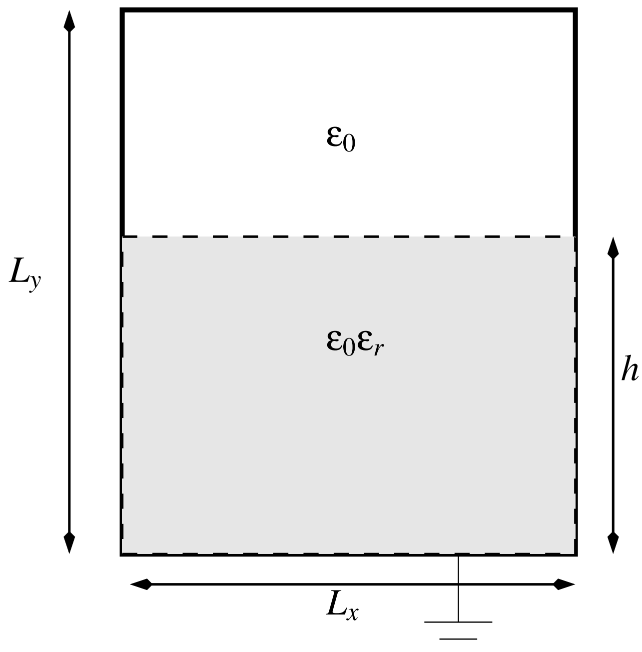
Final Exam (60 Marks) **July 28, 2020**

Dept. of Electrical Engineering, IIT Madras
Applied Programming Lab Jan 2020 session

- ▷ **Lines in red are corrections in the exam, due to queries from students.**
- ▷ **Time duration of exam is 6 hours. You are expected to time yourself and submit within that time. You may take breaks, but keep track of the time spent on the exam (including looking online, at your assignment submissons etc) and keep your time within 6 hours.**
- ▷ **The exam is out of** 60 **marks but will be scaled to** 20 **marks for the final grade.**
- ▷ **pseudocode should be readable and neatly formatted.**
- ▷ **Do not copy. Do not share your code. If I find two codes that are copied from each other, Both will be reported. There is no excuse for either copying, or showing your code.**
- ▷ **vector operations are a must.**
- ▷ **Label all plots. Add legends. Make the plots professional looking.**
- ▷ **Comments are not optional. They are required.**
- ▷ **The report should be a pdf that should be professional in appearance, and I will give marks for it. It should include the code listing, and the output in the form of plots, etc.**
- ▷ **PDF file should be named** ***your-roll-number.pdf***
- ▷ **Python code should be named** ***your-roll-number.py***
- ▷ **Python code should run!!**

1. **A rectangular tank ($L_x = 10$ cm and $L_y = 20$ cm) is partially filled with a fluid of dielectric constant $\varepsilon_r = 2$. (the dielectric constant of air may be taken to be $\varepsilon_r = 1$. The height of the filled portion is h. The walls of the tank are ideal conductors.**



The sides and bottom of the tank are grounded while the top of the tank is connected to the sides via a series RLC circuit (the tank is the C) and an AC source. As h changes, the capacitance seen by the circuit changes and the resonant frequency shifts.

The potential can be solved by numerically solving Laplace's Equation as done in the lab assignment. However, note that ε is not constant and you need to deal with the boundary at the surface of the fluid. For this, mesh the tank so that there is a row of nodes at h. Then using a $M \times N$ grid with the surface at y_k,

$$\phi_{m,n} = \frac{\phi_{m-1,n} + \phi_{m+1,n} + \phi_{m,n-1} + \phi_{m,n+1}}{4} \qquad m \neq k, 0 < m < M, 0 < n < N$$

$$\phi_{k,n} = \frac{\varepsilon_r \phi_{k-1,n} + \phi_{k+1,n}}{(1+\varepsilon_r)} \qquad m = k, 0 < n < N$$

where the surface of the fluid is at y_k. The second equation above is to handle continuity of D_n at the surface.

(a) The report together with properly labelled figures and code listing . . . 10 marks

(b) **Develop your algorithm to determine h from the observed resonant frequency and present it. You can give references to parts of the lab assignments, and derive what else you plan to do. Everything in the rest of the parts must be explained here. .... 5 marks**

(c) **How will you parallelize the computation? The $m = k$ row has to be handled differently. Explain why your algorithm is efficient. .... 5 marks**

(d) **Create a python function that solves Laplace's Equation and obtains ϕ_{mn} on the grid. The function accepts as arguments:**

M, the number of nodes along x, including the boundary nodes.

N, the number of nodes along y, including the boundary nodes.

Δ, the distance between nodes (assumed same along x and along y)

k, the height given as the index k corresponding to h.

δ, The desired accuracy δ.

N_0 the maximum number of iterations to complete.

The top boundary is assumed to be at 1 volt. It should return

`phi[M,N]` The array of solved potential values correct to δ

N, Number of iterations actually carried out

`err[N]`, The vector of errors

Note that the error in phi should be extrapolated to ∞. .... 15 marks

(e) **Run the code for $h/L_y = 0.1, 0.2, ..., 0.9$ and obtain Q_{top} vs h and Q_{fluid} vs h where Q_{top} is the charge per metre on the top plate held at 1 volt, and Q_{fluid} is the charge on the portion of the wall touching the dielectric fluid. Plot the same. Is it linear? Should it be? Explain. .... 10 marks**

(f) **For $h = 0.5L_y$, compute E_x and E_y on the at $(m+0.5, n+0.5)$, i.e., at the centre of mesh cells. Show that D_n is continuous at $m = k$. .... 5 marks**

(g) **Obtain the change in angle of the Electric field at $m = k$. Does this agree with Snell's Law? Should it? Explain. .... 10 marks**

Useful Python Commands (use “?” to get help on these from ipython)

```
from pylab import *
import system-function as name
Note: lstsq is found as scipy.linalg.lstsq
```

Python Commands (continued)

```
ones(List)
zeros(List)
range(N0,N1,Nstep)
arange(N0,N1,Nstep)
linspace(a,b,N)
logspace(log10(a),log10(b),N)
X,Y=meshgrid(x,y)
where(condition)
where(condition & condition)
where(condition | condition)
a=b.copy()
lstsq(A,b) to fit A*x=b
A.max() to find max value of numpy array (similalry min)
A.astype(type) to convert a numpy array to another type (eg int)
trunc(A) to truncate values of A to integer.
def func(args):
  ...
  return List
matrix=c_[vector,vector,...] to create a matrix from vectors
w=0.54+0.46*cos(2*pi*n/(N-1)) to generate a window of N points, where n=linspace(-N/
fftshift(v) to shift a vector from -max to +max to lie on unit circle.
```

Python commands (contd)

```
figure(n) to switch to, or start a new figure labelled n
plot(x,y,style,...,lw=...)
semilogx(x,y,style,...,lw=...)
semilogy(x,y,style,...,lw=...)
loglog(x,y,style,...,lw=...)
contour(x,y,matrix,levels...)
quiver(X,Y,U,V) # X,Y,U,V all matrices
xlabel(label,size=)
ylabel(label,size=)
title(label,size=)
xticks(size=) # to change size of xaxis numbers
yticks(size=)
legend(List) to create a list of strings in plot
annotate(str,pos,lblpos,...) to create annotation in plot
grid(Boolean)
show()
```