

PRoject Report

Topic: Types of Relations on a Set



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**PAPER: CS 404 – DISCRETE MATHEMATICS**

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1. **Reflexive Relation:**

A relation R on a set A is said to be reflexive if –

∀x (x ∈ A → (x, x) ∈ R)

**i.e.** If every element in the set A is related to itself.

**Example:** The relation R = {(1,1), (1,2), (2,2), (3,3)} on set A = {1,2,3}, is reflexive.

1. **Not Reflexive Relation:**

A relation R on set A is said to be not reflexive if –

∃x (x ∈ A ∧(x, x) **∉** R)

**i.e**. If atleast one element in the set A is not related to itself.

**Example:** The relation R = {(1,1), (1,2), (2,2), (3,2)} on set A = {1,2,3}, is not reflexive as ‘3’ ∈ A but (3, 3) **∉** R.

1. **Irreflexive Relation:**

A relation R on set A is said to be irreflexive if –

∀x (x ∈ A → (x, x) **∉** R)

**i.e.** if none of the element in the set A is related to itself.

**Example:** The relation R = {(1,2), (2,1), (3,2)} on set A = {1,2,3}, is irreflexive.

1. **Not Irreflexive Relation:**

A relation R on set A is said to be not irreflexive if –

∃x (x ∈ A **∧** (x, x) ∈ R)

**i.e.** If atleast one element in the set A is related to itself.

**Example:** The relation R = {(1,1), (2,1), (3,2)} on set A = {1,2,3}, is not irreflexive.

1. **Symmetric Relation:**

A relation R on set A is said to be symmetric if –

∀x, y∈ A ((x, y) ∈ R ⇔(y, x) ∈ R)

**Example 1:** The relation R = {(1,1), (1,2), (2,1), (3,2), (2,3)} on set A = {1,2,3}, is symmetric.

**Example 2:** Brotherhood can be considered as a symmetric relation. If A is the brother of B, then this implies B is also the brother of A. (Here, we are not considering who is elder or younger)

1. **Not Symmetric Relation:**

A relation R on A is said to be not symmetric if-

∃x, y ∈ A ((x, y) ∈ R **∧** (y, x) **∉** R)

**Example:** The relation R = {(1,1), (1,2), (2,2), (3,2), (2,3)} on set A = {1,2,3}, is not symmetric.

1. **Asymmetric Relation:**

A relation R on A is said to be asymmetric if-

∀x, y ∈ A ((x, y) ∈ R →(y, x) **∉** R)

**Example:** The relation R = {(1,2), (2,3), (3,1)} on set A = {1,2,3}, is asymmetric.

1. **Not Asymmetric Relation:**

A relation R on A is said to be not asymmetric if-

∃x, y ∈ A ((x, y) ∈ R **∧** (y, x) ∈ R)

**Example:** The relation R = {(1,1), (2,3), (3,1)} on set A = {1,2,3}, is not asymmetric.

1. **Anti-symmetric Relation:**

A relation R on A is said to be anti-symmetric if-

∀x, y ∈ A (((x, y) ∈ R **∧** (y, x) ∈ R) ⇒ (x = y))

**Example:** The relation R = {(1,1), (1,2), (2,3), (3,3)} on set A = {1,2,3}, is anti-symmetric.

1. **Not Anti-symmetric Relation:**

A relation R on A is said to be not anti-symmetric if-

∃x, y ∈ A (((x, y) ∈ R **∧** (y, x) ∈ R) ⇒ (x ≠ y))

**Example:** The relation R = {(1,1), (1,2), (2,1), (3,3)} on set A = {1,2,3}, is not anti-symmetric.

1. **Transitive Relation:**

A relation R on A is said to be transitive if-

∀x, y, z ∈ A (((x, y) ∈ R **∧** (y, z) ∈ R) ⇒ (x, z) ∈ R)

**Example 1:** The relation R = {(1,1), (1,2), (2,3), (1,3)} on set A = {1,2,3}, is transitive.

**Example 2:** Zeroth law of thermodynamics describes a transitive relation on a set of 3 bodies.

**i.e,** Considering A, B, C as three different bodies, if A, B are in thermal equilibrium & B, C are in thermal equilibrium, then A, C has to be in thermal equilibrium. This defines the property ‘Temperature’. We can then say A, B, C share the same temperature.

1. **Not Transitive Relation:**

A relation R on A is said to be not transitive if-

∃x, y, z ∈ A (((x, y) ∈ R **∧** (y, z) ∈ R) → (x, z) **∉** R)

**Example 1:** The relation R = {(1,2), (2,1), (2,3), (1,3)} on set A = {1,2,3}, is not transitive

as (1,2) and (2,1) ∈ R but (1,1) **∉** R.

**Example 2:** Friendship can be considered as a relation which is not transitive.

If A & B are friends, & B&C are friends, then A&C may not be friends.

1. **Equivalence Relation:**

A binary relation R on a set A is said to be an equivalence relation if it satisfies the following properties:

* R on A is reflexive.
* R on A is symmetric.
* R on A is transitive.

**Example 1:** The relation R = {(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2), (1,3), (3,1)} on set A = {1,2,3} is an equivalence relation.

**Example 2:** The relation R = {(1,1), (2,2), (3,3)} on set A = {1,2,3} is also an equivalence relation.

**Example 3:** Let A be a set of students studying MCA and R be a binary relation on A such that any two students studying MCA are related if they are studying MCA from Tezpur University. In this case R is an equivalence relation.

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