INFO-6205
Fall 2023 Project

Fibonacci = Pythagoras

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Introduction:

Aim of the project:

- In this project we use the Fibonacci numbers video by mathologer as a base with the following goals in mind.
- We are to build a tree of Pythagorean triples. For example, 3, 4, 5 or 5, 12, 13..
- The next part would include building a (lazy) tree of Rational Number pairs as described in the video. Given a sequence of branches from the root of the tree {3,4,5}, you should be able to return either a Pythagorean triple or a pair of Rationonals.
- Implement rational multiplication by traversing the tree appropriately (provide the formula in terms of tree branching).

Program:

1. Base code:

Here in the fibonacci number we make use of the notation using u and v as mentioned by the professor. Thus in each node of the tree we will have two variables u and v with the Fibonacci box given by: [v-u, u , v , v+u]

The base class for each node looks like this:

```
public class Node {

//    Value of that particular node
    public int v,u;
    public int level;
//    Three nodes representing the three branches of the tree
    public Node left;
    public Node mid;
    public Node right;
// Supplier for lazy printing of node
    private Supplier<String> rSupplier;
    private Supplier<String> pSupplier;
```

```
// Functional interfaces for lazy evaluation
    private Supplier<Double> aSupplier;
    private Supplier<Double> bSupplier;

//    Getter and setter methods

// Functional interfaces for lazy evaluation
    public Node() {
        // Initialize lazy printing of rational and pythagorean
        rSupplier = () -> "("+(this.v - this.u)+"/ "+(this.v + this.u)+","+u +"/"+v+")";
        pSupplier = () -> "{"+((this.v*this.v )- (this.u*this.u))+","+(2*this.u*this.v)+","+(this.u*(this.v+this.u)
+this.v*(this.v-this.u))+"}";

        // Initialize lazy evaluation for a and b of rational
        aSupplier = () -> (v - u) / (double) (v + u);
        bSupplier = () -> u / (double) v;
    }

    // Getter for lazy-printed rational
    public String getRationalLazy() {
        return rSupplier.get();
    }

    // Getter for lazy-evaluated rational
    public double getRationalALazy() {
        return aSupplier.get();
    }

    // Getter for lazy-evaluated rational
    public double getRationalBALazy() {
        return bSupplier.get();
    }

    // Getter for lazy-printed pythagorean
    public String getPythagoreanLazy() {
        return pSupplier.get();
    }
}
```

Here we make use of the supplier class for lazy evaluation of the Rational numbers an Pythagorean Triples.

Tree Traversal:

The goal of this is to traverse a tree given a array of string which represent a path on the tree and output the resultant Pythagorean triple or Rational numbers or Fibonacci box.

Code snippet:

```
public String pythagoreanTraversal(List<String> path){
        if (path == null || path.size() <= 0 || path.size() > height) return "";

        Node curNode = root;
        String res = "";
        for(String i : path){
            if(i == "l") curNode = curNode.left;
            else if (i == "m") curNode = curNode.mid;
            else if (i == "r") curNode = curNode.right;
        }

        res = "["+((curNode.v*curNode.v )- (curNode.u*curNode.u))+","+(2*curNode.u*curNode.v)+","+
(curNode.u*(curNode.v+curNode.u)+curNode.v*(curNode.v-curNode.u))+"]";

        return res;
    }
```

Generate the values of Pythagorean/Rational tree values :

To output the values of the tree we make use of a BFS approach. Printing the values level by level.

Code Snippet:

```
public void pythagoreanBFS(Node tree){

        res = "";
        if(tree == null) return ;
        List<Node> queue = new ArrayList<>();
        queue.add(tree);
        while(!queue.isEmpty()){
            Node temp = queue.remove(0);
            res = res + temp.getPythagoreanLazy()+", ";
            if(temp.getLeft() != null) {
                queue.add(temp.getLeft());
                queue.add(temp.getMid());
                queue.add(temp.getRight());
            }

        }

    }
```

Multiplication as a Traversal of the tree: (Brute force method)

```
private boolean searchTree(Node node, int targetU, int targetV, StringBuilder path) {
        if (node == null) {
            return false;
        }

        if (node.u == targetU && node.v == targetV || (((node.v - node.u) == targetU) && ((node.v + node.u)
== targetU))){
            return true; // Found a matching node
        }

        // Check left subtree
        path.append("l");
```

```
        if (searchTree(node.getLeft(), targetU, targetV, path)) {
            return true;
        }
        path.deleteCharAt(path.length() - 1);

        // Check middle subtree
        path.append("m");
        if (searchTree(node.getMid(), targetU, targetV, path)) {
            return true;
        }
        path.deleteCharAt(path.length() - 1);

        // Check right subtree
        path.append("r");
        if (searchTree(node.getRight(), targetU, targetV, path)) {
            return true;
        }
        path.deleteCharAt(path.length() - 1);

        return false;
    }
```

Algorithm for multiplication as tree branching: (Non-brute force method)

```
// Let a and b denote the numerator and denominator of the product of two rational numbers.
fact = gcd(a, b)
a = a/fact
b = b/fact
path = stack()
while( a!=0 & b!=0 and b != 2*a) {
            if( 2*a < b ) {
                    if( (b-2*a) < a) {
                            temp = a
                            a = b - 2*a
                            b = temp
                            path.push(“m”)
```

```
                        }
                        else {
                                b = b - 2*a
                                path.push("l")
                        }
                }
                else {
                        temp = a
                        a =  2*a - b
                        b = temp
                        path.push("r")
                }

        }
while( !path.isEmpty()){
                System.out(path.pop)
}
```

Explanation of algorithm:

In this algorithm if we try to move from the root of the tree to our solution there would be multiple paths we need to explore before we reach the target value. This makes it impossible to explore as the size of the tree grows. Thus we start from the target value and move towards the root. This ensures that we are exploring the only route that is possible.

Steps of Algorithm:

1. Find the Greatest Common Divisor of the numerator and denominator obtained from the product of the two rational numbers. Lets use a and b to denote these respectively.
2. Divide both a and b with the gcd to reduce to its lowest form.

3. Initialise a stack
4. While the conditions a!=0 b!=0 and b!=2a repeat the following loop:
    1. If 2a<b
        1. If b-2a < a : push m to the stack and update a to b-2a and b to a
        2. Else if b-2a > a : push l on the stack and update b to b-2a
    2. Else if 2a>b : update a to 2a-b and b to a and push r to the stack
5. Once we exit the loop pop elements of the stack onto the display till stack is not empty.

2. Unit test:

We have added unit tests for testing the generation of the Fibonacci tree, Pythagorean Triples and Rational Number. We have also added the testing for Tree traversal given a path.

Fibonacci Test File:

```
@Test
   public void baseCase0(){

       FibonacciSquares test = new FibonacciSquares();
       test.generateSquares(0);
       String exp = "[1,1,2,3], ";
       assertEquals(exp,test.treeBFS());
       System.out.println("Expected: "+exp+"\nRecieved:"+test.treeBFS());
   }

   @Test
   public void baseCaseN(){

       FibonacciSquares test = new FibonacciSquares();
       test.generateSquares(3);
       String exp = "[1,1,2,3], [3,1,4,5], [3,2,5,7], [1,2,3,5], [5,1,6,7], [5,4,9,13], [3,4,7,11], [7,2,9,11], 
[7,5,12,17], [3,5,8,13], [5,2,7,9], [5,3,8,11], [1,3,4,7], [7,1,8,9], [7,6,13,19], [5,6,11,17], [13,4,17,21], 
[13,9,22,31], [5,9,14,23], [11,4,15,19], [11,7,18,25], [3,7,10,17], [11,2,13,15], [11,9,20,29], [7,9,16,25],
```

```
[17,5,22,27], [17,12,29,41], [7,12,19,31], [13,5,18,23], [13,8,21,29], [3,8,11,19], [9,2,11,13], [9,7,16,23],
[5,7,12,19], [11,3,14,17], [11,8,19,27], [5,8,13,21], [7,3,10,13], [7,4,11,15], [1,4,5,9], ";
        assertEquals(exp,test.treeBFS());
        System.out.println("Expected: "+exp+"\nRecieved:"+test.treeBFS());
    }
```

Pythagorean Triples:

```
@Test
    public void baseCase0(){

        PythagoreanTriples t = new PythagoreanTriples();
        String genRes = t.generateTriples(0);
        String exp = "{3,4,5}, ";
        assertEquals(genRes, exp);

    }

    @Test
    public void baseCaseN(){

        PythagoreanTriples t = new PythagoreanTriples();
        String genRes = t.generateTriples(3);
        String exp = "{3,4,5}, {15,8,17}, {21,20,29}, {5,12,13}, {35,12,37}, {65,72,97}, {33,56,65}, {77,36,85},
{119,120,169}, {39,80,89}, {45,28,53}, {55,48,73}, {7,24,25}, {63,16,65}, {133,156,205}, {85,132,157},
{273,136,305}, {403,396,565}, {115,252,277}, {209,120,241}, {275,252,373}, {51,140,149}, {165,52,173},
{319,360,481}, {175,288,337}, {459,220,509}, {697,696,985}, {217,456,505}, {299,180,349},
{377,336,505}, {57,176,185}, {117,44,125}, {207,224,305}, {95,168,193}, {187,84,205}, {297,304,425},
{105,208,233}, {91,60,109}, {105,88,137}, {9,40,41}, ";
        assertEquals(genRes, exp);

   }
```

Rational Numbers:

```
    @Test
```

```java
    public void baseCase0(){

        RationalPairs p = new RationalPairs();
        String genRes = (p.generatePairs(0));
        String exp = "(1/3, 1/2), ";

        assertEquals(genRes, exp);
    }

    @Test
    public void baseCase1(){

        RationalPairs p = new RationalPairs();
        String genRes = (p.generatePairs(3));
        String exp = "(1/3, 1/2), (3/5, 1/4), (3/7, 2/5), (1/5, 2/3), (5/7, 1/6), (5/13, 4/9), (3/11, 4/7), (7/11, 2/9), (7/17, 5/12), (3/13, 5/8), (5/9, 2/7), (5/11, 3/8), (1/7, 3/4), (7/9, 1/8), (7/19, 6/13), (5/17, 6/11), (13/21, 4/17), (13/31, 9/22), (5/23, 9/14), (11/19, 4/15), (11/25, 7/18), (3/17, 7/10), (11/15, 2/13), (11/29, 9/20), (7/25, 9/16), (17/27, 5/22), (17/41, 12/29), (7/31, 12/19), (13/23, 5/18), (13/29, 8/21), (3/19, 8/11), (9/13, 2/11), (9/23, 7/16), (5/19, 7/12), (11/17, 3/14), (11/27, 8/19), (5/21, 8/13), (7/13, 3/10), (7/15, 4/11), (1/9, 4/5), ";

        assertEquals(genRes, exp);
    }
```

Tree Traversal :

```java
@Test
    public void baseCase0(){

        Traversal t = new Traversal(4);
        List<String> path = new ArrayList<>();
        path.add("l");
        path.add("m");
        String rec = t.getNode(path);
        String exp = "[13,9,4,5]";

        assertEquals(exp, rec);
```

```
}

@Test
public void baseCase1(){

    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    path.add("r");
    String rec = t.getNode(path);
    String exp = "[23,14,9,5]";

    assertEquals(exp, rec);

}

@Test
public void errorCase0(){

    // Length of path > Height of gebnerated tree
    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    path.add("r");
    path.add("l");
    path.add("m");
    path.add("r");
    String rec = t.getNode(path);
    String exp = "";

    assertEquals(exp, rec);

}
```

```
@Test
public void pythagoreanTraversalErrorTest(){

    // Length of path > Height of generated tree
    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    path.add("r");
    path.add("l");
    path.add("m");
    path.add("r");
    String rec = t.pythagoreanTraversal(path);
    String exp = "";

    assertEquals(exp, rec);

}

@Test
public void pythagoreanTraversalTest0(){

    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    String rec = t.pythagoreanTraversal(path);
    String exp = "[65,72,97]";

    assertEquals(exp, rec);

}

@Test
public void rationalTraversalErrorTest(){
```

```
    // Length of path > Height of generated tree
    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    path.add("r");
    path.add("l");
    path.add("m");
    path.add("r");
    String rec = t.rationalTraversal(path);
    String exp = "";

    assertEquals(exp, rec);

  }

  @Test
  public void rationalTraversalTest0(){

    Traversal t = new Traversal(4);
    List<String> path = new ArrayList<>();
    path.add("l");
    path.add("m");
    String rec = t.rationalTraversal(path);
    String exp = "(5/13, 4/9)";

    assertEquals(exp, rec);

  }
```

Rational Number Multiplication:

```
@Test
  public void baseCase0(){
```

```
        FibonacciSquares test = new FibonacciSquares();
        test.generateSquares(6);
        PathSearcher searcher = new PathSearcher();
        String exp = "lllr";
        assertEquals(exp,searcher.searchPath(test.getTreeRoot(), 2, 3, 4, 5));
        System.out.println("Expected: "+exp+"\nRecieved:"+searcher.searchPath(test.getTreeRoot(), 2, 3, 4,
5));
    }

    @Test
    public void baseCaseN(){

        FibonacciSquares test = new FibonacciSquares();
        test.generateSquares(6);
        PathSearcher searcher = new PathSearcher();
        String exp = "lrl";
        assertEquals(exp,searcher.searchPath(test.getTreeRoot(), 1, 3, 4, 5));
        System.out.println("Expected: "+exp+"\nRecieved:"+searcher.searchPath(test.getTreeRoot(), 2, 3, 4,
5));
    }
```

Output and Graphs:

Output generated for 6 levels:

FibonacciSquares:

[1,1,2,3], [3,1,4,5], [3,2,5,7], [1,2,3,5], [5,1,6,7], [5,4,9,13], [3,4,7,11], [7,2,9,11], [7,5,12,17], [3,5,8,13], [5,2,7,9], [5,3,8,11], [1,3,4,7], [7,1,8,9], [7,6,13,19], [5,6,11,17], [13,4,17,21], [13,9,22,31], [5,9,14,23], [11,4,15,19], [11,7,18,25], [3,7,10,17], [11,2,13,15], [11,9,20,29], [7,9,16,25], [17,5,22,27], [17,12,29,41], [7,12,19,31], [13,5,18,23], [13,8,21,29], [3,8,11,19], [9,2,11,13], [9,7,16,23], [5,7,12,19], [11,3,14,17], [11,8,19,27], [5,8,13,21], [7,3,10,13], [7,4,11,15], [1,4,5,9], [9,1,10,11], [9,8,17,25], [7,8,15,23], [19,6,25,31], [19,13,32,45], [7,13,20,33], [17,6,23,29], [17,11,28,39], [5,11,16,27], [21,4,25,29], [21,17,38,55], [13,17,30,47], [31,9,40,49], [31,22,53,75], [13,22,35,57], [23,9,32,41], [23,14,37,51], [5,14,19,33], [19,4,23,27], [19,15,34,49], [11,15,26,41], [25,7,32,39], [25,18,43,61], [11,18,29,47], [17,7,24,31], [17,10,27,37], [3,10,13,23], [15,2,17,19], [15,13,28,41], [11,13,24,37], [29,9,38,47], [29,20,49,69], [11,20,31,51], [25,9,34,43], [25,16,41,57], [7,16,23,39], [27,5,32,37], [27,22,49,71], [17,22,39,61], [41,12,53,65], [41,29,70,99], [17,29,46,75], [31,12,43,55], [31,19,50,69], [7,19,26,45], [23,5,28,33], [23,18,41,59], [13,18,31,49], [29,8,37,45], [29,21,50,71], [13,21,34,55], [19,8,27,35], [19,11,30,41], [3,11,14,25], [13,2,15,17], [13,11,24,35], [9,11,20,31], [23,7,30,37], [23,16,39,55], [9,16,25,41], [19,7,26,33], [19,12,31,43], [5,12,17,29], [17,3,20,23], [17,14,31,45], [11,14,25,39], [27,8,35,43], [27,19,46,65], [11,19,30,49], [21,8,29,37], [21,13,34,47], [5,13,18,31], [13,3,16,19], [13,10,23,33], [7,10,17,27], [15,4,19,23], [15,11,26,37], [7,11,18,29], [9,4,13,17], [9,5,14,19], [1,5,6,11], [11,1,12,13], [11,10,21,31], [9,10,19,29], [25,8,33,41], [25,17,42,59], [9,17,26,43],

[23,8,31,39], [23,15,38,53], [7,15,22,37], [31,6,37,43], [31,25,56,81], [19,25,44,69], [45,13,58,71], [45,32,77,109], [19,32,51,83], [33,13,46,59],
[33,20,53,73], [7,20,27,47], [29,6,35,41], [29,23,52,75], [17,23,40,63], [39,11,50,61], [39,28,67,95], [17,28,45,73], [27,11,38,49], [27,16,43,59],
[5,16,21,37], [29,4,33,37], [29,25,54,79], [21,25,46,71], [55,17,72,89], [55,38,93,131], [21,38,59,97], [47,17,64,81], [47,30,77,107], [13,30,43,73],
[49,9,58,67], [49,40,89,129], [31,40,71,111], [75,22,97,119], [75,53,128,181], [31,53,84,137], [57,22,79,101], [57,35,92,127], [13,35,48,83], [41,9,50,59],
[41,32,73,105], [23,32,55,87], [51,14,65,79], [51,37,88,125], [23,37,60,97], [33,14,47,61], [33,19,52,71], [5,19,24,43], [27,4,31,35], [27,23,50,73],
[19,23,42,65], [49,15,64,79], [49,34,83,117], [19,34,53,87], [41,15,56,71], [41,26,67,93], [11,26,37,63], [39,7,46,53], [39,32,71,103], [25,32,57,89],
[61,18,79,97], [61,43,104,147], [25,43,68,111], [47,18,65,83], [47,29,76,105], [11,29,40,69], [31,7,38,45], [31,24,55,79], [17,24,41,65], [37,10,47,57],
[37,27,64,91], [17,27,44,71], [23,10,33,43], [23,13,36,49], [3,13,16,29], [19,2,21,23], [19,17,36,53], [15,17,32,49], [41,13,54,67], [41,28,69,97],
[15,28,43,71], [37,13,50,63], [37,24,61,85], [11,24,35,59], [47,9,56,65], [47,38,85,123], [29,38,67,105], [69,20,89,109], [69,49,118,167], [29,49,78,127],
[51,20,71,91], [51,31,82,113], [11,31,42,73], [43,9,52,61], [43,34,77,111], [25,34,59,93], [57,16,73,89], [57,41,98,139], [25,41,66,107], [39,16,55,71],
[39,23,62,85], [7,23,30,53], [37,5,42,47], [37,32,69,101], [27,32,59,91], [71,22,93,115], [71,49,120,169], [27,49,76,125], [61,22,83,105], [61,39,100,139],
[17,39,56,95], [65,12,77,89], [65,53,118,171], [41,53,94,147], [99,29,128,157], [99,70,169,239], [41,70,111,181], [75,29,104,133], [75,46,121,167],
[17,46,63,109], [55,12,67,79], [55,43,98,141], [31,43,74,117], [69,19,88,107], [69,50,119,169], [31,50,81,131], [45,19,64,83], [45,26,71,97], [7,26,33,59],
[33,5,38,43], [33,28,61,89], [23,28,51,79], [59,18,77,95], [59,41,100,141], [23,41,64,105], [49,18,67,85], [49,31,80,111], [13,31,44,75], [45,8,53,61],
[45,37,82,119], [29,37,66,103], [71,21,92,113], [71,50,121,171], [29,50,79,129], [55,21,76,97], [55,34,89,123], [13,34,47,81], [35,8,43,51], [35,27,62,89],
[19,27,46,73], [41,11,52,63], [41,30,71,101], [19,30,49,79], [25,11,36,47], [25,14,39,53], [3,14,17,31], [17,2,19,21], [17,15,32,47], [13,15,28,43], [35,11,46,57],
[35,24,59,83], [13,24,37,61], [31,11,42,53], [31,20,51,71], [9,20,29,49], [37,7,44,51], [37,30,67,97], [23,30,53,83], [55,16,71,87], [55,39,94,133],
[23,39,62,101], [41,16,57,73], [41,25,66,91], [9,25,34,59], [33,7,40,47], [33,26,59,85], [19,26,45,71], [43,12,55,67], [43,31,74,105], [19,31,50,81],
[29,12,41,53], [29,17,46,63], [5,17,22,39], [23,3,26,29], [23,20,43,63], [17,20,37,57], [45,14,59,73], [45,31,76,107], [17,31,48,79], [39,14,53,67],
[39,25,64,89], [11,25,36,61], [43,8,51,59], [43,35,78,113], [27,35,62,97], [65,19,84,103], [65,46,111,157], [27,46,73,119], [49,19,68,87], [49,30,79,109],
[11,30,41,71], [37,8,45,53], [37,29,66,95], [21,29,50,79], [47,13,60,73], [47,34,81,115], [21,34,55,89], [31,13,44,57], [31,18,49,67], [5,18,23,41],
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Pythagorean Triples for 6 levels:

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Rational Numbers generated for 6 Levels:

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Tree Visualization:

Collapsed view of a 6 layer tree

Pythagorean Triple Tree (n = 6)


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{35,12,37}
{63,16,65}
{99,20,101}
{143,24,145}
{195,28,197}
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{385,552,673}
{341,420,541}
{1581,820,1781}
{2263,2184,3145}
{583,1344,1465}
{261,380,461}
{1421,780,1621}
{1943,1824,2665}
{423,1064,1145}
{225,272,353}
{1025,528,1153}
{2337,784,2465}
{4387,4884,6565}
{2275,3828,4453}
{1475,1428,2053}
{5487,2584,6065}
{8437,8484,11965}
{2725,5628,6253}
{387,884,965}
{3311,2040,3889}
{4085,3588,5437}
{549,1820,1901}
{161,240,289}
{897,496,1025}
{2145,752,2273}
{3939,4340,5861}
{1955,3348,3877}
{1219,1140,1669}
{4399,2040,4849}
{6837,6916,9725}
{2277,4636,5165}
{259,660,709}
{2479,1560,2929}
evel 5
Leaf Nodes (Level 6)
{39,80,89}
{5,12,13}
{45,28,53}
{55,48,73}
{7,24,25}

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            - {427,1836,1885}
- {85,132,157}
    - {493,276,565}
        - {1189,420,1261}
            - {2173,564,2245}
            - {4551,5320,7001}
            - {2871,4480,5321}
        - {2175,2392,3233}
            - {9075,4508,10133}
            - {13425,13208,18833}
            - {3857,8424,9265}
        - {1071,1840,2129}
            - {6867,3956,7925}
            - {9009,8240,12209}
            - {1649,4560,4849}
    - {663,616,905}
        - {2379,1100,2621}
            - {5063,1584,5305}
            - {9821,11100,14821}
            - {5421,8900,10421}
        - {3705,3752,5273}
            - {14345,6888,15913}
            - {21755,21708,30733}
            - {6747,14204,15725}
        - {1241,2520,2809}
            - {9417,5656,10985}
            - {11899,10620,15949}
            - {1819,5580,5869}
    - {135,352,377}
        - {1323,836,1565}
            - {3479,1320,3721}
            - {6125,6612,9013}
            - {2781,4940,5669}
        - {1593,1376,2105}
            - {5369,2400,5881}
            - {8555,8772,12253}
            - {3051,6020,6749}
        - {185,672,697}
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            - [illegible]
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            - {5015,1872,5353}
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            - {4125,7268,8357}
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            - {8249,3720,9049}
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            - {4587,9116,10205}
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            - {4089,2680,4889}

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- {11607,9976,15305}
- {1287,4816,4985}

- {403,396,565}
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      - {12261,14500,18989}
      - {8085,12412,14813}
    - {6321,7120,9521}
      - {26961,13520,30161}
      - {39603,38804,55445}
      - {11123,24564,26965}
    - {3441,5680,6641}
      - {21201,12080,24401}
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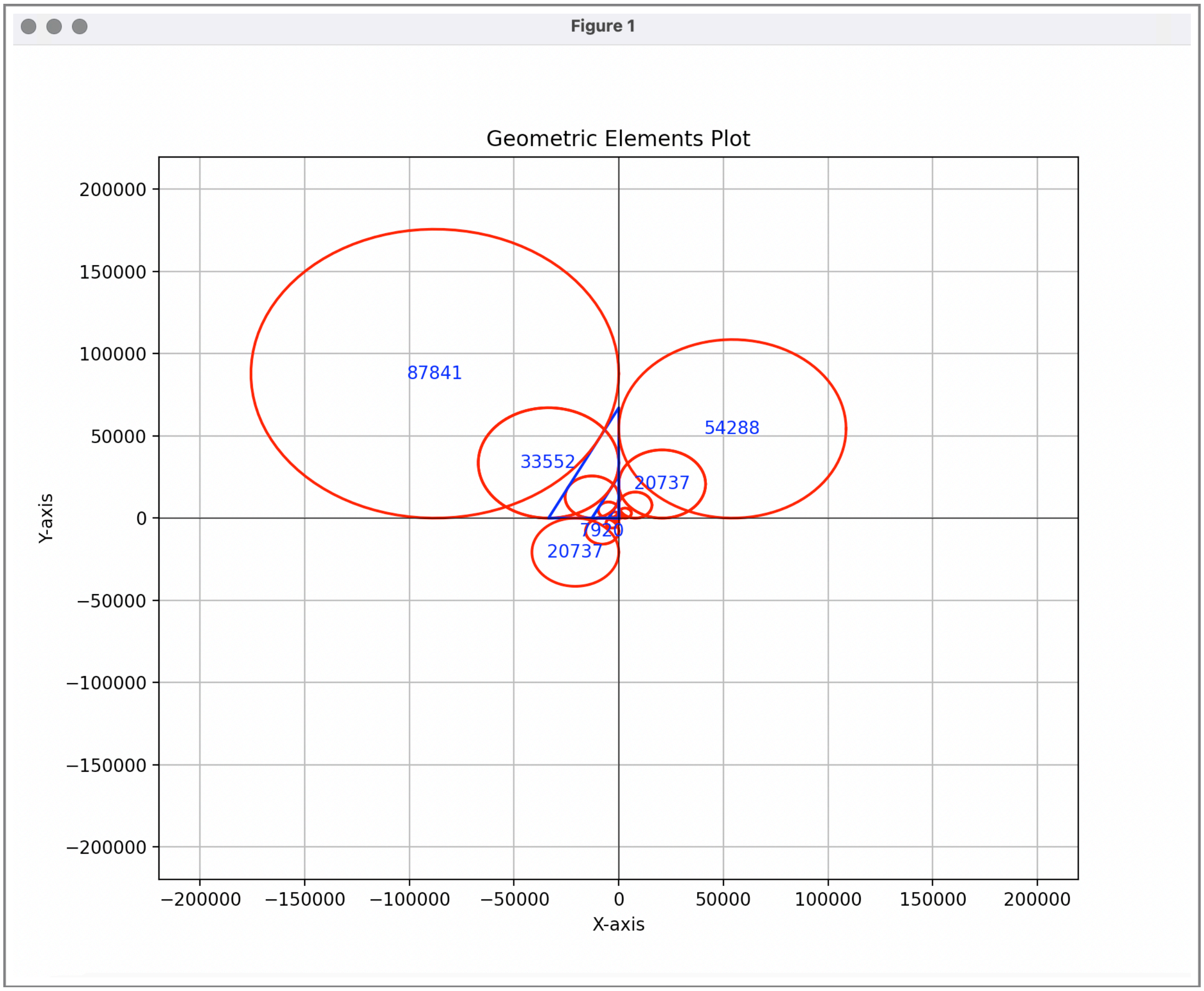
Exocircle Visualisation:



Bonus Task:

Induction Proof :

To be proven:

$$F_n*F_{n+2} + F_{n+1}*F_{n+3} = F_{2n+4}$$

Step 1:

Consider n = 2

$$F_2*F_4 + F_3*F_5$$
$$= 2*5+3*8$$
$$= 34 = F_8$$

Hence the equation to be proven holds true for our base case n = 2

Step 2:

Consider for some value n the equation holds true
Thus we can write it as:

$$F_n*F_{n+2} + F_{n+1}*F_{n+3} = F_{2n+4} \qquad \text{—————————- Eq 1}$$

To prove the equation we must show that if for some value of n the equation holds true then it holds true for n+1

Step 3:

The equation for n+1 :

$$F_{n+1}F_{n+3} + F_{n+2}F_{n+4} = F_{2n+6}. \qquad \text{—————————- Eq 2}$$

For any given value of n in the fibonacci series we know that :

$$F_n = F_{n-2}+F_{n-1} \qquad \text{————————————- Eq 3}$$

$$F_n^2 + 2*F_nF_{n+1} + 2*F_{n+1}^2 = F_{2n+4} \text{————————————- Eq 4 **}$$

Using this in equation 2 LHS

$$F_{n+1}*(F_{n+1} + F_{n+2}) + F_{n+2}*(F_{n+2} + F_{n+3})$$

$$= F_{n+1}^2 + F_{n+1}F_{n+2} + F_{n+2}^2 + F_{n+2}F_{n+3}$$

$$= F_{n+1}^2 + F_{n+1}F_{n+2} + F_{n+2}^2 + F_{n+2}*(F_{n+1} + F_{n+2})$$

$$= F_{n+1}^2 + 2*F_{n+1}F_{n+2} + 2*F_{n+2}^2$$

$$= F_{2n+6}$$

Induction:

Thus by induction we can prove that if Equation 1 holds true for some value of n then it is also true for n+1.

Given that this Equation holds true for n = 2 as shown in Step 1 we can also state that this holds true for all values of n > 2.

This has been proven in the publication **Fibonacci Meets Pythagoras. (Link provided by the mathologer) | **Link** (the equation had to be modified as our values for n begin from 0 while author uses 1)