Multi Domain Surrogate Models for Gravitational Waveforms

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I scienced something, so now you get to read it.

I. INTRODUCTION

I done did it. Some other people that done'd similar things: [1].

II. METHODS

To describe the multi-domain surrogate methodology, we use notation as in [2], where $_S$ is used to denote a surrogate model. We denote the gravitational waveform produced by a fiducial model as $h(t;\boldsymbol{\lambda})$, where t is defined on a domain $T = [t_I, t_F]$. The independent variable t will represent time for the rest of this work, but can in general be a monotonic function of time. We consider a partition $\{t_k \in [t_I, t_F] \mid k \in \{0, 1, \ldots, N\}\}$ of the domain T, where

$$t_I = t_0 < t_1 < \ldots < t_N = t_F. \tag{1}$$

We will refer to the sub-interval

$$T_i = [t_{i-1}, t_i], \qquad i \in \{1, \ldots, N\} \tag{2}$$

as the i^{th} subdomain. Note that this set of subdomains $\{T_i\}_{i=1}^N$ forms a closed cover of the domain $T = [t_I, t_F]$. We would like to independently model the fiducial waveform over each of the N subdomains T_i.

In order to construct a surrogate model of this fiducial waveform family, we first define a new open cover $\{\tilde{T}_i\}_{i=1}^N$ from the original closed cover $\{T_i\}_{i=1}^N$ by extending the elements of the closed cover to form an overlap region of widths Δt_i between adjacent subdomains:

$$\begin{aligned} t_i^+ &= t_i + \frac{\Delta t_i}{2} \\ t_i^- &= t_i - \frac{\Delta t_i}{2} \end{aligned}, \qquad i \in \{1, \ldots, N-1\} \tag{3}$$

$$\tilde{T}_i = \begin{cases} [t_0, t_1^+), & i = 1 \\ (t_{i-1}^-, t_i^+), & i \in \{2, \ldots, N-1\} \\ (t_{N-1}^-, t_N], & i = N \end{cases} \tag{4}$$

Having defined this new open cover of the time domain, we can now use it to construct a multi-domain surrogate model. As mentioned in [2], the fiducial waveform model is evaluated at a sufficiently minimal number of parameter values $\boldsymbol{\lambda}_i$ to produce a dataset $\mathcal{T} = \{h(t;\boldsymbol{\lambda}_i)\}_{i=1}^M$ over a densely sampled time grid, where M is the number of parameter values. To independently model the fiducial waveform over each subdomain $\tilde{T}_i$, we define a set of *masking* functions $\{\phi_i(t)\}_{i=1}^N$, where the function $\phi_i(t)$ is supported[1] on the subdomain $\tilde{T}_i$:

$$\phi_i : T \to [0, 1] \text{ s.t. supp } \phi_i \equiv \{t \in T : \phi_i(t) \neq 0\} \subseteq \tilde{T}_i$$

These functions allow for the construction of N datasets $\mathcal{T}_i = \{\phi_i(t) h(t;\boldsymbol{\lambda}_j)\}_{j=1}^M$, which is each supported entirely on the subdomain $\tilde{T}_i$. For each of these datasets $\mathcal{T}_i$, we can construct an independent surrogate model $(\phi_i h)_S(t;\boldsymbol{\lambda})$. The process of surrogate construction ensures that the surrogate waveform is also supported on the subdomain $\tilde{T}_i$. Finally, we combine these N surrogate waveforms $\{(\phi_i h)_S(t;\boldsymbol{\lambda})\}_{i=1}^N$ to form the multi-domain surrogate waveform as:

$$h_S(t;\boldsymbol{\lambda}) \equiv \sum_{i=1}^N \psi_i(t)(\phi_i h)_S(t;\boldsymbol{\lambda}), \tag{5}$$

Here, $\{\psi_i(t)\}_{i=1}^N$ is a set of functions that we refer to as *gluing* functions, which are defined to have the same properties as the masking functions:

$$\psi_i : T \to [0, 1] \text{ s.t. supp } \psi_i \equiv \{t \in T : \psi_i(t) \neq 0\} \subseteq \tilde{T}_i$$

The functions $\phi_i(t)$ and $\psi_i(t)$ are so far left unspecified, but their choice can not be completely independent. To see this, consider the case of evaluating the multi-domain surrogate waveform at a training paramater value $\boldsymbol{\lambda}_j$. Assuming that the surrogate models over the individual subdomains are accurate, we expect the following approximate equalities to hold:

$$\begin{aligned} &(\phi_i h)_S(t;\boldsymbol{\lambda}_j) \approx \phi_i(t) h(t;\boldsymbol{\lambda}_j), \qquad \forall i \in \{1, \ldots, N\} \\ \implies\ &h_S(t;\boldsymbol{\lambda}_j) \approx \sum_{i=1}^N \psi_i(t)\phi_i(t) h(t;\boldsymbol{\lambda}_j) \end{aligned}$$

Since we want the multi-domain surrogate waveform to be an accurate representation of the fiducial waveform,

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[1] Supports on topological spaces are generally defined as closed sets, but we do not bother with this distinction as it would only result in a further cumbersome construction

this results in the following constraint on the masking and gluing functions:

$$h_S(t;\boldsymbol{\lambda}_j) \approx h(t;\boldsymbol{\lambda}_j) \implies \boxed{\sum_{i=1}^{N} \psi_i(t)\phi_i(t) \equiv 1} \tag{6}$$

Finally, as we would like for the final surrogate waveform to be smooth, the element-wise products $\{\psi_i\phi_i\}_{i=1}^{N}$ of the masking and gluing functions form a *smooth partition of unity subordinate to* the open cover $\{\tilde{T}_i\}_{i=1}^{N}$. A brief overview of smooth partitions of unity is provided in Appendix A.

III. RESULTS

Here's what I found after I done'd it.

And Maxwell said:

$$dF = 0, \tag{7}$$

$$d_\star F = \mu_0\, J. \tag{8}$$

and then there was light.

IV. CONCLUSION

Here's what you should learn now that I done'd it.

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Appendix A: Smooth Partitions of Unity

Partitions of unity are essential tools in the study of smooth manifolds, used to glue together objects defined locally to form global ones when the local objects do not agree on their common domain. Here, we provide a brief overview of smooth partitions of unity, which are used in the construction of the multi-domain surrogate model.

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[3] `https://www.youtube.com/watch?v=Sqt94fX6DSg`.