Performance Analysis of Digital Communications over Rician-TWDP Channels

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Abstract— This paper presents the conceptual behavior of digital communication for the composite Rician-Two Wave Diffuse Power (TWDP) fading-shadowing model. Here, small scale fading is attributed by the Rician channel, whereas shadowing effects are modeled by TWDP. More specifically, in this work the exact closed-form expression is derived for the probability density function (PDF), cumulative distribution function (CDF), joint moments, mean, variance, amount of fading (AF) and outage probability. Also, the effects of various shadowing parameters on system performance have been studied in detail. The obtained results are matched cum verified with special case results available in the literature and simulation results.

***Index Terms*— Rician-TWDP, composite channels, fading, shadowing, amount of fading and outage probability**

I. INTRODUCTION

RADIO signal propagation, in wireless communications, is usually influenced by three independent processes, first shadowing, second is multipath fading, both are described by using fading models and the other process is the distance varying factor called path loss. All of the three processes result in the failure of transmitting signals to reach the receiver perfectly. It happens because signals lost its power after following the basic mechanisms of scattering, diffraction, and reflection between the source and the destination. Small scale fading where power fluctuation occurs for a very short duration is due to rapid constructive and destructive interference of radio signals [1-2]. Shadowing occurs when there is a large obstruction between the sender and receiver. In the case of the indoor wireless environment, having small space structures like open offices, banks, schools, buildings, laboratories, etc., the above mechanisms are usually ignored or absent. More specifically, due to the incapability of many wireless devices in adjusting desired hand-offs, WLAN users have never exhibited large movements. As a result, they experience one or two scattering clusters and acknowledge a fistful of different shadowing values [3-6]. Based on this, Rayleigh, Rician, Weibull, k-μ/gamma composite fading and the newly TWDP shadowing models have been studied in the literature [7-11]. A lot of work on the analysis of system performance metrics such as diversity combining and bit error rate over TWDP has been proposed in [12-13] respectively. No work is reported in literature yet which analyzed the effect of TWDP shadowing on the Rician fading channel. In this paper, an indoor model having a combination of Rician and TWDP distribution is designed in a unified and flexible way. Here, Rician distribution is being attributed for small scale fading in which the dominant line-of-sight component is modeled by TWDP shadowing. The main imparts of this work is to study the performance of the composite Rician-TWDP shadowing model. The paper is outlined below, section II describes the system model of composite Rician-TWDP model and the statistical characteristics of the model are demonstrated in section III. The various results are analyzed in section IV along with a vivid conclude remarks in the last section V.

II. SYSTEM MODEL

It is assumed that multipath fading follows Rician distribution, its PDF is evaluated as,

$$f_R(r) = \frac{r}{P_1}\exp\left(\frac{-r^2}{2P_1} - K_1\right) I_0\left(r\sqrt{\frac{2K_1}{P_1}}\right) \tag{1}$$

where $I_0(.)$ represents the modified Bessel function of the first kind and zero-order, K_1 so-called as Rician K factor given by $K_1 = A^2/2P_1$, where A is the magnitude of the specular component and P_1 is mean squared voltage of the diffused component [3].

Two waves with diffuse power fading consist of two direct specular components along with diffused components. In this work, the shadowing follows the TWDP distribution. The PDF is expressed as in [6],

$$f_Y(y) = \frac{y}{2P_2}\sum_{j=0}^{1}\exp\left(\frac{-y^2}{2P_2} - P_{2ij}\right)\sum_{i=1}^{L} a_i I_0\left(y\sqrt{\frac{2P_{2ij}}{P_2}}\right) \tag{2}$$

where, L is the order of TWDP, such as $L \geq 1/2\,K\Delta$, where Δ is the relative strength of the two specular(LOS) component, as given by $\Delta = 2A_1A_2/A_1^2A_2^2$ and $P_{2ij} = K\left[1+(-1)^j\,\Delta\cos\left(\pi\left(\frac{i-1}{2L-1}\right)\right)\right]$ where K_2 is the TWDP K factor, given by $K_2 = \left(A_1^2 + A_2^2\right)/2P_2$ where A_1, A_2 is the magnitude of two specular components and P_2 is mean squared voltage of the diffused components.

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Different values of the exact coefficient a_i for the first five orders of the approximate TWDP fading PDF given in the table below.

Order	a_1				
1	1	a_2			
2	$\frac{1}{4}$	$\frac{3}{4}$	a_3		
3	$\frac{19}{144}$	$\frac{25}{48}$	$\frac{25}{72}$	a_4	
4	$\frac{751}{8640}$	$\frac{3577}{8640}$	$\frac{49}{320}$	$\frac{2989}{8640}$	a_5
5	$\frac{2857}{44800}$	$\frac{15741}{44800}$	$\frac{27}{1120}$	$\frac{1209}{2800}$	$\frac{2889}{22400}$

III. STATISTICAL CHARACTERISTICS

In this section, we studied the Rician fading channel in which the dominant line-of-sight component is subjected to TWDP shadowing. Using the concept of shadowing, the signal envelope, R, at receiver in a shadowed Rician-TWDP channel can be obtained by solving the conditional probability given in (3) as:

$$f_R(r) = \int_0^\infty f_{(R|Y)}(r|y) f_Y(y)\,dy \tag{3}$$

whereas

$$f_{(R|Y)}(r|y) = \frac{r}{P_1}\exp\left(\frac{-r^2}{2P_1}\right)\exp\left(-K_1 y^2\right) I_0\left(yr\sqrt{\frac{2K_1}{P_1}}\right) \tag{4}$$

Theorem 1. *For K_1, K_2, P_1, and $P_2 > 1$ and $r \in R$ the PDF of Composite Rician-TWDP channel can be written as*

$$f_R(r) = \frac{r}{2P_1(2K_1P_2+1)}\sum_{i=1}^{L} a_i \sum_{j=0}^{1}\exp\left(\frac{-r^2}{2P_1} - P_{2ij}\right)\times \sum_{v=0}^{\infty}\frac{\Gamma(v+1)}{v!^2}\left(\frac{P_{2ij}}{2K_1+1}\right)^2 {}_1F_1\left(v+1, 1, \frac{K_1P_2r^2}{P_1(2K_1P_2+1)}\right) \tag{5}$$

where ${}_1F_1$ (_; _; _; _) is the Gauss hyper geometric function [14, Eq. (9.14)].

Proof: See Appendix A

When this signal is passed into the channel, it is added with Gaussian Noise. As a result, the received instantaneous signal to noise power ratio is raised by R^2. If we define instantaneous signal to noise power ratio per symbol as, $\gamma = R^2 E_s / N_0$ then the PDF of the instantaneous SNR, $f_\gamma(\gamma)$ is obtained from (5) via the transformation of variables $f_\gamma(\gamma) = f_Z\left(\sqrt{\Omega\gamma/\bar{\gamma}}\right) \Big/ \left(2\sqrt{\gamma\bar{\gamma}/\Omega}\right)$ as,

$$f_\gamma(\gamma) = \frac{1}{4P_1(2K_1P_2+1)\bar{\gamma}}\sum_{i=1}^{L} a_i \sum_{j=0}^{1}\exp\left(\frac{-\gamma}{2P_1\bar{\gamma}} - P_{2ij}\right)\times \sum_{v=0}^{\infty}\frac{\Gamma(v+1)}{v!^2}\left(\frac{P_{2ij}}{2K_1P_2+1}\right)^2 {}_1F_1\left(v+1, 1, \frac{K_1P_2\gamma}{P_1(2K_1P_2+1)\bar{\gamma}}\right) \tag{6}$$

Lemma 1. *For K_1, K_2, P_1 and $P_2 > 1$ and $r \in R$ the CDF of Composite Rician-TWDP Channel can be calculated as, $F_r(R) = P_r(R \le r) = \int_0^r f(r)\,dr$, which is given as*

$$F_r(R) = \frac{1}{2(2K_1P_2+1)}\times \sum_{i=1}^{L} a_i \sum_{j=0}^{1}\exp\left(-P_{2ij}\right)\sum_{v=0}^{\infty}\frac{1}{v!^2}\left(\frac{P_{2ij}}{2K_1P_2+1}\right)^v \times \sum_{s=0}^{\infty}\frac{\Gamma(v+s+1)}{\Gamma(s+1)s!}\left(\frac{2K_1P_2}{2K_1P_2+1}\right)^s \Gamma\left(s+1, \frac{r^2}{2P_1}\right) \tag{7}$$

Proof*:* By putting the PDF envelope of a signal (5) in $F_r(R) = P_r(R \le r) = \int_0^r f(r)\,dr$ and after rearranging, we get

$$F_r(R) = \frac{1}{2P_1(2K_1P_2+1)}\sum_{i=1}^{L} a_i \sum_{j=0}^{1}\exp\left(-P_{2ij}\right)\times\sum_{v=0}^{\infty}\frac{\Gamma(v+1)}{v!^2}\left(\frac{P_{2ij}}{2K_1+1}\right) \times\int_0^\infty \exp\left(\frac{-r^2}{2P_1} - P_{2ij}\right) {}_1F_1\left(v+1, 1, \frac{K_1P_2r_2}{P_1(2K_1P_2+1)}\right) dr \tag{8}$$

writing ${}_1F_1$ (_; _; _; _) Gauss hypergeometric function into series expansion [14, Eq. 9.14.1] and using [14, Eq. 3.381.8], we get the final expression for the cumulative distribution function of the Rician TWDP channel as in (7).

Lemma 2. *For K_1, K_2, P_1 and $P_2 > 1$ and $r \in R$ the n^{th} moment of Composite Rician-TWDP Channel can be calculated as $E[R^n] = \int_{-\infty}^{\infty} r^n f_R(r)\,dr$ and is given as*

$$E[R^n] = (2P_1)^{\frac{n}{2}}\,\Gamma\left(\frac{n}{2}+1\right) W^{(n)}\left(v, K_1, a_i, P_{2ij}, P_2\right) \tag{9}$$

where,

$$W^{(n)}(v, K_1, a_i, P_{2ij}, P_2) = \frac{1}{2(2K_1P_2+1)}\sum_{i=0}^{L} a_i \sum_{j=0}^{1}\exp(-P_{2ij})\times \sum_{v=0}^{\infty}\frac{\Gamma(v+1)}{v!^2}\left(\frac{P_{2ij}}{2K1+1}\right)^v {}_1F_1\left(v+1, \frac{n}{2}+1, \frac{2K_1P_2}{2K_1P_2+1}\right)$$

The function $W^{(n)}\left(v, K_1, a_i, P_{2ij}, P_2\right)$ introduced here is for simplicity only.

Proof: Putting expression of $f_R(r)$ from (5), into $E[R^n]=\int_{-\infty}^{\infty} r^n f_R(r)\,dr$ we get,

$$E[R^n]=\frac{1}{2P_1(2K_1P_2+1)}\sum_{i=0}^{L}a_i\sum_{j=0}^{1}\exp\left(-P_{2ij}\right)\sum_{v=0}^{\infty}\frac{\Gamma(v+1)}{v!^2}\left(\frac{P_{2ij}}{2K_1+1}\right) \times\int_0^{\infty} r^{n+1}\exp\left(\frac{-r^2}{2P_1}\right)\,{}_1F_1\left(v+1,1,\frac{K_1P_2r^2}{P_1(2K_1P_2+1)}\right)dr \tag{10}$$

Using [14, Eq. 7.621.4] and change of variables, the above equation can be written as in (9).

***Mean**:* It is represented by $E[R]$. The mean for composite Rician-TWDP channel is calculated using n=1 in (9)

$$E[R]=\sqrt{\frac{\pi P_1}{2}}W^{(1)}(v,K_1,a_i,P_{2ij},P_2) \tag{11}$$

***Second Moment**:* It is represented by $E[R^2]$. The second moment for the composite Rician-TWDP channel is calculated using n=2 in (9)

$$E[R^2]=2P_1W^{(2)}(v,K_1,a_i,P_{2ij},P_2) \tag{12}$$

***Variance**:* The variance is defined as $\sigma^2{}_R=E[R^2]-\{E[R]\}^2$. Using results from (11) and (12), we get variance as

$$\sigma^2{}_R=2P_1W^{(2)}(v,K_1,a_i,P_{2ij},P_2)-\frac{\pi P_1}{2}\left\{W^{(1)}(v,K_1,a_i,P_{2ij},P_2)\right\}^2 \tag{13}$$

***Amount of Fading**:* It defines the extreme level of fading parameters. The AF for the composite Rician-TWDP channel is expressed as

$$AF=\frac{\mathrm{var}[R^2]}{\left\{E[R^2]\right\}^2}=\frac{E[R^4]-\left\{E[R^2]\right\}^2}{\left\{E[R^2]\right\}^2} \tag{14}$$

where $E[R^4]$ represent the fourth moment of the composite Rician-TWDP Channel obtained using n=4 in (9). Finally, *AF* results in

$$AF=\frac{2W^{(4)}(v,K_1,a_i,P_{2ij},P_2)-\left\{W^{(2)}(v,K_1,a_i,P_{2ij},P_2)\right\}^2}{\left\{W^{(2)}(v,K_1,a_i,P_{2ij},P_2)\right\}^2} \tag{15}$$

***Outage Probability**:* It is defined in terms of the cumulative distribution function, namely $f_\gamma(\gamma)$, and calculated at $\gamma=\gamma_{th}$ as,

$$P_{out}=P_r(\gamma\le\gamma_0)=\int_0^{\gamma_{th}} f_\gamma(\gamma)\,d\gamma=F_\gamma(\gamma_0)$$

***Corollary 1.** For K_1, K_2, P_1 and P_2 >1 and $r\in R$ the OP of the Composite Rician-TWDP Channel can be calculated as*

$$P_{out}=\frac{1}{2(2K_1P_2+1)}\sum_{i=1}^{L}a_i\sum_{j=0}^{1}\exp(-P_{2ij})\times\sum_{v=0}^{\infty}\frac{1}{v!^2}\left(\frac{P_{2ij}}{2K_1P_2+1}\right)^v \times\sum_{s=0}^{\infty}\frac{\Gamma(v+s+1)}{\Gamma(s+1)s!}\left(\frac{2K_1P_2}{2K_1P_2+1}\right)^s\Gamma\left(s+1,\frac{\gamma_{th}}{2P_1}\right) \tag{16}$$

where γ_{th} is the threshold SNR.

Proof: Putting (6) into $P_{out}=P_r(\gamma\le\gamma_0)=\int_0^{\gamma_{th}} f_\gamma(\gamma)\,d\gamma=F_\gamma(\gamma_0)$ and using [14, Eq. 3.381.8] we get (16).

IV. Results and Discussion

The analysis of PDF, CDF and outage probability are done by plotting the results in this section. Hereby using eq. (5), (7) and (16) various the graphs are plotted to study the effect of shadowing and fading parameters. The results shown here features the statistics of the composite Rician-TWDP shadowed model.

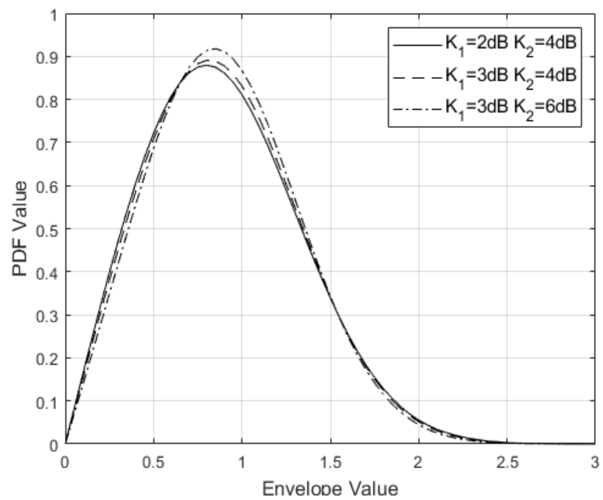

Fig.1: PDF of the model with various K_1, K_2, and $\Delta=0.1$ values.

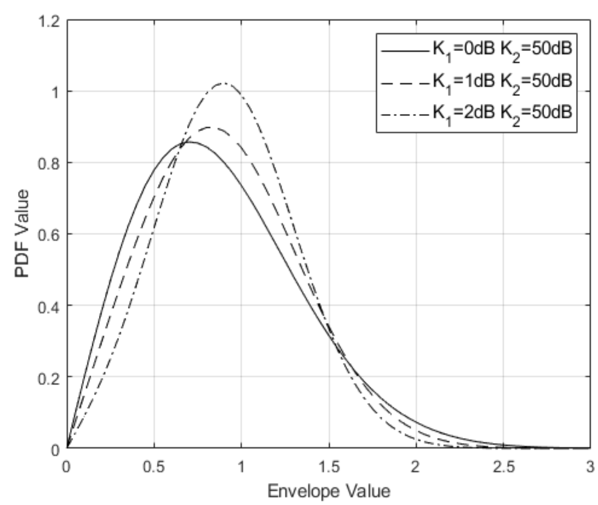

Fig.2: PDF of the model for a special case at $\Delta=0$

In Fig.1 for fixed $\Delta = 0.1$, as Rician factor K_1 is increased from K_1=2dB to K_1=3dB, there is a significant rise in PDF peak due to a drop in shadowing for fixed TWDP factor K_2=4dB. The system behaves similar to the previous one when the TWDP factor K_2 is varied from K_2=4dB to K_2=6dB at fixed K_1=3dB. The curves resemble the special case of Rayleigh and Rician distribution when K_2 made very high (shadowing disappears completely) and $\Delta = 0$, as shown in Fig. 2. At K_1 =0, it behaves as Rayleigh and for other values it acts as Rician. In Fig. 3 the PDF is plotted for $\Delta = 0.5$ value and it is clearly observed that as Δ value increases, the peak density level decreases gradually.

The CDF of the Rician TWDP shadowed fading model with Δ=0.2 value is shown in Fig.4. Fig.5 shows the graph of outage probability w.r.t average SNR at fixed $\gamma_{th} = 10dB$ and Δ =0.2. As shown in the figure, the OP curve decreases for higher values of K_1, K_2 at fixed average SNR. The special case results of K_1=0dB K_2=50dB $\Delta = 0$ (Rayleigh) and K_1=2dB K_2=50dB $\Delta = 0$ (Rician) are illustrated for CDF and OP as shown in Fig. 4 and 5 respectively.

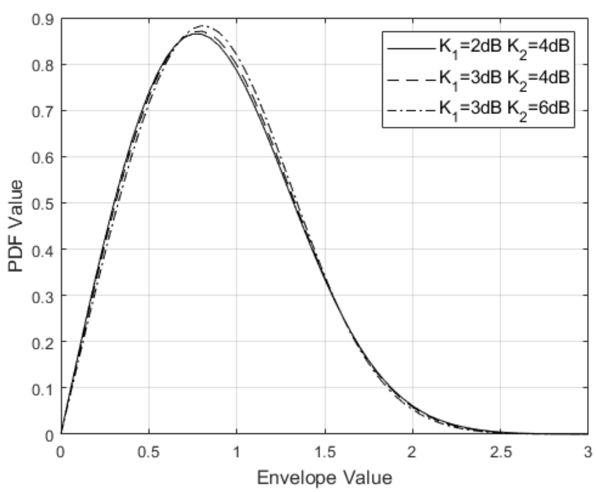

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Fig.3: PDF of the model for parameters K_1, K_2, and $\Delta = 0.5$.

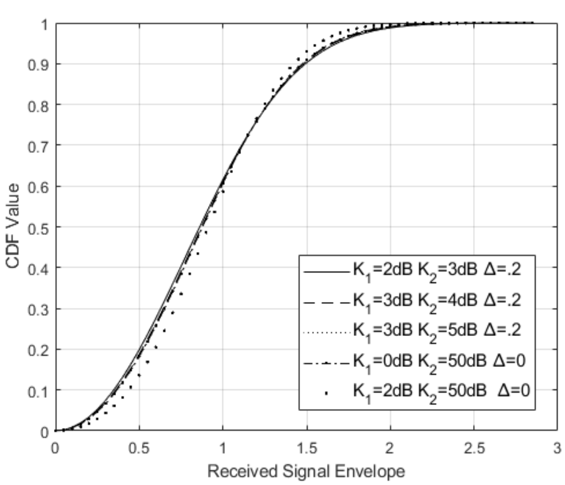

Fig.4: CDF of the model for parameters K_1, K_2, and Δ values.

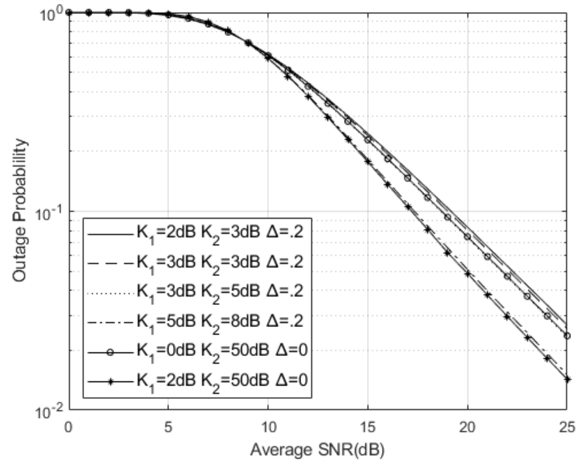

Fig.5: Outage Probability versus Average SNR

V. Conclusion

This paper investigates a composite fading-shadowing model for indoor WLAN applications where the fading channel follows Rician distribution and shadowing is modeled by the TWDP model. More specifically, the expression for PDF, CDF, joint moment, outage probability and amount of fading were derived in simplified form. The above derived analytical expressions are numerically evaluated and compared with special cases of Rician TWDP shadowing model.

The model is best suited for indoor environments, where large obstacles are rarely encountered and users never allow their movements in large areas due to incapability of majority WLAN devices to adjust desired hand-offs. The vivid analysis cum discussion suggests the wider application of Rician–TWDP, to get the best performance from the latest WLAN standards and indoor communication.

Appendix A

Replacing A of K_1 in (1) by Ay making use of conditional probability and then using (2) and (4) in (3) after rearranging, we get

$$f_R(r) = \frac{r}{2P_1P_2}\exp\left(\frac{-r^2}{2P_1}\right)\sum_{i=1}^{L} a_i \sum_{j=0}^{1}\exp\left(-P_{2ij}\right)\times$$
$$\times\int_0^\infty y\exp\left(-y^2\left(K_1+\frac{1}{2P_2}\right)\right) I_0\left(yr\sqrt{\frac{2K_1}{P_1}}\right) I_0\left(y\sqrt{\frac{2P_{2ij}}{P_2}}\right)dy \tag{17}$$

Using [15, Eq. (7)], [14, Eq. (6.643)] and finally using [16, Eq. (3)], PDF of the desired Rician-TWDP fading model can be expressed as in (5).

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