Performance Analysis of Signal Pattern Reducing Techniques for Low probability of Detection

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Abstract—Ad-hoc flexibly deployable networks destined for tactical military or civilian applications often suffer from poor synchronization which may lead to unreliable communication. Quasi-Synchronous (QS) CDMA employing Loosely Synchronous (LS) codes allows weak synchrnonization due to the Zero-Correlation Zone of the code's correlation functions. Apart from reliability, an essential characteristic of military networks is having low probability of detection (LPD) in the presence of adversaries employing sophisticated detection techniques. Such techniques are based on the fact that manmade signals are cyclostationary, meaning that they have some periodic structure (e.g. spreading sequence repetition) that can be exploited for improved detection. Disturbing those recurring patterns can reduce the probability of detection measured in terms of the Degree of Cyclostationarity (DCS). We aim to achieve that by employing some techniques that will perturb the signal structure by randomly selecting spreading sequences, random time dithering or a combination of the two. We study the tradeoff between the communication performance and DCS reduction of such a system under these different techniques and compare the results.

Index Terms—quasi-synchronous, loosely synchronized, low probability of detection (LPD), random sequence selection, dithering

I. INTRODUCTION

Tactical military and civilian ad-hoc networks often require easy and quick deployment in difficult environments. Lack of coordinating infrastructure leads to synchronization issues, especially in GPS-denied scenarios, thus compromising reliable communication. Apart from the timing uncertainty caused by weak synchronization, an important issue in such networks especially for military applications - is covertness; in various scenarios it is essential that the communication system is not easily detectable in the presence of a sophisticated adversary.

To tackle synchronization issues, Quasi-Synchronous (QS) DS-CDMA has been proposed and numerous code designs have been analysed [1]–[6]. QS-DS-CDMA has been often combined/ relies on the use of Loosely-Synchronous (LS) codes [6] that have good auto-correlation and cross-correlation functions, characterized by a Zero-Correlation Zone (ZCZ) which enables maintaining orthogonality up to a certain delay hence allowing weak synchronization. As far as detectability is concerned, CDMA already offers an advantage with respect to power-based detection techniques (such as radiometers) due

to its property of spreading the signal in the frequency domain, rendering it similar to white noise over the transmission bandwidth. However, an advanced adversary employing a socalled cycle detector, can detect the existence of transmission due to the periodic structure of a CDMA signal (spreading sequence repetition). The class of cyclostationary signals, i.e. signals demonstrating such patterns, has been extensively studied in the literature; for a comprehensive review the reader is referred to [7], [8]. In order to disturb this periodic structure, one could avoid repeating the same spreading sequence, for example by randomly selecting a spreading sequence for each data symbol transmitted. The information theoretic capacity reduction due to random sequence selection in multi-user CDMA was studied in [9]. It was shown that although there is an inherent loss due to the spreading, that loss rapidly vanishes for systems where the number of users is large compared to the spreading code length. The CDMA channel with randomly and independently chosen spreading sequences accurately models the situation where pseudonoise (PN) sequences span many symbol periods. The spectral efficiency of such a system under different detection schemes is analysed under the assumption of symbol-level synchronization [10]. Other techniques and waveform designs aiming to enhance low probability of intercept (LPI) or low probability of detection (LPD) communication include [11]-[14].

In this paper, we investigate the performance of three different signal pattern reduction techniques aiming to enhance the LPD characteristics of QS-DS-CDMA measured in terms of the Degree of Cyclostationarity (DCS). We propose and analyze the assignment of multiple random sequences to a user under the assumption that each user has a dedicated set of sequences to select from. We also review the our previously developed technique based on time dithering, presented in [14], and combine it with the aforementioned random sequence selection approach. We study the trade-off between the DCS reduction and BER performance and compare the three techniques. The advantages and drawbacks of each technique are also discussed. Our results show that dithering outperforms the other methods in terms of DCS reduction.

The paper is structured as follows: section II contains the system model we are concerned with. In section III we present three pattern reducing schemes: random spreading sequence selection, random time dithering and random sequence selection with dither. Section IV contains the simulation parameters and results including the metrics used for LPD and communication performance evaluation. In section V we discuss some potential future directions and we conclude in section VI.

II. SYSTEM MODEL

The transmitted signal of a user in a QS-DS-CDMA system is modeled as

$$x(t) = \sum_{n=0}^{N-1} b_n \sum_{\ell=0}^{L-1} a_{\ell} p(t - \ell T_c - nT - \Delta t)$$
 (1)

where N is the number of data symbols transmitted per packet, b_n is the n-th symbol which is spread by the sequence $\{a_0, a_1, ..., a_{L-1}\}$ of length L. The pulse p(t) has duration T_c where T_c and $T = L \cdot T_c$ are the chip and symbol duration, respectively. We further assume that the timing uncertainty, Δt , caused by imperfect synchronization is uniformly distributed in $(0, T_D)$ and is fixed for all N symbols in the packet.

Normally in QS-DS-CDMA each user spreads all their symbols using the same (unique) spreading sequence. We instead propose a system where each user is assigned a set of sequences; spreading is then performed by picking one of those sequences randomly in a per-symbol basis as will be explained in the following section.

The employed spreading sequences are picked from a LS code that is obtained by a recursive construction followed by an insertion of zeros as described in [6]. A LS code is defined by the triplet (M, L_c, Z) , where M is the family size, L_c is the code length not including the zeros and Z is the size of the code's ZCZ. LS codes further satisfy the fundamental orthogonality bound with equality, i.e. $L_c = M \cdot Z$. Practically, a code of ZCZ size equal to Z can accommodate up to a delay of (Z-1) chips without compromising code orthogonality and therefore without causing inter-symbol interference (ISI). Keeping that in mind, in a QS-DS-CDMA system, we can choose a code with a ZCZ size that satisfies the system's time uncertainty requirement.

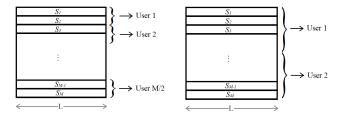
We remark here, that in order to accommodate longer time uncertainties (or, equivalently, looser synchronization requirements) we could increase the number of carriers, hence extending the code's ZCZ in time as in [14]–[16].

III. PATTERN REDUCING SCHEMES

A. Sequence Selection Scheme

In this section, we describe and analyze a scheme for spreading sequence assignment and random selection which we call Random Selection from Mutually Exclusive Code Subsets (RS-MECS).

Assume a QS-DS-CDMA system, as presented in section II, with U users and employing a codebook of size M where $M=c\cdot U$, with $c\in\mathbb{N}^*$. In the quasi-synchronous scenario a Loosely Synchronous (LS) code [6] can be employed to account for the time uncertainty and allow imperfect synchrony. Once the codebook is chosen, each user will be assigned c sequences for exclusive use, as illustrated in Figure 1. We



(a) c=2 sequences per user (b) c=M/2 sequences per user

Fig. 1: RS-MECS examples

note that it is not necessary to make a 'symmetric' sequence assignment, such as the one described above but we will only explore this setting here. Obviously, when c=M, the system can only accommodate 1 user.

Every user then picks a sequence uniformly at random (among the subset assigned to them) to spread each of their data symbols. The transmitted signal model then becomes

$$x(t) = \sum_{n=0}^{N-1} b_n \sum_{\ell=0}^{L-1} a_{\ell,n} p(t - \ell T_c - nT - \Delta t)$$
 (2)

where, the spreading sequence is additionally indexed by the data symbol number.

In such a case detection needs to include an additional correlating step in order to determine the sequence used to spread each symbol. Even though this scheme ensures absence of MAI, there is some communication performance degradation as a consequence of the correlating step.

B. Random Time Dithering

Another method to reduce the periodic patterns of the signal is by introducing random time dithering as proposed in [14]. In this section we review this technique and we will henceforth refer to it as DITH.

We assume the QS-DS-CDMA system presented in section II where each user is assigned a unique sequence to spread their data. Choosing a (M, L_c, Z) LS code, the system can accommodate up to M users who do not experience any MAI as long as their relative delays are within the ZCZ of the code. Given the time uncertainty requirement of the system, one could choose a longer ZCZ than the one required for weak synchronization in order to introduce an additional random delay (dither) that will result in disturbing the periodic structure of the signal.

The transmitted signal of a user in a QS-DS-CDMA system employing dithering becomes

$$x(t) = \sum_{n=0}^{N-1} b_n \sum_{\ell=0}^{L-1} a_{\ell} p(t - \ell T_c - nT - \Delta t - \epsilon_n T_c)$$
 (3)

where the additional term $\epsilon_n T_c$ accounts for random time dithering on a per-symbol basis.

Similar to the random sequence selection scheme, we assume that the receiver has no knowledge of the dithering pattern and therefore needs to correlate the received data

symbols in order to achieve code acquisition (synchronization) and despread thus incurring some performance loss.

C. Random Selection with Dithering

Random sequence selection and random time dithering can also be combined in order to disturb the signal structure even further. In this scheme, that we will refer to as Random Selection with Dithering (RS-DITH), each user is assigned a set of spreading sequences and is also allowed to perform dithering within the ZCZ (assuming that the synchronization requirement permits to do so).

In this case, the transmitted signal model of (1) becomes

$$x(t) = \sum_{n=0}^{N-1} b_n \sum_{\ell=0}^{L-1} a_{\ell,n} p(t - \ell T_c - nT - \Delta t - \epsilon_n T_c)$$
 (4)

thus including both dithering and a symbol-dependent spreading sequence.

For this scenario we expect an additional degradation in BER performance due to the need to correlate both for the code and the delay on a per-symbol basis.

IV. SIMULATION AND RESULTS

A. LPD Evaluation - Cyclic Spectral Analysis

In this section we present the metrics used to quantify the probability of detection reduction, which is based on disturbing the periodic structure of the transmitted signal. Unlike conventional power based detection approaches (such as radiometers), we focus on feature based detection techniques (i.e. cycle detectors) that may be employed by sophisticated adversaries.

Cyclic spectral analysis is a tool that enables improved accuracy and reliability and permits signal separation in the case of a special category of signals, called *cyclostationary* signals (most man-made signals belong to this category) [7]. When a signal or process is stationary, its statistical characteristics (e.g. mean, variance, correlation) remain constant through time. On the other hand, cyclostationary signals have either a periodic or an almost periodic autocorrelation function which, for a signal x(t), can be represented by a Fourier series as

$$R_x(t,\tau) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t},\tag{5}$$

Here, τ is the lag parameter, α is a parameter called *cycle* frequency and the coefficient $R_x^{\alpha}(\tau)$ is a 2-D function called the Cyclic Autocorrelation Function (CAF) and is given by:

$$R_x^{\alpha}(\tau) = \lim_{T \to \infty} \int_{-T/2}^{T/2} x(t - \tau/2) x^*(t + \tau/2) e^{-j2\pi\alpha t} dt,$$
 (6)

for $\alpha=k/T$ where T is a fundamental period of the signal, $k\in\mathbb{N}$ and the superscript * denotes complex conjugation. Note that when $\alpha=0$, $R_x^\alpha(\tau)$ becomes the conventional autocorrelation function.

With the aid of the cyclic autocorrelation we may quantify cyclostationarity by means of a metric called *Degree of Cyclostationarity* (DCS), which can be computed either in time

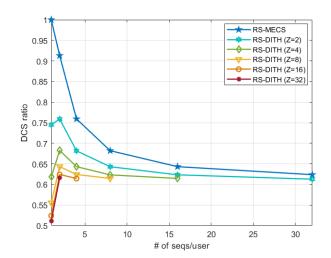


Fig. 2: DCS Reduction for RS-MECS and RS-DITH

or frequency domain. Here we will focus on the time domain. By first defining the temporal correlation coefficient as

$$\gamma_x^{\alpha}(\tau) = \frac{R_x^{\alpha}(\tau)}{R_x(0)},$$

we can then write the time decomposed degree of cyclostationarity as

$$DCS_{\tau}^{\alpha} = \left| \gamma_x^{\alpha}(\tau) \right|^2.$$

Given the above equation, we may compute the cycle-frequency decomposed degree of cyclostationarity as

$$DCS^{\alpha} = \frac{\int_{-\infty}^{\infty} DCS_{\tau}^{\alpha} d\tau}{\int_{-\infty}^{\infty} DCS_{\tau}^{0} d\tau},$$

which allows us to compute the signal's degree of cyclostationarity over all values of α as

$$DCS = \sum_{\alpha \neq 0} DCS^{\alpha}.$$
 (7)

We further define the following DCS ratio that will serve to quantify the DCS reduction after employing the selected technique (RS-MECS, RS-DITH, DITH [14])

$$DCS \ ratio = \frac{DCS \ of \ signal \ using \ selected \ technique}{DCS \ of \ original \ signal} \quad (8)$$

B. Performance Evaluation & Simulation Setup

Communication performance is measured in terms of the Bit-Error Rate (BER) for various values of the signal-to-noise ratio (SNR) - equal to E_b/N_0 . For that, we introduce additive white Gaussian noise (AWGN) of various power levels to the 2- Pulse Amplitude Modulated (PAM) data symbols (bits) and run Monte Carlo simulations to obtain the BER and DCS curves for packets of N=1e6 data symbols.

The employed LS codes, (2, 64, 32), (4, 64, 16), (8, 64, 8), (16, 64, 4), (32, 64, 2), were constructed recursively as in [6].

	Simulation Parameter Values				
Scheme	RS-MECS	DITH	RS-DITH		
C	$1 \le C \le 32$	C = 1	$1 \le C \le 32$		
D	D = 0	$0 \le D \le 32$	$0 \le D \le 32$		
L_c	64				

TABLE I: Simulation Parameters

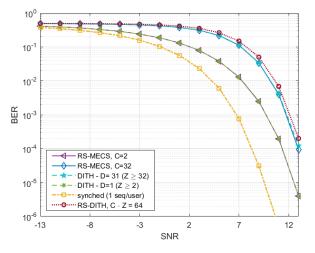


Fig. 3: BER Performance of DITH, RS-MECS and RS-DITH

C. Simulation Results

We first demonstrate the DCS reduction for RS-MECS and RS-DITH for different sizes of the code subset assigned to a user, C, and for different codes in Figure 2. The codes compared have the same base code length $L_c = 64$ but different family and ZCZ sizes. The DCS reduction attained by RS-MECS is the same for a given sequence subset size irrespective of the code used, that is, it depends exclusively on the number of sequences assigned to a user (and therefore M). On the other hand, RS-DITH achieves higher DCS reduction for codes with longer ZCZ and performs better than RS-MECS for a fixed value of C. Note that when only one sequence is assigned for each user, then there is no random selection; the user always uses the unique spreading sequence assigned to them, as in conventional QS-DS-CDMA, and therefore the leftmost points in each of the RS-DITH curves correspond to DITH by up to Z-1 chips. That shows that DITH outperforms both RS-MECS and RS-DITH as far as DCS reduction is concerned for $\Delta t = 0$, which is probably counter intuitive.

In Figure 3 SNR vs BER performance simulation results for DITH, RS-MECS and RS-DITH are presented. We have included the BER that can be achieved by the (2,64,32) and (32,64,2) LS codes, omitting the curves for "intermediate" codes, i.e. (4,64,16), (8,64,8) or (16,64,4), whose performance falls in between as one could have guessed. Interestingly, the BER performance of RS-MECS is very similar to that of DITH in the sense that the BER curves of DITH when only 1 chip of dithering allowed (i.e. $Z \geq 2$) coincides with the BER curve of RS-MECS when utilizing 2 sequences per user $(M \geq 2)$. Likewise, the curve that

RS-MECS		DITH / RS-DITH			
$\Delta t \leq T_c$	$\Delta t = 7T_c$	$\Delta t = 0$	$\Delta t = T_c$	$\Delta t = 7T_c$	
C=2,	C=2	D=1,	D=0,		
$(M \ge 2)$	$(M \ge 2)$	$(Z \ge 2)$	$(Z \ge 2)$		
C=4,	C=4,	D=3,	D=2,		
$(M \ge 4)$	$(M \ge 4)$	$(Z \ge 4)$	$(Z \ge 4)$		
C=8,	C = 8	D=7,	D=6,	D = 0	
$(M \ge 8)$	$(M \ge 8)$	$(Z \ge 8)$	$(Z \ge 8)$	$(Z \ge 8)$	
C = 16,		D = 15,	D = 14,	D=8	
$(M \ge 16)$		$(Z \ge 16)$	$(Z \ge 16)$	$(Z \ge 16)$	
C = 32,		D = 31,	D = 30,	D = 24	
$(M \ge 32)$		$(Z \ge 32)$	$(Z \ge 32)$	$(Z \ge 32)$	

TABLE II: Dither-Seqs/User Given Sync Requirement

corresponds to 31 chips ($Z \geq 32$) of dithering overlaps with that of RS-MECS with 32 sequences assigned to each user ($M \geq 32$). Note that the performance degradation for DITH only depends on the size of the ZCZ, assumming that the receiver always correlates within ZCZ. In particular, the loss in BER performance is a consequence of the code acquisition process (fine synchronization) that needs to be performed before despreading. On the other hand, in the case of RS-MECS BER performance drops with the increase of the number of sequences assigned to each user, C (bounded by M). The BER performance of RS-DITH is lower than both DITH and RS-MECS for the same codes and depends on the product $C \cdot Z$ (and consequently on $M \cdot Z$). That practically means that shorter codes will achieve better BER performance, obviously with reduced DCS reduction.

In Figure 4 the trade-off between DCS ratio and BER is illustrated for a fixed SNR. The points in each line in this figure correspond to the values of ZCZ size/amount of dithering in chips, D, for DITH or family size/codebook subset size, C, for RS-MECS shown in Table II for the colored columns (where $\Delta t = 0$). Table II also demonstrates that the DCS reduction offered by DITH or RS-DITH diminishes as the time uncertainty increases due to the fact that ZCZ chips are allocated to both sync requirement and dithering, while for RS-MECS, as long as the code can satisfy the time uncertainty requirement, the DCS reduction doesn't change. For example, assume the (8, 64, 8) LS Code is used and 7 chips are needed to satisfy the sync requirement. In this case, RS-MECS will provide DCS reduction, while DITH won't since all 7 ZCZ chips have to be allocated for synchronization, as shown in Table II. Meanwhile RS-DITH will provide the same DCS reduction as RS-MECS. However, if a code with longer ZCZ is used, the gain provided by dithering becomes prevalent again.

V. FUTURE DIRECTIONS

In many tactical settings, communication may be infrequent and sparse in time. Assuming we are concerned with a system where transmissions are sporadic, we can allow each user to make use of the whole codebook. That waives the requirement of $M \geq N$ that needs to be true for RS-MECS, but introduces other issues, such as risk of MAI or authentication problems, the implications of which should be carefully studied especially in the case of military applications.

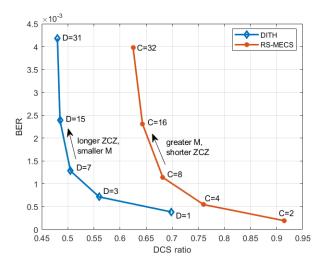


Fig. 4: DCS / BER Trade-off for DITH and RS-MECS

Under such a setting, a correlating step would need to be added anew in order to infer the sequence used to spread each data symbol in the packet thus causing performance degradation. There is now, however, also the possibility of MAI due to the non-zero probability of another user transmitting at the same time using the same sequence. Of course, in a sporadic transmission regime it is quite unlikely that 2 users employing the same sequence will be transmitting simultaneously [17].

Hybrid sequence assignment, i.e. selection from overlapping code subsets, could also be explored in the future. That could mitigate the need for a much larger codebook but MAI and authentication would still remain issues to be addressed. Sequence overuse scenarios in a sporadic transmission context have been investigated in [17], [18].

VI. CONCLUSION

The employment of pattern reducing techniques in the context of QS-DS-CDMA is investigated aiming to reduce the Degree of Cyclostationarity leading to a lower probability of detection. The methods studied are 1) random sequence selection (RS-MECS), 2) random time dithering (DITH), and 3) random sequence selection with dither (RS-DITH).

We have shown that the DCS reduction is better for DITH, RS-DITH follows and RS-MECS offers the smallest reduction among the methods discussed. Periodic pattern reduction comes at the cost of increased receiver complexity as well as some BER performance loss. In particular we have concluded that the BER performance of RS-MECS with C sequences per user, yields the same BER performance with that of DITH with a size-C ZCZ or a base length (L_c) C RS-DITH scheme. However, ignorance of randomization patterns allows secrecy protection and dispensable coordination. Another drawback of RS-MECS and RS-DITH is that they require the employment of a much larger codebook than normally needed in order to maintain immunity to MAI, otherwise the number of users that can be accommodated by the system drops proportionally to the inverse of the number of sequences assigned to each user.

We also studied the trade-off between performance loss and DCS reduction for all 3 schemes. Even though combining random sequence selection with dithering seemed like a promising direction, our work has shown that it does not yield further improvement in DCS reduction. Moreover, the BER performance drops further due to the need to correlate both for the delay and the spreading sequence on a per-symbol basis, while receiver complexity increases significantly.

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