On the Performance of Non-Orthogonal Multiple Access in 5G Systems with Randomly Deployed Users

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Abstract—In this letter, the performance of non-orthogonal multiple access (NOMA) is investigated in a cellular downlink scenario with randomly deployed users. The developed analytical results show that NOMA can achieve superior performance in terms of ergodic sum rates; however, the outage performance of NOMA depends critically on the choices of the users' targeted data rates and allocated power. In particular, a wrong choice of the targeted data rates and allocated power can lead to a situation in which the user's outage probability is always one, i.e. the user's targeted quality of service will never be met.

Index Terms— 5G communications, non-orthogonal multiple access, stochastic geometry.

I. INTRODUCTION

THE fourth generation of mobile networks, such as long term evolution (LTE), are being deployed worldwide, and research to define the next generation mobile network is now receiving considerable attention [1]. Particularly non-orthogonal multiple access (NOMA) has been recognized as a promising multiple access techniques for fifth generation (5G) networks due to its superior spectral efficiency [2].

In this letter, the performance of NOMA is investigated in a downlink network with randomly deployed mobile users. In particular, the performance of NOMA is evaluated in two types of situations. *Firstly* we consider the case in which each user has a targeted data rate that is determined by its quality of service (QoS). In this situation, the outage probability is an ideal metric for performance evaluation since it measures the ability

of NOMA to meet the users' QoS requirements. The developed analytical results demonstrate that the choices of the users' targeted data rates and allocated power are critical to their outage performance. In particular, there is a critical condition about these system parameters, such that the outage probability is always one if the condition is not satisfied. But provided that this condition is satisfied, NOMA can ensure that the multiple users access the shared wireless medium and experience *the same diversity order* as conventional orthogonal multiple access (MA) techniques.

Secondly we consider the case in which users' rates are allocated opportunistically according to their channel conditions. In this situation, the ergodic sum rate achieved by NOMA is studied. Particularly the high signal-to-noise ratio (SNR) approximation of the ergodic rate is developed first, and an asymptotic study of the sum rate is carried out by focusing on the case in which the number of mobile users goes to infinity. The provided analytical and simulation results demonstrate that NOMA can achieve superior performance in terms of ergodic sum rates. For example, the more users that join in cooperation, the larger sum rate the NOMA can achieve, which demonstrates that NOMA is spectrally efficient. In addition asymptotic studies show that NOMA can approach an upper bound on the multi-user system which is achieved by always allocating all the bandwidth resources to the user with the best channel condition.

II. NOMA TRANSMISSION PROTOCOL

Consider a cellular downlink transmission scenario, in which the base station is located at the center of a disc, denoted by $\mathcal{D}$, with radius $\mathcal{R}_D$, and M users are uniformly distributed within the disc. The channel between the m-th user and the base station is denoted by h_m, and $h_m = \frac{\tilde{g}_m}{\sqrt{1+d_m^\alpha}}$, where $\tilde{g}_m$ denotes the Rayleigh fading channel gain, α is the path loss factor, and d_m denotes the distance from the user to the base station. Without loss of generality, the channels are sorted as $|h_1|^2 \leq \cdots \leq |h_M|^2$. According to the NOMA protocol, the base station will send $\sum_{m=1}^{M} \sqrt{a_m P} s_m$, where s_m is the message for the m-th user, P is the transmission power, and a_m is the power allocation coefficient, i.e. $a_1 \geq \cdots \geq a_M$. Therefore the observation at the m-th user is given by

$$y_m = h_m \sum_{i=1}^{M} \sqrt{a_i P} s_i + n_m, \tag{1}$$

where n_m denotes additive noise. Successive interference cancellation (SIC) will be carried out at the users. Therefore the m-th user will detect the i-th user's message, $i < m$, and then remove the message from its observation, in a successive manner. The message for the i-th user, $i > m$, will be treated as noise at

the m-th user. As a result, the data rate achievable to the m-th user, $1 \leq m \leq (M-1)$, is given by

$$R_m = \log\left(1 + \frac{\rho|h_m|^2 a_m}{\rho|h_m|^2 \sum_{i=m+1}^{M} a_i + 1}\right), \qquad (2)$$

conditioned on $R_{j \to m} \geq \tilde{R}_j$, where ρ denotes the transmit SNR, $\tilde{R}_j$ denotes the targeted data rate of the j-th user, and $R_{j \to m}$ denotes the rate for the m-th user to detect the j-th user's message, $j \leq m$, i.e. $R_{j \to m} = \log\left(1 + \frac{\rho|h_m|^2 a_j}{\rho|h_m|^2 \sum_{i=j+1}^{M} a_i + 1}\right)$. Note that the rate at the M-th user is $R_M = \log(1 + \rho|h_M|^2 a_M)$.

In this letter two types of $\tilde{R}_m$ are considered.

1) Case I: $\tilde{R}_m$ is determined by the users' QoS requirements, i.e. each user has a preset $\tilde{R}_m$. In this case, it is important to examine the probability of the following two events. One is that a user can cancel others users' messages, i.e. $R_{j \to m} \geq \tilde{R}_j, j < m$, and the other is that NOMA can ensure the user's QoS requirements to be satisfied, i.e. $R_m \geq \tilde{R}_m$. When both constraints are satisfied, the sum rate of NOMA is simply $\sum_{m=1}^{M} \tilde{R}_m$. Therefore the sum rate will not be of interest in this case, and it is important to calculate the probabilities of the two events, as shown in Section IV.

2) Case II: $\tilde{R}_m$ is determined opportunistically by the user's channel condition, i.e. $\tilde{R}_j = R_j$. Therefore it can be easily verified that the condition $R_{j \to m} \geq \tilde{R}_j$ always holds since $|h_m|^2 \geq |h_j|^2$ for $m > j$. Consequently the sum rate achieved by NOMA is given by

$$R_{sum} = \sum_{m=1}^{M-1} \log(1 + \frac{\rho|h_m|^2 a_m}{\rho|h_m|^2 \tilde{a}_m + 1}) + \log(1 + \rho|h_M|^2 a_M),$$

where $\tilde{a}_m = \sum_{i=m+1}^{M} a_i$. Therefore, it is important to find the ergodic sum rate achieved by NOMA, as shown in Section V.

III. DENSITY FUNCTIONS OF CHANNEL GAINS

The evaluation of the outage probability and ergodic rates requires the density functions of the channel gains. Denote by $\tilde{h}$ an *unordered* channel gain. Conditioned on the fact that the users are uniformly located in the disc, $\mathcal{D}$, and small scale fading is Rayleigh distributed, the cumulative density function (CDF), of the unordered channel gain, $|\tilde{h}|^2$, is given by [3]

$$F_{|\tilde{h}|^2}(y) = \frac{2}{\mathcal{R}_D^2} \int_0^{\mathcal{R}_D} \left(1 - e^{-(1+z^\alpha)y}\right) z\, dz. \qquad (3)$$

This integral can be easily calculated when $\alpha = 2$ [3]. But for many communication scenarios, such as indoor environments, $\alpha > 2$, which makes it challenging to get an insightful expression for the CDF of $|\tilde{h}|^2$. In the following the Gaussian-Chebyshev quadrature will be used to find an approximation for the above integral [4]. By applying Gaussian-Chebyshev quadrature, (3) can be simplified as follows:

$$F_{|\tilde{h}|^2}(y) \approx \frac{1}{\mathcal{R}_D} \sum_{n=1}^{N} w_n g(\theta_n), \qquad (4)$$

where $g(x) = \sqrt{1-x^2}\left(1 - e^{-c_n y}\right)\left(\frac{\mathcal{R}_D}{2}x + \frac{\mathcal{R}_D}{2}\right)$, N is a parameter to ensure a complexity-accuracy tradeoff, $c_n = 1 + \left(\frac{\mathcal{R}_D}{2}\theta_n + \frac{\mathcal{R}_D}{2}\right)^\alpha$, $w_n = \frac{\pi}{N}$, and $\theta_n = \cos\left(\frac{2n-1}{2N}\pi\right)$.

Consequently the probability density function (PDF) of the unordered channel gain can be approximated as follows:

$$f_{|\tilde{h}|^2}(y) \approx \frac{1}{\mathcal{R}_D} \sum_{n=1}^{N} \beta_n e^{-c_n y}, \qquad (5)$$

where $\beta_n = w_n \sqrt{1 - \theta_n^2} \left(\frac{\mathcal{R}_D}{2}\theta_n + \frac{\mathcal{R}_D}{2}\right) c_n$. Compared to the original form in (3), the ones shown in (4) and (5) can be used to simplify the performance analysis significantly since they are linear combinations of exponential functions.

IV. CASE I: OUTAGE PERFORMANCE OF NOMA

The outage events at the m-th user can be defined as follows. First define $E_{m,j} \triangleq \{R_{j \to m} < \tilde{R}_j\}$ as the event that the m-th user cannot detect the j-th user's message, $1 \leq j \leq m$, and $E_{m,j}^c$ as the complementary set of $E_{m,j}$. The outage probability at the m-th user can be expressed as follows:

$$P_m^{out} = 1 - P(E_{m,1}^c \cap \cdots \cap E_{m,m}^c). \qquad (6)$$

The event $E_{M,M}^c$ is defined as $E_{M,M}^c = \{\rho|h_M|^2 a_M > \phi_M\}$, and the other events $E_{m,j}^c$, $1 \leq j \leq m$, can be expressed as follows:

$$E_{m,j}^c = \left\{ \frac{\rho|h_m|^2 a_j}{\rho|h_m|^2 \sum_{i=j+1}^{M} a_i + 1} > \phi_j \right\}$$
$$\overset{(a)}{=} \left\{ \rho|h_m|^2 \left(a_j - \phi_j \sum_{i=j+1}^{M} a_i\right) > \phi_j \right\}, \qquad (7)$$

where $\phi_j = 2^{\tilde{R}_j} - 1$. Note that the step (a) is obtained by assuming the following condition holds:

$$a_j > \phi_j \sum_{i=j+1}^{M} a_i. \qquad (8)$$

Furthermore define $\psi_j \triangleq \frac{\phi_j}{\rho\left(a_j - \phi_j \sum_{i=j+1}^{M} a_i\right)}$ for $j < M$, $\psi_M \triangleq \frac{\phi_M}{\rho a_M}$, and $\psi_m^* = \max\{\psi_1, \ldots, \psi_m\}$. As a result, the outage probability can now be expressed as follows:

$$P_m^{out} = 1 - P(|h_m|^2 > \psi_m^*)$$
$$= \int_0^{\psi_m^*} \frac{M!\left(F_{|\tilde{h}|^2}(x)\right)^{m-1}\left(1 - F_{|\tilde{h}|^2}(x)\right)^{M-m} f_{|\tilde{h}|^2}(x)}{(m-1)!(M-m)!} dx, \qquad (9)$$

which is obtained by analyzing order statistics [5].

Note that when $y \to 0$, the CDF of the unordered channel gains can be approximated as $F_{|\tilde{h}|^2}(y) \approx \frac{1}{\mathcal{R}_D} \sum_{n=1}^{N} \beta_n y$, and the approximation of the PDF is given by

$$f_{|\tilde{h}|^2}(y) \approx \frac{1}{\mathcal{R}_D} \sum_{n=1}^{N} \beta_n (1 - c_n y). \qquad (10)$$

When $\rho \to \infty$, $\psi_m^* \to 0$. Therefore a high SNR approximation of the outage probability is given by

$$P_m^{out} \approx \tau_m \int_0^{\psi_m^*} (\eta x)^{m-1}(1 - \eta x)^{M-m} \frac{1}{\mathcal{R}_D}$$
$$\times \sum_{n=1}^{N} \beta_n (1 - c_n x)\, dx \approx \frac{\tau_m}{m}\eta^m (\psi_m^*)^m, \qquad (11)$$

where $\eta = \frac{1}{\mathcal{R}_D}\sum_{n=1}^{N} \beta_n$ and $\tau_m = \frac{M!}{(m-1)!(M-m)!}$. Therefore the diversity order achieved by NOMA is given by

$$P_m^{out} \to \frac{1}{\rho^m}. \qquad (12)$$

The result in (12) demonstrates that the m-th user will experience a diversity order of m. This is better than a conventional orthogonal MA scheme with a randomly scheduled user whose diversity order is one. Compared to opportunistic user scheduling,

NOMA will achieve better spectral efficiency and user fairness since all the users are served at the same time, frequency and spreading code.

It is worthy pointing out that the superior outage performance achieved by NOMA is conditioned on the constraint in (8). When such a condition is not satisfied, e.g. $a_j \leq \phi_j \sum_{i=j+1}^{M} a_i$, the user's outage probability is always one, i.e. $P_m^{out} = 1$, as shown in Section VI.

V. CASE II: ERGODIC SUM RATE OF NOMA

When $\hat{R}_j = R_j$, the ergodic sum rate is given by

$$
R_{ave} = \sum_{m=1}^{M-1} \int_0^\infty \log\left(1 + \frac{x\rho a_m}{x\rho\tilde{a}_m + 1}\right) f_{|h_m|^2}(x)dx
$$
$$
+ \int_0^\infty \log(1 + \rho x a_M) f_{|h_M|^2}(x)dx. \quad (13)
$$

Even with the approximations in (4) and (5), an exact expression for the ergodic sum rate is still difficult to obtain, and we will focus on the high SNR approximation as well as the asymptotic behavior of the sum rate when $M \to \infty$.

1) High SNR Approximation: When $\rho \to \infty$, the ergodic sum rate can be expressed as follows:

$$
R_{ave} \approx \sum_{m=1}^{M-1} \int_0^\infty \log\left(1 + \frac{a_m}{\tilde{a}_m}\right) f_{|h_m|^2}(x)dx
$$
$$
+ \underbrace{\int_0^\infty \log\left(1 + x\rho a_M\right) f_{|h_M|^2}(x)dx}_{T_1}. \quad (14)
$$

Rewrite T_1 as $T_1 = \frac{\rho a_M}{\ln 2} \int_0^\infty \frac{1 - F_{|h_M|^2}(x)}{1 + x\rho a_M}dx$. Rewrite the CDF $|\tilde{h}|^2$ in the following form:

$$
F_{|\tilde{h}|^2}(x) = \frac{1}{\mathcal{R}_D} \sum_{n=0}^{N} b_n e^{-c_n x}, \quad (15)
$$

where $b_n = -w_n \sqrt{1 - \theta_n^2} \left(\frac{\mathcal{R}_D}{2}\theta_n + \frac{\mathcal{R}_D}{2}\right)$ for $1 \leq n \leq N$, $b_0 = -\sum_{n=1}^{N} b_n$, and $c_0 = 0$. As a result, the CDF of the largest order statistics is $F_{|h_M|^2}(x) = \left(F_{|\tilde{h}|^2}(x)\right)^M$. Now T_1 can be expressed as follows:

$$
T_1 = \frac{\rho a_M}{\ln 2} \int_0^\infty \frac{1}{1 + x\rho a_M} \left(1 - \frac{1}{\mathcal{R}_D^M} \sum_{k_0 + \cdots + k_N = M}\right.
$$
$$
\left.\binom{M}{k_0, \cdots, k_N} \left(\prod_{n=0}^{N} b_n^{k_n}\right) e^{-\sum_{n=0}^{N} k_n c_n x}\right) dx, \quad (16)
$$

where $\binom{M}{k_0, \cdots, k_N} = \frac{M!}{k_0! \cdots k_N!}$. Clearly the integral $\int_0^\infty \frac{\epsilon}{1 + x\rho a_M}dx$ does not exist, where ϵ is a constant. One can first make the following observation:

$$
\binom{M}{k_0, \cdots, k_N} \left(\prod_{n=0}^{N} b_n^{k_n}\right) e^{-\sum_{n=0}^{N} k_n c_n x} = \mathcal{R}_D^M, \quad (17)
$$

when $k_0 = M$, and $k_i = 0$, $1 \leq i \leq N$, since $F_{|\tilde{h}|^2}(\infty) = 1$. This observation can be used to remove the constants in the integral, and T_1 is written as follows:

$$
T_1 = -\frac{\rho a_M}{\mathcal{R}_D^M \ln 2} \int_0^\infty \frac{1}{1 + x\rho a_M} \sum_{\substack{k_0 + \cdots + k_N = \\ M, k_0 \neq M}} \binom{M}{k_0, \cdots, k_N}
$$
$$
\times \left(\prod_{n=0}^{N} b_n^{k_n}\right) e^{-x \sum_{n=0}^{N} k_n c_n} dx. \quad (18)
$$

In the above equation, each term of the sum is an exponential function with a non-zero exponent, i.e. $\sum_{n=0}^{N} k_n c_n \neq 0$. With some algebraic manipulations, the ergodic sum rate achieved by NOMA can be obtained as follows:

$$
R_{ave} \approx \sum_{m=1}^{M-1} \log\left(1 + \frac{a_m}{\tilde{a}_m}\right) - \frac{1}{\mathcal{R}_D^M \ln 2}
$$
$$
\times \sum_{\substack{k_0 + \cdots + k_N = \\ M, k_0 \neq M}} \binom{M}{k_0, \cdots, k_N} \left(\prod_{n=0}^{N} b_n^{k_n}\right) e^{\frac{\sum_{n=0}^{N} k_n c_n}{2\rho a_M}}
$$
$$
\times \left(\frac{\sum_{n=0}^{N} k_n c_n}{\rho a_M}\right)^{-\frac{1}{2}} W_{-\frac{1}{2}, 0}\left(\frac{\sum_{n=0}^{N} k_n c_n}{\rho a_M}\right), \quad (19)
$$

where $W_{k,u}(\cdot)$ denotes the Whittaker function.

2) Asymptotic Study with $M \to \infty$: In this subsection, we focus on the asymptotic performance of NOMA when $M \to \infty$. First define the growth function as $G(x) \triangleq \frac{1 - F_{|\tilde{h}|^2}(x)}{f_{|\tilde{h}|^2}(x)}$. Note that the condition needed in order to apply the extreme value theorem is that the limit, $\lim_{x \to \infty} G(x)$, exists. This condition holds for the addressed distribution as shown in the following:

$$
\lim_{x \to \infty} G(x) = \lim_{x \to \infty} \frac{1 - \frac{1}{\mathcal{R}_D} \sum_{n=0}^{N} b_n e^{-c_n x}}{\frac{1}{\mathcal{R}_D} \sum_{n=1}^{N} \beta_n e^{-c_n x}} \overset{(a)}{=} \frac{-b_N}{\beta_N}, \quad (20)
$$

where the step (a) is obtained because $\frac{1}{\mathcal{R}_D} b_0 e^{-c_0 x} = 1$ and $c_N \leq c_i$ for all $1 \leq i \leq N$.

The evaluation of the asymptotic behavior of $|h_M|^2$ needs u_M, the unique solution of $1 - F_{|\tilde{h}|^2}(u_M) = \frac{1}{M}$. This equation can be first rewritten as follows:

$$
-\frac{1}{\mathcal{R}_D} \sum_{n=1}^{N} b_n e^{-c_n u_M} = \frac{1}{M}. \quad (21)
$$

When $u_M \to \infty$, (21) can be approximated as $-\frac{b_N e^{-c_N u_M}}{\mathcal{R}_D}\left(1 + O\left(\frac{1}{u_M}\right)\right) = \frac{1}{M}$. It is worth pointing out that the terms $e^{-c_n u_M}$, $n < N$, are decreasing at a rate faster than $\frac{1}{u_M}$, but the use of the above expression can ensure that the existing results in [5] and [6] can be applied straightforwardly. Particularly, following steps similar to those used in [6], the solution u_M is given by

$$
u_M = \frac{1}{c_N} \log(M) + O(\log \log M). \quad (22)
$$

Similarly we observe $G^{(m)}(u_M) = O\left(\frac{1}{u_M^m}\right)$. By applying Corollary A1 in [6], it is straightforward to show that $P(\log M - c_N \log \log M \leq |h_M|^2 \leq \log M + c_N \log \log M) \geq 1 - O\left(\frac{1}{\log M}\right)$, where $\frac{-b_N}{\beta_N} = c_N$.

Therefore we can conclude that NOMA can achieve the following ergodic sum rate:

$$
R_{ave} \to \log(\rho \log \log M), \quad (23)
$$

with a probability approaching one when $M \to \infty$ and $\rho \to \infty$. Consider an opportunistic MA approach that allocates all the bandwidth resource to the user with the best channel condition. It is easy to verify that this opportunistic scheme achieves the upper bound of the system throughput with an asymptotic behavior of $\log(\rho \log \log M)$. Therefore NOMA can achieve the same asymptotic performance as the opportunistic scheme, but NOMA can offer better fairness since all the users are served simultaneously.
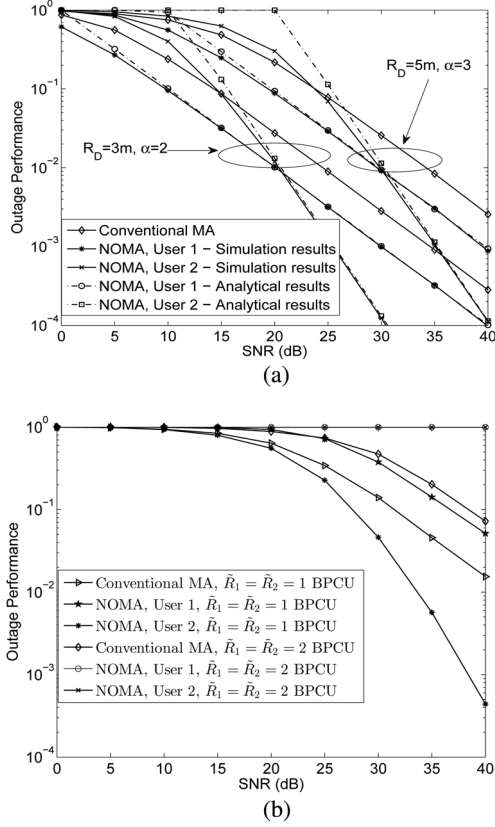
Fig. 1. Outage performance of the multiple access technologies (a) $\tilde{R}_1 = 0.1$ BPCU and $\tilde{R}_2 = 0.5$ BPCU (b) Different values of R_m with $\mathcal{R}_D = 5$ m and $\alpha = 3$.

Fig. 2. Ergodic sum rates achieved by the multiple access technologies (a) $M = 2$ (b) Impact of M with $\mathcal{R} = 5$ m and $\alpha = 2$.

$\times 10^{-4}$	Case 1	Case 2	Case 3	Case 4
Conventional	10.59	15.21	20.54	29.6
User 1-simul.	3.734	7.956	7.294	15.33
User 1-anal.	3.781	7.99	7.3852	15.61
User 2-simul.	0.1662	0.2812	0.6772	1.002
User 2-anal.	0.1837	0.2848	0.7007	1.086

Fig. 3. The outage performance of NOMA. The transmit power and the noise power are 10 Watt and -40 dBm, respectively. $\alpha = 3$. Case 1 denotes $\mathcal{R}_D = 80$ m, $\tilde{R}_1 = 0.1$ BPCU and $\tilde{R}_2 = 0.5$ BPCU. Case 2 denotes $\mathcal{R}_D = 80$ m, $\tilde{R}_1 = 0.2$ BPCU and $\tilde{R}_2 = 0.6$ BPCU. Case 3 denotes $\mathcal{R}_D = 100$ m, $\tilde{R}_1 = 0.1$ BPCU and $\tilde{R}_2 = 0.5$ BPCU. Case 4 denotes $\mathcal{R}_D = 100$ m, $\tilde{R}_1 = 0.2$ BPCU and $\tilde{R}_2 = 0.6$ BPCU.

VI. NUMERICAL RESULTS

In this section, the performance of NOMA is evaluated by using computer simulations, where a conventional orthogonal MA approach with *a randomly scheduled user* is used for benchmarking. The power allocation coefficients are $a_1 = \frac{4}{5}$ and $a_2 = 1 - a_1$ for $M = 2$. For $M > 2$, $a_m = \frac{M-m+1}{\mu}$ and μ is to ensure $\sum_{m=1}^{M} a_m = 1$. $N = 10$. In Fig. 1 the outage performance is shown as a function of SNR, where the targeted rate for the conventional scheme is $\sum_{m=1}^{M} \tilde{R}_m$ bit per channel use (BPCU). As can be observed from Fig. 1(a), NOMA outperforms the comparable scheme, and the diversity order of the users is a function of their channel conditions, which is consistent to (11). However, with an incorrect choice of $\tilde{R}_j$ and a_m, the outage probability will always be one, as shown in Fig. 1(b). In Fig. 2, the ergodic sum rate achieved by NOMA is shown as a function of SNR. The two figures in Fig. 2 demonstrate that NOMA can achieve a larger sum rate than the orthogonal MA scheme, and approach the upper bound of the system throughput which is achieved by the opportunistic MA scheme. It is worth pointing out that the provided simulation results shown in Fig. 1(a) and Fig. 2(a) match the analytical results developed at (11) and (19). The above numerical studies are based on the normalized SNR model, and more practical system parameters are used in Table III which shows that the developed analytical results are still close to the simulation ones. In addition, the results in Table III are also consistent to those results in Fig. 1, e.g. the
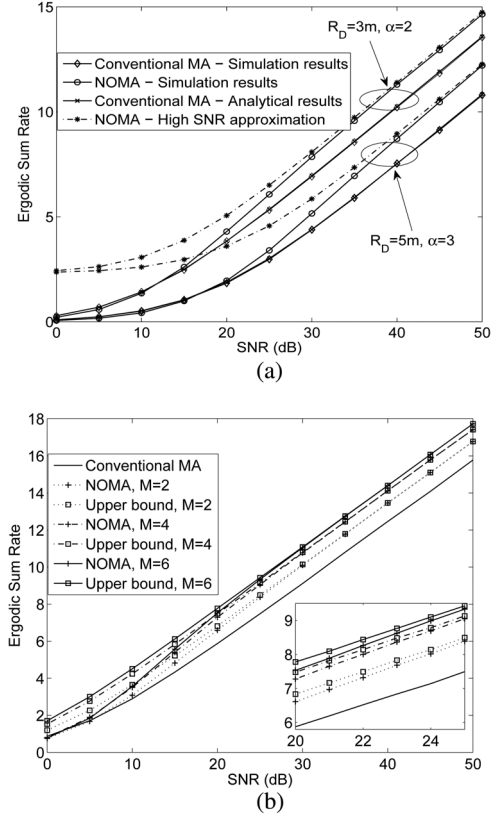
conventional scheme suffers some loss of the outage performance compared to NOMA.

VII. CONCLUSIONS

In this paper we have first demonstrated that NOMA can achieve better outage performance than the orthogonal MA techniques, under the condition that the users' rates and power coefficients are carefully chosen. In addition, we have shown that NOMA can achieve a superior ergodic sum rate, and is asymptotically equivalent to the opportunistic MA technique. However, there are two potential drawbacks to NOMA. One is that NOMA introduces additional complexity due to the use of SIC, and the other is that the performance gain of NOMA at low SNR is insignificant. Therefore it is important to study how to achieve a tradeoff of performance and complexity at different values of SNR.

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