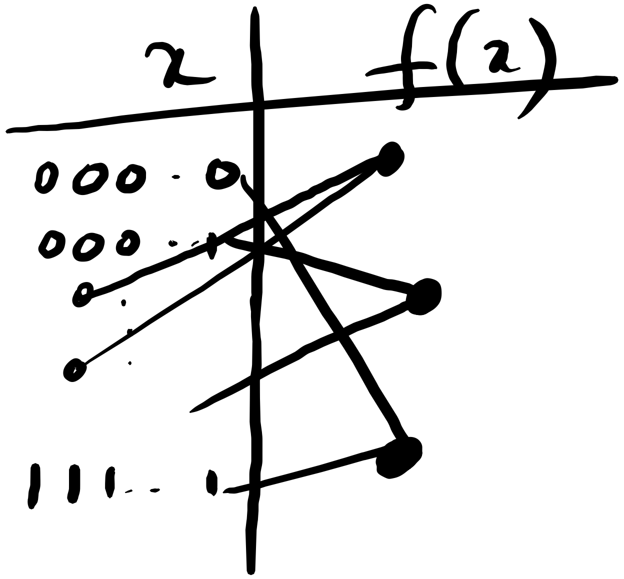
Day 5

Recap

$$f : \{0,1\}^n \longrightarrow \{0,1\}^n$$



$x, y \quad f(x) = f(y)$

$$\boxed{x = y \oplus s}$$

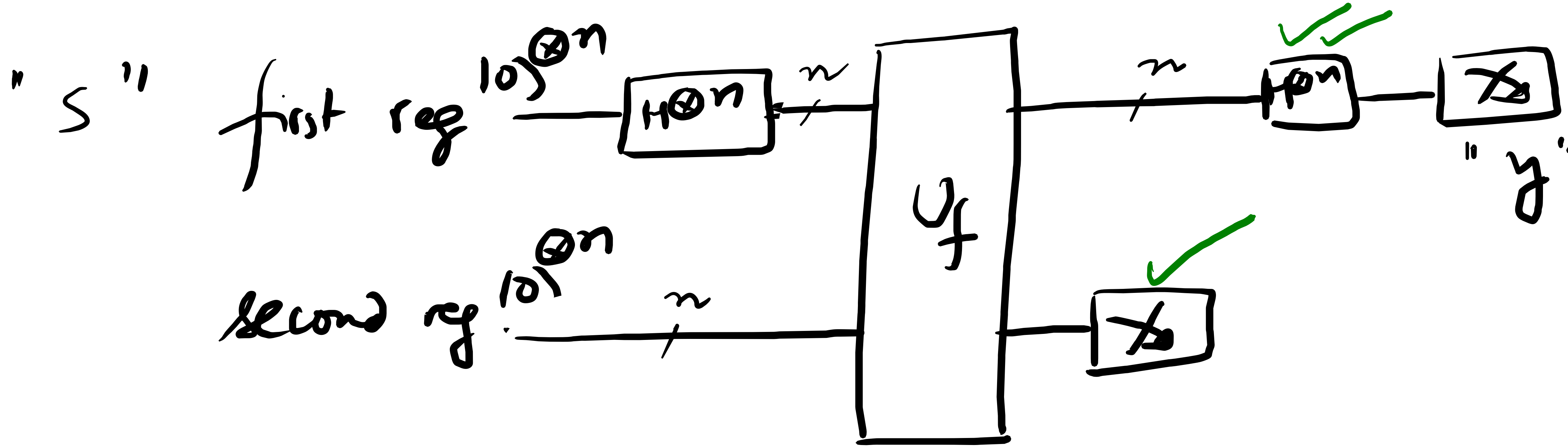
$x, \; x \oplus s$

Goal - find $s \in \{0,1\}^n$.

$2^{n-1} + 1$ # queries

$f \longrightarrow U_f$

$n \approx 20$



$y \cdot s = 0$

10101

$s_1 \oplus s_3 \oplus s_5 = 0$

$y = (y_1, y_2, \ldots, y_n)$

$s = (s_1, s_2, \ldots, s_n)$

$n-1$ linearly independent equations

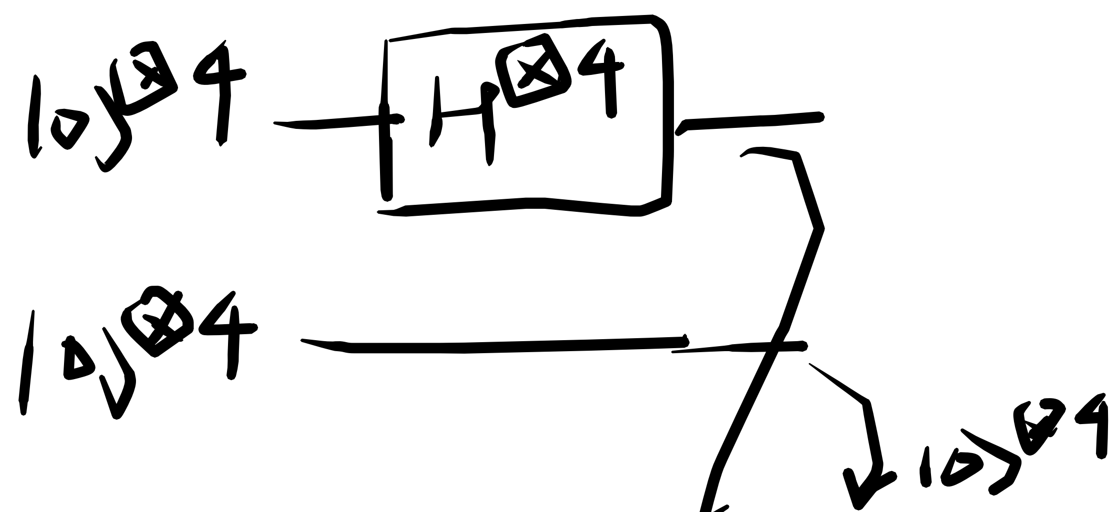
eg:

$$\{0,1\}^4 \longrightarrow \{0,1\}^4$$

x	$f(x)$
0000, 1001	1111
0001, 1000	0001
0010, 1011	1110
0011, 1010	1101
0100, 1101	0000
0101, 1100	0101
0110, 1111	1010
0111, 1110	1001

$$\boxed{s = 1001}$$

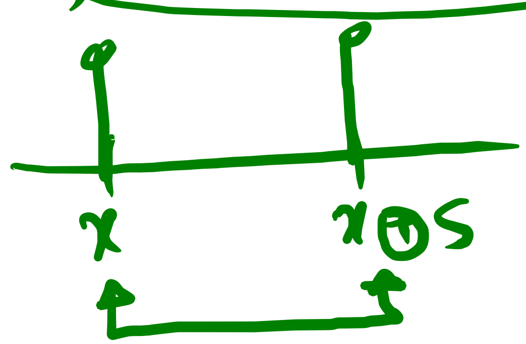
n is large



$$\frac{1}{\sqrt{2^4}}\left(|0000\rangle + |0001\rangle + \dots + |1111\rangle\right)$$

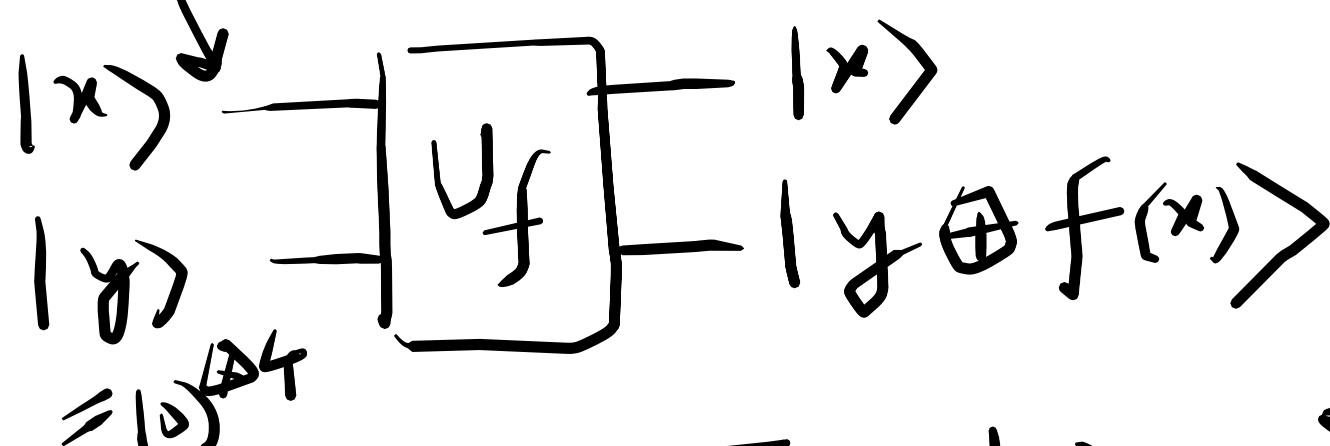
$$\otimes\ |0\rangle^{\otimes 4}$$

$$f(x) = f(x \oplus s)$$



$$\frac{1}{\sqrt{2^4}}\left(|0000\rangle + \cdots + |1111\rangle\right) \otimes |0\rangle^{\otimes 4}$$

$$\xrightarrow{U_f} \frac{1}{\sqrt{2^4}}\Big(|0000\rangle\,|\underline{f(0000)}\rangle + |0001\rangle\,|\underline{f(0001)}\rangle + \cdots + |1111\rangle\,|\underline{f(1111)}\rangle\Big)$$



$$\equiv \begin{matrix} |x\rangle \\ |f(x)\rangle \end{matrix}$$

$$\sum |x\rangle|y\rangle \longrightarrow \sum |x\rangle|f(x)\rangle$$

Measure the second register

$$|f(0000)\rangle, \; |f(0001)\rangle, \; \ldots, \; |f(1111)\rangle$$

$s_2 \oplus s_3 = 0$

$$|f(\sigma)\rangle = |1010\rangle$$

0110 or 1111

Contents of the 1st register $\boxed{\frac{1}{\sqrt{2}}|0110\rangle + \frac{1}{\sqrt{2}}|1111\rangle}$

$$H^{\otimes 4}\left[\frac{1}{\sqrt{2}}|0110\rangle + \frac{1}{\sqrt{2}}|1111\rangle\right] \xrightarrow{\text{measure}} \text{"}y\text{"}$$

$y \cdot s = 0$

$$H^{\otimes 4}\left[\frac{1}{\sqrt{2}}|0110\rangle + \frac{1}{\sqrt{2}}|1111\rangle\right]$$

$$= \frac{1}{\sqrt{2}} H^{\otimes 4}|0110\rangle + \frac{1}{\sqrt{2}} H^{\otimes 4}|1111\rangle$$

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2^4}}\sum_{z}(-1)^{z\cdot(0110)}|z\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2^4}}\sum_{z'}(-1)^{z'\cdot 1111}|z'\rangle\right]$$

$$= \frac{1}{\sqrt{8}}\Big[|0000\rangle + |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle\Big]$$

1001

$s_2 = 0$

$s_3 = 0$

$s_1 \oplus s_4 = 0$

$s_1 = 1 \quad s_4 = 1$

$y \cdot s = 0$

$0010 \Rightarrow s_3 = 0$ $\qquad s_2 = 0$

$1101 \Rightarrow s_1 \oplus s_2 \oplus s_4 = 0$

1001

$\boxed{s_1 \oplus s_4 = 0}$

$$s = (1 \quad 0 \quad 0 \quad 1)$$

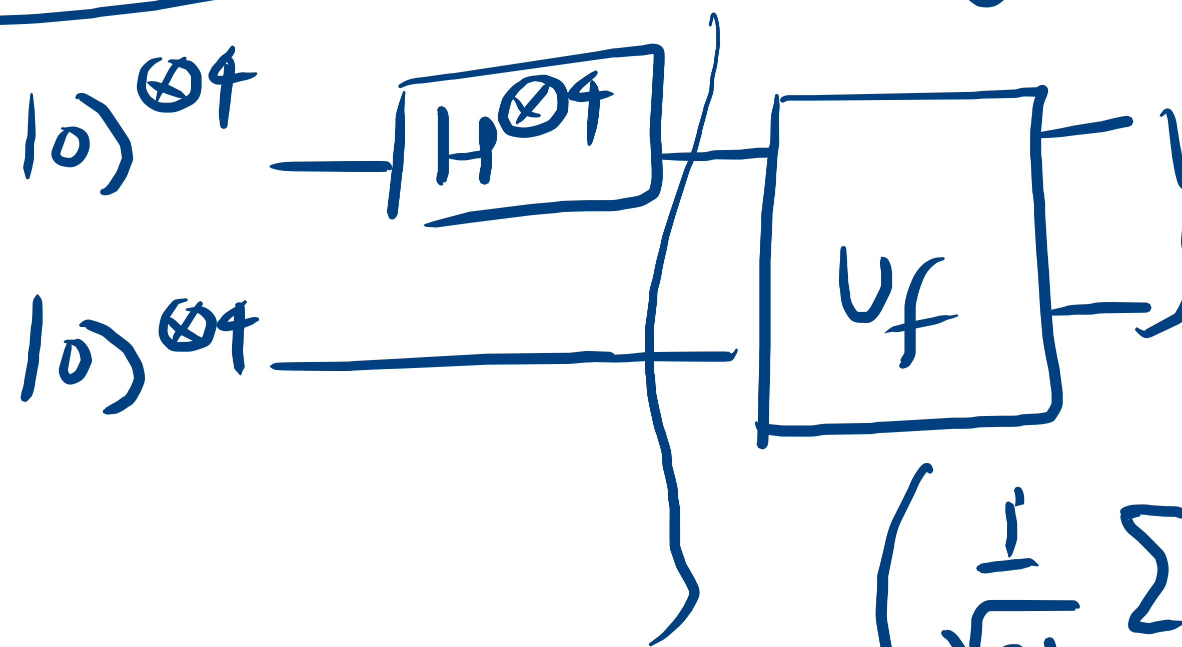
0

Simon's algo.

$$|0000\rangle|f(0000)\rangle + |0001\rangle|f(0001)\rangle + \cdots$$

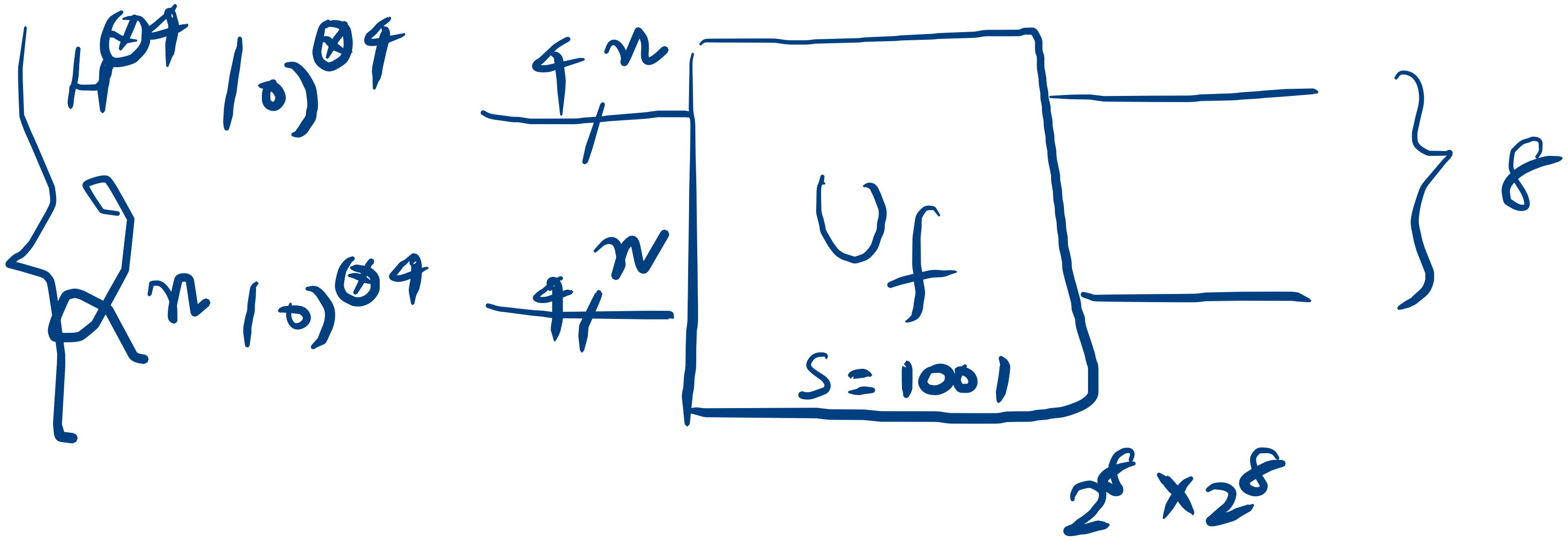
$$s = 1 \quad 0 \quad 0 \quad 1$$

Can you make U_f

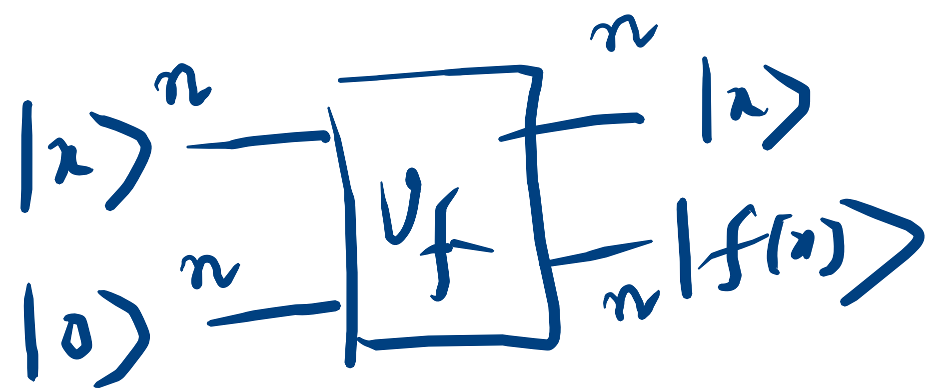


$$\sum_{x \in \{0,1\}^4} |x\rangle|f(x)\rangle$$

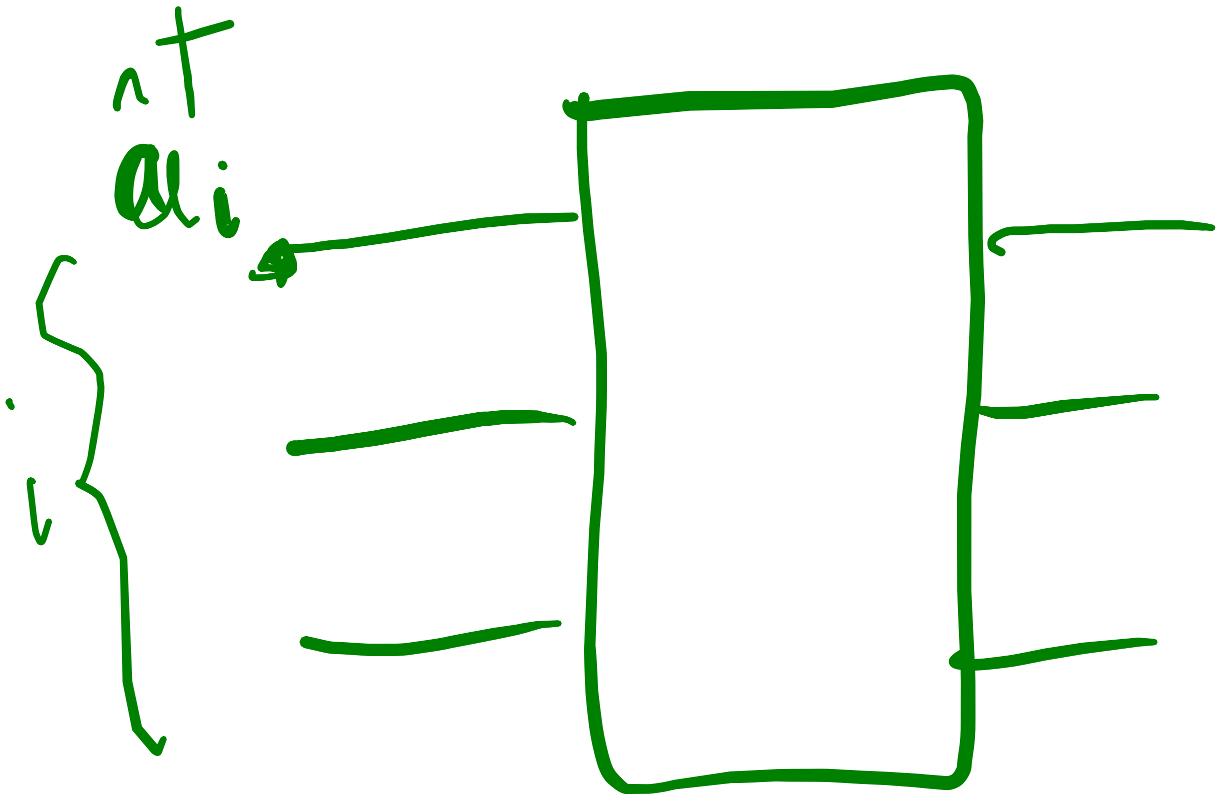
$$\left(\frac{1}{\sqrt{2^4}} \sum |x\rangle\right)|0\rangle^{\otimes 4} \longrightarrow \frac{1}{\sqrt{2^4}} \sum_x |x\rangle|f(x)\rangle$$

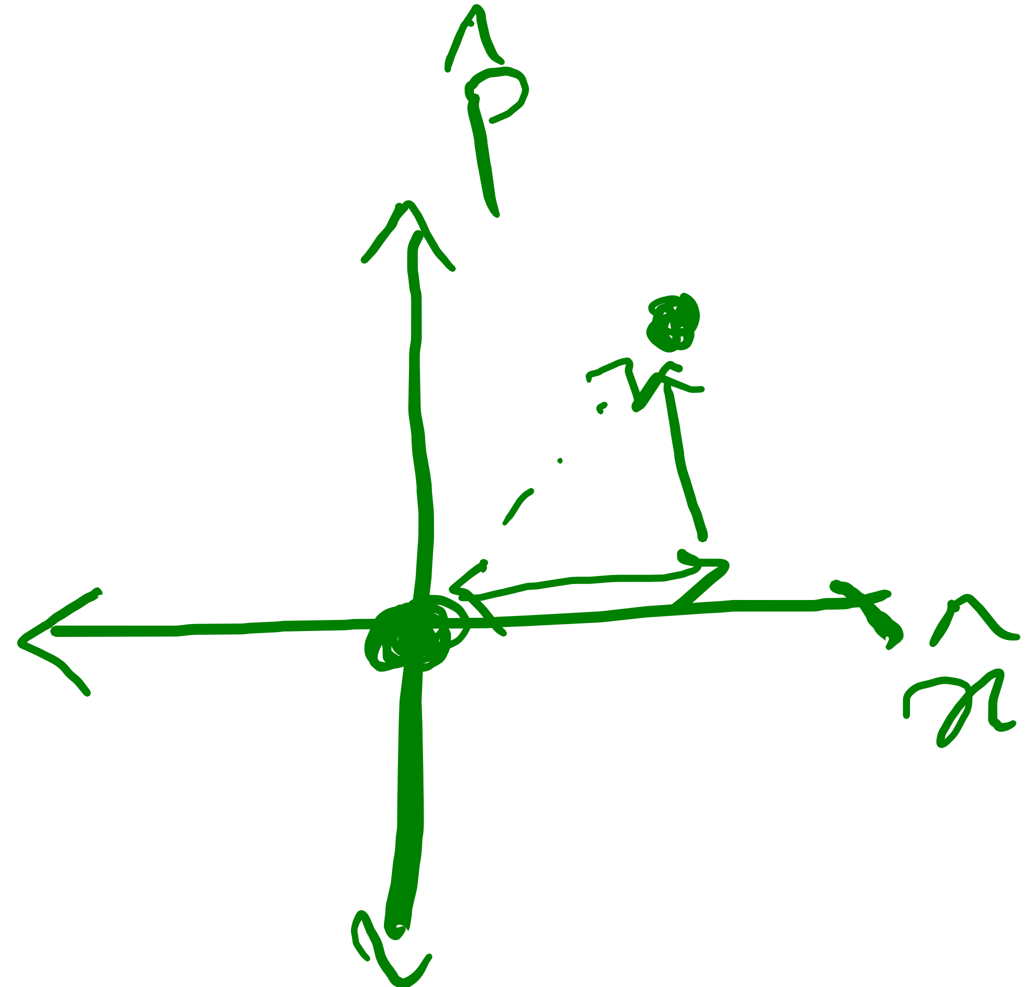


oracle (s)



s	U_f
2	16×16
3	64×64
4	256×256
n	$2^{2n} \times 2^{2n}$





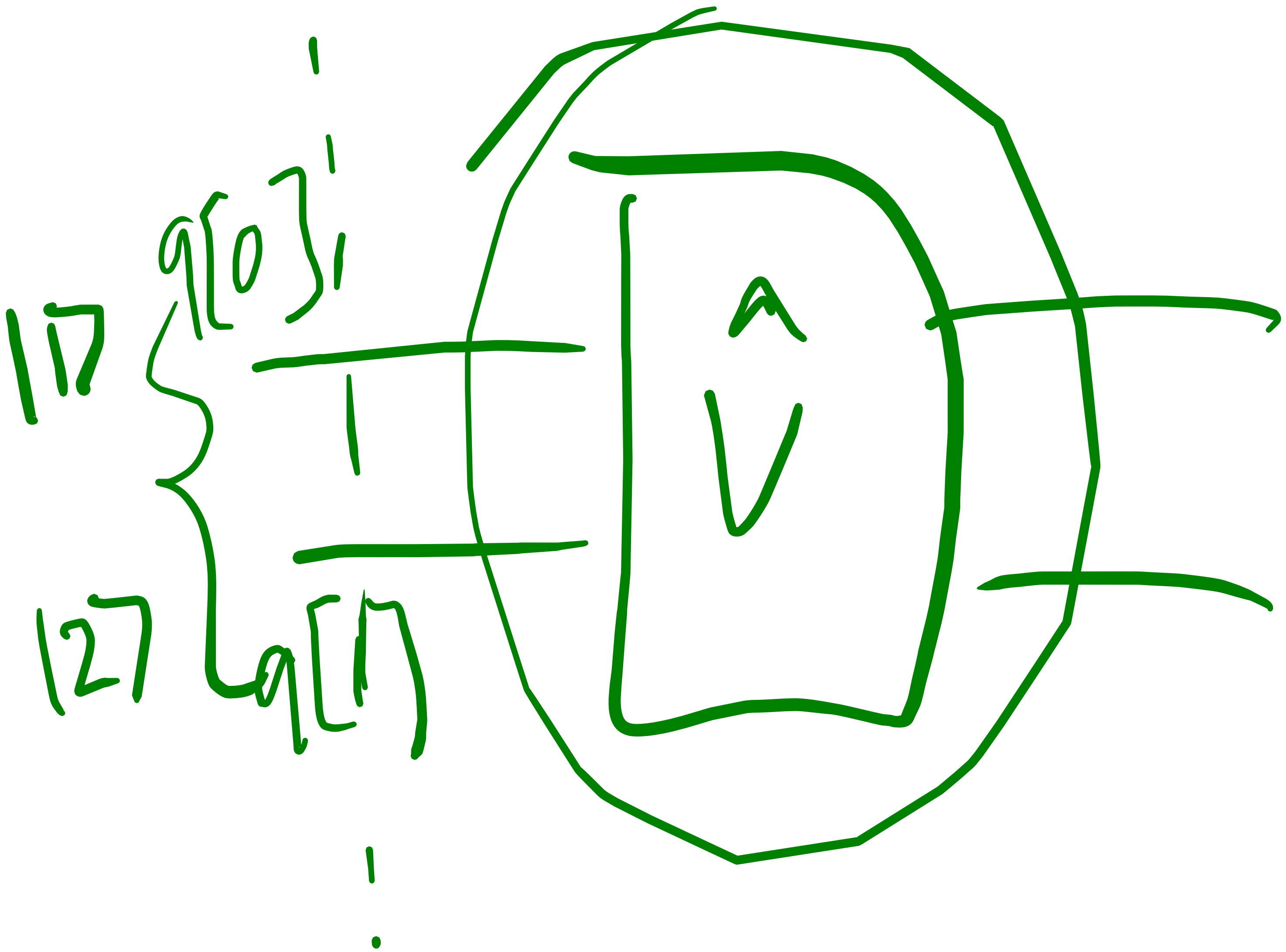
$$\hat{a}_i^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$\hat{a}_i |n\rangle = \sqrt{n}\,|n-1\rangle$$

$|n\rangle$

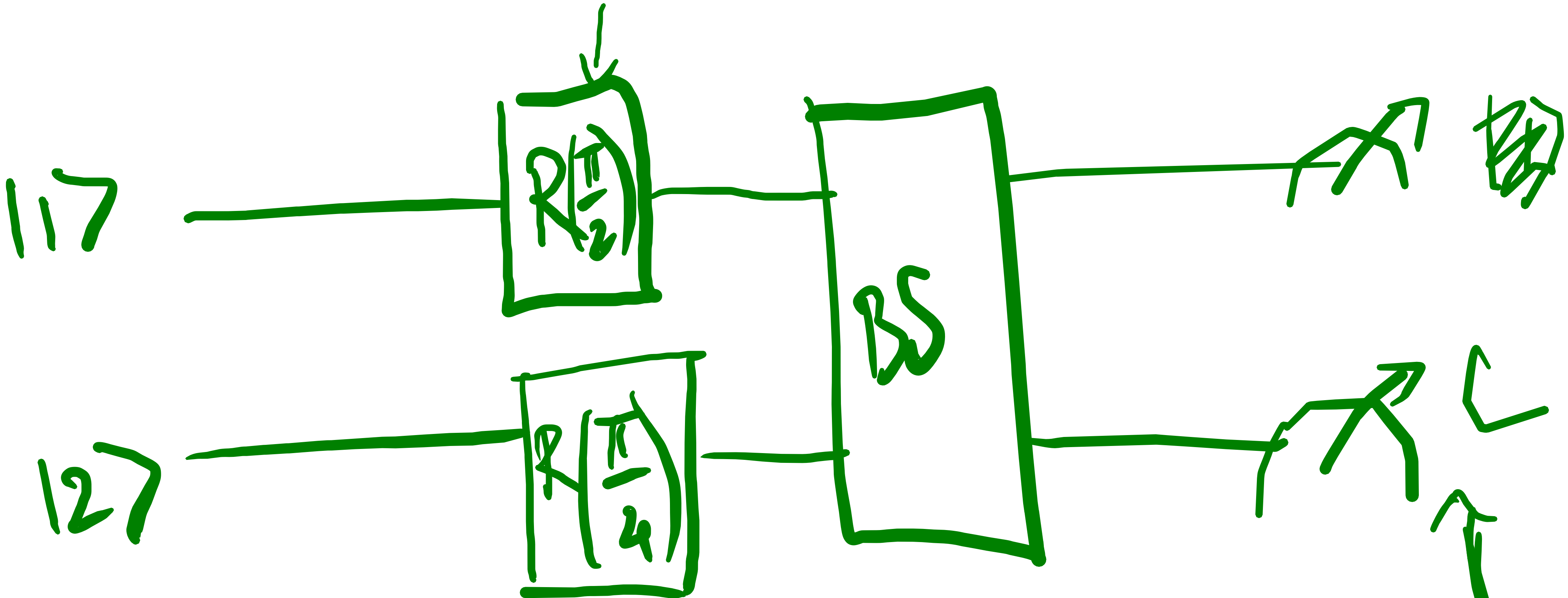
$|\psi\rangle = |2\rangle$

State Operation



$|1\rangle$

$|2\rangle$



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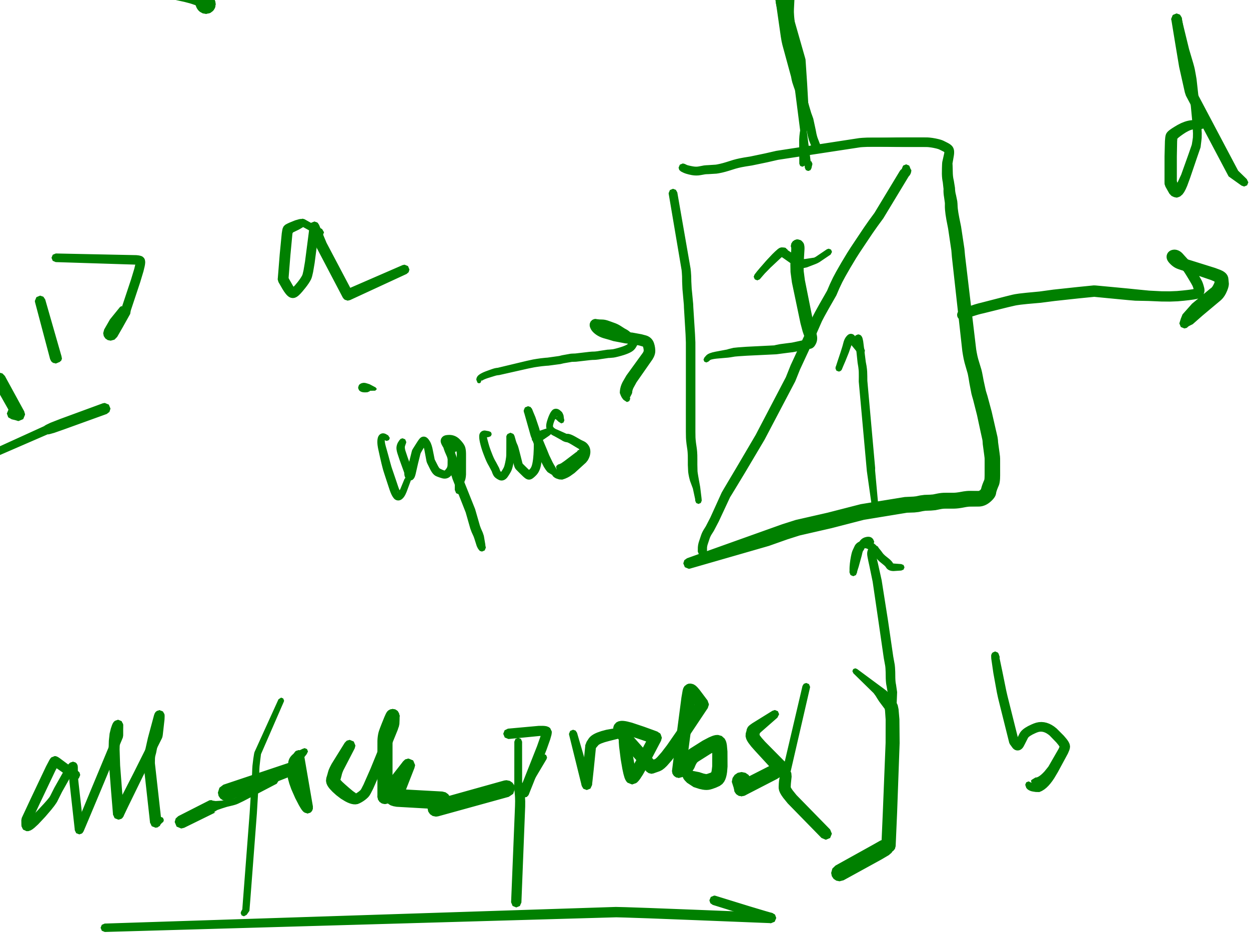
$|1,2\rangle \to |2,1\rangle$, $|0,3\rangle$, $|3,0\rangle$



time

0 → tmat

$U = e^{-iHt/\hbar}$

$E = h\nu$

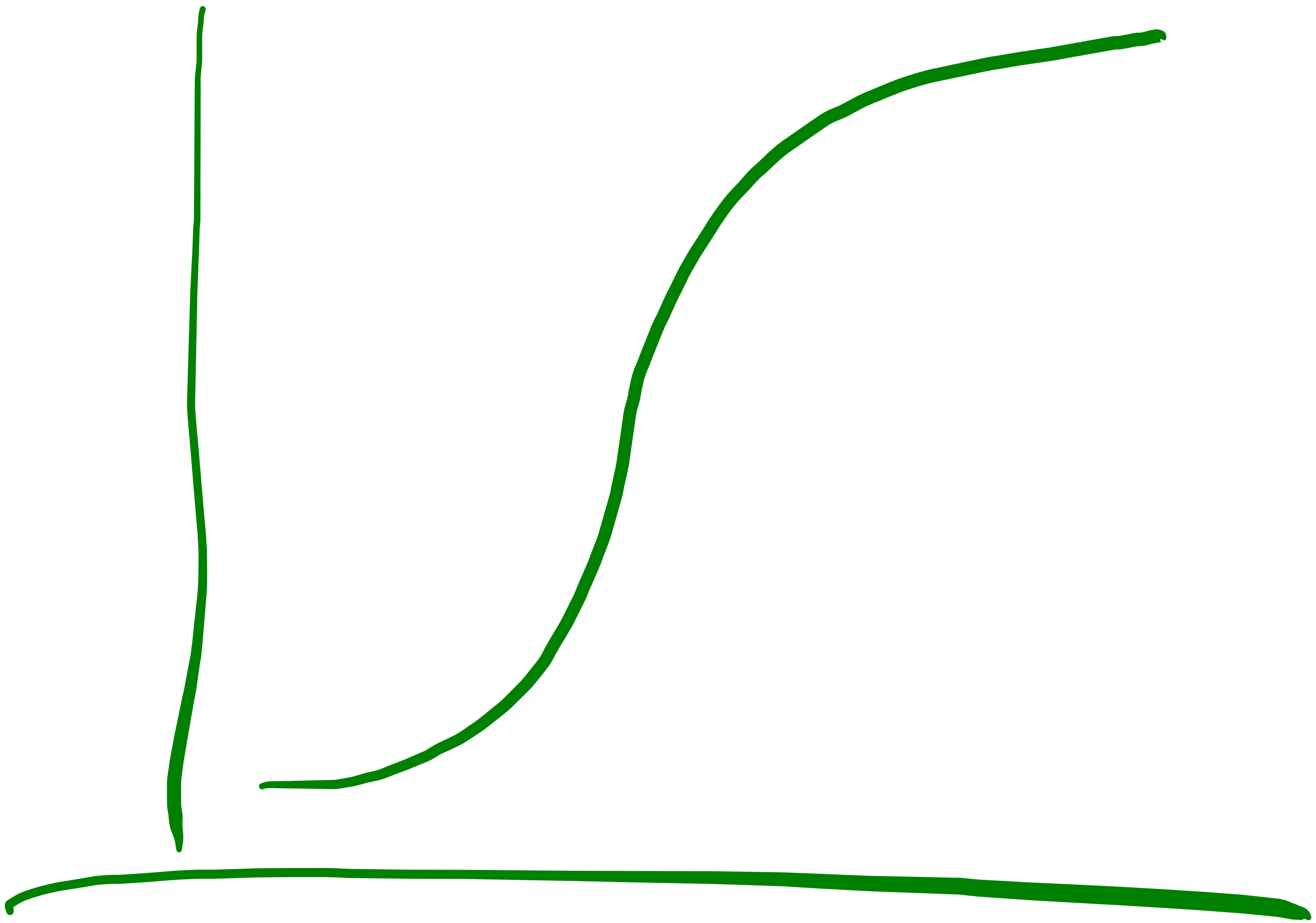
H : Light Atom

Jax → Jap

{ "Key1" : " " ,

"Key2" : " " , " }

dict[Key1] = return the key value



detering curve

Day 8

Bob = ABC

Unitary

Hadamard Gate

= HEFGH

= ABC

Unitary

ABC = HEFGH

MNO

Solve that

Hint: Z = HXH

A year B don

A BC = H(TPØ)H

HH

~~HH = I~~

$\cancel{HAHHBHHC}$ H = TPØ

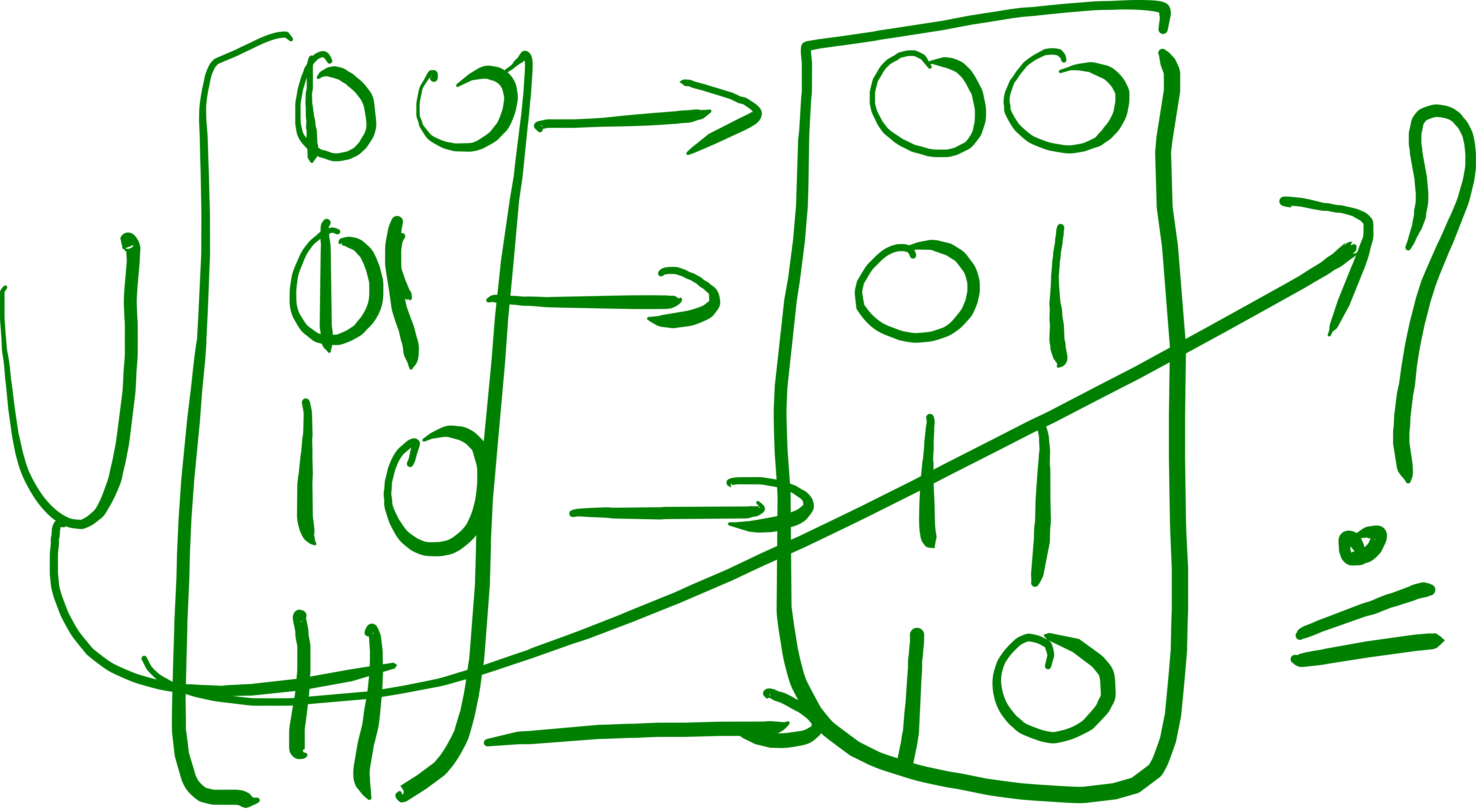
MNO @

E = H@M@H

F = H@N@H

G = H@O@H

1010 $\longrightarrow$ 1111



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