Develop a mathematical model of D.C Motor and design PID controller

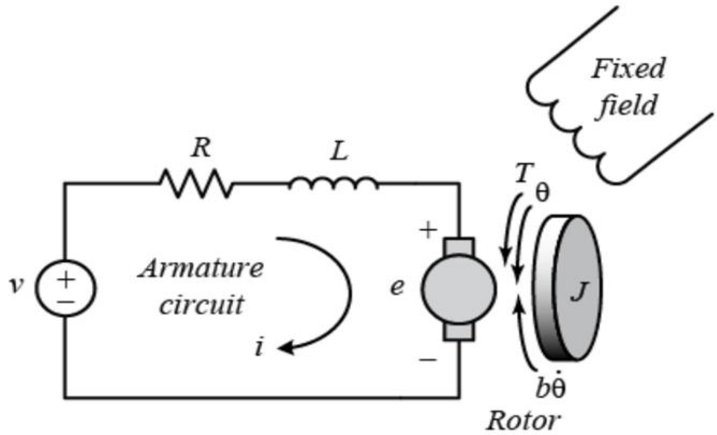
Introduction:

The idea here is to develop controller to control the position of a D.C motor. First, designed the mathematical model of motor and derive the transfer function of D.C motor with input as applied voltage and output as angular position of the shaft. Create the Simulink model of the derived transfer function and develop the PID controller.

Project Description:

The above-mentioned project is divided into two sub-sections one is deriving the mathematical model and transfer function of motor and other is to design a Simulink model of D.C motor with PID controller. Here first, we will analyse the circuit diagram of D.C motor and find the differential equation to derive transfer function of the motor.

The electric circuit of the stator and the free-body diagram of the rotor are as shown:



The parameters and the respective values are given below:

- Input: voltage applied to the motor's armature (rotor) (V),
- Model Output: angular position of the shaft (θ).
- The physical parameters for our example are:
  - J: moment of inertia of the rotor 3.2284E-6 kg.m^2
  - b: motor viscous friction constant 3.5077E-6 Nms
  - K_e: electro-motive force constant 0.0274 V/rad/sec
  - K_t: motor torque constant 0.0274 Nm/Amp
  - R: electric resistance 4 ohm
  - L: electric inductance 2.75E-6H

Mathematical model of D.C motor:

- The motor torque is proportional to the armature current i by a constant factor K_T as shown in the relation below.

$$T = K_T\, i$$

- The back emf is proportional to the angular velocity of the shaft by a constant factor as follows:

$$e = K_b \dot{\theta}$$

- Applying Newton's second law:

$$J\frac{d^2\theta}{dt^2} = T - b\frac{d\theta}{dt} \Longrightarrow \frac{d^2\theta}{dt^2} = \frac{1}{J}(K_t i - b\frac{d\theta}{dt})$$

- Applying Kirchhoff's law:

$$L\frac{di}{dt} = -Ri + V - e \Longrightarrow \frac{di}{dt} = \frac{1}{L}(-Ri + V - K_b\frac{d\theta}{dt})$$

Simulink model and PID controller:

- Create block diagram of the above derived differential equations using Simulink and analyses the output.
- Develop the PID controller to control the angular position of the D.C motor.
- Write a program to run the Simulink model with PID controller using M-script, define all the required parameters of the motor.

Conclusion:

As per the required specification I have designed the Simulink model for the D.C motor and also developed the PID controller. Finally, the Simulink model and the PID controller was tested and it runs successfully.