2010 Sixth International Conference on Natural Computation (ICNC 2010)

Simulation of Covariance Analysis Describing Equation Technique (CADET) in Missile Hit Probability Calculation

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***Abstract*—Monte Carlo method is always used in calculating the probability of a missile hit, but it usually cost a lot of time to get accurate statistics. This paper makes use of covariance analysis describing equation technique (CADET) by performance analysis of rapid and comprehensive of missile system. First we elaborate on the primary coverage of CADET. Then we show the application of CADET in the assessment of missile hit probability in combination with a non-linear model of missile. At last, we analyze the pros and cons of CADET by comparing with the result of Monte Carlo method.**

Keywords-missile hit probability; Monte Carlo method; covariance analysis describing equation technique

I. INTRODUCTION

Effectiveness of missile weapon systems is the ability of missiles which accomplish the settled tasks in the modern warfare conditions; the ultimate aim which we pursue in the development and employment of the systems; the most important index which we evaluate the weapon systems. Reference [1] shows that the analysis of effectiveness of missile weapon systems is a process, which integrate numerous monomial tactical and technical indicators into composite performance index that represent the weapon to complete one aspect of combat mission capability, then re-integrated into the weapon system performance . We can judge the weapon system through comparing the system performance value. Hit probability is the most important part of the weapons performance evaluation and the condition-prerequisite to destroy the target as in reference [2]. It can provide effective and quantitative analysis of combat effectiveness of weapon system. Accurate calculation of the hit probability has substantive significance for assessment the efficacy of weapon systems.

Accuracy of weapons systems reveals the spreading characteristics of the point of fall of weapons systems' ammunition relative to the target. It is a measure of ability to hit the target, and an important performance indicator of weapon system. In addition, we usually use hit probability in token of system accuracy while guided-missile system is outside of the protection zone. The estimate of missile hit probability can provide valid data for the study and decision of military operations theories.

Monte Carlo method is often used in counting of missile hit probability. Monte Carlo method can be defined a statistical analysis method aiming at a system affected by random factors in the required on-site or accurate simulation environment, obtains a mass of statistics materials through experiment to evaluate performance indicators as shown in reference [3]. Monte Carlo method can not only get the assessed value performance indicators, but also shows the impact of the performance of weapons systems and the rules of war and other factors on hit probability index, thereby furnishing foundation of quantitative analysis for improve the weapons systems' property and rules of warfare, and it's results are fairly accurate. But Monte Carlo method as shown in reference [4] generally needs hundreds or even thousands of times simulation to get accurate statistics, which will take a lot of computing time. CADET is a method put forward by American ASC Inc. to analyze precision guided of nonlinear guidance system the seventies of last century as shown in reference [5]. This method with the advantage of high-precision and less time-consuming, which combined Describe Equation theory with Covariance Analysis theory, is a better way to analyze and calculate arms fore performance.

This paper will combine a missile non-linear model to apply CADET to hit probability evaluation, and compare with the results of Monte Carlo method to analyze the pros and cons of CADET.

II. THEORY OF CADET

Equation for general nonlinear systems based

$$\dot{x}(t) = f(x,t) + G(t)w(t) \tag{1}$$

Where $x(t)$ is a n-dimensional state vector; $w(t)$ is a p-dimensional random force vector effected on the system interference and control input. $G(t)$ is the corresponding dimension matrix.

Random state vector $x(t)$ is composed of a mean $m(t)$ and the random component $r(t)$, that is

$$\begin{cases} x(t) = m(t) + r(t) \\ m(t) = E[x(t)] \\ P(t) = E[r(t) \cdot r^T(t)] \end{cases} \tag{2}$$

Random force vector $w(t)$ is also composed of the mean $b(t)$ and the random component $u(t)$, that is

$$\begin{cases} w(t) = b(t) + u(t) \\ b(t) = E[w(t)] \\ E[u(t) \cdot u^T(t)] = Q(t) \cdot \delta(t-\tau) \end{cases} \tag{3}$$

Where $\delta(t-\tau)$ is as δ function; $G(t)$ is white noise spectral density matrix.

According to statistical linearization principle, we can get

$$f(x,t) \approx \hat{f} + Nr \tag{4}$$

Substitute into the above formula

$$\begin{aligned} \dot{x}(t) &= \hat{f} + Nr + G(t)w(t) \\ &= Nx(t) + G(t)w(t) + R(t) \end{aligned} \tag{5}$$

Where

$$R(t) = \hat{f} - Nm \tag{6}$$

Since $\hat{f}$ and N have not random variable $x(t)$, Equation (5) is obvious linear differential equations.

According to covariance analysis theory of linear system under the action of the white noise disturbance, the same has been a mean vector $m(t)$ of random state vector $x(t)$ and the propagation equation of covariance matrix $P(t)$

$$\dot{m}(t) = \hat{f} + G(t)b(t) \tag{7}$$

$$\dot{P}(t) = N(t)P(t) + P(t)N^T(t) + G(t)Q(t)G^T(t) \tag{8}$$

As $\hat{f}$ and N generally contain the mean $m(t)$ and covariance $P(t)$ of $x(t)$, so equation (7) and equation (8) are generally nonlinear.

So long as given the probability density function $p(x)$ of random variable $x(t)$, we can compute corresponding describing function. In particular, when $x(t)$ is of joint normal distribution, then

$$\hat{f} = E[f(x,t)] = \int_{-\infty}^{\infty} f(x,t)p(x)dx \tag{9}$$

$$N = \frac{d\hat{f}}{dm} \tag{10}$$

III. THE ANALYSIS OF MISSILE HIT PROGRESS

Reference [6] shows that the short-cycle perturbation equations of missiles lateral channel are:

$$\begin{cases} \ddot{\psi} + b_1\dot{\psi} + b_2\beta = -b_3\delta \\ \dot{\psi}_c - b_4\beta = 0 \\ \psi = \psi_c + \beta \end{cases}$$

(11)

Where ψ is missile yaw angle, ψ_c is velocity declination.

Missile kinematics equation

$$\begin{cases} \dot{x} = V \cdot \cos\psi_c \\ \dot{z} = -V \cdot \sin\psi_c \end{cases} \tag{12}$$

Where V is missile velocity.

Proportional navigation control equation

$$\delta = K \cdot \dot{q} \tag{13}$$

Where $\dot{q}$ is rate of change of the sight angle high and low; K is proportional navigation coefficient。

According Equations (11) and (12), the missile lateral channel is a nonlinear system. When only considering the measurement noise, we may impose noise e on $\dot{q}$, where e is a zero-mean Gaussian white noise corresponded with the CADET hypothesis, and its spectral density value is Q.

Selecting the system state variables:

$$x_1 = \dot{\psi}, \quad x_2 = \psi, \quad x_3 = \psi_c, \quad x_4 = x, \quad x_5 = z$$

State variables above are random variables, then the mean and covariance propagation equation are following

$$\begin{cases} \dot{m}_1 = -b_1m_1 - b_2m_2 + b_1m_3 - b_3m_\delta \\ \dot{m}_2 = m_1 \\ \dot{m}_3 = b_4m_2 - b_4m_3 \\ \dot{m}_4 = V \cdot e^{-\frac{1}{2}P_{33}} \cos m_3 \\ \dot{m}_5 = -V \cdot e^{-\frac{1}{2}P_{33}} \sin m_3 \end{cases} \tag{14}$$

$$\dot{P}(t) = N(t)P(t) + P(t)N^T(t) + G(t)Q(t)G^T(t) \tag{15}$$

Where

$$N = \begin{bmatrix} -b_1 & -b_2 & b_2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & b_4 & -b_4 & 0 & 0 \\ 0 & 0 & -V \cdot e^{-\frac{1}{2}P_{33}} \sin m_3 & 0 & 0 \\ 0 & 0 & -V \cdot e^{-\frac{1}{2}P_{33}} \cos m_3 & 0 & 0 \end{bmatrix}$$

This will be the mean and covariance equations of the system state variables. If the initial conditions both $m(t_0)$ and $P(t_0)$ are known, we can get the control systems' mean and covariance of the state variables at any time under the effect of random interference and noise. x_4 and x_5 are missile coordinates, through analyzing their value in the end of simulation moment in line for the mean miss distance.

IV. SIMULATION AND ANALYSIS

Equation (14) and (15) can be directly solved by Runge-Kutta method. Only m_δ as the mean value of input function needs to solve continuously with the approach of the simulation time. As the zero mean noise signal, combining with equation (13), we will seek the solution of m_δ by the solution of $\dot{q}$.

In order to find the theoretical value $\dot{q}$ of divert the attention ,we can make the X_s axis vector in the coordinate system projected onto the inertial coordinate system, then compared with the projection that the unit vector in the sight lines onto the inertial reference frame.

$$\tan q = -\frac{r_{zd}}{r_{xd}} \tag{16}$$

Take inverse trigonometric functions, and find the first-order derivative, we can get the sight conversion rate $\dot{q}$

$$\dot{q} = \frac{z_r \dot{x}_r - x_r \dot{z}_r}{x_r^2 + z_r^2} \tag{17}$$

Where x_r and z_r are respectively the components of relative position of missile and target in the inertial coordinate system corresponding r_{xd} and r_{zd} ; $\dot{x}_r$ and $\dot{z}_r$ respectively represent the component of relative velocity of missile and target.

To sum up, we can get the mean and covariance of each state variables of CADET equation. x_4 and x_5 are the x and z coordinates of the missile, it may constitute a missile - target collision curve.

Figures 1 to 6 shows the simulation results based on the CADET and the results in comparison with Monte-Carlo method.

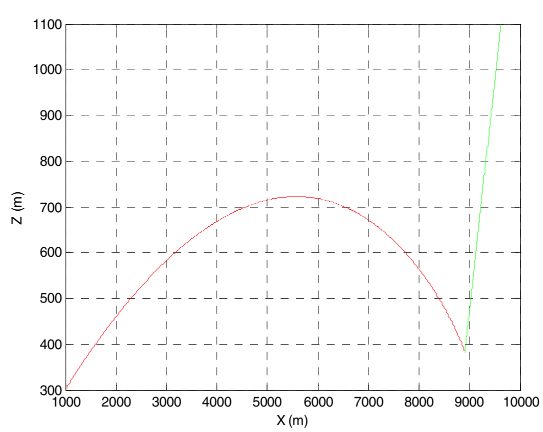

Fig. 1. The missile - target collision curve of CADET

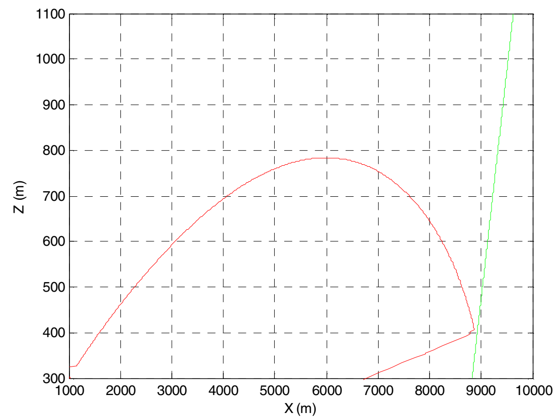

Fig. 2. The missile - target collision mean curve of Monte-Carlo method

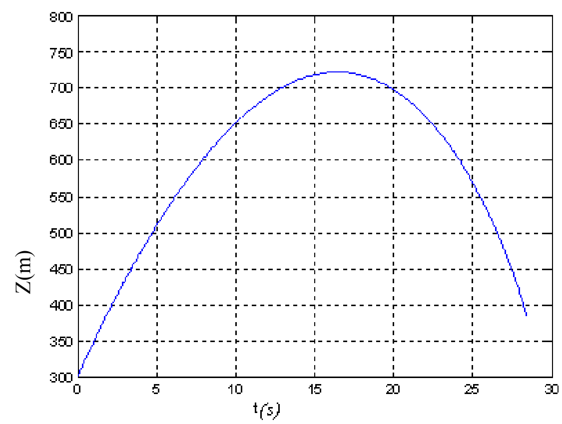

Fig. 3. CADET Method - the mean of missile in z coordinates

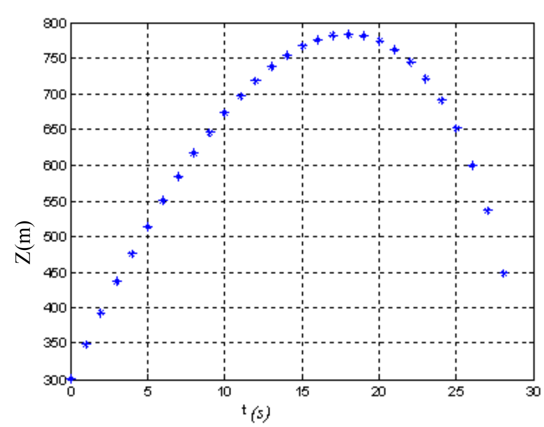

Fig. 4. Monte-Carlo Method - the missile mean of z coordinates

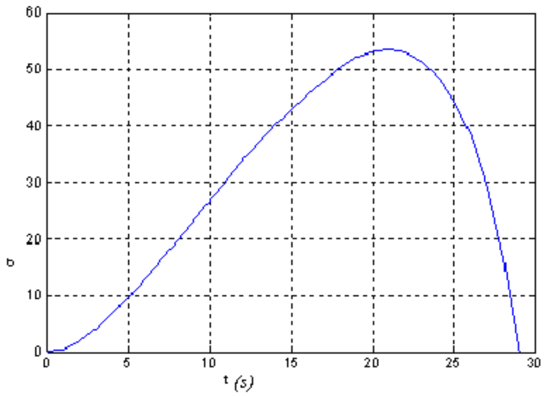

Fig. 5. CADET Method - the mean square error (MSE) of missile in z coordinates

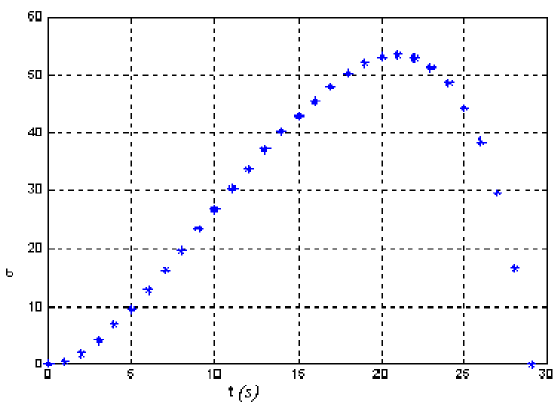

Fig. 6. Monte-Carlo Method - the missile mean square error of z coordinates

It can be seen from the diagrams:

(1) This two kinds of methods are very close at the result of mean area, this is because the analysis of covariance for linear systems analysis is accurate, and the nonlinearity of this example is not serious, the effect is not big.

(2) In the mean-variance, although the results were different, they were very close. Therefore, the computed results of CADET also reflect the performance of the system.

(3) The CADET is based on the equations of motion, so CADET method has the same ability of shown the effect of navigation ratio and other parameters to hit probability as Monte-Carlo method. Thus CADET method provides a way for the actual use of high efficiency and scientific accurate method of calculating the hit probability.

V. Discussion

(1) The CADET method can effectively calculate the mean value and variance of the system. But statistics test method is the proximate results of limited times statistical simulation. With demands of the precision increasing, statistical experiment method has to increase statistical samples and it takes a long time. Facing the increasing of random factors, it will take more time because of the decline of the program efficiency. Therefore, the statistical properties of CADET method is not only science available, but also very effective.

(2) The precision of CADET method is approximate the same as that of simulate 100 times by Monte Carlo method while time-consuming is one of dozens as that of the latter. Therefore, CADET provides a convenient and effective method to optimize the system parameters.

(3) In the condition of system nonlinearity being very serious, expected vector and the linear system dynamic matrix solution will be very complex, even can't work out analytical formula. In this case, the precision of CADET method is greatly reduced. But the simulation and numerous references shows that even if expecting precision doesn't accord with the statistical test results, the regularity is the same. Therefore, in choosing the system parameters, CADET method can also reflect control system characteristics.

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