Weak Signal Detection from Noisy Signal Using Stochastic Resonance with Particle Swarm Optimization Technique

Sumit Kumar, Rajib Kumar Jha
SIPL, Department of Electrical Engineering
Indian Institute of Technology Patna, Bihta, 801103
Email: (sumitphd13, jharajib)@gmail.com

***Abstract*—In this paper, we have investigated a novel stochastic resonance & particle swarm optimization (PSO) technique for weak signal detection from noisy signal (weak signal + internal noise). PSO technique is used to determine the optimal amount of noise for weak signal detection. Our proposed work is in Neyman-Pearson framework which maximizes the probability of detection P_D for a fixed value of probability of false alarm P_{FA}. With the equality and in-equality constraints, we have explored the penalty function method to design unconstrained objective function. In the proposed technique, we have considered 2 different, 3 different and 4 different noises separately and observed that the probability of detection P_D gets increased. Simulations are performed to investigate the result with a numerical example to exhibit the practicality of the proposed technique.**

***Index Terms*—Signal detection, receiver operating characteristic (ROC), probability distribution function (pdf), hypothesis.**

I. Introduction

The term noise in a wide sense is associated with the word hindrance. It was considered that the presence of noise can only worsen performance of the system. But, it has been studied that noise can contribute positively rather than reducing the performance of the system. Therefore, the effective action of the noise is to enhance the weak signal detection performance. This concept was thoroughly studied and reviewed by Namara et al. and Gammaitoni et al. [1], [2]. Stochastic Resonance (SR) is a non-linear phenomenon in which weak input signals [3]– [6] can be improved by the addition of an appropriate amount of external noise [7]. It is one of the most promising and relatively simple example of non-linear systems under the influence of noise. More technically, SR occurs if the signal to noise ratio (SNR), input/output correlation have a well marked maximum at a certain noise level.

In order to show SR, a system must own three basic properties: *a non-linearity with respect to threshold*, *a sub-threshold signal* like a signal with small amplitude and *a source of additive noise like Gaussian*. This stochastic resonance phenomenon occurs frequently in bistable systems. The behaviour of SR mechanism proves that at lower noise intensities the weak signal can not cross the threshold, thus producing a small value of SNR. For large noise intensities the output is prevailed by the noise which leads to a low SNR. But for moderate noise intensities, the noise allows the signal to cross the threshold giving maximum SNR at certain optimum noise level. As far as weak signal detection is concerned, SNR is directly related to the probability of detection in case of Gaussian noise, but for non-Gaussian noise, SNR is no longer related with the detection performance. In [9], [10], area under receiver operating characteristic (ROC) curve has been adopted as a performance measure for its simplicity and robustness for the non-Gaussian noise whose probability density function (pdf) must be symmetric and uni-modal. An adaptive stochastic learning mechanism performing a stochastic gradient ascent on the output SNR to evaluate the optimal noise [11] has been well performed. The performance parameter can be discussed in various ways such as increment in output signal-to-noise-ratio, increment in probability of detection under constant value of false alarm probability, decrement in probability of error [12], or increment in entropy-based bit count [13], area under region of convergence curve measure *etc*. Performance of weak signal detectors can be improved by adding independent noise [8] externally to the noisy signal, as shown in Fig. 1. Computation of stochastic resonance noise for different applications is always a challenging issue because the nature of the internal noise associated with the input signal is unknown. The selection of threshold is very crucial issue as the probability of detection, P_D and probability of false alarm, P_{FA} are monotonically increasing function, as shown in Fig. 2.

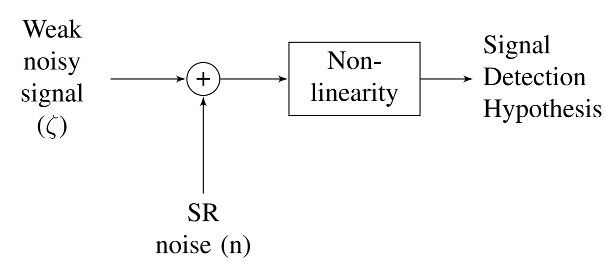


Fig. 1: The noise benefited detector, where input ζ is weak noisy signal & n is externally added stochastic resonance noise

II. Mathematical Framework

This section has been divided into two subsection. In the first subsection, we have discussed the theory of detection and

978-1-5090-2760-6/17/$31.00 ©2017 IEEE

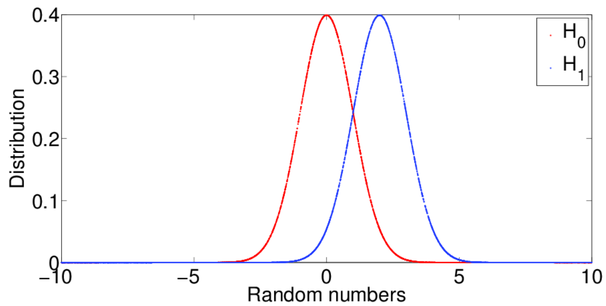


Fig. 2: Distribution function showing that how difficult to separate the data belonging to two curve

in second subsection PSO is discussed.

A. *Mathematics of Detection with SR*

$$\begin{cases} H_0 : p_\zeta(\zeta; H_0) = p_0(\zeta) \\ H_1 : p_\zeta(\zeta; H_1) = p_1(\zeta). \end{cases} \tag{1}$$

For any noisy signal ζ, the test statistics T selects the hypothesis, which is a function of ζ with the probability $\phi(\zeta)$, where $\phi(\zeta)$ is a critical function or decision function *i.e.,* $0 \leq \phi(\zeta) \leq 1$. Test on the data ζ with certain decision threshold value η is written as

$$T(\zeta) \underset{H_1}{\overset{H_0}{\lesseqgtr}} \eta. \tag{2}$$

Now in the data, which kind of noise generates the better probability of detection can be written mathematically as follows

$$y = \zeta + n. \tag{3}$$

To define probability of detection after noise realization

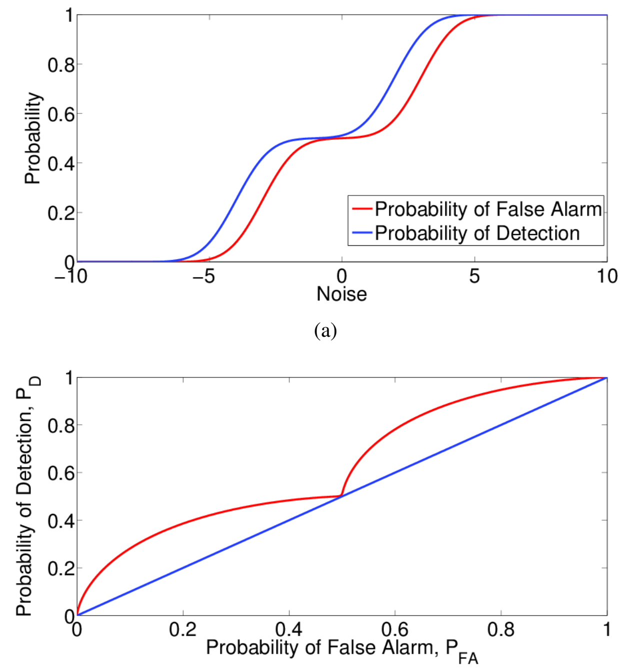
$$P_D^{f_n} = \int_{R^N} P_D\,(n)\, f_N\,(n)\,\mathrm{d}n. \tag{4}$$

and false alarm probability after realization of noise

$$P_{FA}^{f_n} = \int_{R^N} P_{FA}\,(n)\, f_N\,(n)\,\mathrm{d}n. \tag{5}$$

Let $f_{N_{optimal}}$ be the pdf of the required optimal noise, N_{opt}, that can be added externally to the noisy signal ζ to maximize the probability of detection P_D with the point $P_{FA} \leq C$, where C is predefined value. Let U denote the set of all probability density functions on $\mathbf{R}$. So, probability density function needs to be evaluated for which optimum P_D can be obtained. It can be represented as follows.

$$f_{N_{optimal}} = argmax_{f_N \in U} \int_{R^N} P_D\,(n)\, f_N\,(n)\,\mathrm{d}n. \tag{6}$$



(a)

(b)

Fig. 3: (a) Probability as a function of noise (b) P_D vs. P_{FA} for SR detection. See paper [8] for more detail.

such that

$$\begin{aligned} &f_{N_{optimal}} \geq 0 \qquad for\ \ all\ \ n \\ &\int_{R^N} f_N\,(n)\,\mathrm{d}n = 1. \end{aligned} \tag{7}$$

$$P_{FA}\left(f_{N_{optimal}}\right) = \int_{R^N} P_{FA}\,(n)\, f_{N_{optimal}}\,(n)\,\mathrm{d}n \leq \alpha. \tag{8}$$

Eq. (7) represents the basic property of probability density function whereas Eq. (6) and Eq. (8) confirm the conditions for the Neyman-Pearson (NP) criteria to get the optimal stochastic resonance noise probability density function which provides maximum probability of detection, P_D. The SR noise finding algorithm [15] has been discussed to optimize the term P_D. If the noise is independent then it obeys the condition $f\,(n \mid \zeta, H_j) = f_N\,(n)$ considering the hypothesis H_0 or H_1 and the received noisy signal ζ.

That is,

$$\begin{aligned} &max\ P_D(f_N) \\ subject\ to\ \ & \\ &P_{FA}(f_N) = C, a\ given\ value \\ &\sum_{m=1}^{k} \lambda_m = 1 \\ &\sum_{m=1}^{k} \lambda_m^2 \leq 1 \\ &\lambda_m \geq 0 \quad \forall\ \ m. \end{aligned} \tag{9}$$

where $\boldsymbol{n} = [\boldsymbol{n_1}, \boldsymbol{n_2}, ..., \boldsymbol{n_k}]$ and $\boldsymbol{\lambda} = [\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2}, ..., \boldsymbol{\lambda_k}]$, where λ_i is probability of the particular i^{th} noise value.

B. Formulation of Particle Swarm Optimization

The well description of the PSO is given in the paper [16].

$$\min \quad f(x) \tag{10}$$

such that

$$h(x) = \begin{bmatrix} h_1(x) \\ . \\ . \\ h_M(x) \end{bmatrix} = \begin{bmatrix} 0 \\ . \\ . \\ 0 \end{bmatrix}, \quad g(x) = \begin{bmatrix} g_1(x) \\ . \\ . \\ g_L(x) \end{bmatrix} \leq \begin{bmatrix} 0 \\ . \\ . \\ 0 \end{bmatrix}$$

Applying Penalty function method,

$$\min l(x) = f(x) + \sum_{i=1}^{M} p_i h_i^2(x) + \sum_{i=1}^{M} u_i \phi(g_i(x)) \tag{11}$$

$$v_r^{m+1} = \alpha \left(\omega v_r^m + d_1 \beta_{r1}^m \left(P_r^m - x_n^m \right) + d_2 \beta_{r2}^m \left(P_g^m - x_r^m \right) \right) \tag{12}$$

$$y_r^{m+1} = y_r^m + v_r^{m+1}. \tag{13}$$

where $r = 1, ..., N$ is the particle size, m is the iteration index, α is the constriction factor, ω is inertia weighting coefficient which controls the velocities of the particle, d_1 and d_2 are cognitive and social parameters respectively. β_{r1}^m and β_{r2}^m represents uniformly distributed random variable. Eq. (12) and Eq. (13) represent the velocity and position equation of the PSO.

III. Illustration by Example

This section illustrates an examples for weak signal detection [8], [14] using stochastic resonance technique,

$$\begin{aligned} H_0 &: \zeta(j) = w(j) \\ H_1 &: \zeta(j) = A + w(j), \end{aligned} \tag{14}$$

for j = 0,1,... , N-1, $A \geq 0$ is a known DC signal, and $w(j)$ is symmetric Gaussian mixture noise as a disturbance which is already present in the signal.

A. Performance with Asymmetric Noise

$$f_N(n) = \sum_{m=1}^{V} \lambda_m \delta(n - n_m) \tag{15}$$

1) For V=2:

$$P_{D,optimal}^y = \lambda_1 F_1(n_1) + \lambda_2 F_1(n_2) \tag{16}$$

and

$$P_{FA}^y = \lambda_1 F_0(n_1) + \lambda_2 F_0(n_2) \tag{17}$$

Particle swarm optimization is applied ($swarmsize = 64$, $maximumiteration = 1000$, $\omega = 1.0$, $d_1 = 1.0$ and $d_2 = 1.0$) for the calculation of stochastic resonance noises and weights evaluation. We have $n_1 = 2.5070$, $n_2 = -3.4930$, $\lambda_1 = 0.6900$ and $\lambda_2 = 0.3100$. For these noises, we get $P_D = 0.6920$, at a fixed $P_{FA} = 0.5005$.

2) For V=3:

$$P_{D,optimal}^y = \lambda_1 F_1(n_1) + \lambda_2 F_1(n_2) + \lambda_3 F_1(n_3) \tag{18}$$

and

$$P_{FA}^y = \lambda_1 F_0(n_1) + \lambda_2 F_0(n_2) + \lambda_3 F_0(n_3) \tag{19}$$

With the same particle swarm optimization parameters (V=2), we obtained the noises as $n_1 = 2.4986$, $n_2 = -3.4611$, $n_3 = 2.5300$, $\lambda_1 = 0.3205$, $\lambda_2 = 0.3205$ and $\lambda_3 = 0.3627$. At these values of noises and certain probabilities, we have $P_D = 0.6926$ at $P_{FA} = 0.5005$.

3) For V=4:

$$P_{D,optimal}^y = \lambda_1 F_1(n_1) + \lambda_2 F_1(n_2) + \lambda_3 F_1(n_3) + \lambda_4 F_1(n_4) \tag{20}$$

and

$$P_{FA}^y = \lambda_1 F_0(n_1) + \lambda_2 F_0(n_2) + \lambda_3 F_0(n_3) + \lambda_4 F_0(n_4) \tag{21}$$

With the same particle swarm optimization parameters, we obtained the noises as $n_1 = 2.5271$, $n_2 = 2.6605$, $n_3 = 2.5397$, $n_4 = -3.4798$, $\lambda_1 = 0.2363$, $\lambda_2 = 0.2324$, $\lambda_3 = 0.2133$ and $\lambda_4 = 0.3183$. For these noises and weight values, we obtained $P_D = 0.6969$ at $P_{FA} = 0.5005$. So increasing number of noises increases the mathematical complexity but it increases the probability of detection P_D slightly.
The simulation time also been discussed in the Fig. 4. As the number of asymmetric noise increases, the simulation time gets increased.

TABLE I: Probability of detection P_D and probability of false alarm P_{FA} using Particle swarm optimization based methods with number of noises and its weights

Item	SR (2 noise)	SR (3 noise)	SR (4 noise)
P_D^y	0.6920	0.6926	0.6969
P_{FA}^y	0.5005	0.5005	0.5005
n_1	2.5070	2.4986	2.5271
n_2	-3.4930	-3.4611	2.6605
n_3	NA	2.5300	2.5397
n_4	NA	NA	-3.4798
λ_1	0.6900	0.3205	0.2363
λ_2	0.3100	0.3205	0.2324
λ_3	NA	0.3627	0.2133
λ_4	NA	NA	0.3183

B. Performance with Symmetric Noise

Here, we considered the same noise as discussed in [8] and saw the performance of the PSO. The results are tabulated in

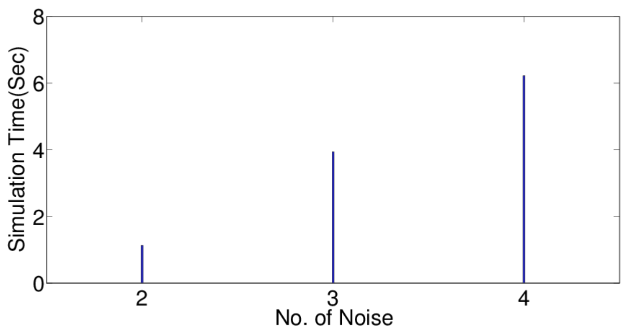


Fig. 4: Simulation time (sec) variation with respect to the number of noises.

Table II. The comparative study shows that it almost produces the same result as in [8]. Here, we considered the same noise as discussed in [8] and saw the performance of the PSO. The results are tabulated in Table II. The comparative study shows that it almost produces the same result as in [8].

TABLE II: Comparison of probability of detection calculated by proposed PSO & Hao Chen et. al [8]

SR noise	P_n^{opt}	$P_{symmetric}^{opt}$	$P_{uniform}^{opt}$	$P_{Gaussian}^{opt}$	$No\ \ noise$
$P_D^y(PSO)$	0.6920	0.6707	0.5901	0.5807	0.5114
$P_D^y($ [8]$)$	0.6915	0.6707	0.6011	0.5807	0.5114

IV. Conclusion

The Particle swarm optimization is applied in the signal detection application under Neyman-Pearson criteria. The constraint optimization function has been dealt using penalty function method. We have found almost same result as in [8] for two point asymmetric noise and symmetric noise. This fundamental theory of stochastic resonance and particle swarm optimization method can be incorporated for the signal detection problems like watermark detection, logo detection, edge detection, cancer detection in medical images *etc*. Future scope identifies work to be carried out in the direction of improvement in stochastic resonance, how Neyman-Pearson framework can be improved.

References

[1] B. Mc. Namara and K. Wiesenfeld, "Theory of Stochastic Resonance," Physical Review A, vol. 39, no. 9, pp. 4854-4869, May 1989.
[2] L. Gammaitoni, P. Hanggi, P. Jung and F. Marchesoni, "Stochastic resonance," Reviews of Modern Physics, vol. 70, pp. 223-287, 1998.
[3] S. M. Bezrukov and I. Vodyanov, "Noise-induced enhancement of signal transduction across voltage-dependent ion channels," Nature, vol. 378, pp. 362-364, Nov. 1995.
[4] J. K. Douglass, L. Wilkens, E. Pantazelou and F. Moss, "Noise enhancement of information transfer in crayfish mechanoreceptors by stochastic resonance," Nature, vol. 365, pp. 337-340, Sep. 1993
[5] J. E. Levin and J. P. Miller, "Broadband neural encoding in the cricket cercal sensory system enhanced by stochastic resonance," Nature, vol. 380, pp. 165-168, Mar. 1996.
[6] K. Wiesenfeld and F. Moss, "Stochastic resonance and the benefits of noise: From ice ages to crayfish and squids," Nature, vol. 373, pp. 33-36, Jan. 1995
[7] B. Kosko, Noise, New York: Viking/Penguin, 2006.
[8] H. Chen, P. K. Varshney, S. M. Kay and J. H. Michels, "Theory of stochastic resonance effects in signal detection: Part I-fixed detectors," IEEE Trans. Signal Process. , vol. 55, no. 7, pp. 3172-3184, Jul. 2007.
[9] G. Guo, M. Mandal and Y. Jing, "A robust detector of known signal in non-Gaussian noise using threshold system," Journal of Signal Processing, vol. 92 Issue 11, pp. 2676-2688, Nov. 2012.
[10] G. Guo, M. Mandal and Y. Jing, "Optimal Design of Noise-Enhanced Binary Threshold Detector Under AUC Measure," IEEE Signal Process. Lett., vol. 20, no. 2, Feb. 2013.
[11] S. Mitaim and B. Kosko, "Adaptive stochastic resonance," Proc, IEEE, vol. 86, pp. 2152-2183, 1998.
[12] D. Rousseau, G. V. Anand and F. Chapeau-Blondeau, "Noise enhanced nonlinear detector to improve signal detection in non-Gaussian noise," Signal Process. , vol. 86, no. 11, pp. 3456-3465, Nov. 2006.
[13] A. Patel and B. Kosko, "Stochastic resonance in noisy spiking retinal and sensory neuron models," Neural Netw. , vol. 18, pp. 467-478, Aug. 2005.
[14] S. Kay, "Can detectability be improved by adding noise?," IEEE Signal Process. Lett., vol. 7, no. 1, pp. 8-10, Jan. 2000.
[15] A. Patel and B. Kosko, "Optimal noise benefits in Neyman Pearson and inequality-constrained signal detection," IEEE Trans. Signal Process., vol. 57, no. 5, pp. 1655-1669, May 2009.
[16] A. I. F. Vaz and E. M. G. P. Fernandes, "Optimization of nonlinear constrained particle swarm," Baltic J. Sustainability, vol. 12, no. 1, pp. 30-36, 2006.