# A Probabilistic Approach to Radiated Electromagnetic Interference

Michael Haider and Johannes A. Russer
Department of Electrical and Computer Engineering
Technical University of Munich
Arcisstr. 21, 80333 Munich, Germany
michael.haider@tum.de

Abstract—Radiated electromagnetic interference is an ubiquitous problem that needs to be accounted for in the design of electronic devices. Typically, restrictions on the maximum permissible field level for a device to cause over a certain frequency range are imposed, based on characterizing the device's radiations in the far-field. We present a probabilistic approach to characterize and model radiated interference, based on stochastic electromagnetic fields. This allows for computationally predicting the probability for exceeding the emission threshold, set by international electromagnetic compatibility regulations.

### I. INTRODUCTION

High-frequency currents in modern electronic devices give rise to the unintentional emission of electromagnetic radiation. This radiated electromagnetic interference (EMI) interacts with other electronic devices close-by, disturbing their operation. In order to limit these effects, different regulations have been put in place in the context of electromagnetic compatibility (EMC). However, with given design constraints and high performance demands, these regulations are, usually difficult to fulfill, especially in the early design stage of new electronic devices, where engineers typically lack detailed information about the actual electromagnetic emissions of their prototypes still in development. Additionally, the evergrowing demand for increased data-rate and bandwidth, as well as the soaring number of electronic devices per area, renders accurate computer-aided modeling of electromagnetic interference indispensable. In particular, with respect to the signal-to-noise ratio of communication processes within an electronic device, and the associated signal-integrity issues, an EMI-aware design can aid in reducing the overall power consumption substantially.

Quantitative modeling of radiated EMI requires the identification of the essential parameters, along with a suitable modeling technique, in order to find the transformation of these parameters and to predict the radiated EMI. Therefore, different modeling techniques, which account different aspects of electromagnetic interference have been developed [1]. In this contribution, we approach the problem of characterizing and propagating radiated EMI from a probabilistic perspective [2] considering noisy EM field sources. In this approach, the electromagnetic field is modeled in terms of a random field, that originates from a multitude of different random processes, i.e. random source currents and random source voltages

within an electronic device. In this model, it is impossible to specify accurate values for the electric or magnetic field at a certain location, at a certain time. However, we can specify probabilities for the field intensities being below a certain threshold for all spatial locations and for all times. From the probability measures of the random source processes one can then calculate the probability measures of the random electromagnetic fields at arbitrary positions. This makes it possible, to predict the result of a radiated EMI measurement, i.e. whether a device fulfills certain EMC regulations or not, given a probabilistic description of the device itself and its sources.

For this, a probabilistic model for the distribution of the electromagnetic field needs to be assumed or estimated from measured data. A lot of investigations have been carried out for stationary Gaussian-distributed electromagnetic fields, both, in the frequency-domain [3] and in the time-domain [4]. Measurements are being performed with a two-probe scanning system. The framework has since been expanded to include cyclostationary stochastic electromagnetic fields, again with Gaussian probability distribution [5]. The statistics of the electromagnetic field are governed by the first- and secondorder statistical moments. Hence, the theory describes the propagation of spatial field-field correlations using Green's functions, neglecting the mean values without loss of generality. The assumption of Gaussian statistics seems to be well justified in a lot of scenarios due to the central limit theorem. Our objective is to provide a more fundamental probabilistic approach, that is independent of the underlying probability distribution [2].

In section II an overview of a generalized probabilistic approach to the treatment of radiated EMI is presented. The necessary experimental input for the prediction of far-field EMC measurement results is described in section III.

### II. STOCHASTIC ELECTROMAGNETIC FIELDS

The mathematical model for the description of stochastic electromagnetic fields in terms of a generalized framework has been discussed in [2]. It has been pointed out, that a probability measure in function space of a separable [6] random field with a continuous index set can be constructed from families of finite-dimensional joint probability distributions that satisfy certain compatibility conditions [7]. Furthermore, it has been

shown in [8], that for a given measure  $\mu$  in function space X, it is equivalent to postulate a characteristic functional  $\chi$  with

$$\chi_{\mathbf{X}}(l) = \int_{X} e^{\mathrm{i}l(x)} \,\mu(\mathrm{d}x) \;, \tag{1}$$

where the integration is performed over all measurable functions  $x \in X$ . Restricting ourselves to electromagnetic fields with finite energy, which seems to be a realistic assumption, the function space  $X \subseteq L^2$ , modeling the electric or magnetic field, respectively, becomes the Hilbert space of square integrable functions with the associated inner product. By the Riesz representation theorem, the linear functionals l in (1) can be represented by inner products with Hilbert space elements  $z \in X$ , i.e.  $l(x) = \langle z, x \rangle$ ,  $\forall X \in X$ .

Suppose now, that the function space X with the characteristic functional  $\chi_X$  model the random emission processes within some electronic device, i.e. enclosed within a boundary area surrounding the device. For the prediction of the stochastic electromagnetic field, originating from the device, we take a second random field into account, described by a characteristic functional  $\chi_Y$  over the space of vector-valued functions  $y \in Y$ . We say that the realizations of the random fields y arise from the realizations of the random source field x under the action of a linear propagation operator  $\hat{M}$ , which is a manifestation of Maxwell's equations for a certain scenario. This means that all observed realizations y can be obtained from all source realizations x by

$$y = \hat{M}x. (2)$$

The characteristic functional  $\chi_Y$ , and hence the probability measure on the space of observed fields Y can be easily obtained by [2]

$$\chi_{Y}(z) = \int_{Y} e^{i\langle z, y \rangle} \mu(dy) = \int_{X} e^{i\langle z, \hat{M}x \rangle} \mu(dx)$$
$$= \int_{Y} e^{i\langle \hat{M}^{\dagger}z, x \rangle} \mu(dx) = \chi_{X} \left( \hat{M}^{\dagger}z \right). \tag{3}$$

This provides us with a general procedure for the propagation of stochastic electromagnetic fields, regardless of the type of the underlying distribution.

# III. EXPERIMENTAL INPUT

In the probabilistic approach, described in section II, the characteristic functional of the source field at a surface enclosing the device under test remains to be determined. Fig. 1 schematically depicts the random electromagnetic spatial intensity distribution above a printed circuit board (PCB). In general, the formalism requires experimental input for estimating the characteristic functional  $\chi_{\rm X}$  of the source field at this point. We distinguish between parametric and non-parametric estimation, is typically more accurate but it requires some a-priori knowledge of the general form of the characteristic functional. In a non-parametric estimation procedure, the characteristic functional can be reconstructed by summing over the n-th order statistical moments in terms of a Taylor polynomial in the functional derivative sense [9].

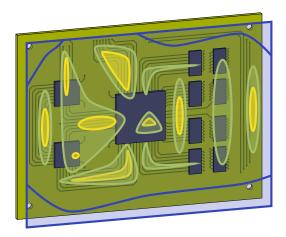


Fig. 1. Radiated EM emissions in the near-field above a printed circuit board.

In terms of parametric estimation, one identifies the key parameters for a prescribed form of the characteristic functional. For a Gaussian stochastic electromagnetic field, this is e.g. the mean function and the correlation dyadic, recovering the theoretical description from [4].

Evaluating the probability of exceeding a certain field threshold at a certain far-field position, both set by the EMC standard's requirements, from the characteristic functional  $\chi_Y$ , one can predict whether a device passes or fails an EMC test.

## IV. CONCLUSION

We have presented a probabilistic approach for modeling radiated electromagnetic interference in terms of stochastic electromagnetic fields. Depending on the actual probability distribution one can experimentally determine a characteristic functional, describing the electromagnetic sources on a given device under test. The characteristic functional can be propagated to any arbitrary point, and can thus be used to predict an EMC test result in the near- and far-field.

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