Frequency Shift Chirp Modulation: The LoRa Modulation

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***Abstract*—Low power wide area networks (LPWAN) are emerging as a new paradigm, especially in the field of Internet of Things (IoT) connectivity. LoRa is one of the LPWAN and it is gaining quite a lot of commercial traction. The modulation underlying LoRa is patented and has never been described theoretically. The aim of this letter is to give the first rigorous mathematical signal processing description of the modulation and demodulation processes. We provide as well a theoretical derivation of the optimum receiver entailing a low-complexity demodulation process, resorting to the Fast Fourier Transform. We compare then the performance of the LoRa modulation and the frequency-shift keying modulation both in an additive white gaussian noise channel and a frequency selective channel, showing the superiority of the LoRa modulation in the frequency selective channel. The results of this letter will enable a further assessment of the LoRa based networks, much more rigorous than what has been done until now.**

***Index Terms*—Chirp modulation, frequency shift chirp modulation (FSCM), Internet of Things (IoT), LoRa, low power wide area networks (LPWAN).**

I. Introduction

Low power wide area networks (LPWAN) are emerging as a new paradigm, especially in the field of Internet of Things connectivity [1]–[3].

LoRa is one of the LPWAN (see [3]) and it is gaining quite a lot of commercial traction. Strictly speaking, LoRa is the physical layer of the LoRaWAN system, whose specification is maintained by the LoRa Alliance.[1] The LoRa modulation is patented and has never been described theoretically. The patent [4], indeed, does not provide the details, in term of equations and signal processing. Paper [5] gives a high level description of the LoRa modulation, providing some basic equations and relying on the intuition of the reader for the decoding process. Papers [6] and [7] get more in detail with the signal, modulation and demodulation description, but still lack a mathematical, signal theory based definition of the modulation and demodulation processes, partly because the analysis is limited to the analog domain. For example, in [7] it is said that "*For a spreading factor S,* $\log_2(S)$ *bits define* f_0," i.e., the initial frequency shift, but there is no explanation of how this is done.

Actually the LoRa modulation is often referred to as a "chirp modulation" [8]–[10]. A close inspection of LoRa reveals that the information bearing element is the frequency shift at the beginning of the symbol and the chirp is similar to a kind of a carrier. For this reason, in our opinion, LoRa is better described as a Frequency Shift Chirp Modulation (FSCM).

The rest of the paper is organised as follows. In Section II, we provide a description of the modulation process and identify the orthogonal signals basis characterising the modulation; in Section III, we provide a description of the optimum demodulator and an efficient implementation of it making use of the Fast Fourier Transform; in Section IV, we provide the results of some computer simulations experiments on the link level performances, comparing as well the FSCM modulation with an frequency-shift keying (FSK) modulation with the same cardinality. Finally in Section V, we draw the conclusions of the paper.

II. Frequency Shift Chirp Modulation

Let us suppose that the bandwidth of the channel we use for the transmission is B so we transmit a sample every $T = \frac{1}{B}$.

A symbol $s(nT_s)$ is sent at the input of the modulator every $T_s = 2^{\text{SF}} \cdot T$. The symbol $s(nT_s)$ is a real number formed using a vector $\mathbf{w}(nT_s)$ of SF binary digits, with SF an integer parameter called, in the context of LoRa, *Spreading Factor* (usually taking values in $\{7, 8, 9, 10, 11, 12\}$) i.e.,

$$s(nT_s) = \sum_{h=0}^{\text{SF}-1} \mathbf{w}(nT_s)_h \cdot 2^h. \tag{1}$$

We can see that $s(nT_s)$ takes values in $\{0, 1, 2, \ldots, 2^{\text{SF}} - 1\}$.

The transmitted waveform, of duration T_s, for a certain $s(nT_s)$ is then

$$c(nT_s + kT) = \frac{1}{\sqrt{2^{\text{SF}}}}\, e^{j2\pi\left[(s(nT_s)+k)_{\bmod 2^{\text{SF}}}\right]kT\frac{B}{2^{\text{SF}}}} \tag{2}$$

$$= \frac{1}{\sqrt{2^{\text{SF}}}}\, e^{j2\pi\left[(s(nT_s)+k)_{\bmod 2^{\text{SF}}}\right]\frac{k}{2^{\text{SF}}}} \tag{3}$$

$$\text{for } k = 0 \ldots 2^{\text{SF}} - 1.$$

We can see that the modulated signal is a chirp waveform, as the frequency increases linearly with k, which is the time index; we note that each waveform differs from a base waveform having

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[1] https://www.lora-alliance.org/

initial frequency equal to 0 by a frequency shift $s(nT_s)$. That is why we call it FCSM.

We remark that all of the analysis of the FCSM modulation in this letter will remain in the discrete domain $\mathbb{Z}(T) = \{\ldots, -3T, -2T, -T, 0, T, 2T, 3T, \ldots\}$, i.e., the fundamental interval for the frequency analysis is $[0, B = \frac{1}{T}]$. As a matter of fact any signal in the discrete domain $\mathbb{Z}(T)$ has a frequency representation periodic with period $B = \frac{1}{T}$ [12]. Therefore, if one prefers to have the FCSM described in the frequency interval $[-B/2, B/2]$, e.g., to deal with the analytic signal, the signal base (4) just need to be multiplied by $e^{-\jmath 2\pi \frac{B}{2} kT} = -1^k$, with no consequences on the derivations and findings of the current letter.

A. On the Orthogonality of the FCSM Waveforms

Having established the possible waveforms of the FCSM, we investigate their orthogonality. To this end we need to check

$$\left\langle c(nT_s + kT)|_{s(nT_s)=i}\,, c(nT_s + kT)|_{s(nT_s)=q} \right\rangle = 0$$

$$i \neq q,\ i, q \in \{0, \ldots, 2^{\mathrm{SF}} - 1\} \tag{4}$$

$$\left\langle c(nT_s + kT)|_{s(nT_s)=i}\,, c(nT_s + kT)|_{s(nT_s)=q} \right\rangle \tag{5}$$

$$= \sum_{k=0}^{2^{\mathrm{SF}}-1} c(nT_s + kT)|_{s(nT_s)=i} \cdot c^*(nT_s + kT)|_{s(nT_s)=q}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi \left[(i+k)_{\bmod 2^{\mathrm{SF}}}\right] \frac{k}{2^{\mathrm{SF}}}} e^{-\jmath 2\pi \left[(q+k)_{\bmod 2^{\mathrm{SF}}}\right] \frac{k}{2^{\mathrm{SF}}}}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi \left[(i+k)_{\bmod 2^{\mathrm{SF}}} - (q+k)_{\bmod 2^{\mathrm{SF}}}\right] \frac{k}{2^{\mathrm{SF}}}}. \tag{6}$$

Equation (6) can be further elaborated, supposing (without loss of generality) $i > q$, by splitting it in three parts, getting rid of the $\bmod$ operator in in each of them.

$$\left\langle c(nT_s + kT)|_{s(nT_s)=i}\,, c(nT_s + kT)|_{s(nT_s)=q} \right\rangle$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1-i} e^{\jmath 2\pi [(i+k)-(q+k)] \frac{k}{2^{\mathrm{SF}}}}$$

$$+ \frac{1}{2^{\mathrm{SF}}} \sum_{k=2^{\mathrm{SF}}-i}^{2^{\mathrm{SF}}-1-q} e^{\jmath 2\pi \left[\left(i+k-2^{\mathrm{SF}}\right)-(q+k)\right] \frac{k}{2^{\mathrm{SF}}}}$$

$$+ \frac{1}{2^{\mathrm{SF}}} \sum_{k=2^{\mathrm{SF}}-q}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi \left[\left(i+k-2^{\mathrm{SF}}\right)-\left(q+k-2^{\mathrm{SF}}\right)\right] \frac{k}{2^{\mathrm{SF}}}} \tag{7}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1-i} e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}}$$

$$+ \frac{1}{2^{\mathrm{SF}}} \sum_{k=2^{\mathrm{SF}}-1-i}^{2^{\mathrm{SF}}-1-q} e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}} \cdot e^{-\jmath 2\pi \left[2^{\mathrm{SF}}\right] \frac{k}{2^{\mathrm{SF}}}}$$

$$+ \frac{1}{2^{\mathrm{SF}}} \sum_{k=2^{\mathrm{SF}}-1-q}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}}. \tag{8}$$

Now we consider first the case of $i - q$ a odd number. In this case, we can write the following:

$$\left\langle c(nT_s + kT)|_{s(nT_s)=i}\,, c(nT_s + kT)|_{s(nT_s)=q} \right\rangle$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}-1}-1} e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}} + e^{\jmath 2\pi [i-q] \frac{k+2^{\mathrm{SF}-1}}{2^{\mathrm{SF}}}}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}-1}-1} e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}} + e^{\jmath 2\pi [i-q] \left(\frac{k}{2^{\mathrm{SF}}} + \frac{1}{2}\right)}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}-1}-1} e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}} - e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}} = 0.$$

If $i - q$ is even then it can always be written as $i - q = 2^d \cdot r$, where d is an integer, such that $0 \leq d < \mathrm{SF}$ and r an odd integer. We tackle first the subcase $r \neq 1$

$$\left\langle c(nT_s + kT)|_{s(nT_s)=i}\,, c(nT_s + kT)|_{s(nT_s)=q} \right\rangle$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}} = \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi [2^q \cdot r] \frac{k}{2^{\mathrm{SF}}}}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi r \frac{k}{2^{\mathrm{SF}-q}}}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{h=0}^{2^q-1} \sum_{p=0}^{2^{\mathrm{SF}-q}-1} e^{\jmath 2\pi r \frac{h \cdot 2^{\mathrm{SF}}+p}{2^{\mathrm{SF}-q}}}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{h=0}^{2^q-1} \sum_{p=0}^{2^{\mathrm{SF}-q}-1} e^{\jmath 2\pi r \frac{p}{2^{\mathrm{SF}-q}}}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{h=0}^{2^q-1} \sum_{p=0}^{2^{\mathrm{SF}-q}-1} e^{\jmath 2\pi r \frac{p}{2^{\mathrm{SF}-q}}} = 0$$

since $\sum_{p=0}^{2^{\mathrm{SF}-q}-1} e^{\jmath 2\pi r \frac{p}{2^{\mathrm{SF}-q}}} = 0$ being r odd.

We tackle now the last remaining case $r = 1$.

$$\left\langle c(nT_s + kT)|_{s(nT_s)=i}\,, c(nT_s + kT)|_{s(nT_s)=q} \right\rangle$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi [i-q] \frac{k}{2^{\mathrm{SF}}}} = \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi 2^q \cdot r \frac{k}{2^{\mathrm{SF}}}}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{k=0}^{2^{\mathrm{SF}}-1} e^{\jmath 2\pi \frac{k}{2^{\mathrm{SF}-q}}} = \frac{1}{2^{\mathrm{SF}}} \sum_{h=0}^{2^q-1} \sum_{p=0}^{2^{\mathrm{SF}-q}-1} e^{\jmath 2\pi \frac{h \cdot 2^{\mathrm{SF}-q}+p}{2^{\mathrm{SF}-q}}}$$

$$= \frac{1}{2^{\mathrm{SF}}} \sum_{h=0}^{2^q-1} \sum_{p=0}^{2^{\mathrm{SF}-q}-1} e^{\jmath 2\pi \frac{p}{2^{\mathrm{SF}-q}}} = 0.$$

As a conclusion for this section, we established the orthogonality of the basis $c(nT_s + kT)|_{s(nT_s)=i}$, i.e., that

$$\left\langle c(nT_s + kT)|_{s(nT_s)=i}\, , \, c(nT_s + kT)|_{s(nT_s)=q} \right\rangle = 0$$
$$i \neq q, \; i, q \in \{0 \ldots 2^{\text{SF}} - 1\}. \tag{9}$$

III. Optimum Detection of FSCM Signals in Additive White Gaussian Noise Channel (AWGN) Channels

Since we have equal energy signals and we suppose perfect time and frequency synchronization as well as a source emitting equally probable symbols, the optimum receiver for FSCM signals in an AWGN channel can be derived easily a described in [11, Sec. 6.1].

The received signal is

$$r(nT_s + kT) \triangleq c(nT_s + kT) + w(nT_s + kT) \tag{10}$$

where $w(nT_s + kT)$ is a zero mean white gaussian noise, with $\sigma^2_{w(nT_s+kT)} \triangleq \sigma^2_w$ independent of $(nT_s + kT)$. The optimum demodulator consists of projecting $r(nT_s + kT)$ onto the different signals $c(nT_s + kT)|_{s(nT_s)=q}, q = 0 \ldots 2^{\text{SF}} - 1$ and choosing the signal $c(nT_s + kT)|_{s(nT_s)=l}$ such that the (square) modulus of the projection is maximum as the best estimate of the transmitted signal. This process is providing the best estimate $\hat{s}(nT_s) = l$ of the transmitted signal $s(nT_s)$.

A. Computationally Efficient Implementation

To calculate the projection, we perform the usual computation

$$\begin{aligned}
&\left\langle r(nT_s + kT), \, c(nT_s + kT)|_{s(nT_s)=q} \right\rangle \\
&= \sum_{k=0}^{2^{\text{SF}}-1} r(nT_s + kT) \cdot c^*(nT_s + kT)|_{s(nT_s)=q} \\
&= \sum_{k=0}^{2^{\text{SF}}-1} r(nT_s + kT) \cdot \frac{1}{\sqrt{2^{\text{SF}}}} e^{-\jmath 2\pi \left[(q+k) \bmod 2^{\text{SF}}\right] \frac{k}{2^{\text{SF}}}} \\
&= \sum_{k=0}^{2^{\text{SF}}-1} r(nT_s + kT) e^{-\jmath 2\pi \frac{k^2}{2^{\text{SF}}}} \frac{1}{\sqrt{2^{\text{SF}}}} \underbrace{e^{-\jmath 2\pi \left[(q+k) \bmod 2^{\text{SF}} - k\right] \frac{k}{2^{\text{SF}}}}}_{\varphi(q,k)}.
\end{aligned} \tag{11}$$

We now turn our attention to the function

$$\varphi(q, k) \triangleq e^{-\jmath 2\pi \left[(q+k) \bmod 2^{\text{SF}} - k\right] \frac{k}{2^{\text{SF}}}}, \quad q, k = 0, 1, \ldots 2^{\text{SF}} - 1. \tag{12}$$

We can see that for $q + k < 2^{\text{SF}}$, i.e., for $k < 2^{\text{SF}} - q$, we have

$$\varphi(q, k) = e^{-\jmath 2\pi [(q+k)-k] \frac{k}{2^{\text{SF}}}} = e^{-\jmath 2\pi q \cdot k \frac{1}{2^{\text{SF}}}}. \tag{13}$$

Instead for $q + k \geq 2^{\text{SF}}$, i.e., for $k \geq 2^{\text{SF}} - q$, we have

$$\begin{aligned}
\varphi(q, k) &= e^{-\jmath 2\pi \left[\left(q+k-2^{\text{SF}}\right)-k\right] \frac{k}{2^{\text{SF}}}} \\
&= e^{-\jmath 2\pi q \cdot k \frac{1}{2^{\text{SF}}}} \cdot e^{\jmath 2\pi \, 2^{\text{SF}} \frac{1}{2^{\text{SF}}}} \\
&= e^{-\jmath 2\pi q \cdot k \frac{1}{2^{\text{SF}}}}.
\end{aligned} \tag{14}$$

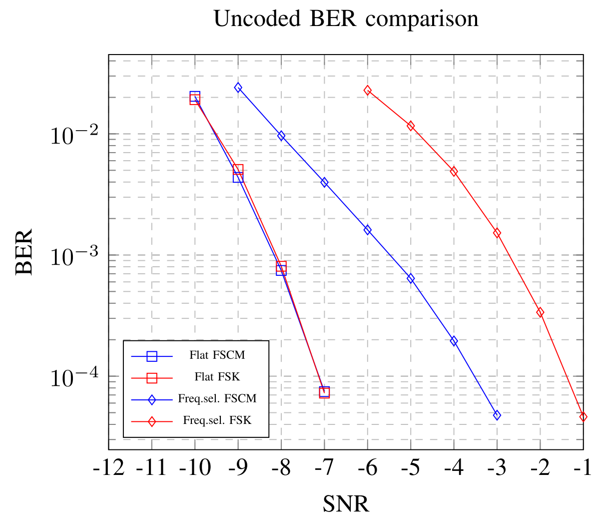

Fig. 1. Link level performance of FCSM and FSK modulations in the AWGN (Flat) and frequency selective (Freq.Sel) channels.

We can now rewrite the inner product in (11) as

$$\left\langle r(nT_s + kT), \, c(nT_s + kT)|_{s(nT_s)=p} \right\rangle \tag{15}$$

$$= \sum_{k=0}^{2^{\text{SF}}-1} \underbrace{r(nT_s + kT) \cdot e^{-\jmath 2\pi \frac{k^2}{2^{\text{SF}}}}}_{d(nT_s+kT)} \cdot \frac{1}{\sqrt{2^{\text{SF}}}} e^{-\jmath 2\pi pk \frac{1}{2^{\text{SF}}}}. \tag{16}$$

From (16) we can see that the process of projecting the signal $r(nT_s + kT)$ onto the signal basis element $c(nT_s + kT)|_{s(nT_s)=p}$ consists of following two steps:

1) multiplying the signal $r(nT_s + kT)$ sample by sample by the signal $e^{-\jmath 2\pi \frac{k^2}{2^{\text{SF}}}}$ (the so called *down-chirp*), obtaining the signal $d(nT_s + kT_s) \triangleq r(nT_s + kT) \cdot e^{-\jmath 2\pi \frac{k^2}{2^{\text{SF}}}}$;
2) taking the Discrete Fourier Transform of the vector $\mathbf{d}(nT_s)$, whose component $\mathbf{d}(nT_s)_k$ is $[\mathbf{d}(nT_s)]_k \triangleq d(nT_s + kT)$ and selecting the output of index p.

IV. Link Level Performance Analysis

In this section, we provide the results of some computer simulations experiments where we compare the FSCM modulation of cardinality 2^7 (i.e., with SF $= 7$) against an FSK modulation with the same cardinality. This means transmitting the waveform

$$c(nT_s + kT) = \frac{1}{\sqrt{2^{\text{SF}}}} e^{\jmath 2\pi s(nT_s)kT \frac{B}{2^{\text{SF}}}} \tag{17}$$

instead of the one in (4) for the input symbol $s(nT_s)$.

We have considered a flat frequency additive white gaussian noise channel (AWGN) as well as a unit energy frequency selective AWGN multipath channel with impulse response $h(nT) = \sqrt{0.8}\delta(nT) + \sqrt{0.2}\delta(nT - T)$. The results are depicted in Fig. 1.

We can see that, while for the AWGN channel the performance of the FCSM and FSK modulations are the same, for a frequency selective channel FCSM outperforms FSK. The reason is that any FCSM is sweeping the all frequency range and kind of averaging the noise. Instead, the FSK signals falling the area of the channel where there is a larger attenuation generate an effect on the bit error rate, which is more adverse that the beneficial effect of the FSK signals falling in the area of the channel where there is actually an amplification.

V. Conclusion

In this letter, we have – to the author's knowledge – provided the first complete mathematically rigorous description of the LoRa modulation (i.e., FCSM) and demodulation processes, based on the (discrete time) signal theory and detection. We derived the optimum demodulator and its efficient version. Finally, we made a performance comparison of the LoRa Modulation and the FSK modulation in terms of uncoded bit error rate. We noted that, while the two modulations exhibit the same performance in an AWGN channel, in a frequency selective channel the LoRa (i.e., FCSM) modulation performs better.

The results presented in this letter will eventually enable a further assessment of the LoRa based networks much more rigorous than what has been done until now.

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