

CSPB 3155: Lab Assignment 1

The purpose of this assignment is to warm-up with functional programming in Scala and to refresh preliminaries from prior courses.

Learning Goals. The primary learning goals of this lab are to build intuition for the following:

- thinking in terms of types, values, and expressions;
- representing data structures as algebraic data types;
- representing programs as abstract syntax trees.

PL Ideas Abstract syntax.

FP Skills Expressions conforming to types and evaluating to values. Data structures as algebraic terms.

General Guidelines. The instructor will randomly assign partners for this lab assignment (should be different for every lab assignment). You will work on this assignment closely with your partner. However, note that **each student is individually responsible** for completing the assignment so that you can do well in your interview.

You are welcome to talk about these questions beyond your teams. However, we ask that you code in pairs. See the collaboration policy for details, including the following:

*Bottom line, feel free to use resources that are available to you as long as the use is **reasonable** and you **cite** them in your submission. However, copying answers directly or indirectly from solution manuals, web pages, or your peers is certainly unreasonable.*

Also, recall the evaluation guideline from the course syllabus.

Both your ideas and also the clarity with which they are expressed matter—both in your English prose and your code!

We will consider the following criteria in our grading:

- How well does your submission answer the questions? *For example, a common mistake is to give an example when a question asks for an explanation. An example may be useful in your explanation, but it should not take the place of the explanation.*

- How clear is your submission? *If we cannot understand what you are trying to say, then we cannot give you points for it. Try reading your answer aloud to yourself or a friend; this technique is often a great way to identify holes in your reasoning. For code, not every program that “works” deserves full credit. We must be able to read and understand your intent. Make sure you state any preconditions or invariants for your functions (either in comments, as assertions, or as **require** clauses as appropriate).*

Try to make your code as concise and clear as possible. Challenge yourself to find the most crisp, concise way of expressing the intended computation. This may mean using ways of expression computation currently unfamiliar to you.

Finally, make sure that your file compiles and runs via `sbt test`. A program that does not compile will *not* be graded—no interview will be conducted.

Submission Instructions. We are using the following tools:

- Github for assignment distribution and submission;
- INGINious for auto-testing submission;

1. You will be editing and submitting the the following files to Github:

- `src/main/scala/jsy/student/Lab1.scala` with your solution to the coding exercises;
- `src/test/scala/jsy/student/Lab1Spec.scala` with your additional tests; and
- `lab1-writeup.pdf` or `lab1-writeup.md` for a pdf or a Markdown document that should be pushed to the root directory of your repository with your response to the written questions (scanned, clearly legible handwritten write-ups are acceptable).

You will not get credit for write-ups in any other file format.

You are also likely to edit `src/main/scala/jsy/student/Lab1Worksheet.sc` for any scratch work.

You need to have a Github identity and should have your full name in your Github profile so that we can associate you with your submissions if there are any confusions.

Following good git practice, please make commits in small bits corresponding to completing small conceptual parts and push often so that your progress is evident. We expect that you have some familiarity with git from prior courses or experience. If not, please discuss with your classmates and the course staff (e.g., via Piazza).

2. At any point, you may submit your `Lab1.scala` file to INGINious for auto-testing. You need to submit to INGINious for the auto-testing part of your score, as well as to continue to the interview.
3. Sign-up for an interview slot for an evaluator. To fairly accommodate everyone, the interview times are strict and **will not be rescheduled**. Missing an interview slot means missing the interview evaluation component of your lab score. Please take

advantage of your interview time to maximize the feedback that you are able receive. Arrive at your interview ready to show your team's implementation and your written responses. Implementations that do not compile and run will not be evaluated.

Getting Started. You must work in teams of two, randomly assigned by the instructor. Log into moodle and follow the Github Classroom link for setting up your Lab 1 repository with your team name. For our bookkeeping, please prefix your team name with **lab1-** (e.g., **lab1-anatomists**). The first person will create the team, and the second person will select the team name from the existing team names.

If you would like to look at the code before getting your own copy for submission, you may go to <https://github.com/csci3155/pppl-lab1>.

Once you have the project files, follow the instructions in the **README.md** for setting up your development environment.

1. **Feedback.** Complete the survey linked from the moodle after completing this assignment. Any non-empty answer will receive full credit.
2. **Scala Basics: Binding and Scope.** For each the following uses of names, give the line where that name is bound. Briefly explain your reasoning (in no more than 1–2 sentences).

(a) Consider the following Scala code.

```
1  val pi = 3.14
2  def circumference(r: Double): Double = {
3      val pi = 3.14159
4      2.0 * pi * r
5  }
6  def area(r: Double): Double =
7      pi * r * r
```

The use of **pi** at line 4 is bound at which line? The use of **pi** at line 7 is bound at which line?

(b) Consider the following Scala code.

```
1  val x = 3
2  def f(x: Int): Int =
3      x match {
4          case 0 => 0
5          case x => {
6              val y = x + 1
7              ({
8                  val x = y + 1
9                  y
10                 } * f(x - 1))
11          }
12      }
13  val y = x + f(x)
```

The use of `x` at line 3 is bound at which line? The use of `x` at line 6 is bound at which line? The use of `x` at line 10 is bound at which line? The use of `x` at line 13 is bound at which line?

3. **Scala Basics: Typing.** In the following, I have left off the return type of function `g`. The body of `g` is well-typed if we can come up with a valid return type. Is the body of `g` well-typed?

```
1  def g(x: Int) = {  
2    val (a, b) = (1, (x, 3))  
3    if (x == 0) (b, 1) else (b, a + 2)  
4  }
```

If so, give the return type of `g` and explain how you determined this type. For this explanation, first, give the types for the names `a` and `b`. Then, explain the body expression using the following format:

$e : \tau$ because
 $e_1 : \tau_1$ because
 \dots
 $e_2 : \tau_2$ because
 \dots

where e_1 and e_2 are subexpressions of e . Stop when you reach values (or names).

As an example of the suggested format, consider the `plus` function:

```
def plus(x: Int, y: Int) = x + y
```

Yes, the body expression of `plus` is well-typed with type `Int`.

$x + y : \text{Int}$ because
 $x : \text{Int}$
 $y : \text{Int}$

4. **Run-Time Library.** Most languages come with a standard library with support for things like data structures, mathematical operators, string processing, etc. Standard library functions may be implemented in the object language (perhaps for portability) or the meta language (perhaps for implementation efficiency).

For this question, we will implement some library functions in Scala, our meta language, that we can imagine will be part of the run-time for our object language interpreter. In actuality, the main purpose of this exercise is to warm-up with Scala.

(a) Write a function `abs`

```
def abs(n: Double): Double
```

that returns the absolute value of `n`. This is a function that takes a value of type `Double` and returns a value of type `Double`. This function corresponds to the JavaScript library function `Math.abs`.

Instructor Solution: 1 line.

- (b) Write a function `xor`

```
def xor(a: Boolean, b: Boolean): Boolean
```

that returns the exclusive-or of `a` and `b`. The exclusive-or returns `true` if and only if exactly one of `a` or `b` is `true`. For practice, do not use the Boolean operators. Instead, only use the `if-else` expression and the Boolean literals (i.e., `true` or `false`).

Instructor Solution: 4 lines (including 1 line for a closing brace).

5. Run-Time Library: Recursion.

- (a) Write a recursive function `repeat`

```
def repeat(s: String, n: Int): String
```

where `repeat(s, n)` returns a string with `n` copies of `s` concatenated together. For example, `repeat("a", 3)` returns `"aaa"`. This function corresponds to the function `goog.string.repeat` in the Google Closure library.

Instructor Solution: 4 lines (including 1 line for a closing brace).

- (b) In this exercise, we will implement the square root function—`Math.sqrt` in the JavaScript standard library. To do so, we will use Newton's method (also known as Newton-Raphson).

Recall from Calculus that a root of a differentiable function can be iteratively approximated by following tangent lines. More precisely, let f be a differentiable function, and let x_0 be an initial guess for a root of f . Then, Newton's method specifies a sequence of approximations x_0, x_1, \dots with the following recursive equation:¹

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} .$$

The square root of a real number c for $c > 0$, written \sqrt{c} , is a positive x such that $x^2 = c$. Thus, to compute the square root of a number c , we want to find the positive root of the function:

$$f(x) = x^2 - c .$$

Thus, the following recursive equation defines a sequence of approximations for \sqrt{c} :

$$x_{n+1} = x_n - \frac{x_n^2 - c}{2x_n} .$$

- i. First, implement a function `sqrtStep`

```
def sqrtStep(c: Double, xn: Double): Double
```

¹The following link is a refresher video on this algorithm: <http://www.youtube.com/watch?v=1uN8cBGVpfs>, January 2012

that takes one step of approximation in computing \sqrt{c} (i.e., computes x_{n+1} from x_n).

Instructor Solution: 1 line.

- ii. Next, implement a function `sqrtN`

```
def sqrtN(c: Double, x0: Double, n: Int): Double
```

that computes the n th approximation x_n from an initial guess x_0 . You will want to call `sqrtStep` implemented in the previous part.

Challenge yourself to implement this function using recursion and no mutable variables (i.e., **vars**)—you will want to use a recursive helper function. It is also quite informative to compare your recursive solution with one using a **while** loop.

Instructor Solution: 7 lines (including 2 lines for closing braces and 1 line for a **require**).

- iii. Now, implement a function `sqrtErr`

```
def sqrtErr(c: Double, x0: Double,
            epsilon: Double): Double
```

that is very similar to `sqrtN` but instead computes approximations x_n until the approximation error is within ε (**epsilon**), that is,

$$|x_n^2 - c| < \varepsilon.$$

You can use your absolute value function **abs** implemented in a previous part. A wrapper function `sqrt` is given in the template that simply calls `sqrtErr` with a choice of `x0` and **epsilon**.

Again, challenge yourself to implement this function using recursion and compare your recursive solution to one with a **while** loop.

Instructor Solution: 5 lines (including 1 line for a closing brace and 1 line for a **require**).

6. Data Structures Review: Binary Search Trees.

In this question, we will review implementing operations on binary search trees from Data Structures. Balanced binary search trees are common in standard libraries to implement collections, such as sets or maps. For example, the Google Closure library for JavaScript has `goog.structs.AvlTree`. For simplicity, we will not worry about balancing in this question.

Trees are important structures in developing interpreters, so this question is also critical practice in implementing tree manipulations.

A binary search tree is a binary tree that satisfies an ordering invariant. Let n be any node in a binary search tree whose data value is d , left child is l , and right child is r . The ordering invariant is that all of the data values in the subtree rooted at l must be $< d$, and all of the data values in the subtree rooted at r must be $\geq d$.

We will represent a binary trees containing integer data using the following Scala **case classes** and **case objects**:

```
sealed abstract class SearchTree
case object Empty extends SearchTree
case class Node(l: SearchTree, d: Int, r: SearchTree) extends SearchTree
```

A `SearchTree` is either `Empty` or a `Node` with left child `l`, data value `d`, and right child `r`.

For this question, we will implement the following four functions.

- (a) The function `repOk`

```
def repOk(t: SearchTree): Boolean
```

checks that an instance of `SearchTree` is valid binary search tree. In other words, it checks using a traversal of the tree the ordering invariant. This function is useful for testing your implementation. A skeleton of this function has been provided for you in the template.

Instructor Solution: 7 lines (including 2 lines for closing braces).

- (b) The function `insert`

```
def insert(t: SearchTree, n: Int): SearchTree
```

inserts an integer into the binary search tree. Observe that the return type of `insert` is a `SearchTree`. This choice suggests a functional style where we construct and return a new output tree that is the input tree `t` with the additional integer `n` as opposed to destructively updating the input tree.

Instructor Solution: 4 lines (including 1 line for a closing brace).

- (c) The function `deleteMin`

```
def deleteMin(t: SearchTree): (SearchTree, Int)
```

deletes the smallest data element in the search tree (i.e., the leftmost node). It returns both the updated tree and the data value of the deleted node. This function is intended as a helper function for the `delete` function. Most of this function is provided in the template.

Instructor Solution: 9 lines (including 2 lines for closing braces and 1 line for a require).

- (d) The function `delete`

```
def delete(t: SearchTree, n: Int): SearchTree
```

removes the first node with data value equal to `n`. This function is trickier than `insert` because what should be done depends on whether the node to be deleted has children or not. We advise that you take advantage of pattern matching to organize the cases.

Instructor Solution: 10 lines (including 2 lines for closing braces).

7. JavaScripty Interpreter: Numbers.

JavaScript is a complex language and thus difficult to build an interpreter for it all at once. In this course, we will make some simplifications. We consider subsets of JavaScript and incrementally examine more and more complex subsets during the course of the semester. For clarity, let us call the language that we implement in this course JAVASCRIPTY.

For the moment, let us define JAVASCRIPTY to be a proper subset of JavaScript. That is, we may choose to omit complex behavior in JavaScript, but we want any programs that we admit in JAVASCRIPTY to behave in the same way as in JavaScript.

In actuality, there is not one language called JavaScript but a set of closely related languages that may have slightly different semantics. In deciding how a JAVASCRIPTY program should behave, we will consult a reference implementation that we fix to be Google's V8 JavaScript Engine. We will run V8 via Node.js, and thus, we will often need to write little test JavaScript programs and run it through Node.js to see how the test should behave.

In this lab, we consider an arithmetic sub-language of JavaScript (i.e., an extremely basic calculator). The first thing we have to consider is how to represent a JAVASCRIPTY *program as data* in Scala, that is, we need to be able to represent a program in our object/source language JAVASCRIPTY as data in our meta/implementation language Scala.

To a JAVASCRIPTY programmer, a JAVASCRIPTY program is a text file—a string of characters. Such a representation is quite cumbersome to work with as a language implementer. Instead, language implementations typically work with trees called *abstract syntax trees* (ASTs). What strings are considered JAVASCRIPTY programs is called the *concrete syntax* of JAVASCRIPTY, while the trees (or *terms*) that are JAVASCRIPTY programs is called the *abstract syntax* of JAVASCRIPTY. The process of converting a program in concrete syntax (i.e., as a string) to a program in abstract syntax (i.e., as a tree) is called *parsing*.

For this lab, a parser is provided for you that reads in a JAVASCRIPTY program-as-a-string and converts into an abstract syntax tree. We will represent abstract syntax trees in Scala using **case classes** and **case objects**. The correspondence between the concrete syntax and the abstract syntax representation is shown in Figure 1.

(a)

Interpreter 1. Implement the `eval` function

```
def eval(e: Expr): Double
```

that evaluates a JAVASCRIPTY expression `e` to the Scala double-precision floating point number corresponding to the *value* of `e`.

Consider a JAVASCRIPTY program `e`; imagine `e` stands for the concrete syntax or text of the JAVASCRIPTY program. This text is parsed into a JAVASCRIPTY AST `e`, that is, a Scala value of type `Expr`. Then, the result of `eval` is a Scala number of type `Double` and should match the interpretation of `e` as a JavaScript expression. These distinctions can


```

sealed abstract class Expr
case class N(n: Double) extends Expr
  N(n)  n
case class Unary(uop: Uop, e1: Expr) extends Expr
  Unary(uop, e1)  uop e1
case class Binary(bop: Bop, e1: Expr, e2: Expr) extends Expr
  Binary(bop, e1, e2)  e1 bop e2

sealed abstract class Uop
case object Neg extends Uop
  Neg  -

sealed abstract class Bop
case object Plus extends Bop
  Plus  +
case object Minus extends Bop
  Minus  -
case object Times extends Bop
  Times  *
case object Div extends Bop
  Div  /

```

Figure 1: Representing in Scala the abstract syntax of JAVASCRIPTY. After each **case class** or **case object**, we show the correspondence between the representation and the concrete syntax.

be subtle but learning to distinguish between them will go a long way in making sense of programming languages.

At this point, you have implemented your first language interpreter!