INFO 6205 Fall 2023 Team Project

Fibonacci, Pythagoras' Theorem and Rational Numbers

November 7, 2023

Introduction

First, watch this video (45 minutes long) by Mathologer. It's very entertaining—you'll be glad you did. You will build a (lazy) tree of Pythagorean triples (the process starts at around 7:05). For example, 3, 4, 5 or 5, 12, 13...

You will then build a (lazy) tree of Rational Number pairs as described at 12:22 in the video. Given a sequence of branches from the root of the tree {3,4,5}, you should be able to return either a Pythagorean triple or a pair of Rationals.

Laziness is not something that comes naturally to Java, but you can implement it by replacing what would be an (eager) value with a function that takes no parameters and will yield the value when invoked.

Note that this is a fairly easy project so I really want you to generate a report with brilliant graphics and a codebase that is elegant and very well-tested with unit tests.

Basics

Task 1: (a) Draw as large a triples tree as you can (such that it will be visible in your report); (b) draw as large a rational pairs tree as you can (ditto).

Task 2: Draw as large a set of exocircles as you can for some triple. Demonstrate with lines and circles all the features described by Mathologer.

Task 3: Implement rational arithmetic (multiplication and addition) by traversing the tree appropriately (provide the formula in terms of tree branching). It's your job to figure out these two operations (use the subscripts described below). Both addition and multiplication will sometimes require normalization (there are no unnormalized rationals in the tree). Let me know if there are any surprises that you think Mathologer might be interested in!

Bonus Task: Prove, by induction on n, that $F_n F_{n+2} + F_{n+1} F_{n+3} = F_{2n+4}$. (Note: this isn't what Mathologer says). For instance: $n = 2 \Rightarrow 2 * 5 + 3 * 8 = 34$.

Suggestions

My recommendation is to use the Fibonacci box in the form that Mathologer develops at 30:45...

$$\begin{bmatrix} v - u & u \\ v + u & v \end{bmatrix}$$

Where u and v have opposite parity, no common factor, and $v > u$.

If this represents the parent node P_{00}, then the three children will be at P_{10} (on the right—the Pythagorean family), P_{01} (on the left—the Platonic family), and P_{11} (in the middle—the Fermat family).

Given the vector $x_{00} = \begin{matrix} u \\ v \end{matrix}$ that is formed from the right column of the Fibonacci box,

the corresponding child vector $x_{mn} = \begin{matrix} u_{mn} \\ v_{mn} \end{matrix}$ is derived by pre-multiplying by the following matrix:

$$\begin{bmatrix} 1-m & m \\ 2n-m & m+1 \end{bmatrix}$$

Thus, any node in either tree can be identified by a series of nm pairs in the subscript. For example, the Pythagorean triple {33,56,65} can be identified as $P_{01,10}$.