<u>Proof by Induction</u>

To be proven:

$$F_n*F_{n+2} + F_{n+1}*F_{n+3} = F_{2n+4}$$

Step 1:

Consider n = 2

$$F_2*F_4 + F_3*F_5$$
$$= 2*5+3*8$$
$$= 34 = F_8$$

Hence the equation to be proven holds true for our base case n = 2

Step 2:

Consider for some value n the equation holds true
Thus we can write it as:

$$F_n*F_{n+2} + F_{n+1}*F_{n+3} = F_{2n+4} \qquad \text{—————————- Eq 1}$$

To prove the equation we must show that if for some value of n the equation holds true then it holds true for n+1

Step 3:

The equation for n+1 :

$$F_{n+1}F_{n+3} + F_{n+2}F_{n+4} = F_{2n+6.} \qquad \text{—————————- Eq 2}$$

For any given value of n in the fibonacci series we know that :

$$F_n = F_{n-2}+F_{n-1} \qquad \text{————————————- Eq 3}$$

$$F_n\text{^}2 + 2*F_nF_{n+1} +2*F_{n+1}\text{^}2 = F_{2n+4} \text{————————————- Eq 4 **}$$

Using this in equation 2 LHS

$$F_{n+1}*(F_{n+1} +F_{n+2}) + F_{n+2}*(F_{n+2}+ F_{n+3})$$
$$= F_{n+1}\text{^}2 + F_{n+1}F_{n+2} +F_{n+2}\text{^}2 +F_{n+2}F_{n+3}$$
$$= F_{n+1}\text{^}2 + F_{n+1}F_{n+2} +F_{n+2}\text{^}2 +F_{n+2}*(F_{n+1} +F_{n+2})$$
$$= F_{n+1}\text{^}2 + 2*F_{n+1}F_{n+2} +2*F_{n+2}\text{^}2$$
$$= F_{2n+6}$$

This has been proven in the publication **Fibonacci Meets Pythagoras. (Link provided by the mathologer) | **<u>Link</u>** (the equation had to be modified as our values for n begin from 0 while author uses 1)

Induction:

Thus by induction we can prove that if Equation 1 holds true for some value of n then it is also true for n+1.

Given that this Equation holds true for n = 2 as shown in Step 1 we can also state that this holds true for all values of n > 2.