Optimization of Spreading Factor Distribution in High Density LoRa Networks

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***Abstract*—LoRa is a promising wireless technology for various sensing and positioning applications in Smart Cities. LoRa uses Chirp Spread Spectrum (CSS) with different Spreading Factors (SF) to handle varying intensities of multipath reflections and interference. However, the standard LoRaWAN uses the pure ALOHA algorithm that suffers from both Intra-SF collisions and Inter-SF collisions which limit it to mostly low density environment. In this work, we optimize the transmission parameters of a LoRaWAN system in *high density* Smart City traffic environment using golden section search and parabolic interpolation. Our approach of optimum distribution of spreading factors not only significantly improves the success rate, but also enable more nodes to use lower spreading which results in lower delay.**

***Index Terms*—LoRa, LoRaWAN, Smart City, High Density Communications, High Traffic, Spreading Factor, Collision.**

I. Introduction

LoRa (short for Long Range) wireless technology has the capability for long range, low bit rate, low power communications which, makes it an ideal solution in many Smart City and Industrial Internet of Things (IIoT) applications including sensing and positioning. LoRa uses Chirp Spread Spectrum (CSS) technology [1]. The broadband chirp pulses have good resistance against disturbances such as noise, narrow and wideband interference, scattering, diffraction as well as multipath fading with low transmission power [2], [3], [4].

However, different types of traffics in Smart City applications have different Quality of Service (QoS) requirements such as deterministic latency, low energy consumption, reliability (high packet success delivery), and secure data transmissions [5], [6]. This may not be effectively addressed by the widely adopted open source network standard LoRaWAN (LoRa Wide Area Network).

LoRaWAN uses pure ALOHA transmission protocol [7] which is more appropriate for low traffic scenarios where, each node (or sensor) is expected to transmit few messages per day [8]. In high node density and traffic intensity environments such as Smart Cities, LoRaWAN will experience much higher collision probability and lower throughput because of the pure ALOHA. In this paper, we propose a novel approach to improve the LoRa network capacity by optimizing Spread Factor (SF) assignments in a frequent transmission, high node density Smart City scenario.

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II. Key Attributes of LoRa Network

In LoRa, various parameters can be tuned to get the optimum performance depending on the environment and the number of LoRa nodes served by a LoRa gateway. The following are some of key LoRa parameters that can be optimized for better performance:

1) Spreading Factor (SF)
2) Transmission Power (TP)
3) Band Width (BW)
4) Coding Rate (CR)
5) Pay Load (PL)

LoRaWAN is a pure ALOHA system where, the nodes do not listen-before-talk or use CSMA to avoid collision. Therefore, the LoRaWAN system is highly susceptible to frame collisions that leads to packet loss.

Frame collision is a major issue in LoRa system [8]. When a collision happens, LoRaWAN will wait and re-transmit the data with a higher SF. This will further worsen the situation, since longer SF will occupy the channel for longer duration increasing the collision probability. For example, Figure 1 shows the SF12 packet takes approximately 1.8 seconds to transmit 40 bytes of payload in the 125 kHz bandwidth channel. This time period is significantly long which will further increase the collisions. Hence, the chances of frame collision goes very high in Smart Cities where the node density is large and/or packet transmission rate is high. Successful Frame delivery is especially critical in Smart City applications that require high Quality of Service (QoS) and near real time delivery of data.

III. Packet Transmission Time Calculation

In LoRa systems, the symbol transmission time, T_{sym}, is given by [9]:

$$T_{sym} = \frac{2^{SF}}{BW} \tag{1}$$

where SF is Spread Factor and BW is signal bandwidth.

In order to calculate the time, T_{pkt}, needed to transmit a data packet via the LoRa radio interface, LoRa modulation details also need to be considered. Figure 2 shows the LoRa signal frame [10] in which, the data is followed by a preamble sequence. So, T_{pkt} includes the time required to transmit both the preamble and the physical message, denoted as T_{pre} and T_{phy} respectively [9].

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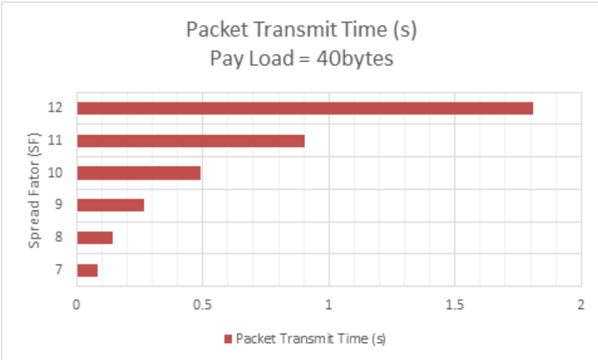

Fig. 1. Packet Transmit Time vs Spread Factor (SF) for 40 bytes pay load in 125 KHz Channel

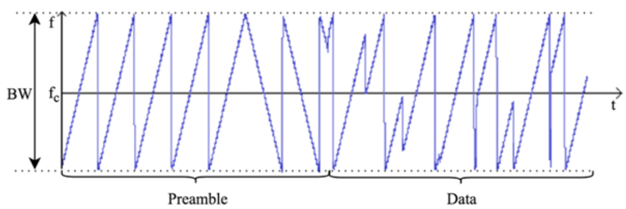

Fig. 2. LoRa Packet Format

$$T_{pkt} = T_{pre} + T_{phy} \tag{2}$$

Preamble transmission time, T_{pre}, is, [10]

$$T_{pre} = (N_{pre} + 4.25)T_{sym} \tag{3}$$

Therefore,

$$T_{pre} = (N_{pre} + 4.25)(\frac{2^{SF}}{BW}) \tag{4}$$

where, N_{pre} is the programmable preamble symbols.

Let us define, a temporary variable $TEMP$ as,

$$TEMP = \frac{(28 + 8PL + 16CRC - 4SF - 20IH)}{4(SF - 2DE)} \tag{5}$$

Then the total physical layer payload size, N_{pl} is given by [9],

$$N_{pl} = 8 + max[ceil[TEMP](CR + 4), 0] \tag{6}$$

Physical message transmission time, T_{phy} can be obtained as,

$$T_{phy} = (8 + max[ceil[TEMP](CR + 4), 0])(\frac{2^{SF}}{BW}) \tag{7}$$

Hence, the total LoRa packet transmission time, T_{pkt}, is given by,

$$T_{pkt} = (N_{pre} + 4.25 + (8 + max[ceil[TEMP](CR + 4), 0])) (\frac{2^{SF}}{BW})$$

where, PL is the payload size (bytes) and SF is the spreading factor (7-12). CRC takes the value of '1' if used and '0' otherwise. Implicit Header value IH is '1' in implicit header mode and, '0' otherwise. DE is '1' if data rate optimization value is used and '0' otherwise. The CR, code rate lies between 1 and 4 (4 / CR + 4).

IV. Receive Power Calculation and LoRa Receiver Sensitivity

Data is successfully received when the received power, P_{rx}, is higher than the receiver sensitivity, RX_{sen}.

$$P_{rx} > RX_{sen}. \tag{8}$$

RX_{sen} is depended on the SF and BW combination [9]. The receiver sensitivity values are shown in the Table I[1].

TABLE I
LoRa Receiver Sensitivities

		Receiver sensitivity, RX_{sen} (dBm)		
SF	DR	BW=125KHz	BW=250KHz	BW=500KHz
7	5	-123	-120	-117
8	4	-126	-123	-120
9	3	-129	-126	-123
10	2	-132	-129	-126
11	1	-134.5	-131.5	-128.5
12	0	-137	-134	-131

P_{rx} can also be written as,

$$P_{rx} = P_{tx} + G - L - L_{pl} \tag{9}$$

where, P_{tx} is the LoRa node transmission power, G is the total antenna gains and L is the signal losses at the transmitter. L_{pl} is the total path loss between transmitter and receiver.

In this work, LoRaWAN networks deployed in the 902 - 928 MHz ISM band is considered to calculate the path loss. Log-Distance path loss model is selected for the wireless channel. The total power loss L_{pl}, is a function of the distance d between the transmitter and receiver.

$$L_{pl} = L_{pl}^{d_0} + 10\gamma \ln(\frac{d}{d_0}) + X_\sigma \tag{10}$$

where X_σ is the variance and γ is the path-loss coefficient.

V. Frame Success Probability Analysis

Packet collision in a LoRa network can be divided into two types: Intra-SF collisions and Inter-SF collisions [11]:

1) Intra-SF collisions: a collision occurs between two LoRa frames with the same frequency and SF. In this case, only the LoRa frame with the highest power can be decoded. This happens when its power at the gateway is at least 6 dB more than the other one.
2) Inter-SF collisions: a collision occurs between two LoRa frames with the same frequency and different SF. In this case, the first frame is demodulated only if the difference between the received power of the first frame and the second frame is higher than the Signal-to-Interference Ratio (SIR) of the first one (Refer Table II) [12]. Each element in the table is the required SIR margin that a packet sent at SF_i shall have in order to be decoded correctly, assuming the colliding packet is sent at SF_j .

[1]These are calculated using Lora Modem Calculator Tool.

TABLE II
MINIMUM SIR REQUIRED TO DEMODULATE SF_i

	Interferer					
SIR Margin (dB)	SF7	SF8	SF9	SF10	SF11	SF12
SF7		-8	-9	-9	-9	-9
SF8	-11		-11	-12	-13	-13
SF9	-15	-13		-13	-14	-15
SF10	-19	-18	-17		-17	-18
SF11	-22	-22	-21	-20		-20
SF12	-25	-25	-25	-24	-23	

Next, we study the maximum node density a LoRa gateway can cover with a given frequency, assuming that nodes are distributed uniformly over a linear segment with distance d centered at the gateway as illustrated in 3.

It is assumed that the nodes are operating in the same frequency but with different SFs (7 to 12). Total number of nodes in the segment are N. Density of the nodes, D, is given by:

$$D = \frac{N}{2d} \tag{11}$$

It is assumed that the data transmissions follow a Poisson distribution with rate $\theta 2dD$ where, θ is packet transmission intensity (packets per second).

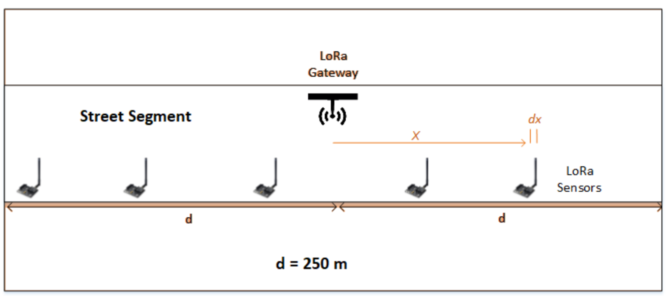

Fig. 3. Typical Smart City Street Segment with one Gateway

Now, let us consider a node, transmitting a packet at distance x from the gateway. The transmission is successful if no intra or inter-SF collisions happen.

A. Intra-SF Collisions

Considering fixed conditions and all nodes (N) have the same transmission parameters, the received power level only depends on the distance. Hence, the potential intra-SF interferes are those whose distance from the gateway is below xR with

$$R = e^{\frac{6}{10\gamma}} \tag{12}$$

The number of potential interferes depends on the number of nodes with the same SF and BW. We denote α_i as the percentage of nodes in N having SF equal to SF_i, where $i \in [7, 12]$. It holds that :

$$\sum_{i=1}^{7} \alpha_i = 1 \tag{13}$$

D_i denotes the density of the nodes having SF equal to SF_i, with

$$D_i = \frac{\alpha_i N}{2d} \tag{14}$$

B. Inter-SF collisions

On the other hand, some nodes with the same SF_i may interfere with nodes having different SF but transmitting in the same frequency. The potential inter-SF interferers are those whose distance from the gateway is below xQ.

$$Q = e^{\frac{SIR_{[i,j]}}{10\gamma}} \tag{15}$$

where, $SIR_{[i,j]}$ is the corresponding SIR of transmissions conducted using SF_i and interferer SF_j.

Therefore, the total number of potential interferers is given by the following equations:

$$\text{Total Interferers} = D_i.R.x + D.Q.x \tag{16}$$

Since the transmission from nodes follow a Poisson distribution, the probability of observing k events in an interval is given by,

$$P[k] = e^{\lambda} \frac{\lambda^k}{k!} \tag{17}$$

Probability of successful transmission $P_{success}(x)$ is therefore the probability that within a vulnerable period of duration $2T_{pkt}$, none of those potential interfering nodes started a transmission. Hence, the success probability is given by:

$$P_{success}(x) \approx e^{-2T_{pkt}\theta(D_i.R.x + D.Q.x)} \tag{18}$$

Therefore, the average success probability for a specific SF among all nodes in the $2d$ IIoT segment is given by the following equation,

$$P^i_{avearge} \approx \frac{1}{D_i d} \int_{x=0}^{d} D_i.e^{-2T_{pkt}\theta(D_i.R.x + D.Q.x)} dx \tag{19}$$

The above equation can be rewritten as,

$$P^i_{avearge} \approx (\frac{1}{d})(\frac{1 - e^{-2T_{pkt}\theta(D_i.R.d + D.Q.d)}}{2T_{pkt}\theta(D_i.R.d + D.Q.d)}) \tag{20}$$

where $i \in [7, 12]$.

VI. OPTIMIZATION

Then we formulated the probability model into a optimization problem to maximize the average probability. Following items show the objective function, variables and constrain.

1) Objective Function:

$$P^i_{avearge} \approx (\frac{1}{d})(\frac{1 - e^{-2T_{pkt}\theta(D_i.R.d + D.Q.d)}}{2T_{pkt}\theta(D_i.R.d + D.Q.d)}) \tag{21}$$

where $i \in [7, 12]$.

2) Variables: SF_i, T_{pkt}, α_i, D_i , Q

3) Constrains:

$$0 \leq \alpha_i \leq 1 \tag{22}$$

$$\alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10} + \alpha_{11} + \alpha_{12} = 1 \tag{23}$$

Success probabilities were evaluated using the optimization toolbox available in MatLab^{TM}. The algorithm used is based on golden section search and parabolic interpolation.

VII. Numerical Results

The above probability model was evaluated using the parameters in Table III. It was assumed that all the nodes are located within 250 m from a single gateway and SFs are equally distributed among the nodes.

TABLE III
LoRa Network Parameters

Parameter	Value
Number of Nodes (N)	1200
Node Transmission Interval	10 min
Code Rate (CR)	4/5
Bandwidth (BW)	125KHz
Spreading Factor (SF)	7-12
Payload(PL)	40bytes
Low data rate optimization (DE)	1
Header(H)	0
Preamble symbols(n_p)	8
Transmission power (P_{tx})	14 dBm
Gains minus losses (GL)	0
Path loss exponent (γ)	4

Success probabilities were first evaluated without any optimization. Figure 4 (a) shows the success probability in each SF node while Figure 4(b) shows the number of nodes using each SF. Based on the results, average probability of success is $\mathbf{58.1}\%$.

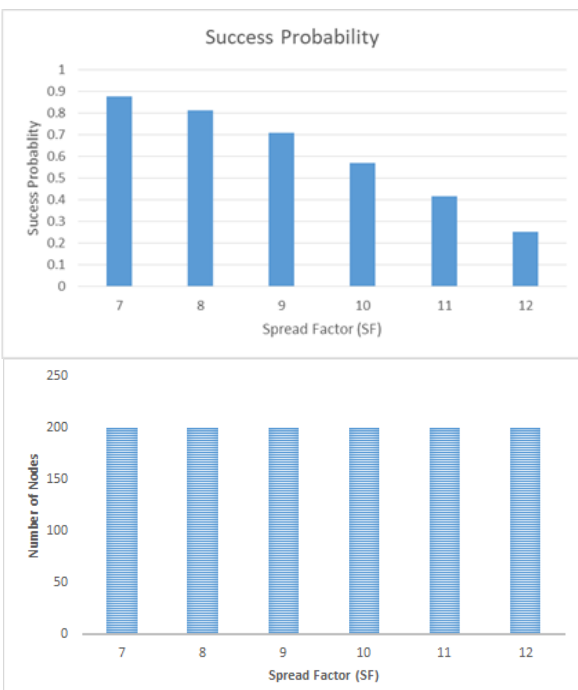

Fig. 4. Success Probability without Optimization

Then the optimized algorithm is deployed. From the algorithm, the optimized values for Alpha values (α_i) are given in Table IV. Figure 5(a) shows the success probability in each SF nodes while Figure 5(b) shows the number of nodes in each SF.

Based on the optimization results, average probability of success is $\mathbf{82.0}\%$. This results indicate that LoRa capacity can be increased significantly by assigning appropriate SF to the nodes. Similarly, other parameters, such as node transmission power, code rate and pay load, can also be optimized for better success probability.

TABLE IV
Alpha Values

SF	α_i
7	.76
8	.24
9	0
10	0
11	0
12	0

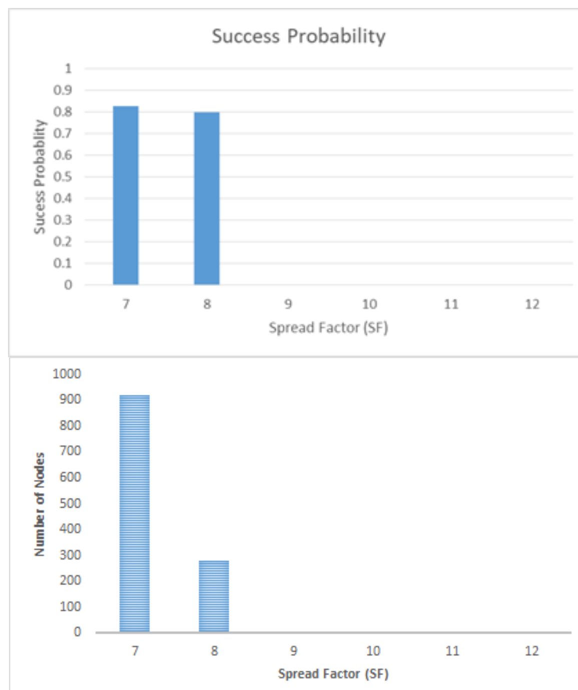

Fig. 5. Success Probability and Number of Nodes with Optimization

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