,4,15,19], [11,7,18,25], [3,7,10,17], [11,2,13,15], [11,9,20,29], [7,9,16,25], [17,5,22,27], [17,12,29,41], [7,12,19,31], [13,5,18,23], [13,8,21,29], [3,8,11,19], [9,2,11,13], [9,7,16,23], [5,7,12,19], [11,3,14,17], [11,8,19,27], [5,8,13,21], [7,3,10,13], [7,4,11,15], [1,4,5,9], [9,1,10,11], [9,8,17,25], [7,8,15,23], [19,6,25,31], [19,13,32,45], [7,13,20,33], [17,6,23,29], [17,11,28,39], [5,11,16,27], [21,4,25,29], [21,17,38,55], [13,17,30,47], [31,9,40,49], [31,22,53,75], [13,22,35,57], [23,9,32,41], [23,14,37,51], [5,14,19,33], [19,4,23,27], [19,15,34,49], [11,15,26,41], [25,7,32,39], [25,18,43,61], [11,18,29,47], [17,7,24,31], [17,10,27,37], [3,10,13,23], [15,2,17,19], [15,13,28,41], [11,13,24,37], [29,9,38,47], [29,20,49,69], [11,20,31,51], [25,9,34,43], [25,16,41,57], [7,16,23,39], [27,5,32,37], [27,22,49,71], [17,22,39,61], [41,12,53,65], [41,29,70,99], [17,29,46,75], [31,12,43,55], [31,19,50,69], [7,19,26,45], [23,5,28,33], [23,18,41,59], [13,18,31,49], [29,8,37,45], [29,21,50,71], [13,21,34,55], [19,8,27,35], [19,11,30,41], [3,11,14,25], [13,2,15,17], [13,11,24,35], [9,11,20,31], [23,7,30,37], [23,16,39,55], [9,16,25,41], [19,7,26,33], [19,12,31,43], [5,12,17,29], [17,3,20,23], [17,14,31,45], [11,14,25,39], [27,8,35,43], [27,19,46,65], [11,19,30,49], [21,8,29,37], [21,13,34,47], [5,13,18,31], [13,3,16,19], [13,10,23,33], [7,10,17,27], [15,4,19,23], [15,11,26,37], [7,11,18,29], [9,4,13,17], [9,5,14,19], [1,5,6,11], [11,1,12,13], [11,10,21,31], [9,10,19,29], [25,8,33,41], [25,17,42,59], [9,17,26,43], [23,8,31,39], [23,15,38,53], [7,15,22,37], [31,6,37,43], [31,25,56,81], [19,25,44,69], [45,13,58,71], [45,32,77,109], [19,32,51,83], [33,13,46,59], [33,20,53,73], [7,20,27,47], [29,6,35,41], [29,23,52,75], [17,23,40,63], [39,11,50,61], [39,28,67,95], [17,28,45,73], [27,11,38,49], [27,16,43,59], [5,16,21,37], [29,4,33,37], [29,25,54,79], [21,25,46,71], [55,17,72,89], [55,38,93,131], [21,38,59,97], [47,17,64,81], [47,30,77,107], [13,30,43,73], [49,9,58,67], [49,40,89,129], [31,40,71,111], [75,22,97,119], [75,53,128,181], [31,53,84,137], [57,22,79,101], [57,35,92,127], [13,35,48,83], [41,9,50,59], [41,32,73,105], [23,32,55,87], [51,14,65,79], [51,37,88,125], [23,37,60,97], [33,14,47,61], [33,19,52,71], [5,19,24,43], [27,4,31,35], [27,23,50,73], [19,23,42,65], [49,15,64,79], [49,34,83,117], [19,34,53,87], [41,15,56,71], [41,26,67,93], [11,26,37,63], [39,7,46,53], [39,32,71,103], [25,32,57,89], [61,18,79,97], [61,43,104,147], [25,43,68,111], [47,18,65,83], [47,29,76,105], [11,29,40,69], [31,7,38,45], [31,24,55,79], [17,24,41,65], [37,10,47,57], [37,27,64,91], [17,27,44,71], [23,10,33,43], [23,13,36,49], [3,13,16,29], [19,2,21,23], [19,17,36,53], [15,17,32,49], [41,13,54,67], [41,28,69,97], [15,28,43,71], [37,13,50,63], [37,24,61,85], [11,24,35,59], [47,9,56,65], [47,38,85,123], [29,38,67,105], [69,20,89,109], [69,49,118,167], [29,49,78,127], [51,20,71,91], [51,31,82,113], [11,31,42,73], [43,9,52,61], [43,34,77,111], [25,34,59,93], [57,16,73,89], [57,41,98,139], [25,41,66,107], [39,16,55,71], [39,23,62,85], [7,23,30,53], [37,5,42,47], [37,32,69,101], [27,32,59,91], [71,22,93,115], [71,49,120,169], [27,49,76,125], [61,22,83,105], [61,39,100,139], [17,39,56,95], [65,12,77,89], [65,53,118,171], [41,53,94,147], [99,29,128,157], [99,70,169,239], [41,70,111,181], [75,29,104,133], [75,46,121,167], [17,46,63,109], [55,12,67,79], [55,43,98,141], [31,43,74,117], [69,19,88,107], [69,50,119,169], [31,50,81,131], [45,19,64,83], [45,26,71,97], [7,26,33,59], [33,5,38,43], [33,28,61,89], [23,28,51,79], [59,18,77,95], [59,41,100,141], [23,41,64,105], [49,18,67,85], [49,31,80,111], [13,31,44,75], [45,8,53,61], [45,37,82,119], [29,37,66,103], [71,21,92,113], [71,50,121,171], [29,50,79,129], [55,21,76,97], [55,34,89,123], [13,34,47,81], [35,8,43,51], [35,27,62,89], [19,27,46,73], [41,11,52,63], [41,30,71,101], [19,30,49,79], [25,11,36,47], [25,14,39,53], [3,14,17,31], [17,2,19,21], [17,15,32,47], [13,15,28,43], [35,11,46,57], [35,24,59,83], [13,24,37,61], [31,11,42,53], [31,20,51,71], [9,20,29,49], [37,7,44,51], [37,30,67,97], [23,30,53,83], [55,16,71,87], [55,39,94,133], [23,39,62,101], [41,16,57,73], [41,25,66,91], [9,25,34,59], [33,7,40,47], [33,26,59,85], [19,26,45,71], [43,12,55,67], [43,31,74,105], [19,31,50,81], [29,12,41,53], [29,17,46,63], [5,17,22,39], [23,3,26,29], [23,20,43,63], [17,20,37,57], [45,14,59,73], [45,31,76,107], [17,31,48,79], [39,14,53,67], [39,25,64,89], [11,25,36,61], [43,8,51,59], [43,35,78,113], [27,35,62,97], [65,19,84,103], [65,46,111,157], [27,46,73,119], [49,19,68,87], [49,30,79,109], [11,30,41,71], [37,8,45,53], [37,29,66,95], [21,29,50,79], [47,13,60,73], [47,34,81,115], [21,34,55,89], [31,13,44,57], [31,18,49,67], [5,18,23,41], [19,3,22,25], [19,16,35,51], [13,16,29,45], [33,10,43,53], [33,23,56,79], [13,23,36,59], [27,10,37,47], [27,17,44,61], [7,17,24,41], [23,4,27,31], [23,19,42,61], [15,19,34,53], [37,11,48,59], [37,26,63,89], [15,26,41,67], [29,11,40,51], [29,18,47,65], [7,18,25,43], [17,4,21,25], [17,13,30,43], [9,13,22,35], [19,5,24,29], [19,14,33,47], [9,14,23,37], [11,5,16,21], [11,6,17,23], [1,6,7,13], [13,1,14,15], [13,12,25,37], [11,12,23,35], [31,10,41,51], [31,21,52,73], [11,21,32,53], [29,10,39,49], [29,19,48,67], [9,19,28,47], [41,8,49,57], [41,33,74,107], [25,33,58,91], [59,17,76,93], [59,42,101,143], [25,42,67,109], [43,17,60,77], [43,26,69,95], [9,26,35,61], [39,8,47,55], [39,31,70,101], [23,31,54,85], [53,15,68,83], [53,38,91,129], [23,38,61,99], [37,15,52,67], [37,22,59,81], [7,22,29,51], [43,6,49,55], [43,37,80,117], [31,37,68,105], [81,25,106,131], [81,56,137,193], [31,56,87,143], [69,25,94,119],

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Pythagorean Triples for 6 levels:

{3,4,5}, {15,8,17}, {21,20,29}, {5,12,13}, {35,12,37}, {65,72,97}, {33,56,65}, {77,36,85}, {119,120,169}, {39,80,89}, {45,28,53}, {55,48,73}, {7,24,25}, {63,16,65}, {133,156,205}, {85,132,157}, {273,136,305}, {403,396,565}, {115,252,277}, {209,120,241}, {275,252,373}, {51,140,149}, {165,52,173}, {319,360,481}, {175,288,337}, {459,220,509}, {697,696,985}, {217,456,505}, {299,180,349}, {377,336,505}, {57,176,185}, {117,44,125}, {207,224,305}, {95,168,193}, {187,84,205}, {297,304,425}, {105,208,233}, {91,60,109}, {105,88,137}, {9,40,41}, {99,20,101}, {225,272,353}, {161,240,289}, {589,300,661}, {855,832,1193}, {231,520,569}, {493,276,565}, {663,616,905}, {135,352,377}, {609,200,641}, {1155,1292,1733}, {611,1020,1189}, {1519,720,1681}, {2325,2332,3293}, {741,1540,1709}, {943,576,1105}, {1173,1036,1565}, {165,532,557}, {513,184,545}, {931,1020,1381}, {451,780,901}, {975,448,1073}, {1525,1548,2173}, {517,1044,1165}, {527,336,625}, {629,540,829}, {69,260,269}, {285,68,293}, {615,728,953}, {407,624,745}, {1363,684,1525}, {2001,1960,2801}, {561,1240,1361}, {1075,612,1237}, {1425,1312,1937}, {273,736,785}, {999,320,1049}, {1917,2156,2885}, {1037,1716,2005}, {2665,1272,2953}, {4059,4060,5741}, {1275,2668,2957}, {1705,1032,1993}, {2139,1900,2861}, {315,988,1037}, {759,280,809}, {1357,1476,2005}, {637,1116,1285}, {1305,592,1433}, {2059,2100,2941}, {715,1428,1597}, {665,432,793}, {779,660,1021}, {75,308,317}, {221,60,229}, {455,528,697}, {279,440,521}, {851,420,949}, {1265,1248,1777}, {369,800,881}, {627,364,725}, {817,744,1105}, {145,408,433}, {391,120,409}, {765,868,1157}, {429,700,821}, {1161,560,1289}, {1755,1748,2477}, {539,1140,1261}, {777,464,905}, {987,884,1325}, {155,468,493}, {247,96,265}, {429,460,629}, {189,340,389}, {345,152,377}, {555,572,797}, {203,396,445}, {153,104,185}, {171,140,221}, {11,60,61}, {143,24,145}, {341,420,541}, {261,380,461}, {1025,528,1153}, {1475,1428,2053}, {387,884,965}, {897,496,1025}, {1219,1140,1669}, {259,660,709}, {1333,444,1405}, {2511,2800,3761}, {1311,2200,2561}, {3195,1508,3533}, {4905,4928,6953}, {1577,3264,3625}, {1947,1196,2285}, {2409,2120,3209}, {329,1080,1129}, {1189,420,1261}, {2175,2392,3233}, {1071,1840,2129}, {2379,1100,2621}, {3705,3752,5273}, {1241,2520,2809}, {1323,836,1565}, {1593,1376,2105}, {185,672,697}, {1073,264,1105}, {2291,2700,3541}, {1491,2300,2741}, {4895,2448,5473}, {7205,7068,10093}, {2037,4484,4925}, {3807,2176,4385}, {5029,4620,6829}, {949,2580,2749}, {3283,1044,3445}, {6321,7120,9521}, {3441,5680,6641}, {8925,4268,9893}, {13575,13568,19193}, {4247,8904,9865}, {5757,3476,6725}, {7239,6440,9689}, {1079,3360,3529}, {2419,900,2581}, {4305,4672,6353}, {2001,3520,4049}, {4029,1820,4421}, {6375,6512,9113}, {2231,4440,4969}, {2013,1316,2405}, {2343,1976,3065}, {215,912,937}, {945,248,977}, {1971,2300,3029}, {1235,1932,2293}, {3871,1920,4321}, {5733,5644,8045}, {1653,3604,3965}, {2911,1680,3361}, {3813,3484,5165}, {693,1924,2045}, {2067,644,2165}, {4017,4544,6065}, {2225,3648,4273}, {5917,2844,6565}, {8967,8944,12665}, {2775,5848,6473}, {3901,2340,4549}, {4935,4408,6617}, {759,2320,2441}, {1395,532,1493}, {2449,2640,3601}, {1105,1968,2257}, {2109,940,2309}, {3367,3456,4825}, {1207,2376,2665}, {989,660,1189}, {1127,936,1465}, {87,416,425}, {437,84,445}, {1007,1224,1585}, {735,1088,1313}, {2747,1404,3085}, {3977,3864,5545}, {1065,2408,2633}, {2331,1300,2669}, {3145,2928,4297}, {649,1680,1801}, {3055,1008,3217}, {5781,6460,8669}, {3045,5092,5933}, {7521,3560,8321}, {11523,11564,16325}, {3683,7644,8485}, {4641,2840,5441}, {5763,5084,7685}, {803,2604,2725}, {2623,936,2785}, {4773,5236,7085}, {2325,4012,4637}, {5073,2336,5585}, {7923,8036,11285}, {2675,5412,6037}, {2769,1760,3281}, {3315,2852,4373}, {371,1380,1429}, {1739,420,1789}, {3737,4416,5785}, {2457,3776,4505}, {8165,4092,9133}, {11999,11760,16801}, {3375,7448,8177}, {6405,3652,7373}, {8479,7800,11521}, {1615,4368,4657}, {5785,1848,6073}, {11115,12508,16733}, {6027,9964,11645}, {15543,7424,17225}, {23661,23660,33461}, {7421,15540,17221}, {9975,6032,11657}, {12525,11132,16757}, {1853,5796,6085}, {4345,1608,4633}, {7755,8428,11453}, {3627,6364,7325}, {7383,3344,8105}, {11661,11900,16661}, {4061,8100,9061}, {3735,2432,4457}, {4365,3692,5717}, {413,1716,1765}, {1419,380,1469}, {2937,3416,4505}, {1817,2856,3385}, {5605,2772,6253}, {8319,8200,11681}, {2415,5248,5777}, {4165,2412,4813}, {5439,4960,7361}, {975,2728,2897}, {2745,848,2873}, {5355,6068,8093}, {2987,4884,5725}, {8023,3864,8905}, {12141,12100,17141}, {3741,7900,8741}, {5335,3192,6217}, {6765,6052,9077}, {1053,3196,3365}, {1785,688,1913}, {3115,3348,4573}, {1387,2484,2845}, {2583,1144,2825}, {4141,4260,5941}, {1501,2940,3301}, {1175,792,1417}, {1325,1092,1717}, {93,476,485}, {357,76,365}, {799,960,1249}, {559,840,1009}, {1995,1012,2237}, {2905,2832,4057}, {793,1776,1945}, {1643,924,1885}, {2201,2040,3001}, {441,1160,1241}, {1887,616,1985}, {3589,4020,5389}, {1909,3180,3709}, {4785,2272,5297}, {7315,7332,10357}, {2323,4836,5365}, {2993,1824,3505}, {3731,3300,4981}, {531,1700,1781}, {1551,560,1649}, {2805,3068,4157}, {1349,2340,2701}, {2881,1320,3169}, {4515,4588,6437}, {1539,3100,3461}, {1537,984,1825}, {1827,1564,2405}, {195,748,773}, {667,156,685}, {1449,1720,2249}, {969,1480,1769}, {3285,1652,3677}, {4815,4712,6737}, {1343,2976,3265}, {2613,1484,3005}, {3471,3200,4721}, {671,1800,1921}, {2537,816,2665}, {4859,5460,7309}, {2619,4340,5069},

{6695,3192,7417}, {10205,10212,14437}, {3213,6716,7445}, {4263,2584,4985}, {5341,4740,7141}, {781,2460,2581}, {1961,720,2089}, {3515,3828,5197}, {1659,2900,3341}, {3431,1560,3769}, {5405,5508,7717}, {1869,3740,4181}, {1767,1144,2105}, {2077,1764,2725}, {205,828,853}, {475,132,493}, {969,1120,1481}, {585,928,1097}, {1749,860,1949}, {2607,2576,3665}, {767,1656,1825}, {1269,740,1469}, {1647,1496,2225}, {287,816,865}, {713,216,745}, {1403,1596,2125}, {795,1292,1517}, {2183,1056,2425}, {3293,3276,4645}, {1005,2132,2357}, {1479,880,1721}, {1885,1692,2533}, {301,900,949}, {425,168,457}, {731,780,1069}, {315,572,653}, {551,240,601}, {893,924,1285}, {333,644,725}, {231,160,281}, {253,204,325}, {13,84,85}, {195,28,197}, {481,600,769}, {385,552,673}, {1581,820,1781}, {2263,2184,3145}, {583,1344,1465}, {1421,780,1621}, {1943,1824,2665}, {423,1064,1145}, {2337,784,2465}, {4387,4884,6565}, {2275,3828,4453}, {5487,2584,6065}, {8437,8484,11965}, {2725,5628,6253}, {3311,2040,3889}, {4085,3588,5437}, {549,1820,1901}, {2145,752,2273}, {3939,4340,5861}, {1955,3348,3877}, {4399,2040,4849}, {6837,6916,9725}, {2277,4636,5165}, {2479,1560,2929}, {2997,2596,3965}, {357,1276,1325}, {2365,588,2437}, {5031,5920,7769}, {3255,5032,5993}, {10611,5300,11861}, {15633,15344,21905}, {4433,9744,10705}, {8211,4700,9461}, {10833,9944,14705}, {2033,5544,5905}, {6887,2184,7225}, {13277,14964,20005}, {7245,11948,13973}, {18857,9024,20905}, {28667,28644,40525}, {8955,18788,20813}, {12201,7360,14249}, {15355,13668,20557}, {2299,7140,7501}, {5015,1872,5353}, {8909,9660,13141}, {4125,7268,8357}, {8249,3720,9049}, {13067,13356,18685}, {4587,9116,10205}, {4089,2680,4889}, {4747,3996,6205}, {427,1836,1885}, {2173,564,2245}, {4551,5320,7001}, {2871,4480,5321}, {9075,4508,10133}, {13425,13208,18833}, {3857,8424,9265}, {6867,3956,7925}, {9009,8240,12209}, {1649,4560,4849}, {5063,1584,5305}, {9821,11100,14821}, {5421,8900,10421}, {14345,6888,15913}, {21755,21708,30733}, {6747,14204,15725}, {9417,5656,10985}, {11899,10620,15949}, {1819,5580,5869}, {3479,1320,3721}, {6125,6612,9013}, {2781,4940,5669}, {5369,2400,5881}, {8555,8772,12253}, {3051,6020,6749}, {2553,1696,3065}, {2923,2436,3805}, {235,1092,1117}, {1665,328,1697}, {3811,4620,5989}, {2755,4092,4933}, {10191,5200,11441}, {14773,14364,20605}, {3973,8964,9805}, {8591,4800,9841}, {11573,10764,15805}, {2373,6164,6605}, {10947,3604,11525}, {20737,23184,31105}, {10945,18288,21313}, {27117,12844,30005}, {41527,41664,58825}, {13255,27528,30553}, {16781,10260,19669}, {20855,18408,27817}, {2919,9440,9881}, {9315,3332,9893}, {16929,18560,25121}, {8225,14208,16417}, {17869,8220,19669}, {27927,28336,39785}, {9447,19096,21305}, {9709,6180,11509}, {11607,9976,15305}, {1287,4816,4985}, {5695,1368,5857}, {12261,14500,18989}, {8085,12412,14813}, {26961,13520,30161}, {39603,38804,55445}, {11123,24564,26965}, {21201,12080,24401}, {28083,25844,38165}, {5363,14484,15445}, {19397,6204,20365}, {37247,41904,56065}, {20175,33368,38993}, {51947,24804,57565}, {79097,79104,111865}, {24825,51968,57593}, {33291,20140,38909}, {41785,37128,55897}, {6169,19320,20281}, {14645,5412,15613}, {26159,28440,38641}, {12255,21488,24737}, {25019,11340,27469}, {39497,40296,56425}, {13737,27416,30665}, {12699,8260,15149}, {14857,12576,19465}, {1417,5856,6025}, {4543,1224,4705}, {9381,10900,14381}, {5781,9100,10781}, {17745,8768,19793}, {26355,25988,37013}, {7667,16644,18325}, {13137,7616,15185}, {17139,15620,23189}, {3059,8580,9109}, {8453,2604,8845}, {16511,18720,24961}, {9231,15080,17681}, {24875,11988,27613}, {37625,37488,53113}, {11577,24464,27065}, {16587,9916,19325}, {21049,18840,28249}, {3289,9960,10489}, {5429,2100,5821}, {9455,10152,13873}, {4191,7520,8609}, {7739,3420,8461}, {12425,12792,17833}, {4521,8840,9929}, {3483,2356,4205}, {3913,3216,5065}, {265,1392,1417}, {1505,312,1537}, {3395,4092,5317}, {2403,3596,4325}, {8687,4416,9745}, {12629,12300,17629}, {3429,7700,8429}, {7215,4048,8273}, {9685,8988,13213}, {1957,5124,5485}, {8611,2820,9061}, {16353,18304,24545}, {8673,14464,16865}, {21645,10268,23957}, {33111,33200,46889}, {10535,21912,24313}, {13485,8228,15797}, {16791,14840,22409}, {2375,7632,7993}, {7171,2580,7621}, {12993,14224,19265}, {6273,10864,12545}, {13485,6188,14837}, {21111,21440,30089}, {7175,14472,16153}, {7245,4628,8597}, {8631,7400,11369}, {935,3552,3673}, {3551,840,3649}, {7685,9108,11917}, {5109,7820,9341}, {17201,8640,19249}, {25235,24708,35317}, {7059,15620,17141}, {13617,7744,15665}, {18067,16644,24565}, {3475,9348,9973}, {12901,4140,13549}, {24735,27808,37217}, {13359,22120,25841}, {34251,16340,37949}, {52185,52208,73817}, {16409,34320,38041}, {21867,13244,25565}, {27417,24344,36665}, {4025,12648,13273}, {9877,3636,10525}, {17679,19240,26129}, {8319,14560,16769}, {17115,7772,18797}, {26985,27512,38537}, {9353,18696,20905}, {8763,5684,10445}, {10281,8720,13481}, {1001,4080,4201}, {2655,728,2753}, {5445,6308,8333}, {3317,5244,6205}, {10033,4944,11185}, {14931,14740,20981}, {4371,9460,10421}, {7345,4272,8497}, {9555,8692,12917}, {1683,4756,5045}, {4389,1340,4589}, {8607,9776,13025}, {4847,7896,9265}, {13195,6372,14653}, {19929,19840,28121}, {6105,12928,14297}, {8875,5292,10333}, {11289,10120,15161}, {1785,5368,5657}, {2709,1060,2909}, {4687,5016,6865}, {2047,3696,4225}, {3675,1612,4013}, {5929,6120,8521}, {2185,4248,4777}, {1595,1092,1933}, {1769,1440,2281}, {105,608,617}, {621,100,629}, {1495,1848,2377}, {1159,1680,2041}, {4611,2380,5189}, {6625,6408,9217}, {1729,3960,4321}, {4067,2244,4645}, {5537,5184,7585}, {1185,3008,3233}, {6231,2080,6569}, {11725,13068,17557}, {6109,10260,11941}, {14841,7000,16409}, {22795,22908,32317}, {7339,15180,16861},

{9017,5544,10585}, {11147,9804,14845}, {1515,4988,5213}, {5607,1976,5945}, {10269,11300,15269}, {5069,8700,10069}, {11305,5232,12457}, {17595,17812,25037}, {5883,11956,13325}, {6313,3984,7465}, {7611,6580,10061}, {891,3220,3341}, {5395,1332,5557}, {11505,13552,17777}, {7473,11536,13745}, {24477,12236,27365}, {36039,35360,50489}, {10199,22440,24649}, {19005,10868,21893}, {25095,23048,34073}, {4727,12864,13705}, {16241,5160,17041}, {31283,35244,47125}, {17043,28124,32885}, {44255,21168,49057}, {67301,67260,95149}, {21045,44132,48893}, {28575,17248,33377}, {35941,31980,48109}, {5365,16692,17533}, {11921,4440,12721}, {21203,23004,31285}, {9843,17324,19925}, {19775,8928,21697}, {31301,31980,44749}, {10965,21812,24413}, {9855,6448,11777}, {11461,9660,14989}, {1045,4452,4573}, {4819,1260,4981}, {10065,11752,15473}, {6321,9880,11729}, {19869,9860,22181}, {29415,28952,41273}, {8471,18480,20329}, {14973,8636,17285}, {19623,17936,26585}, {3575,9912,10537}, {10769,3360,11281}, {20915,23652,31573}, {11571,18980,22229}, {30719,14760,34081}, {46565,46452,65773}, {14421,30380,33629}, {20223,12136,23585}, {25573,22836,34285}, {3925,12012,12637}, {7313,2784,7825}, {12851,13860,18901}, {5811,10340,11861}, {11135,4968,12193}, {17765,18228,25453}, {6357,12524,14045}, {5247,3496,6305}, {5989,4980,7789}, {469,2220,2269}, {2679,520,2729}, {6157,7476,9685}, {4477,6636,8005}, {16665,8512,18713}, {24139,23460,33661}, {6475,14628,15997}, {14105,7872,16153}, {19019,17700,25981}, {3915,10148,10877}, {18285,6028,19253}, {34615,38688,51913}, {18247,30504,35545}, {45123,21364,49925}, {69121,69360,97921}, {22081,45840,50881}, {27875,17052,32677}, {34625,30552,46177}, {4833,15656,16385}, {15645,5588,16613}, {28455,31208,42233}, {13847,23904,27625}, {30163,13884,33205}, {47121,47800,67121}, {15921,32200,35921}, {16435,10452,19477}, {19665,16912,25937}, {2193,8176,8465}, {10057,2424,10345}, {21627,25564,33485}, {14235,21868,26093}, {47367,23744,52985}, {69597,68204,97445}, {19565,43188,47413}, 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Rational Numbers generated for 6 Levels:

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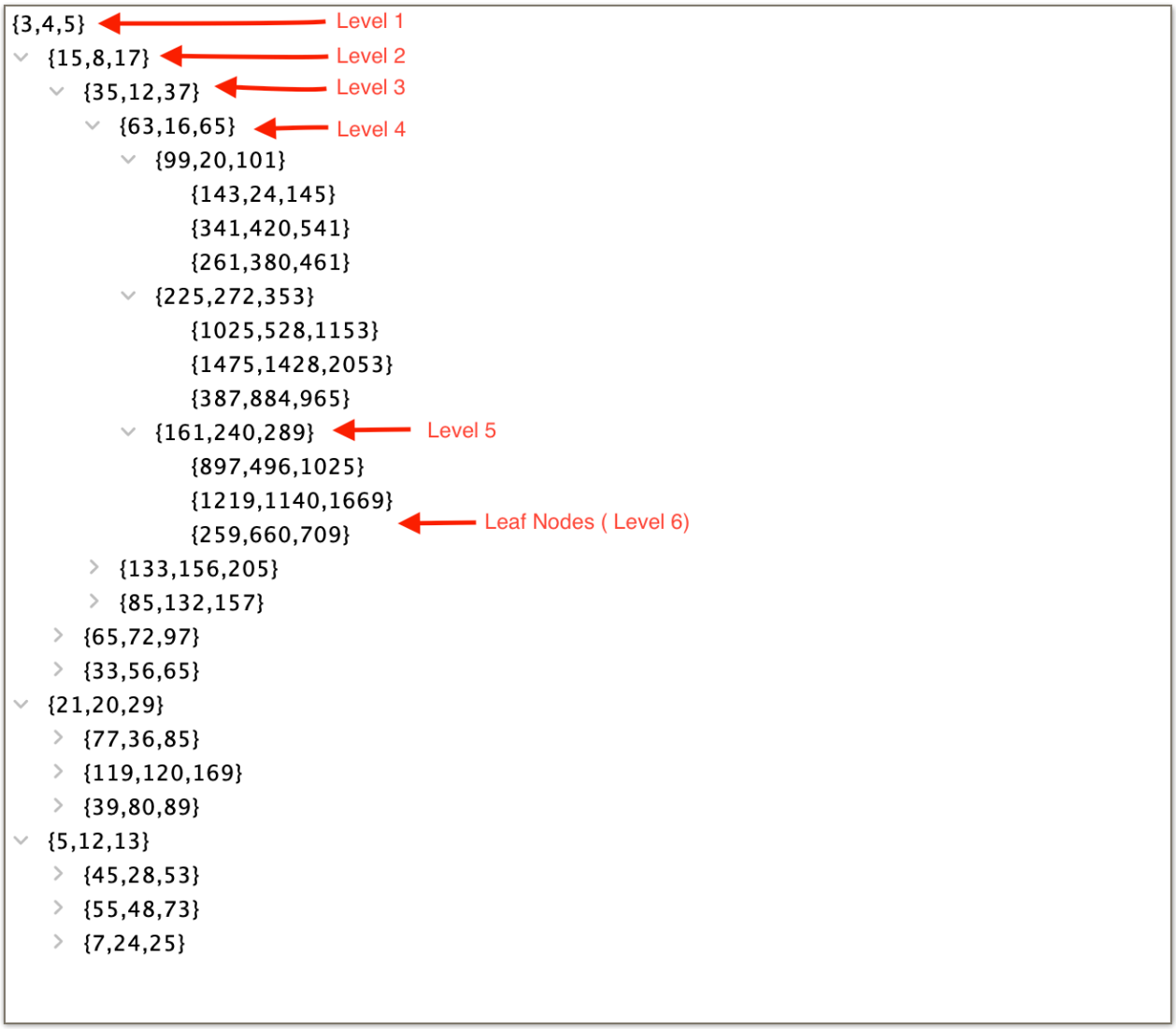
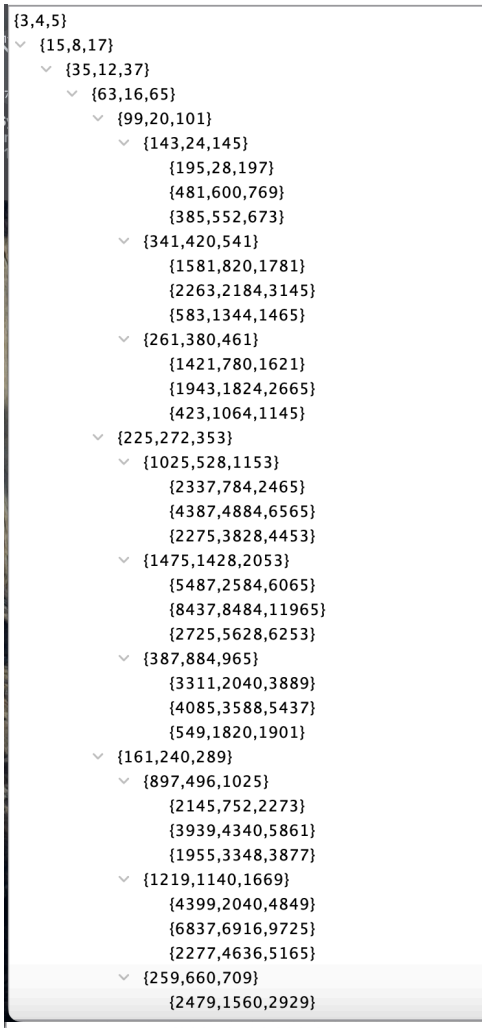
Tree Visualization:

Collapsed view of a 6 layer tree



Pythagorean Triple Tree (n = 6)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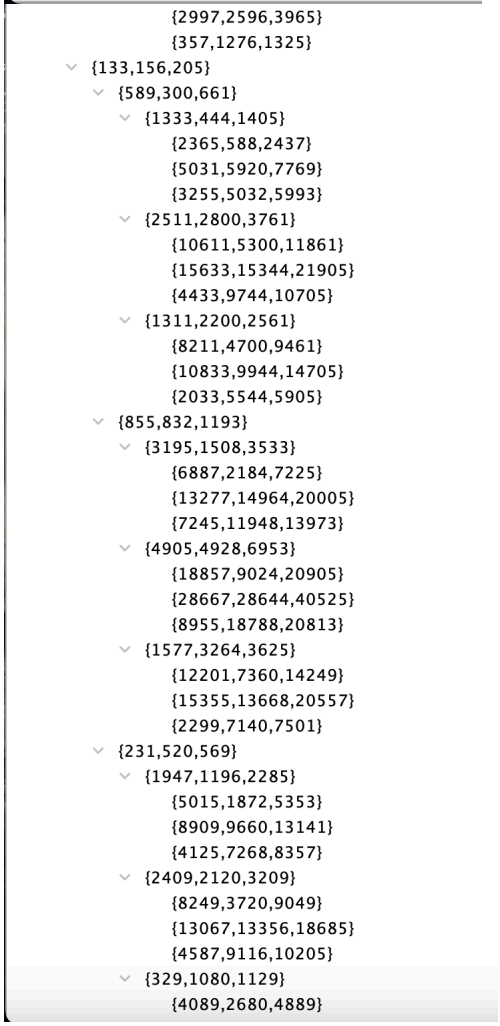
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Collapsed View:

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  - (5/7, 1/6)
    - (7/9, 1/8)
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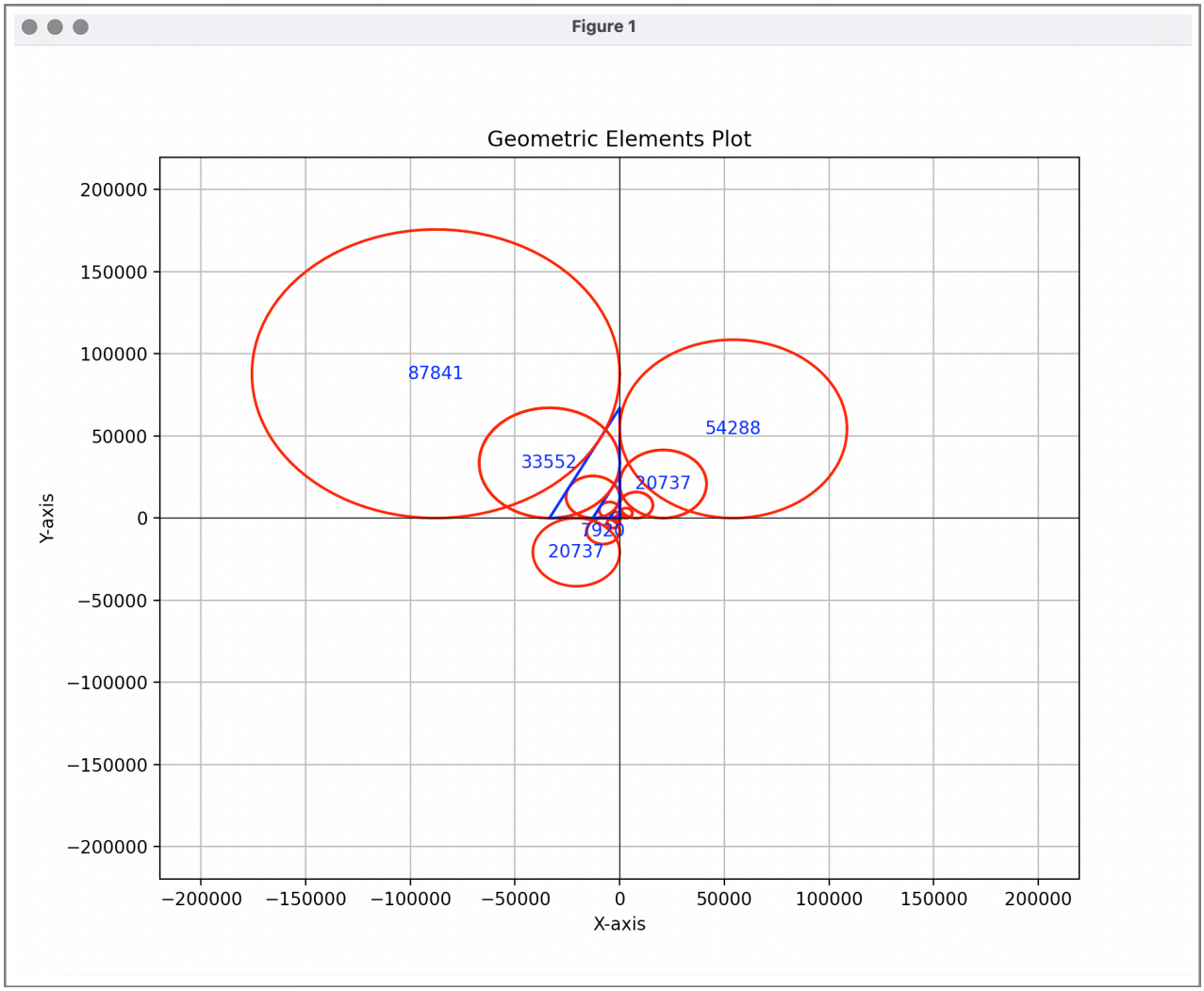
Exocircle Visualisation:



Bonus Task:

Induction Proof :

To be proven:

$$F_n*F_{n+2} + F_{n+1}*F_{n+3} = F_{2n+4}$$

Step 1:

Consider n = 2

$$F_2*F_4 + F_3*F_5$$
$$= 2*5+3*8$$
$$= 34 = F_8$$

Hence the equation to be proven holds true for our base case n = 2

Step 2:

Consider for some value n the equation holds true
Thus we can write it as:

$$F_n*F_{n+2} + F_{n+1}*F_{n+3} = F_{2n+4} \quad \text{———————- Eq 1}$$

To prove the equation we must show that if for some value of n the equation holds true then it holds true for n+1

Step 3:

The equation for n+1 :

$$F_{n+1}F_{n+3} + F_{n+2}F_{n+4} = F_{2n+6}. \quad \text{———————- Eq 2}$$

For any given value of n in the fibonacci series we know that :

$$F_n = F_{n-2}+F_{n-1} \quad \text{———————- Eq 3}$$

$$F_n^2 + 2*F_nF_{n+1} +2*F_{n+1}^2 = F_{2n+4} \quad \text{———————- Eq 4 **}$$

Using this in equation 2 LHS

$$F_{n+1}*(F_{n+1} +F_{n+2}) + F_{n+2}*(F_{n+2}+ F_{n+3})$$

$$= F_{n+1}^2 + F_{n+1}F_{n+2} + F_{n+2}^2 + F_{n+2}F_{n+3}$$

$$= F_{n+1}^2 + F_{n+1}F_{n+2} + F_{n+2}^2 + F_{n+2}*(F_{n+1} +F_{n+2})$$

$$= F_{n+1}^2 + 2*F_{n+1}F_{n+2} + 2*F_{n+2}^2$$

$$= F_{2n+6}$$

Induction:

Thus by induction we can prove that if Equation 1 holds true for some value of n then it is also true for n+1.

Given that this Equation holds true for n = 2 as shown in Step 1 we can also state that this holds true for all values of n > 2.

This has been proven in the publication **Fibonacci Meets Pythagoras. (Link provided by the mathologer) | **Link** (the equation had to be modified as our values for n begin from 0 while author uses 1)