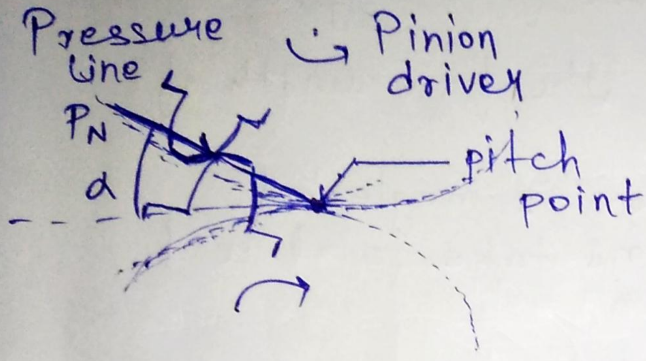
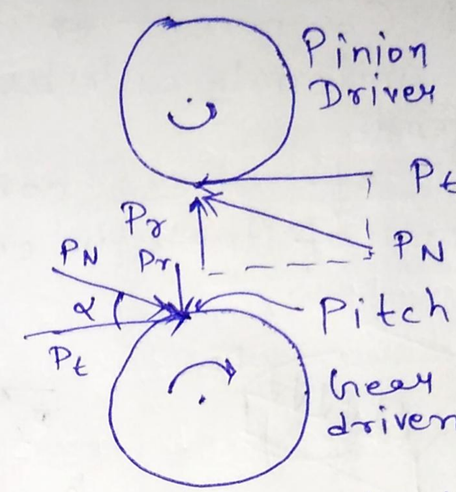
Force Analysis





$$\boxed{M_t = \frac{60 \times 10^6 \, (kW)}{2\pi n}}$$

M_t = torque transmitted by gears
kW = Power transmitted by gears
n = speed of rotations

$$\boxed{P_t = \frac{2M_t}{d'}}$$ $$\boxed{P_r = P_t \tan\alpha}$$

Assumptions

(I) Magnitude of P_N changes. It is neglected
(II) Only one pair of teeth in contact.
(III) Analysis is valid under static conditions.

* In practice, the optimum range of ~~gear teeth~~ the face width is

$$\boxed{8m < b < 12m}$$

In preliminary stages of gear design, the face width is assumed as <u>10 times of module</u>.

* Beam Strength of Gear

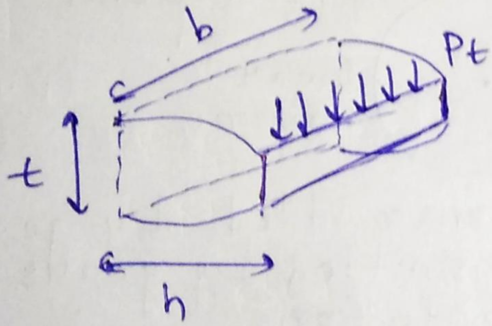
The lewis equation is based on the following assumptions:-

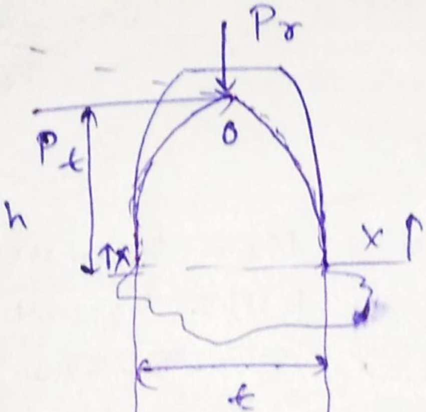
(i) Radial Component neglected

(ii) P_t is uniformly distributed over the face width of the gear.

(iii) Effect of stress concentration is neglected.

(iv) one pair of teeth in contact and take total load.





The weakest section of gear tooth is at XX.

At the Section XX

$$M_b = P_t \times h$$

$$I = \frac{1}{12} b t^3$$

$$y = \frac{t}{2}$$

Bending stress $\sigma_B = \dfrac{M_b y}{I} = \dfrac{(P_t \times h) \times \frac{t}{2}}{\frac{1}{12} b t^3}$

$$\Rightarrow P_t = b \sigma_b \left(\frac{t^2}{6h} \right)$$

Multiplying numerator and denominator by m

$$P_t = m b \sigma_b \left(\frac{t^2}{6hm} \right)$$

Defining $Y = \frac{t^2}{6hm}$

$$\boxed{P_t = m b \sigma_b Y} \qquad \boxed{Y = \frac{t^2}{6hm}} \Rightarrow \text{Lewis form factor}$$

P_t = tangential Force

σ_b = Corresponding stress

* The beam strength is the maximum value of tangential force that the tooth can transmit without bending failure.

Replacing (P_t) by S_b

$S_b \rightarrow$ beam strength of gear tooth

$\sigma_b \rightarrow$ permissible bending stress N/mm²

$$\boxed{S_b = m b \sigma_b Y}$$

To aveide breaking of gear tooth due to bending

$$S_b \geq P_{eff}.$$

* m and b are same for gear and pinion when different materials used for pinion and gear $(Y \times \sigma_b)$ decides the weaker between them.

For 20° pressure Angle

$$\boxed{Y = 0.154 - \frac{0.912}{z}}$$

z = no. of teeth.

So, for same material Y is less for pinion as compared to gear.

$\boxed{\text{Table} \Rightarrow 17.3}$

* Earle Buckingham has suggested that the endurance limit stress of gear tooth is approximately one-third of the ultimate tensile strength of the material.

$$\sigma_b = S_e = \left(\frac{1}{3}\right) S_{ut}$$

* Calculation

$$\boxed{M_t = \frac{60 \times 10^6 \,(kW)}{2\pi n}} \qquad \boxed{P_t = \frac{2M_t}{d'}}$$

→ Service factor (C_s)

$$C_s = \frac{\text{Maximum torque}}{\text{Rated tourque}} = \frac{(M_t)_{Max.}}{M_t} = \frac{P_{t\,max.}}{P_t}$$

$$\Rightarrow \boxed{P_{t\,max.} = C_s \times P_t}$$

→ Velocity factor

(I) For ordinary and commercially cut gears
$V < 10$ m/sec.

$$C_v = \frac{3}{3+V}$$

(II) Accurately hobbed and generated gears
$V < 20$ m/sec.

$$C_v = \frac{6}{6+V}$$

(III) precision gears with shaving, griding and lapping operation

$V > 20$ m/se.

$$C_v = \frac{5.6}{5.6+\sqrt{V}}$$

The pitch line velocity $V = \frac{\pi d' n}{60 \times 10^3}$

$$\boxed{P_{eff} = \frac{C_s P_t}{C_v}}$$

Dynamic load (P_d)

$$P_{eff} = C_s P_t + P_d$$

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{Ceb + P_t}}$$

* **Estimation of module based on beam strength**

$S_b = P_{eff} \times (f_s)$ $\qquad$ $f_s \rightarrow$ factor of safety

$\Downarrow$

$$P_{eff.} = \frac{C_s}{C_v} \times P_t = \frac{60 \times 10^6}{\pi} \left\{ \frac{(kW)\, C_s}{m z n C_v} \right\}$$

$$S_b = m^2 \left(\frac{b}{m}\right) \times \left(\frac{S_{ut}}{3}\right) \times y$$

$$\boxed{m = \left[\frac{60 \times 10^6}{\pi} \left\{ \frac{(kW)(C_s)(f_s)}{z n C_v \left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right) y} \right\} \right]^{1/3}}$$

* **Estimation of module based on wear strength**

$P_t = b\, Q\, d'_p\, K$

$\rightarrow$ Wear strength of gear tooth

Replacing P_t by S_w

$$S_w = b\, Q\, d'_p\, K$$

* $Q \rightarrow$ Ratio factor

$$\boxed{Q = \frac{2 z_g}{z_g + z_p}}$$

for internal gear

$$Q = \frac{2 z_g}{z_g - z_p}$$

* $K \rightarrow$ load stress factor

$$K = \frac{\sigma_c^2 \sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4}$$

For steel with 20° pressure angle

$$\boxed{K = 0.156\left(\frac{BHN}{100}\right)^2} \rightarrow \text{Brinel Hardness number}$$

$S_w > P_{eff}$ → to avoid failure of gear tooth.

$$S_w = P_{eff} \times (f_s)$$

$$P_{eff} = \frac{60 \times 10^6}{\pi}\left(\frac{(kW) \times C_s}{m z n C_v}\right)$$

$$S_w = m^2\left(\frac{b}{m}\right) Q z_p K$$

$$\boxed{m = \left[\frac{60 \times 10^6}{\pi}\left\{\frac{(kW)(C_s)(f_s)}{2p^2 n_p C_v\left(\frac{b}{m}\right) Q K}\right\}\right]^{1/3}}$$