Improving Error Probability Performance of Digital Communication Systems with Compact Nyquist Pulses

Uche A.K. Chude-Okonkwo, BT Maharaj, and Reza Malekian
Dept. of Electrical, Electronic and Computer Engineering
University of Pretoria
South Africa

***Abstract*—Designing pulse shaping filters that satisfy the Nyquist condition for minimum intersymbol interference (ISI) is crucial to the performance of almost all digital transceiver systems. In this paper, a method of improving the error probability performance of various Nyquist pulses, by multiplying them with a specific compactly supported function, is proposed. The resultant pulses are less sensitive to timing error and with smaller maximum distortion than the original pulses. Tabulated results that compare the error probability performance of the improved pulse technique to some conventional pulses are provided.**

Keywords—Nyquist pulse, compactly supported function, intersymbol inteference (ISI), error probability

I. INTRODUCTION

An important goal in the design of practical digital transceiver is reduction of the system's sensitivity to timing errors. Typically, if the transceiver processes the received data on a sample-by-sample basis, timing error may occur. The effect of the timing error is exacerbated by the intersymbol interference (ISI) phenomenon, which arises due to the band-limited nature of the channel. Hence, to combat the arising error probability due to the combine effects of ISI and timing error, the basic idea is to design pulse-shaping filters that minimize the effects of ISI as well as noise. Consider the baseband pulse amplitude modulation system, which is represented by

$$x(t) = \sum_{m=-\infty}^{\infty} d_m g(t - mT_{SYM}) + w(t) \qquad (1)$$

$$g(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm 1, \pm 2, ... \end{cases} \qquad (2)$$

where T_{SYM} is the symbol duration, d_m is the transmitted symbols at the rate $1/T_{\text{SYM}}$, $g(t)$ is the overall channel impulse response, and $w(t)$ is the additive white Gaussian noise. Pulses that satisfy (1) are termed *Nyquist pulses* (NP). The most widely known ISI-free NP is the raised-cosine (rcos) pulse.

Nyquist showed that to satisfy (2) means designing pulses with odd symmetric spectrum about $1/(2T_{\text{SYM}})$. This implies that the design of a number of pulses that satisfy (2) and improve on the performance of pioneer pulses like the rcos is possible. The concept of improved Nyquist pulse is pioneered by the work of Beaulieu et al. [1]. The so-called better than raised-cosine (btrc) [1] and other pulses [2-8] with superior performances compared to the rcos pulse have been proposed.

Of importance in the characteristic properties of these pulses are their performance in the presence of timing jitter, maximum distortion due to noise, error probability and bandwidth occupancy. These performance indices are influenced by the decay rate of the particular pulse employed. In the case of multicarrier communication, it has been shown in [9] that to reduces peak-to-average power ratio, a pulse with a reduced tail size is required. Typically, in real applications, a balance or optimum solution has to be sort between bandwidth and ISI mitigation (reduced pulse tail) due to the constraint placed by the uncertainty principle. We want a pulse that has time domain response that allows for the transmission of data train with no interference, and a frequency response that suppresses noise and is sufficiently selective [10]. To achieve low error probability without sacrificing bandwidth, interesting results based on the linear combination of two pulses (that completely overlap in spectral domain) is proposed in [4]. Some other works on the linear combination of pulses can be found in [7, 9, 11].

Traditional method of pulse design targets the frequency response of the pulse. A different approach is to target the impulse response function without having to compromise much bandwidth [12]. In this paper, we target the impulse response function and proposed the linear multiplication of some existing Nyquist pulses with a compactly supported function. The product of the multiplication results in a compactly supported Nyquist pulse whose bandwidth can be controlled by a factor termed the *scaling parameter*. By 'compactly supported' we mean that the product pulse approaches zero very quickly as time approaches infinity than the original pulse. This approach reduces timing error. The performance results of the proposed pulse in terms of noise margin and error probability are obtained and compared with similar results for some conventional pulses such as the rcos, btrc, the linear combination of rcos and btrc (rcos/btrc) [4] and the second-order continuous window (socw) pulses [5]. The comparison is also extended to the case of the flipped-hyperbolic secant [8] and the Nyquist filters with piece-wise rectangular-polynomial frequency characteristics [2].

978-1-7281-3618-9/19/$31.00 ©2019 IEEE

The rest of the paper is organized as follows. In Section II, the problem definition that indicates the aim of this paper is presented. The proposed compact Nyquist pulse is derived in Section III. The performance comparison of the proposed pulse compared to some other pulses are presented in Section IV.

II. Problem Definition

Let the expectation of the inter symbol interference (ISI) error probability P_e for a Nyquist pulse $p_N(t)$ be expressed as

$$\mathrm{E}[P_e] = \int P_e(\varepsilon) f_e(\varepsilon) d\varepsilon \qquad (3)$$

where $f_e(\varepsilon)$ is the probability density function of the time error ε. Considering the case of binary antipodal signaling and AGWN, $P_e(\varepsilon)$ can be evaluated as [13]

$$P_e(\varepsilon) = \frac{1}{2} - \frac{2}{\pi} \sum_{m=1, m=odd}^{M} \left\{ \frac{\exp(-m^2\omega^2/2)\sin(m\omega g_0)}{2} \right\} \cdot \prod_{k=N_1, k\neq 0}^{N_2} \cos(m\omega g_k) \qquad (4)$$

In (4), M represents the number of coefficients considered in the approximate Fourier series of noise complementary distribution, $\omega = 2\pi / T_f$ where T_f is the period used in the series, N_1 and N_2 represent the number of interfering symbols before and after the transmitted symbol and $g_k = p_N(kT_{SYM} + \varepsilon)$ is the sample version of $g(t)$, where $p_N(t)$ is the pulse shape used and T_{SYM} is the symbol duration. Our objective is to define a product pulse that minimizes $\mathrm{E}[p_e(\varepsilon)]$. The method that generates such pulse is presented in the next section.

III. Proposed Compactly Supported Nyquist Pulses

The linear multiplication of pulses proposed in this work is theorized in this section. Basically, this method proposes the multiplication of a conventional pulse $p(t)$ with a compactly supported pulse $p_C(t)$ to produce a pulse $p_N(t)$ that has interesting impulse response with low tails. The time and frequency characteristics of the compactly supported pulse and the product pulse are presented.

Definition: Let U be a nonempty set in $\Re^n$ for some positive integer n, and let $C^{\infty}_{com}(U)$ denote the space of compactly supported smooth functions on U. If for some pulse $p(t)$ the partial derivatives exist for all possible orders defined on its domain, then the product of $p(t)$ and some function $p_C(t)$ $\in C^{\infty}_{com}(U)$ is such that

$$p_N(t) = p(t) p_C(t) \qquad (5)$$

where $p_C(t)$ is a compactly supported smooth function. This definition follows from the theory of distribution [14], which generalizes the notion of functions. The expression in (5) is valid as long as the following time domain characteristics hold

$$\langle p_N(t), \varphi(t) \rangle = \langle p_C(t), p(t)\varphi(t) \rangle \qquad (6)$$

where $\varphi(t)$ is some compactly supported test function. This implies that the inner product of the composite pulse, and the product pulse with the test function is preserved.

By the above definition, and in extension to the approximate distribution theory, it is quite obvious that $p_N(t)$ will also be compactly supported. More precisely, the mapping that sends $p_C(t)$ to $(p(t)p_C(t))$ is a linear mapping from $C^{\infty}_{com}(U)$ into itself. Hence, $p(t)$ can be localized by multiplying it by smooth functions with compact support. The result is a pulse with tails that approach zero very quickly as time approaches infinity compared to the original pulse.

Having established the time domain characteristic of the product pulse, we go further to ascertain its frequency characteristics by considering the following Lemma.

Lemma 1: Let $z(t)$, $v(t)$ and $h(t)$ be some functions whose Fourier transforms are $Z(\omega)$, $V(\omega)$ and $H(\omega)$, respectively. If $z(t) = v(t)h(t)$, where $B_v = \mathrm{supp}\{V(\omega)\}$ and $B_h = \mathrm{supp}\{H(\omega)\}$ are the frequency supports (bandwidths) of $v(t)$ and $h(t)$ respectively, then it is true that $B_z = \mathrm{supp}\{Z(\omega)\}$ is such that

$$B_z = B_v + B_h \qquad (7)$$

Proof: To prove (7) we consider the two extreme cases where the supports (in time) of $h(t)$ tend to zero, i.e. $h(t) = \delta(t)$, and where the supports of $h(t)$ tend to infinity, i.e. $h(t) = u(t-t_0)$. Let $z_1(t) = v(t-t_0)u(t-t_0)$ and $z_2(t) = v(t)\delta(t)$. The Fourier transform of $z_1(t)$ is expressed as:

$$\begin{aligned} F\{z_1(t)\} &= F\{v(t-t_0)u(t-t_0)\} = e^{-j\omega t_0} V(\omega) \\ &= e^{-j\omega t_0} V(\omega)\Big|_{t_0 \to 0} = V(\omega) \end{aligned} \qquad (8)$$

The result in (8) implies that as the time support of $h(t)$ tend to infinity, then $Z(\omega) = V(\omega)$ and $B_z = B_v$. Therefore, $B_z = B_v + B_h$ holds since, $B_h = 0$.

To evaluate the Fourier transform of $z_2(t)$, we consider the fact that the delta function and a constant $1/\sqrt{2\pi}$ are Fourier-transforms of each other, thus:

$$\tilde{\delta}(\omega) = \int_{-\infty}^{\infty} dt (2\pi)^{-1/2} e^{j\omega t} \delta(t) \qquad (9)$$

$$\delta(t) = \int_{-\infty}^{\infty} d\omega (2\pi)^{-1} e^{-j\omega t} \qquad (10)$$

If we let $v(t)$ be e^t, then it can easily be shown that within the limits of integration from $-\infty$ to ∞,

$$F\{z_2(t)\} = F\{\delta(t)v(t)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega \frac{e^{-j\omega t}}{2\pi} e^t e^{-j\omega t} dt = \infty \qquad (11)$$

In relation to (7), since the bandwidth of $\delta(t)$ approaches infinity, the expression $B_z = B_v + B_h$ holds since, $B_z = B_h \to \infty$. That concludes the proof. Hence, from Lemma 1 we can say that if the bandwidth of $p_C(t)$ is much less than that of $p(t)$, then that of $p_N(t)$ will approximately be equal to that of $p(t)$, otherwise the bandwidth of $p_N(t)$ will always be greater than that of $p(t)$.

The $p_C(t)$ introduced and used in this work, which satisfies (5) and (6) is defined as the following generalized pulse

$$p_C(t) = (1 - x_1(t))\sum_{k=0}^{\infty}\left(-\frac{0.5x_2(t)}{k!}\right)^k \tag{12}$$

where $x_1(t) = (t/\Gamma)^2$, $x_2(t) = (t/\Gamma)^2$ and Γ, $(\Gamma > 0) \in \Re^+$ is a scaling parameter that defines the bandwidth of $p_C(t)$ and subsequently the excess bandwidth of $p_N(t)$ in relation to that of $p(t)$. Its value defines how quickly the tails of $p_N(t)$ taper off. For practical purposes, (12) has to be truncated, which may affect the error probability performance [8]. The truncation challenges are addressed by considering an approximation of (12), which is a continuous function expressed as

$$p_C(t) = (1 - x_1(t))\exp(-x_2(t)) \tag{13}$$

IV. Time and Bandwidth Characteristics of the Compactly Supported Nyquist Pulses

The time domain of the compactly supported pulse $p_C(t)$ in (13) is used in this work and is shown in Fig. 1 for $\Gamma = 1$.

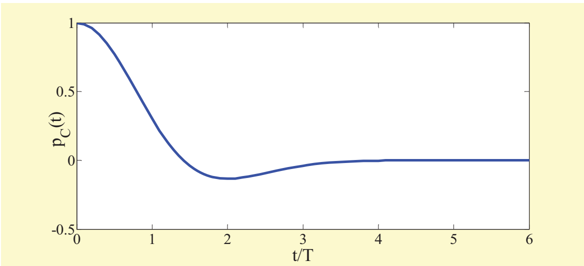


Fig. 1. Impulse response of $p_C(t)$ for $\Gamma = 1$.

Basically, as Γ decreases, the tails of $p_N(t)$ tends to zero more quickly thereby resulting in better error probability. The dependency of the time and bandwidth characteristics of $p_N(t)$ on Γ are illustrated in Fig. 2 and 3 for the cases of the rcos. It is important to note that the choice of the function in (7) provides us with a way of controlling the bandwidth and time characteristics of $p_N(t)$ at integer values of Γ.

Figures 2 and 3 illustrate the time and frequency characteristics of the proposed pulse $p_N(t)$. The time characteristics explicitly shown in Fig. 2 indicates that as Γ decreases, the tails of $p_N(t)$ tend to zero more quickly while the pulse shape is preserved. Figures 3 show that there is no bandwidth compromise at 3-dB bandwidth, and negligible null-to-null bandwidth price paid for tapering off of the pulse tails. Hence, for systems and application for which data rate is not *very crucial* compared to error probability, the improved pulse technique provides an interesting option. The optimal choice of Γ depends on the particular application for which the pulse is employed. For systems in which bandwidth resources is scarce, high values of Γ is ideal, and for systems in which error performance is more important compared to bandwidth resource, low values of Γ will be more adequate.

V. Performance Evaluation and Numerical Results

In this section, we evaluate the performance of the proposed compactly supported pulses from different practical perspectives. We begin by considering and comparing the eye diagrams of the proposed pulse at Γ = 4, 6, 8 with those for which $p(t)$ are rcos and btrc, respectively. The comparison affords us a way of visually assessing the vulnerability of the transmission systems to the problem of ISI.

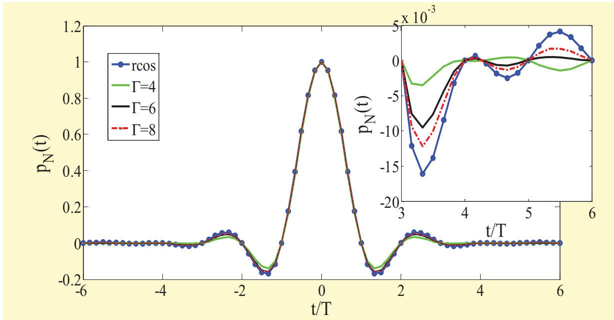


Fig. 2. Time domain characteristics of $p_N(t)$ for rcos at $\alpha = 0.35$.

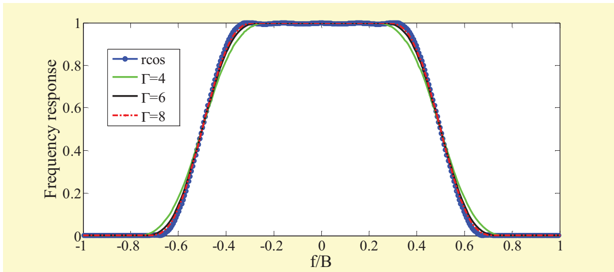
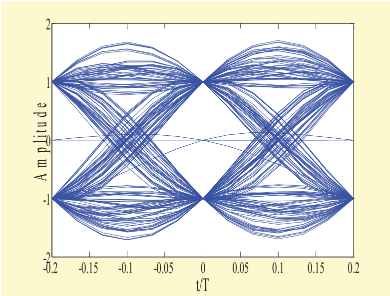


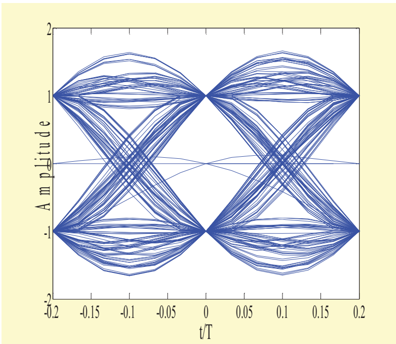
Fig. 3. Frequency domain characteristics of $p_N(t)$ for rcos at $\alpha = 0.35$.

From Fig. 4 and 5, compared to the rcos and btrc, respectively, the proposed pulse technique performs better in terms of timing jitter and with more open eyes. The simulated noise margin in the case of the rcos ($p(t)$) are rcos (0.91), Γ = 8 (0.64), Γ = 8 (0.78), Γ = 8 (0.83). And for the case of the btrc, the noise margin are btrc (0.62), Γ = 8 (0.36), Γ = 8 (0.51), Γ = 8 (0.56). These values indicate that the proposed pulse technique outperforms the respective conventional pulses with respect to noise margin.

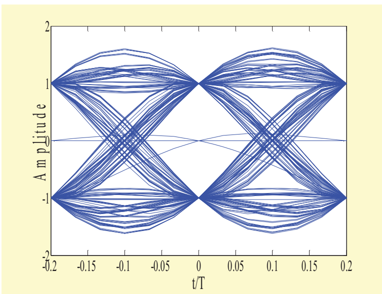
While pulse performance analysis using the eye diagram provides assessable visual platform for analysis, the ultimate metric of performance assessment is the ISI error probability [15]. In Table 1 and 2, the comparative results for the error probability of six different pulses and the proposed pulse technique computed using the expression for P_e in (4) are shown. The results are computed using T_f = 40 and M = 61. A signal-to-noise ratio of 15dB has been assumed while 2^{10} interfering symbols are generated. Three timing offset values (t/T) of 0.05, 0.1 and 0.2 are considered. The conventional pulses considered for the comparison are the rcos, btrc [1], the combination of the btrc and rcos [4], socw [5], fsech [8] and H3 [2]. From the results in the tables, it is clear that the proposed pulse technique outperforms each conventional pulse in terms of P_e. The performance margin gets wider with increase in timing jitter. Generally speaking, timing jitter raises error rate values since the ISI is a result of the receiver eye being sampled off the center [1]. Hence, the results are interestingly consistent with the effect of tapering off the tails of $p(t)$ by $p_C(t)$, which results in lower error at increased timing jitter. To achieve a significantly lower error probability potential, the improved pulse technique with Γ value that tends to unity is optimal. However, the achievable low error probability is obtained at the expense of a slight bandwidth increase. For systems in which bandwidth resources is scarce, high values of Γ is better suited, which will results is a slightly lower error probability without bandwidth compromise.



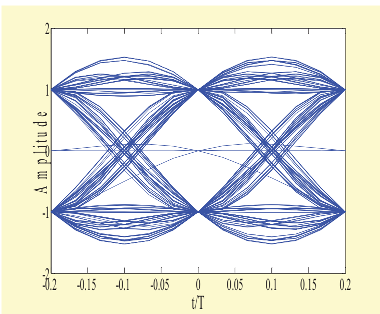
(a)



(b)



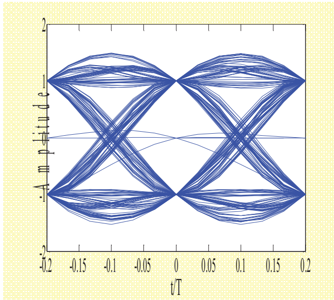
(c)



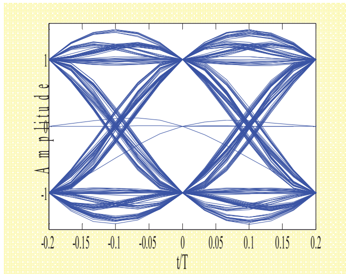
(b)

Fig. 4. Eye diagrams of the at α = 0.35 for (a) rcos (b) $p_N(t)$ at Γ = 8 (c) $p_N(t)$ at Γ = 6 (d) $p_N(t)$ at Γ = 4.

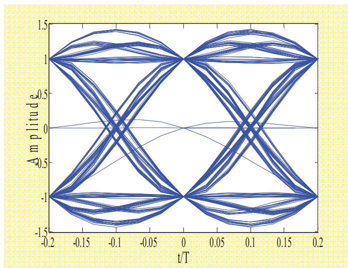
The conventional pulses considered for the comparison are the rcos, btrc [1], the combination of the btrc and rcos [4], socw [5], fsech [8] and H3 [2]. From the results in the tables, it is clearly observable that the proposed pulse technique outperforms each conventional pulse in terms of P_e. The performance margin gets wider with increase in timing jitter.



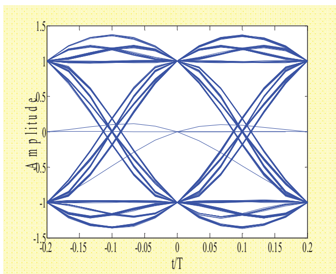
(a)



(b)



(c)



(d)

Fig. 5. Eye diagrams of the at α = 0.35 for (a) btrc (b) $p_N(t)$ at Γ = 8 (c) $p_N(t)$ at Γ = 6 (d) $p_N(t)$ at Γ = 4 .

Generally speaking, timing jitter raises error rate values since the ISI is a result of the receiver eye being sampled off the center [1]. Hence, the results are interestingly consistent with the effect of tapering off the tails of $p(t)$ by $p_C(t)$, which results in lower error at increased timing jitter. To achieve a significantly lower error probability potential, the improved pulse technique with Γ value that tends to unity is optimal. However, the achievable low error probability is obtained at the expense of a slight bandwidth increase. For systems in which bandwidth resources is scarce, high values of Γ is better suited, which will results is a slightly lower error probability without bandwidth compromise.

TABLE I. ERROR PROBABILITIES FOR $N = 2^{10}$ AND SNR=15 dB AT $\alpha = 0.35$

Pulse		$t/T = \pm 0.05$	$t/T = \pm 0.1$	$t/T = \pm 0.2$
rcos		**8.219e-8**	**2.818e-6**	**9.746e-4**
$\times p_C$	Γ = 4	4.689e-8	8.019e-7	1.806e-4
	Γ = 5	5.956e-8	1.371e-6	3.850e-4
	Γ = 6	6.490e-8	1.668e-6	5.012e-4
	Γ = 7	6.875e-8	1.894e-6	5.925e-4
	Γ = 8	7.147e-8	2.066e-6	6.528e-4
btrc		**5.812e-8**	**1.298e-6**	**3.567e-4**
$\times p_C$	Γ = 4	4.224e-8	6.324e-7	1.269e-4
	Γ = 5	4.661e-8	7.871e-7	1.747e-4
	Γ = 6	4.941e-8	8.965e-7	2.108e-4
	Γ = 7	5.131e-8	9.756e-7	2.379e-4
rc/btrc	a_{opt}= 1.82	**5.119e-8**	**1.027e-6**	**2.697e-4**
$\times p_C$	Γ = 4	3.663e-8	4.595e-7	7.857e-5
	Γ = 5	3.957e-8	5,437e-7	1.006e-4
	Γ = 6	4.166e-8	6.116e-7	1.204e-4
	Γ = 7	4.325e-8	6.689e-7	1.385e-4
socw		**5.149e-8**	**7.982e-7**	**2.495e-4**
$\times p_C$	Γ = 4	3.466e-8	4.074e-7	6.579e-5
	Γ = 5	3.719e-8	4.745e-7	8.222e-5
	Γ = 6	3.925e-8	5.394e-7	1.009e-4
	Γ = 7	4.104e-8	6.030e-7	1.212e-4
fsech		**7.558e-8**	**2.334e-6**	**7.720e-4**
$\times p_C$	Γ = 4	5.035e-8	9.377e-7	2.255e-4
	Γ = 5	5.746e-8	1.263e-6	3.434e-4
	Γ = 6	6.227e-8	1.513e-6	4.392e-4
	Γ = 7	6.560e-8	1.700e-6	5.131e-4
H3	α_1= 0.13, b = 0.63	**4.526e-8**	**8.343e-7**	**2.157e-4**
$\times p_C$	Γ = 4	3.1144e-8	3.195e-7	4.499e-5
	Γ = 5	3.347e-8	3.770e-7	5.842e-5
	Γ = 6	3.517e-8	4.250e-7	7.115e-5
	Γ = 7	3.643e-8	4.640e-7	8.235e-5

CONCLUSION

In this paper a method of improving the performance of various conventional Nyquist pulses has been proposed. The improved pulse is the product of a conventional function with a compactly supported smooth function. Simulation results show that the proposed pulse technique outperforms the conventional pulses in terms of error probability in the presence of timing jitter and noise margin.

ACKNOWLEDGMENT

This research is supported by the Sentech Chair in Broadband Wireless and Multimedia Communication, University of Pretoria, South Africa.

REFERENCES

[1] N. C. Beaulieu, C. C. Tan and M. O. Damen, "A "Better than" Nyquist Pulse," *IEEE Communications Letters*, vol. 5, no. 9, 2001, pp. 367-368.

[2] N. D. Alexandru and A. L. O. Balan, " Two improved Nyquist filters with piece-wise rectangular-polynomial frequency characteristics," *AEU-Intern J. Electronics and Commun.,* vol. 66, no. 11, 2012, pp. 880-883.

[3] A. Eghbali, T. Saramaki, and H. Johansson, "On Two-Stage Nyquist Pulse Shaping Filters," *IEEE Trans. Signal Process.*, vol. 60, no. 1, 2012, pp. 483-488.

[4] P. Sandeep, S. Chandan, A. K. Chaturvedi, "ISI-free Pulses with Reduced Sensitivity to Timing Errors," *IEEE Communication Letters*, vol. 9, no. 4, 2005, pp. 292-294.

TABLE II. ERROR PROBABILITIES FOR $N = 2^{10}$ AND SNR=15 dB AT $\alpha = 0.5$

Pulse		$t/T = \pm 0.05$	$t/T = \pm 0.1$	$t/T = \pm 0.2$
rcos		**3.972e-8**	**5.489e-7**	**1.022e-4**
$\times p_C$	Γ = 4	3.062e-8	3.091e-7	4.333e-5
	Γ = 5	3.472e-8	4.079e-7	6.568e-5
	Γ = 6	3.611e-8	4.451e-7	7.484e-5
	Γ = 7	3.701e-8	4.698e-7	8.111e-5
btrc		**2.413e-8**	**1.858e-7**	**2.088e-5**
$\times p_C$	Γ = 4	2.106e-8	1.320e-7	1.186e-5
	Γ = 5	2.197e-8	1.464e-7	1.400e-5
	Γ = 6	2.253e-8	1.559e-7	1.549e-5
	Γ = 7	2.290e-8	1.625e-7	1.658e-5
rc/btrc	a_{opt}=1.59	**2.208e-8**	**1.612e-7**	**1.962e-5**
$\times p_C$	Γ = 4	1.726e-8	8.509e-8	7.367e-6
	Γ = 5	1.831e-8	1.016e-7	1.027e-5
	Γ = 6	1.906e-8	1.145e-7	1.283e-5
	Γ = 7	1.960e-8	1.243e-7	1.494e-5
socw		**2.366e-8**	**2.221e-7**	**4.848e-5**
$\times p_C$	Γ = 4	1.679e-8	8.208e-8	8.403e-6
	Γ = 5	1.832e-8	1.068e-7	1.376e-5
	Γ = 6	1.947e-8	1.277e-7	1.898e-5
	Γ = 7	2.031e-8	1.444e-7	2.354e-5
fsech		**3.495e-8**	**4.119e-7**	**6.601e-5**
$\times p_C$	Γ = 4	2.997e-8	2.923e-7	3.908e-5
	Γ = 5	3.188e-8	3.350e-7	4.700e-5
	Γ = 6	3.303e-8	3.623e-7	5.400e-5
	Γ = 7	3.377e-8	3.805e-7	5.818e-5
H3	α_1= 0.29, b = 0.7	**1.942e-8**	**1.304e-7**	**2.106e-5**
$\times p_C$	Γ = 4	1.534e-8	6.203e-8	4.910e-6
	Γ = 5	1.591e-8	6.951e-8	6.073e-6
	Γ = 6	1.630e-8	7.478e-8	6.960e-6
	Γ = 7	1.660e-8	7.904e-8	7.737e-6

[5] R. Song and S. H. Leung, "A Novel OFDM Receiver with Second Order Polynomial Nyquist Window Function," *IEEE Communication Letters*, vol. 9, no. 5, 2005, pp 391-393.

[6] S. D. Assimonis, M. Matthaiou and G. K Karagiannidid, "Two-Parameter Nyquist Pulses with Better Performance," *IEEE Communication Letters*, vol. 12, no. 11, 2008, pp. 12: 1-3.

[7] N. D. Alexandru and A. L. Balan, "Optimization of a Square-root Nyquist Filter with Staircase Frequency Characteristic," *2013 4th Intern. Symp. on Elect and Elect Engineering (ISEEE)*, 11-13 Oct. 2013, pp. 1-6.

[8] A. Assalini, and A. M. Tonello, "Improved Nyquist Pulses," *IEEE Communication Letters*, vol. 8, no. 2, 2004, pp 87-89.

[9] C. A Azurdia-Meza, K. Lee, and K. Lee, "PAPR Reduction in SC-FDMA by Pulse Shaping using Nyquist Linear Combination Pulses," *IEEE communications letters*, vol. 16, no. 12, 2012, pp. 2008-2011.

[10] H. Baher, *Signal Processing and Integrated Circuits*, UK: John Wiley, 2012.

[11] C. Azurdia-Meza, K. Lee and K. Lee, "ISI-Free Linear Combination Pulses with Better Performance," *IEICE Trans. on communications*, vol. 96, no. 2, 2013, pp. 635-638.

[12] J. P. Fonseka and E. M. Dowling, "Compact Pulse Shaping for Efficient PAM/QAM Signaling," *IEEE Communications Letters*, vol. 15, no. 6, 2011, pp. 593-595.

[13] N. C. Beaulieu, "The Evaluation of Error Probability for Intersymbol and Cochannel Interference," *IEEE Trans. Commun*, vol. 39, no. 9, 2004, pp. 1740-9.

[14] F. G. Friedlander, *Introduction to the theory of distributions*, UK: Cambridge University Press, 1998.

[15] C. C. Tan and N. C. Beaulieu, "Transmission Properties of Conjugate-root Pulses," *IEEE Trans. Commun*, vol. 52, no. 4, 2004, pp. 553-8.