Detection Probability Calculations for Fluctuating Targets under Clutter

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***Abstract*—Recent computational advantages in electromagnetic (EM) modeling, along with the growing demand to detect low observable moving targets, have brought about a renewed interest in the field of target detection. Real-life applications, such as flight path optimization, motivate a dynamical approach where the radar cross section (RCS) is not only stochastic by nature, but also significantly influenced by the target aspect angle. The classic techniques that modeled target RCS fluctuations as a random variable are generally found unsuitable for such tasks and make way to aspect-dependent RCS models produced by advanced EM softwares. The random nature of the echo target signals can then be simulated by introducing variations in aspect angle between detections.**

This paper describes a method for generating single-pulse detection probabilities of aspect-dependent RCS models in the presence of noise and clutter. As an example, we compare various Weibull clutter models to emphasize the importance of clutter parameters selection when analyzing radar performance along specific flight paths.

***Index Terms*—Detection probability, RCS fluctuation, Aircraft target, Clutter**

I. Introduction

RCS is a common measure of how detectable an object is by radar. RCS is often visualized as a target's equivalent area as seen by the radar, so that larger aircrafts are more easily detected than smaller ones. However, real aircrafts tend to have complex structures and their RCS are highly dependent on the aspect angle. An accurate calculation of such RCS is a complex task which requires computational EM methods. To reduce the computational burden, several RCS fluctuation models have been suggested and used since the 1950s with a common assumption that the angular variation in RCS will translate to a time variation of RCS that would appear random. Addressing the RCS as a random variable with probability density allowed a statistical treatment of pulse detection in the presence of noise. What is essentially a fundamental parameter of an aircraft could now be discussed in terms of signal-to-noise, detection probability and false alarm probability which are key factors in determining the detector threshold.

While classical models, such as the known Swerling models [1], provide a reasonably good representation of certain types of complex targets, the basic assumptions from which they are derived are not always valid. Moreover, a classical RCS model is no longer applicable when the RCS variation with aspect angle needs to be known accurately as possible, such as in path planning for aerial vehicles. Since an aircraft in flight is expected to experience turbulences and vibrations, the actual dynamic RCS may slightly deviate from the computed static RCS model at a given aspect angle. Considering also the effect of atmospheric refraction turbulences on wave propagation, the resulting changes in phase and amplitude can be translated to randomly distributed changes in aspect angle, so that the proposed statistical model may assume distributed RCS values around a given aspect angle.

A method to calculate single-pulse detection probability for RCS fluctuations targets based on model generation is described by [2]. To calculate the detection probability the authors assume thermal noise, so the envelope of the complex Gaussian noise follows the Rayleigh distribution. However, background noise may consist also of clutter, the undesired scatter from objects within the radar beam that are not targets. Since ground clutter fluctuations observed by a low resolution radar approximately follow the Rayleigh distribution, the total background is often conveniently treated also as such. However, for high-resolution radars and radars at small grazing angle this assumption may no longer be valid. The probability density function (PDF) of clutter then tends to have a long tail which is more accurately modeled by compound-Gaussian clutter models such as the Weibull distribution, log-normal distribution and K-distribution. Heavy-tailed clutter models make target detection more challenging compared with Rayleigh distributed clutter due to the higher likelihood of target-like outliers which can increase the probability of false alarm [3]. This challenge grows further for fluctuating targets, emphasizing the need for an accurate target fluctuations model.

II. Methodology

A new RCS model generation method for calculating the detection probability for a given aspect angle has been proposed by [2]. We follow their analysis while adding a clutter component, which was ignored in the original analysis, and provide a general recipe for addressing detection in the presence of clutter. A discussion of methods for simulating accurate clutter maps can be found in the literature and is beyond the scope of this paper (see for instance [4]). To simplify the discussion we use predetermined values of the mean signal-clutter plus noise power ratio ($\bar{S}$CNR) and clutter-noise power ratio (CNR) to illustrate the impact of clutter distribution on probability detection at given aspect angles. As further explained below, $\bar{S}$ is used when the RCS is averaged over all azimuth angles whereas S represents RCS at a specific azimuth angle. To simplify the discussion we focus hereinafter on the calculation of single-pulse detection probability. However, note that for coherent target detection, where both target and clutter maintain constant phases between pulses, the multi-pulse detection probability can be similarly calculated by dividing the receiver noise by $\sqrt{M}$, where M is the number of pulses.

A. Target's RCS model generation

To demonstrate our method we present a generic aircraft model as shown in fig. 1. The target RCS data was obtained using WIPL-D (pure MOM commercial software) for sampled azimuth angles in the range $0^\circ < \theta^i < 180^\circ$ and elevation angles in the range $-3^\circ < \phi^i < 3^\circ$ in angular resolution of 0.1°. A typical RCS plot of the aircraft, calculated at zero elevation, is presented in fig. 2. Assuming the actual aspect angle of the cruising aircraft fluctuates around a mean angle, we can calculate the mean RCS at any given azimuth angle θ^i following the steps outlined by [2].

First, treating each sampled azimuth angle θ^i as the mean of a normal distribution and setting the standard deviation of fluctuation at $\Delta\theta = 1^\circ$, we generate a series of N random values of azimuth angles $\Theta^{i,j}$ $(j = 1, ..., N)$ from the normal distribution function

$$p(\Theta^{i,j}|\theta^i, \Delta\theta^i) = \frac{1}{\sqrt{2\pi\Delta\theta^i}} exp\left[\frac{(\Theta^{i,j} - \theta^i)^2}{2\Delta\theta^i}\right]$$

while limiting the results within $\theta^i \pm 3^\circ$. For each θ^i we follow the same procedure to generate a series of N random values of of elevation angles $\Phi^{i,j}$ with a mean elevation angle $\phi^i = 0$ selected for simplicity. The standard deviation value selected here is suggestive and should be estimated according to dynamical parameters such as the expected magnitude of instantaneous changes in aircraft body position. We found that $N \simeq 30,000$ is sufficient to ensure a fair sample selection that reproduces the probability detection results in repeated simulations, as was also indicated by [2].

Second, using the RCS model we generate for each azimuth angle θ^i a series of N target power values

$$\sigma_t^{i,j} = \mathrm{RCS}(\Theta^{i,j}, \Phi^{i,j})$$

from which we obtain the target amplitude according to

$$\sigma_t^{i,j} = \frac{1}{2}\left(A_t^{i,j}\right)^2.$$

The mean RCS at a given azimuth angle θ^i is then calculated by

$$S^i = \overline{\sigma_t^i} = \frac{1}{N}\sum_{j=1}^{N}\sigma_t^{i,j}.$$

B. Clutter RCS model generation

We consider for our analysis the Weibull distribution which is a simple two-parameter distribution commonly used in clutter statistics ([5]). The clutter amplitude A_c follows the Weibull distribution according to

$$p(A_c|\alpha, \beta) = \frac{\beta}{\alpha}\left(\frac{A_c}{\alpha}\right)^{b-1} exp\left[\left(-\frac{A_c}{\alpha}\right)^b\right]$$

Fig. 1. A generic aircraft model.

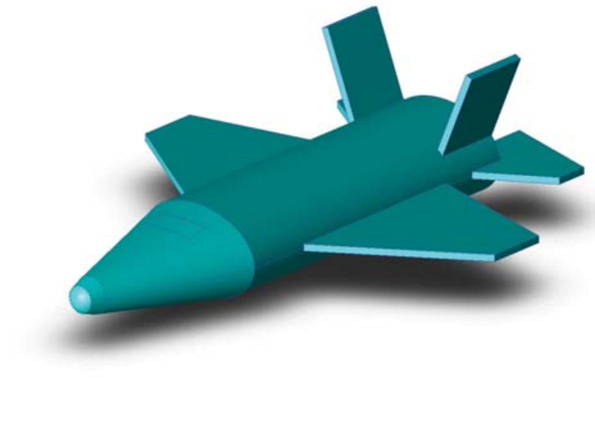
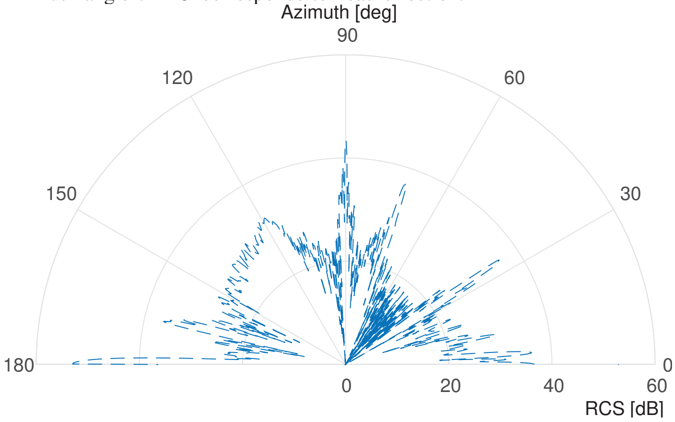


Fig. 2. A typical RCS plot of the generic aircraft calculated at zero elevation. Azimuth angle $\theta = 0^\circ$corresponds to head direction.



where α is the scale parameter, which is related to the clutter power, and β is the shape parameter, which dictates the tail of the distribution. Note that when comparing clutter models of different parameters the mean clutter power $\overline{\sigma_c} = \frac{1}{2}\overline{A_c^2}$ should be kept constant. Therefore, given $\overline{\sigma_c}$ and α, β needs to be fixed for each parameter selection according to

$$\overline{A_c^2} = \alpha^2\Gamma\left(1 + \frac{2}{\beta}\right),$$

where $\Gamma(x)$ is the Gamma function.

We generate from the Weibull distribution a series of N random values of A_c^j $(j = 1, ..., N)$ and from the uniform distribution a series of N random values representing the phase angle of the returning clutter echo ψ_c (limits at $[0, 2\pi)$) . In addition, to simulate in-phase and quadratic noise inside the receiver we draw N random noise amplitudes A_n from Rayleigh distribution

$$p(A_n) = \frac{A_n}{\sigma_n^2} exp\left(-\frac{A_n^2}{2\sigma_n^2}\right)$$

where σ_n^2 is the characteristic noise power in each channel, and draw corresponding phase angles ψ_n from a uniform distribution.

The mean clutter and noise power from which the numbers are drawn are determined in this specific example by the ratios

$$\bar{S}\text{CNR} = \frac{\bar{S}}{\overline{\sigma_c} + \sigma_n^2} = 10\,dB$$

and

$$\text{CNR} = \frac{\overline{\sigma_c}}{\sigma_n^2} = 0\,dB,$$

where $\bar{S}$ is an average over the RCS values for aspect angles in the range $0 \leq \theta^i \leq 180°$ and $\phi^i = 0$.

C. Detection probability

Following the steps above, we obtain for each azimuth angle θ^i a set of N clutter amplitudes, clutter phase angles, noise amplitudes, noise phase angles and target amplitudes. A set of N input signal amplitudes $A_s^{i,j}$ $(j = 1, ..., N)$ can then calculated by adding these components in the complex plane:

$$Q^{i,j} = A_t^{i,j} \times cos\left(\psi_t^{i,j}\right) + A_c^j \times cos(\psi_c^j) + A_n^j \times cos(\psi_n^j)$$

$$I^{i,j} = A_t^{i,j} \times sin\left(\psi_t^{i,j}\right) + A_c^j \times sin(\psi_c^j) + A_n^j \times sin(\psi_n^j)$$

$$A_s^{i,j} = \sqrt{\left(Q^{i,j}\right)^2 + \left(I^{i,j}\right)^2}$$

where $Q^{i,j}$ and $I^{i,j}$ represent the quadrature receiver components. The input signal amplitudes for each azimuth angle θ^i need to be compared to a predetermined detection threshold T_d to determine the relative part of successful detection from which the detection probability is derived:

$$p_d^i = \frac{1}{N}\sum_{j=1}^{N}\left(A_s^{i,j} > T_d\right).$$

The detection threshold can be calculated from the PDF of the clutter-plus-noise distribution given the required false-alarm probability p_{fa} ([6]). However, we do not attempt to formulate an analytical expression of a specific PDF and alternatively use the same set of equations to obtain a discrete clutter-plus-noise distribution assuming no target is present. This is done by setting a constant $A_t^i = 0$ so that a new set of random input signals amplitudes A_s^k $(k = 1, ..., L)$ is obtained independently of aspect angle. The number of generated samples L should clearly be much larger than N to achieve sufficient repeatability, thus for $p_{fa} = 10^{-6}$ we generate $L = 10^9$ clutter-plus-noise samples. Arranging all samples in descending order we set the threshold value as that of the $L \times p_{fa}$ position sample of the sequence.

III. Results

Fig. 3 presents the variation of SCNR with azimuth angle θ^i, where

$$\text{SCNR}^i = \frac{S^i}{\overline{\sigma_c} + \sigma_n^2}.$$

While we adopt $\bar{S}\text{CNR} = 10\,dB$, fig. 3 implies that target detection may be more challenging in some aspect angles compared to others. To study the impact of clutter model on p_d we adopt clutter models with various shape parameters $\beta = 2.0, 1.5, 1.0$ (see fig. 4) and compare the resulting p_d plots in fig. 5. Note that $\beta = 2.0$ reproduces the Rayleigh distribution so it can alternatively be viewed as a case where the entire background noise originates from thermal noise as discussed in [2].

Fig. 3. Signal-clutter plus noise (SCNR) variation with aspect angle. The mean signal-clutter plus noise ($\bar{S}$CNR) is assumed to be $10\,dB$.

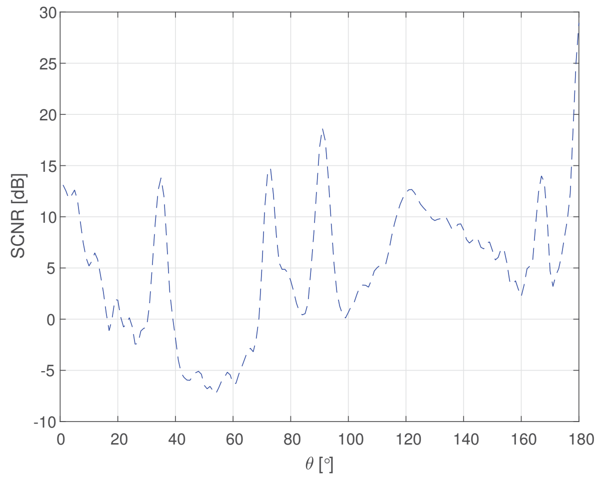
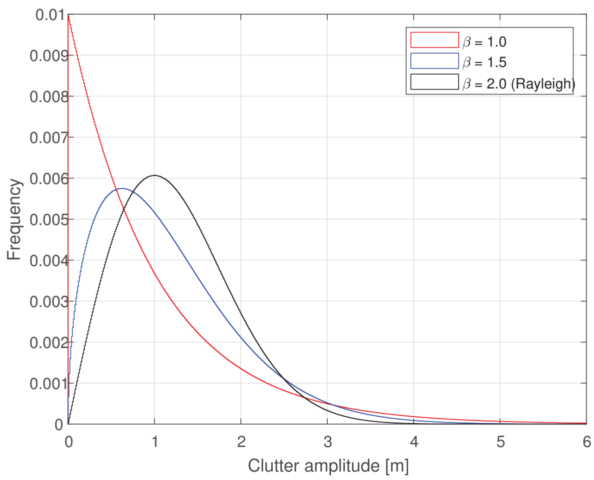
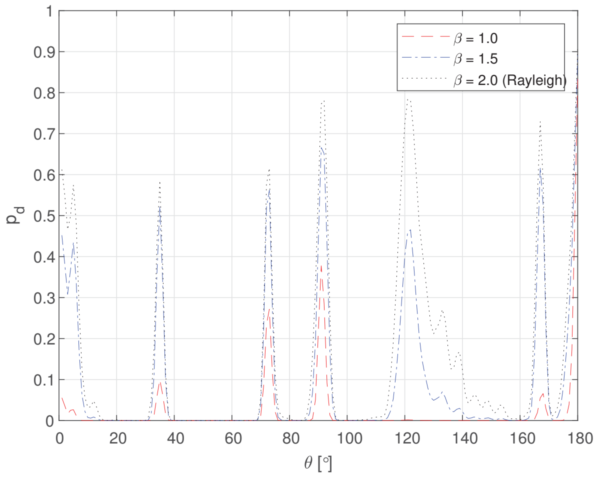


Fig. 4. Weibull distribution PDFs for various shape and scale parameters. The mean of the clutter power distributions is kept constant.



Taking as a reference the classic Swerling I model together with the Gaussian background (β =2.0), the simplified single-pulse detection probability is expected to be low with $p_d \sim 0.28$. However, closely examining fig. 5 shows that the target is more likely to be detected ($p_d > 0.5$) in the head, tail and wing directions, and in specific fuselage side directions, while likely being lost in the background noise in other directions. Lowering β corresponds to a heavier-tail environment which can result in more frequent occurrences of target-like outliers, especially for low SCNR. To minimize false detection in such

Fig. 5. Detection probability (p_d) variation with aspect angle for the various clutter distributions.



environments the threshold level must be increased, which consequently decreases the detection probability. In fact, as shown in fig. 5, for $\beta = 1.0$ detection is only likely when viewing the aircraft from the tail direction. To accurately evaluate the detection performance of an aircraft in motion along a specific track therefore requires some prior knowledge of β for the specific terrain type.

IV. Conclusions

We presented a method to calculate the detection probability of an aerial target by combining a newly proposed approach for modeling aspect-dependent RCS fluctuation together with classic statistical clutter models. Different clutter models are taken as an example to demonstrate the impact of parameter estimation on the resulting detection probabilities. The proposed scheme aims to provide basis for a more reliable simulation for analyzing the radar detection performance of a flight path of interest.

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