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by Jonathan Chiu, Mohammad Davoodalhosseini, Janet Jiang and Yu Zhu

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Central Bank Digital Currency and Banking

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Abstract

This paper builds a model with imperfect competition in the banking sector. In the model, banks issue deposits and make loans, and deposits can be used as payment instruments by households. We use the model to assess the general equilibrium effects of introducing a central bank digital currency (CBDC). We identify a new channel through which the CBDC can improve the efficiency of bank intermediation and increase lending and aggregate output even if its usage is low, i.e., the CBDC serves as an outside option for households, thus limiting banks' market power in the deposit market. We then calibrate the model to the US economy and find that with a proper interest rate, CBDC can raise bank lending by around 7% and increase output by around 1%. The quantitative results are sensitive to parameters governing the acceptance of different means of payments and the degree of competition in the deposit market.

Bank topics: Digital currencies and fintech; Monetary policy; Monetary policy framework; Market structure and pricing

JEL codes: E50, E58

1 Introduction

Many central banks are assessing the possibility of introducing a widely accessible Central Bank Digital Currency (CBDC) into their economies.¹ Issuing a CBDC would have implications for monetary policy and banking. Of concern is that a CBDC might crowd out bank deposits, increase banks' funding costs, and reduce lending and investment, i.e., bank disintermediation. This concern is highlighted in the report on CBDC from the Bank for International Settlements (BIS), which argues that "... a flow of retail deposits into a CBDC could lead to a loss of low-cost and stable funding for banks, with the size of such a loss in normal times depending on the convenience and costs of the CBDC. Banks could try to prevent a loss of deposits by raising interest rates or seek funding to replace such outflows, e.g., through wholesale funds and term deposits, which would likely be more costly."² The International Monetary Fund (IMF) staff discussion note by Mancini-Griffoli et al. (2018) also argues that "as some depositors leave banks in favor of CBDC, banks could increase deposit interest rates to make them more attractive. But the higher deposit rates would reduce banks' interest margins. As a result, banks would attempt to increase lending rates, though at the cost of loan demand." Also, Meaning et al. (2018) suggest an important topic for future research: "Even abstracting from bank runs, at what point do the benefits of a new competitive force for the banking sector get outweighed by the negative consequences of the central bank disintermediating a large part of banks business models?"

The goal of our paper is to formally assess the effects of CBDC on banking. We build on Keister and Sanches (2018) to develop a tractable model where cash and bank deposits are both used as means of payments. Banks take deposits from households and offer loans to firms to finance projects. Banks' liabilities (deposits)

¹For reasons and arguments for issuing a CBDC, see Engert and Fung (2017) and references therein.

²See the BIS report by the Committee on Payments and Market Infrastructures (2018).

serve as means of payments. In other words, banks perform liquidity transformation by converting illiquid loans into liquid deposits.³ We then introduce a widely accessible CBDC into this economy and study its implications on bank funding costs, lending, investment, and output. We study the effects of various designs of CBDC and consider the following: whether the CBDC is interest bearing or not; whether it is a substitute for bank deposits (deposit-like) or substitute for cash (cash-like); whether it can be held by banks; and, if so, whether it can be used to satisfy reserve requirements. We show that the effects of a CBDC on the banking system depend on the competition level in the deposit market and the interest rate on the CBDC.

In general, a deposit-like CBDC with a sufficiently high interest rate has a negative effect on lending. Intuitively, if the CBDC rate is sufficiently high, banks can only compensate for the rising funding cost by raising the lending rate, which would lead to a reduction in lending. This happens regardless of the competition level in the deposit market. If the deposit market is not perfectly competitive, a moderate interest rate on the CBDC increases bank deposits and lending. In this case, the CBDC limits banks' market power and forces them to offer depositors better rates, which attract more deposits and reduce bank profits per unit of deposits. Banks are willing to accommodate the increased deposit demand because they still make a profit per unit of deposits. This leads to more bank funding and in turn leads to an increased supply of loans and lower lending rates.

Interestingly, the CBDC can improve lending even if it is not used in equilibrium. CBDC serves as a viable outside option to bank deposits and hence disciplines the behavior of banks. As a result, the demand for bank deposits becomes more elastic with respect to the deposit rate. Our model has the extreme version: deposits

³Such a market structure can arise due to information and commitment frictions. Because of these frictions, credit is imperfect. Households need to bring means of payment for consumption. Similarly, firms cannot directly borrow from households because they cannot commit to repay. Banks have enforcement technology and can commit to honor their obligations. Therefore, they can channel funds from households to firms and provide payment services. We abstract from consumption loans and focus on commercial loans.

are perfectly elastic at the point of the CBDC rate. Therefore, the CBDC rate becomes a floor on the deposit rate regardless of whether the CBDC is used or not.⁴ This insight is closely related to Lagos and Zhang (2018), who show that monetary policy can be effective even if money is not used in the equilibrium because monetary policy changes the value of the outside option and disciplines the equilibrium. The implication of this is that the effectiveness of a CBDC should be assessed based on its effect on deposits or deposit rate instead of its usage.

We then calibrate the model to the US economy and assess the importance of our mechanism. Results from our benchmark calibration show that (1) with a proper interest rate, CBDC can raise bank lending and investment substantially (7%); (2) there is a wide interval of CBDC rates (between 0.05% and 2.55%) at which introducing CBDC increases lending; and (3) introducing CBDC can result in a maximum 1% output increase. We also study the welfare implications of CBDC. An alternative calibration shows that (1) and (2) are qualitatively robust to calibration methods.

There are a few papers that study the effects of CBDC on banking. Keister and Sanches (2018) consider a perfectly competitive banking sector and show that CBDC can only reduce bank lending. In contrast, our model extends theirs to allow for an imperfectly competitive banking sector and shows that CBDC can increase bank lending.⁵ Andolfatto (2018), in contrast, considers an economy with a monopolistic

⁴This also suggests that the CBDC rate may be a better monetary policy tool because it transmits directly to the deposit rate. The current monetary policy instruments do not seem to affect deposit rates much. For example, the policy rate in the US has increased by 2% from 2016 to 2018, but the commercial banks have barely moved the deposit rates, measured as the national rate on non-jumbo deposits (less than \$100,000) for money market, savings, or interest checking accounts. Relatedly, Berentsen and Schar (2018) argue that CBDC makes the monetary policy more transparent. In particular, if a CBDC is publicly accessible, its interest rate would be the lowest rate in the economy.

⁵Perfect competition is a limiting case where the number of banks goes to infinity. The key insight is that the market structure is important when assessing the effect of CBDC on bank intermediation. If the banking sector is imperfectly competitive, then a CBDC increases bank intermediation if its rate is in an intermediate range but causes disintermediation if its rate is above that intermediate range. This range shrinks as the number of banks increases, and we obtain the same result as the number of banks approaches infinity.

commercial bank and shows that CBDC may have a positive effect on bank deposits but no impact on bank lending if the central bank lends to the commercial bank. Our paper shows that CBDC can have a positive impact on both deposits and lending even if the central bank does not lend to the commercial banks and there are multiple banks. In terms of methodology, Andolfatto (2018) uses an overlapping generation model, while our paper follows Lagos and Wright (2005).

Brunnermeier and Niepelt (2018) derive conditions under which the issuance of inside money and outside money are equivalent, even if inside money and outside money have liquidity or return differences. Their results imply that introducing CBDC does not necessarily change macroeconomic outcomes. Barrdear and Kumhof (2016) use a rich DSGE model and estimate that issuing a CBDC could increase GDP by up to 3% through reducing real interest rates.

In general, our paper contributes to the literature on New Monetarist models. Berentsen, Camera, and Waller (2008) first incorporate banking into the Lagos and Wright (2005) model. Our model differs from Berentsen, Camera, and Waller (2008) in two dimensions. First, banks in our model engage in imperfect competition. Second, our banks create inside money by taking deposits. We provide conditions under which a monetary equilibrium exists and is unique. We also show that the model can have multiple equilibria under certain parameters, implying that the banking sector can introduce instability into the economy. This point is also made in Gu et al. (2018).

The key mechanism of our paper depends crucially on the market power of banks in the deposit market. Dreschler, Savov, and Schnabl (2017) provide empirical evidence that banks have market power in the deposit market and explore the implication of this on monetary policy transmission. Dreschler, Savov, and Schnabl (2018) study the effect of this market power on maturity transformation and interest rate risk. Kurlat (2018) shows that this market power raises the cost of inflation.

Other papers on e-money and digital currency include Davoodalhosseini (2018) and Zhu and Hendry (2019). Davoodalhosseini (2018) studies a model where a CBDC allows balance-contingent transfers as opposed to cash, but the CBDC is more costly for agents to use than cash (because cash offers anonymity). He shows that the co-existence of cash and the CBDC may not be optimal, because cash can serve as an outside option for agents, restricting the central bank’s power in implementing monetary policy. Our paper has many similarities with Zhu and Hendry (2019), who study the behavior of a monopolistic e-money issuer. Our paper studies behaviors of banks that have monopoly power and issue deposits that can be used for payment.⁶

This rest of the paper is organized as follows. Section 2 introduces the baseline model, where there is no CBDC. Section 3 derives the equilibrium of the baseline model. Section 4 considers a CBDC with three different designs and studies its implications on the equilibrium under each design. Section 5 calibrates the model and assesses the quantitative implications, and Section 6 concludes. Extensions and some omitted proofs are collected in the appendix.

2 Environment

The model follows a version of Lagos and Wright (2005) that is studied in Zhu and Hendry (2019). The role of banks in our model is similar to that in Keister and Sanches (2018). Time is discrete and continues forever. There is a continuum of households with measure 2, a continuum of entrepreneurs with measure N^E , and N bankers. Here, N is an integer. As in the standard New Monetarist model, at each date t , agents interact sequentially in two settings: a frictional decentralized market

⁶For further reference on e-money and digital currency, see Agur, Ari, and Dell’Ariccia (2019); Chapman and Wilkins (2019); Chiu and Wong (2015); Davoodalhosseini and Rivadeneyra (2018); Engert and Fung (2017); Fung and Halaburda (2016); Kahn, Rivadeneyra, and Wong (2018); Mancini-Griffoli et al. (2018); Schilling and Uhlig (2018); Zhu and Hendry (2019); and references therein.

(DM) and a frictionless centralized market (CM). There are two types of good: a numeraire good x produced in the CM and a good y produced in the DM. Both goods are perishable. There is a durable and intrinsically worthless object issued by the government, i.e., fiat money. Its supply at time t is M_t . The bankers also issue deposits to the households.

Households are divided into two permanent types, i.e., buyers and sellers, each with measure 1. In the CM, both types work, consume x , and determine their positions in fiat money and bank deposits. Their labor h is translated into x one-for-one. In the DM, the buyers want to consume y , which can be produced on the spot by the sellers. Then, buyers and sellers meet and trade bilaterally. Because of anonymity and lack of commitment, credit is not viable. To facilitate the trade, buyers can use fiat money. In addition, banks can commit to repay, and as a result, their deposits can also be used as a means of payment.

There are three types of DM meetings. With α_1 probability, a buyer gets into a type 1 meeting, where only fiat money can be used. With α_2 probability, a buyer gets into a type 2 meeting, where only bank deposits can be used. With α_3 probability, a buyer gets into a type 3 meeting, where both can be used. These meetings can be thought of in the following way. Type 1 meetings are transactions in local stores that do not have access to debit cards. Type 2 meetings are online transactions where the buyers and sellers are spatially separated and can only use debit cards or bank transfers for payment. Type 3 meetings are at local stores with point-of-sale (POS) machines, and hence both payment methods are accepted. The types of meeting are not revealed until the start of the DM each period. Therefore, buyers hold portfolios of fiat money and bank deposits.

Each period, a measure of N^E entrepreneurs are born and they will die in the next CM. They consume x in the next CM and have a linear utility. They are born with investment opportunities and can transform x current CM goods to $f(x)$ CM

goods in the next CM. Here $f'(0) = \infty$, $f'(\infty) = 0$, $f' > 0$, and $f'' < 0$. However, they do not have resources and cannot commit to repay households. Therefore, they need to borrow but cannot directly borrow from the households. Similarly, N bankers are born, who will die in the next CM. Bankers want to consume x in the next CM with a linear utility. They do not have resources but can commit to repayment and can also enforce payment. As a result, they can issue deposits and use deposits to finance the investment opportunities. One unit of deposit is a promise to pay back purchase power next period that is worth one unit of CM good. Bankers also have the technology to provide payment services, i.e., bank deposits can be used in type 2 and type 3 meetings.

In every CM, the newly born bankers issue some deposits to the households in exchange for fiat money and also issue some deposits to entrepreneurs as loans. Then the entrepreneurs use the deposits to buy x from buyers for investment. In the next CM, entrepreneurs sell some of the investment output to obtain cash or deposits, which are used to pay back the loans, and retain some output for their own consumption. Bankers then use the loan payments to redeem the deposits held by the households and retain some payments from the entrepreneurs for their own consumption.

The government injects or contracts the money supply in the CM. We focus on the case where money supply growth has a constant gross rate $\mu = M_{t+1}/M_t$. Using the Fisher's equation, we can write the nominal interest rate as $\iota = (1 + \mu) / \beta - 1$.

2.1 Households

A buyer has period utility

$$U(x, y, h) = U(x) - h + u(y).$$

In the CM, the buyer solves

$$\begin{aligned} W^B(Z, D) &= \max_{x, h, \hat{Z}, \hat{D}} \left\{ U(x) - h + \beta V^B(\hat{Z}, \hat{D}) \right\} \\ \text{st. } x &= h + Z + D + T - \frac{\phi}{\hat{\phi}} \hat{Z} - \psi \hat{D}, \end{aligned}$$

where ϕ and $\hat{\phi}$ are the prices of money in terms of the CM good, \hat{D} is the real value of deposits tomorrow, and $\psi \hat{D}$ is the real value of deposits today. Therefore, the real interest rate on deposits is $1/\psi - 1$. As standard, we can substitute out h using the budget equation and obtain

$$W^B(Z, D) = Z + D + T + \max_x [U(x) - x] + \max_{\hat{D}, \hat{Z}} \left\{ -\frac{\phi}{\hat{\phi}} \hat{Z} - \psi \hat{D} + \beta V^B(\hat{Z}, \hat{D}) \right\}.$$

This shows that $W^B(Z, D)$ is linear in Z and D . The first-order conditions (FOCs) are

$$\begin{aligned} x &: U'(x) = 1 \\ \hat{Z} &: \frac{\phi}{\hat{\phi}} \geq \beta V_1^B(\hat{Z}, \hat{D}), \text{ equality if } \hat{Z} > 0 \\ \hat{D} &: \psi \geq \beta V_2^B(\hat{Z}, \hat{D}), \text{ equality if } \hat{D} > 0, \end{aligned}$$

where the subscripts mean the derivative with respect to corresponding arguments. Notice that the first-order conditions imply that \hat{Z} and \hat{D} are the same for all buyers.

The DM problem can be written as

$$\begin{aligned} V^B(Z, D) &= \alpha_1 [u \circ Y(Z) - P(Z)] + \alpha_2 [u \circ Y(D) - P(D)] \\ &\quad + \alpha_3 [u \circ Y(Z + D) - P(Z + D)] + W^B(Z, D), \end{aligned} \tag{1}$$

where $Y(\cdot)$ and $P(\cdot)$ are terms of trade, which is discussed later.

A seller has period utility,

$$\mathbf{U}(x, y, h) = U(x) - h - v(y).$$

Because the seller does not need to consume in the DM, we can assume that $Z = D = 0$ without loss of generality. Therefore, the seller's CM problem is

$$\begin{aligned} W^S(Z, D) &= \max_{x, h} \{U(x) - h + \beta V^S(0, 0)\} \\ \text{st. } x &= h + Z + D + T. \end{aligned}$$

Again W^S is linear in Z and D . The seller's DM problem is

$$\begin{aligned} V^S(0, 0) &= \alpha_1 \left[P(\tilde{Z}) - v \circ Y(\tilde{Z}) \right] + \alpha_2 \left[P(\tilde{D}) - v \circ Y(\tilde{D}) \right] \\ &\quad + \alpha_3 \left[P(\tilde{Z} + \tilde{D}) - v \circ Y(\tilde{Z} + \tilde{D}) \right] + W^S(0, 0), \end{aligned}$$

where \tilde{D} and \tilde{Z} are the holdings of the buyer in the meetings.

Upon a meeting, the buyer makes a take-it-or-leave-it offer, which determines the terms of trade $Y(\cdot)$ and $P(\cdot)$. Let z be the total real value of all available payment instruments in a meeting; then the buyer offers

$$\max_{y, p \leq z} [u(y) - p] \text{ s.t. } p = v(y).$$

For simplicity, assume $v(y) = y$. Then one can show that

$$Y(z) = \begin{cases} z & \text{if } z < y^* \\ y^* & \text{if } z > y^* \end{cases}, \quad P(z) = \begin{cases} z & \text{if } z < y^* \\ y^* & \text{if } z > y^* \end{cases}, \quad (2)$$

where $u'(y^*) = 1$.

Combing the FOCs of buyers and (1) and (2), we obtain the Euler's equations,

$$\frac{\phi}{\beta \hat{\phi}} \geq \alpha_1 \lambda(\hat{Z}) + \alpha_3 \lambda(\hat{Z} + \hat{D}) + 1, \text{ equality iff } \hat{Z} > 0, \quad (3)$$

$$\frac{\psi}{\beta} \geq \alpha_2 \lambda(\hat{D}) + \alpha_3 \lambda(\hat{Z} + \hat{D}) + 1, \text{ equality iff } \hat{D} > 0, \quad (4)$$

where $\lambda(D) = \max[u'(D) - 1, 0]$ is the liquidity premium. At the steady state, (3) and (4) reduce to

$$\iota \geq \alpha_1 \lambda(Z) + \alpha_3 \lambda(Z + D), \text{ equality iff } Z > 0, \quad (5)$$

$$\frac{\psi}{\beta} - 1 \geq \alpha_2 \lambda(D) + \alpha_3 \lambda(Z + D), \text{ equality iff } D > 0. \quad (6)$$

Here, (5) defines the aggregate demand for Z as a function of D . Given this, (6) defines the aggregate inverse demand function for D given ι , i.e., $\psi = \Psi(D)$. We suppress the dependence of Ψ on ι to ease notations.

Lemma 1 $\Psi(D)$ is decreasing in D and increasing in ι .

Proof. After some straightforward algebra, one can show that

$$\begin{aligned}\Psi'(D) &= \alpha_2 \beta \lambda'(D) + \frac{\alpha_1 \alpha_3 \beta \lambda'(Z+D) \lambda'(Z)}{\alpha_1 \lambda'(Z) + \alpha_3 \lambda'(Z+D)} \leq 0, \\ \frac{\partial \Psi(D)}{\partial \iota} &= \frac{\alpha_3 \beta \lambda'(Z+D)}{\alpha_1 \lambda'(Z) + \alpha_3 \lambda'(Z+D)} \geq 0.\end{aligned}$$

Notice that the first inequality holds strictly if $D < y^*$ and the second inequality holds strictly if $\alpha_3 > 0$ and $Z + D < y^*$. ■

2.2 Entrepreneurs

The entrepreneurs decide their demand for loans given the loan rate ρ . Their problem is

$$\max_l \{f(l) - (1 + \rho)l\}.$$

This implies that the inverse loan demand for a firm is $f'(l) = 1 + \rho$. This defines an aggregate inverse loan demand function,

$$L^d(\rho) = N^E f'^{-1}(1 + \rho).$$

Obviously $L^d(\cdot)$ is decreasing.

2.3 Bankers

For now, we assume that the lending market is perfectly competitive, and banks engage in a Cournot competition in the deposit market. We later on consider the case where the loan market and the deposit market are not perfectly competitive with a competitive interbank market. Bankers face a reserve requirement. At the

end of each CM, the real value of a banker's cash holding must exceed χ fractions of the total deposits, where χ is set exogenously by the government. In addition, the bankers incur a deposit handling cost c per real value of deposit.

Then, banker j chooses d_j , ℓ_j , and z_j to maximize its utility, taking ρ and $d_{-j} = \sum_{i \neq j} d_i$ as given:

$$\begin{aligned} \max_{z_j, \ell_j, d_j} & \left\{ (1 + \rho) \ell_j + \frac{z_j}{\mu} - [d_j + \Psi(d_{-j} + d_j) d_j c] \right\} \\ \text{st} \quad & \ell_j + z_j = \Psi(d_{-j} + d_j) d_j, \\ & z_j \geq \chi \Psi(d_{-j} + d_j) d_j. \end{aligned} \quad (7)$$

The banker gets back the loan plus interest $(1 + \rho) \ell_j$, gets the post-inflation value of money holdings, redeems the deposits d_j , and pays the deposit handling cost. The first equation in the constraint is the balance sheet identity of the bank at the end of the first CM. The right-hand side is the liability, which is the real value of deposits, and the left-hand side is the asset, which includes money and loans. The second constraint reflects the reserve requirement. Using the balance sheet identity and the reserve requirement to substitute out z_j , one can rewrite the problem as

$$\begin{aligned} \max_{z_j, \ell_j, d_j} & \left\{ \left(1 + \rho - \frac{1}{\mu}\right) \ell_j - \left[d_j + \Psi(d_{-j} + d_j) d_j c - \frac{\Psi(d_{-j} + d_j) d_j}{\mu} \right] \right\} \\ \text{st} \quad & \ell_j \leq (1 - \chi) \Psi(d_{-j} + d_j) d_j. \end{aligned} \quad (8)$$

Given each ρ , this defines a best response function that maps d_{-j} to d_j . We look for a symmetric equilibrium where $d_{-j} = (N_b - 1) d_j$ for every j . Once we solve for d_j , we can compute the loan supply at ρ . Throughout the paper, we assume that the following holds.

Assumption 1 *a) Given any $d_{-j} \in [0, y^*)$ and $\kappa > \beta$, either there exists a unique $d_j > 0$ such that $\Psi'(d_{-j} + d) d + \Psi(d_{-j} + d) \geq \kappa$ if $d \leq d_j$, or $\Psi'(d_{-j} + d) d + \Psi(d_{-j} + d) < \kappa$ for all $d \geq 0$.*

b) In addition, $\Psi'(Nd) d + \Psi(Nd)$ decreases with d on $[0, y^/N)$.*

Part (a) of this assumption is similar to the single crossing condition and states that $\Psi'(d_{-j} + d)d + \Psi(d_{-j} + d)$ as a function of d should cross the horizontal axis from the above and at most once. We need part (a) to ensure that the best response of banker j to any amount of deposits (less than y^*) created by other banks is unique. We need part (b) to ensure that there is at most one symmetric Nash equilibrium of the Cournot game. One can show that this assumption holds if v is linear, u is CRRA utility with a coefficient less than 1, and $\alpha_3 = 0$. By continuity, this would hold if α_3 is sufficiently small and if the utility function is given by

$$u(y) = \frac{(y + \varepsilon)^{1-\sigma} - \varepsilon^{1-\sigma}}{1 - \sigma},$$

where $\sigma < 1$ and ε is sufficiently small. Define

$$\begin{aligned}\bar{\mu} &= 1 / [\Psi(0)^{-1} + c], \\ \tilde{\rho} &= \left[\frac{1}{\Psi(0)} + c - \frac{1}{\mu} \right] (1 - \chi)^{-1} + \frac{1}{\mu} - 1, \\ \bar{\rho} &= \frac{c + 1/\beta - 1 + \chi(1 - 1/\mu)}{1 - \chi}.\end{aligned}$$

Here, $\bar{\mu}$ is the lowest inflation level at which bankers are unwilling to operate without lending opportunities. If $\mu < \bar{\mu}$, the bankers are willing to supply deposits even if they cannot lend and can only hold money. If $\mu \geq \bar{\mu}$, they stop operating without lending opportunities because holding cash is too costly. Similarly, $\tilde{\rho}$ is the highest lending rate that bankers are unwilling to operate if they are forced to lend up to the reserve requirement. If $\rho > \tilde{\rho}$, bankers supply positive deposits. Notice that if $\mu < \bar{\mu}$, then $\tilde{\rho} < 1/\mu - 1$. Lastly, $\bar{\rho}$ is the highest interest at which bankers earn a finite profit. If $\rho > \bar{\rho}$, bankers can have an unbounded profit by having an infinite amount of deposits and loans.

Proposition 1 *Under Assumption 1, the following claims hold:*

1. *There is a unique symmetric pure strategy equilibrium in the Cournot game if $\rho < \bar{\rho}$.*

2. Suppose $\Psi(0) < \infty$ and $\Psi'(0) < \infty$. The total deposit $D(\rho)$ satisfies

$$D(\rho) = \begin{cases} 0 & \text{if } \mu > \bar{\mu} \text{ and } \rho < \tilde{\rho} \\ Nd^*(\rho) & \text{if } \rho < \bar{\rho} \text{ and } \beta < \mu < \bar{\mu} \\ & \text{or } \rho > \tilde{\rho} \text{ and } \mu > \bar{\mu} \\ \left[\frac{N}{N-1}y^*, \infty\right] \cup \{Nd^*(\rho)\} & \text{if } \rho = \bar{\rho} \\ \infty & \text{if } \rho > \bar{\rho} \end{cases},$$

where

$$\begin{aligned} \Psi'(Nd^*(\rho))d^*(\rho) + \Psi(Nd^*(\rho)) &= 1/\xi, \\ \xi &= \max\left\{\left(1 + \rho - \frac{1}{\mu}\right)(1 - \chi), 0\right\} + 1/\mu - c. \end{aligned}$$

3. $D(\rho)$ is weakly increasing in ρ .

Proof. First notice that in the equilibrium there is no deposit iff the best response to $d_{-j} = 0$ is $d_j = 0$. This is true iff

$$\left[(1 - \chi) \max\left\{1 + \rho, \frac{1}{\mu}\right\} + \frac{\chi}{\mu} - c\right] \Psi(0) - 1 < 0.$$

One can check that this is true iff $\mu > \bar{\mu}$ and $\rho < \tilde{\rho}$. This proves the second claim.

Now suppose that either $\mu > \bar{\mu}$ or $\rho < \tilde{\rho}$ does not hold. For any $\rho < \bar{\rho}$, banker j 's problem can be rewritten as

$$\max_{d_j} \xi \Psi(d_{-j} + d_j) d_j - d_j,$$

where $\xi = 1/\mu - c$ if $\rho < 1/\mu - 1$ and $\xi = \left(1 + \rho - \frac{1}{\mu}\right)(1 - \chi) + 1/\mu - c$ if $\rho > 1/\mu - 1$.

By Assumption 1, there is a unique solution to this problem. This solution satisfies $\Psi'(d_{-j} + d_j) d_j + \Psi(d_{-j} + d_j) = 1/\xi$ or $d_j = 0$. Then the symmetric pure strategy Nash equilibrium d must satisfy $\Psi'(Nd) d + \Psi(Nd) = 1/\xi$. Notice that $\Psi(0) > 1/\xi$ and $\Psi(y^*) = \beta < 1/\xi$. Therefore, $\Psi'(Nd) d + \Psi(Nd) < 1/\xi$ for all d that are smaller than but sufficiently close to y^*/N . Because $\Psi'(Nd) d + \Psi(Nd)$ is continuously decreasing on $[0, y^*/N)$, there exists a unique solution. Therefore, there is a unique symmetric pure strategy equilibrium $d^*(\rho)$. The total supply is $D(\rho) = Nd^*(\rho)$.

If $\rho = \bar{\rho}$, there is a unique solution to $\Psi'(Nd) d + \Psi(Nd) = 1/\xi = \beta$ on $[0, y^*/N)$.

In addition, this equation holds if $d > y^*/(N - 1)$. But not all $d > y^*/(N - 1)$ can

be an equilibrium. This is because if $d_{-j} = d(N-1) < y^*$, banker j 's best response is $d < y^* - d(N-1)$, which yields a strictly positive profit, while $d > y^* - d(N-1)$ leads to 0 profit. However, $d > y^*/(N-1)$ can be a symmetric pure strategy equilibrium because no banker has a profitable deviation. Therefore, $D(\rho) = d^*(\rho)$ or $D(\rho) > \frac{N}{N-1}y^*$. If $\rho > \bar{\rho}$, the best response of banker j is $d_j = \infty$. Therefore, $D(\rho) = \infty$. To see the last claim, just notice that by Assumption 1, $d^*(\rho)$ is increasing. ■

Corollary 1 *The loan supply function is*

$$L^s(\rho) = \begin{cases} 0 & \text{if } \rho < \frac{1}{\mu} - 1 \\ [0, (1-\chi) D(\rho) \Psi(D(\rho))] & \text{if } \rho = \frac{1}{\mu} - 1 \\ (1-\chi) D(\rho) \Psi(D(\rho)) & \text{if } \rho \in \left(\frac{1}{\mu} - 1, \bar{\rho}\right) \\ \{(1-\chi) N d^*(\rho) \Psi(N d^*(\rho))\} \cup \left[\frac{(1-\chi) N \beta y^*}{N-1}, \infty\right] & \text{if } \rho = \bar{\rho} \\ \infty & \text{if } \rho > \bar{\rho} \end{cases}.$$

Proof. Obviously, if $1+\rho < 1/\mu$, then $\ell_j = 0$ because it is more profitable for the bankers to hold the fiat money than to make loans. As a result, $L^s(\rho) = N\ell_j = 0$. If $1+\rho = 1/\mu$, the banks are indifferent on $\ell_j \in [0, (1-\chi) \Psi(d_{-j} + d_j) d_j]$, where d_j solves

$$\max_{d_j} \left\{ \left(\frac{1}{\mu} - c \right) \Psi(d_{-j} + d_j) d_j - d_j \right\}, \quad (10)$$

because the inequality constraint is not binding. Then $D(\rho) = N d_j$ and $L^s(\rho) = N\ell_j \in [0, (1-\chi) D(\rho) \Psi(D(\rho))]$. If $1/\mu < 1+\rho$, bankers lend out all they can. Then the loan supply can be computed from $D(\rho)$. ■

This result shows that L^s is single valued and continuous if $1/\mu - 1 < \rho < 1/\beta - 1$ or $\rho < 1/\mu - 1$. If $\rho = 1/\mu - 1$, then L^s may be set valued but convex. But if $\rho > 1/\mu - 1$, then L^s may be set valued and not convex. Also notice that, because $D(\rho)$ can be 0 even if $\rho > 1/\mu - 1$ as shown in Proposition 1, $L^s(\rho)$ may be 0 even if $\rho > 1/\mu - 1$. However, this only happens if the money growth rate is too high, i.e., $\mu > \bar{\mu}$. If $\mu < \bar{\mu}$, then $L^s(\rho) > 0$ if $\rho > 1/\mu - 1$.

3 Equilibrium

We focus on the steady state equilibrium. Then the inflation rate equals the money growth rate μ . Any ρ that solves $L^d(\rho) = L^s(\rho)$ constitutes an equilibrium. We can plot the loan supply and loan demand curves in ρ - L space, as in Figure 1, to analyze the equilibrium. For illustration, we focus on the case where both $\Psi(D)D$ and $D(\rho)$ are increasing.

The loan demand curve is strictly decreasing as shown by the solid red line. The loan supply curve is illustrated by the solid blue line. As in the proof, it has two cases. The first case is shown in Figure 1(a), where $\tilde{\rho} < 1/\mu - 1$. If $\rho < 1/\mu - 1$, then the loan supply curve coincides with the horizontal axis. If $\rho = 1/\mu - 1$, then banks have positive deposits, i.e., $D > 0$. They are indifferent between lending and holding cash. Therefore, the loan supply curve is vertical between 0 and $(1 - \chi) D(\rho) \Psi(D(\rho))$. The loan supply curve is strictly increasing on $\left(\frac{1}{\mu} - 1, \bar{\rho}\right)$. And if $\rho = \bar{\rho}$, it is vertical again. Obviously, the demand and the supply have one and only one intersection, which implies the existence and uniqueness of the steady state equilibrium. Notice that if $\rho = 1/\mu - 1$ at the intersection, banks hold voluntary reserves. This is because firms are not very productive and banks choose to not lend up to the limit.

Figure 1(b) plots the second case, where $\tilde{\rho} > 1/\mu - 1$. The only difference is that the supply curve coincides with the horizontal line if $\rho < \tilde{\rho}$. In this case, the demand for liquidity is low. As a result, if the lending rate is too low, the banks are not willing to take deposits and hence the loan supply is 0. Again, a steady state equilibrium exists and is unique.

In general, an equilibrium may not exist because $L^s(\rho)$ may be non-convex valued. And there can be multiple equilibria because $L^s(\rho)$ may decrease in some regions. However, one can show that at least one equilibrium exists as long as the productivity is not too high.

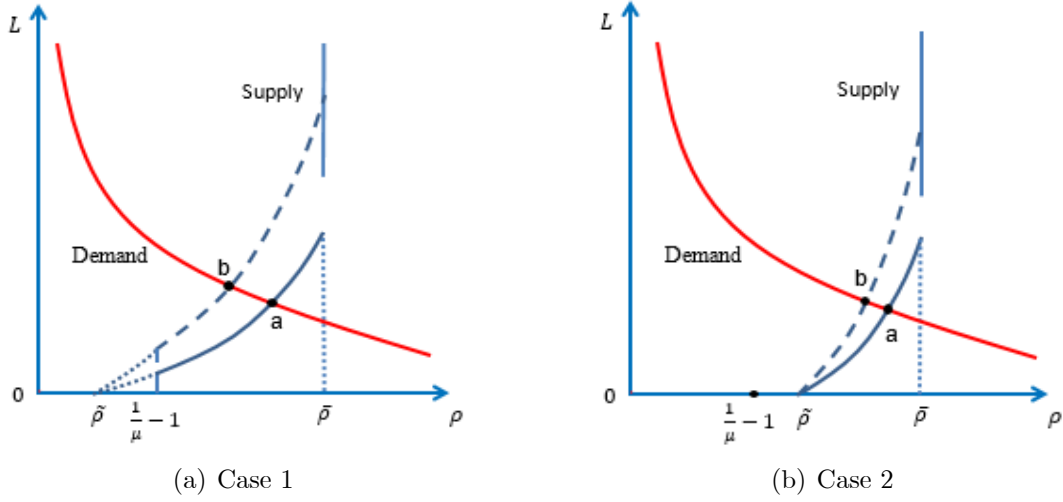


Figure 1: Equilibrium with Imperfect Competition

Assumption 2 *The production function f satisfies $f' \left(\frac{(1-\chi)Nd^*(\bar{\rho})\Psi(Nd^*(\bar{\rho}))}{N^E} \right) < 1 + \bar{\rho}$.*

Assumption 2 requires that the productivity not be too high so that L^d passes through L^s at its continuous part, which guarantees the existence of an equilibrium. We also believe this is the relevant case because if the productivity is too high so that Assumption 2 fails, the liquidity premium on deposits vanishes and bankers pay high interest on deposits.

Proposition 2 *Under Assumptions 1 and 2, there exists at least one monetary equilibrium if ι is not too big. If, in addition, $D\Psi(D)$ is increasing in D , the equilibrium is unique. The equilibrium loan rate is then less than $\bar{\rho}$.*

Proof. Notice that $L^s(\rho)$ is continuous if $D(\rho)$ is continuous. In addition, $L^s(\rho) = 0$ for sufficiently small ρ and $\min L^s(\bar{\rho}) = (1 - \chi)Nd^*(\bar{\rho})\Psi(Nd^*(\bar{\rho}))$. On the other hand, $L^d(\rho)$ is decreasing and positive for any $\rho > 0$ because $f'(0) = \infty$. As a result, $L^d(\rho) - L^s(\rho) > 0$ for sufficiently small ρ and $L^d(\bar{\rho}) - L^s(\bar{\rho}) < 0$ by assumption. In addition, both L^d and L^s are continuous on $[0, \bar{\rho})$. To see the uniqueness, just notice that on $[0, \bar{\rho})$, $L^s(\rho)$ is increasing and $L^s(\rho)$ is strictly

decreasing. By the intermediate value theorem, there exists at least one equilibrium with the loan rate less than $\bar{\rho}$. ■

For the existence of the equilibrium, we require that entrepreneurs not be too productive such that the loan demand and loan supply curves intersect to the left of $\bar{\rho}$. If this is the case, the buyers are liquidity constrained at least in type 2 meetings.

We can also use the diagrams to analyze comparative statics. For example, if N increases, the solid blue line rotates counter-clockwise to the dashed blue line in Figure 1. Then the equilibrium changes from point a to point b . In both cases, the demand intersects the supply in its increasing region. Therefore, loan supply increases and equilibrium ρ decreases. If, however, the intersection is in the vertical part of the supply, equilibrium ρ and L do not change.

Now we analyze the effect of higher inflation μ or equivalently higher ι focusing on the case where $\mu < \bar{\mu}$. First consider the case where $\alpha_3 = 0$ and $\chi > 0$. If μ increases, the loan demand curve stays unchanged while the loan supply curve changes to the dashed line. In this case, an increase in μ decreases the demand for fiat money but does not increase the demand for deposits because fiat money and deposits are not substitutes. However, an increase in μ increases the cost of holding fiat money for the banks. As a result, bankers are willing to lend at a lower loan rate because holding fiat money becomes a less attractive alternative.

But for lending rates at which bankers are willing to lend before the increase in the inflation rate μ , the loan supply decreases. This is because it is more costly to satisfy the reserve requirement, and the bankers reduce the supply of deposit and lending. In Figure 2(a), the equilibrium changes from a to b , and there is less lending and a higher lending rate. However, if, before the change, the bankers hold voluntary reserves as shown in Figure 2(b), an increase in μ increases lending and reduces loan rates because now bankers are more willing to lend.

Now consider the case with $\chi = 0$ and $\alpha_3 > 0$. An increase in μ induces

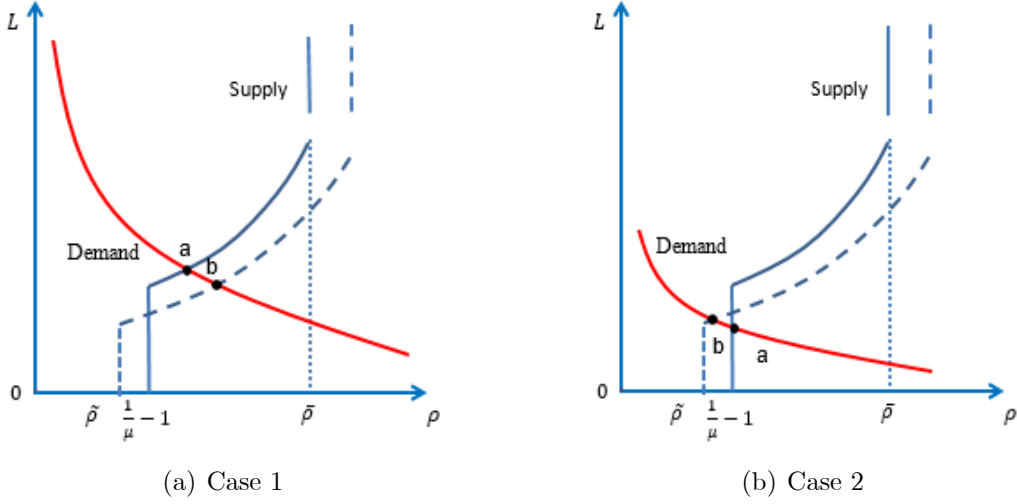


Figure 2: Unconstrained in Type 3 Meetings and $\chi > 0$

households to substitute out to hold more deposits. This drives up the demand for deposits. Because the bankers do not need to hold reserves, they are willing to increase the supply of deposits and hence increase the supply of loans. This unambiguously increases the equilibrium loan quantity and reduces the loan rate, as shown in Figure 3(a) and Figure 3(b).

In general, if both α_3 and χ are positive, higher inflation can have positive or negative effects on loans depending on which of the above effects dominates. But one can show the following is true.

Proposition 3 *If bankers hold voluntary reserves, an increase in μ increases lending and decreases the loan rate.*

Proof. In this case, if μ increases, the loan demand and loan supply curves can only intersect at a point to the left of the current equilibrium on the ρ - L space. Because loan demand is decreasing, this implies that the loan rate is lower and loan quantity is higher. ■

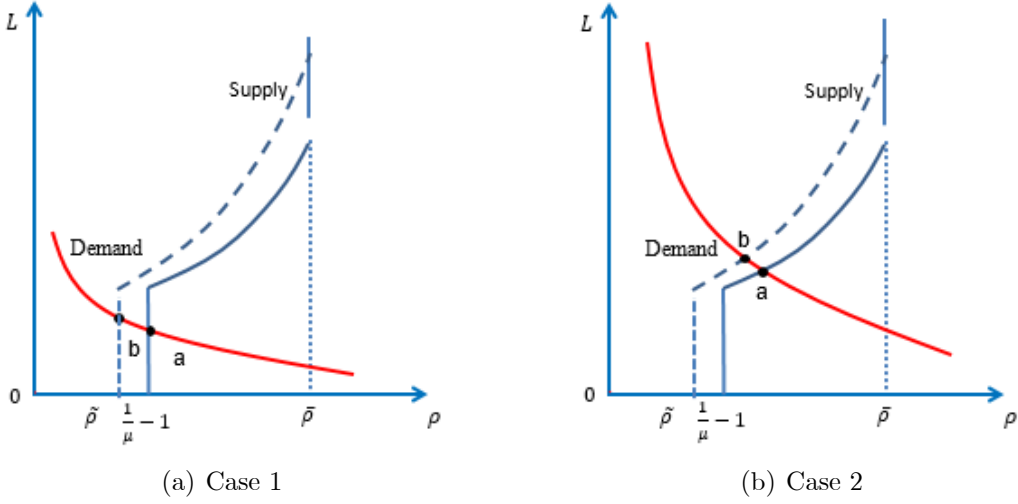


Figure 3: $\alpha_3 > 0$ and $\chi = 0$

4 Effects of CBDC

This section analyzes the effects of an interest-bearing CBDC on bank lending and the real economy. Obviously, the effects of a CBDC depend on its design. There are several design dimensions that we can consider using our model: whether CBDC is interest bearing, in what type of meetings CBDC can be used, and whether banks can hold CBDC and for what purpose (i.e., whether or not banks can hold it against their reserve requirements). We focus on the case where a CBDC is interest bearing and designed to be a perfect substitute for bank deposits, i.e., it can be used in type 2 and type 3 meetings. This is likely the case where CBDC would have the most significant impacts on banking. At the end of this section, we briefly discuss the implications of some other types of CBDC.

We consider three cases in this section. First, the bankers are not allowed to hold a CBDC. Second, the banks are allowed to hold a CBDC but the CBDC does not count as reserves. Third, the CBDC can be used as reserves. Throughout this section, we only consider the parameters under which the equilibrium lending rate of the CBDC is above $1/\mu - 1$, i.e., we focus on the cases in Figure 1 where L^d

intersects L^s in its strictly increasing region. We then evaluate whether introducing the CBDC increases or decreases lending. If, instead, the equilibrium lending rate is $1/\mu - 1$ before introducing the CBDC, then the CBDC only weakly decreases lending. It is worth pointing out that the results in the sections show that one should not judge the usefulness of the CBDC based on how much it is used in the daily transactions of consumers. Its effect on both deposit and lending rates is a better measure of its usefulness. In the rest of the section, we use ρ^* to denote the equilibrium lending rate before the CBDC is introduced, and we focus on the case where $\mu < \bar{\mu}$, i.e., bankers are willing to operate without lending opportunities if there is no CBDC. All the results carry over to the case with $\mu > \bar{\mu}$.

4.1 CBDC Not Accessible by Bankers

Suppose that a CBDC grows at a gross rate μ_m and pays a nominal interest i_m . Both μ_m and i_m are policy instruments of the central bank. Let Z_E be the real balances in the CBDC. Then the CM problem of the buyers changes to

$$\begin{aligned} W^B(Z, Z_E, D) = & Z + D + Z_E + T + \max_x [U(x) - x] \\ & + \max_{\hat{D}, \hat{Z}_E, \hat{Z}} \left\{ -\frac{\phi}{\hat{\phi}} \hat{Z} - \frac{\phi_E}{\hat{\phi}_E} \frac{1}{1 + i_m} \hat{Z}_E - \psi \hat{D} + \beta V(\hat{Z}, \hat{Z}_E, \hat{D}) \right\}, \end{aligned}$$

where ϕ_E is the price of the CBDC in terms of the numeraire good. It can be different from ϕ in the equilibrium because the CBDC may pay interest or have a different growth rate.

Following the same calculation, one can obtain the steady-state demand for all three payment instruments given the deposit rate and policy rates:

$$\iota \geq \alpha_1 \lambda(Z) + \alpha_3 \lambda(Z + Z_E + D), \text{ equality iff } Z > 0 \quad (11)$$

$$\frac{\psi}{\beta} - 1 \geq \alpha_2 \lambda(D + Z_E) + \alpha_3 \lambda(Z + Z_E + D), \text{ equality iff } D > 0 \quad (12)$$

$$\frac{\mu_m}{\beta(1 + i_m)} - 1 \geq \alpha_2 \lambda(D + Z_E) + \alpha_3 \lambda(Z + Z_E + D), \text{ equality iff } Z_E > 0. \quad (13)$$

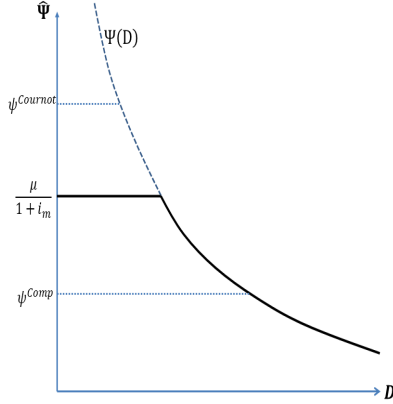


Figure 4: Inverse Demand of Bank Deposits

From the last two equations, we can see that if $\psi > \frac{\mu_m}{1+i_m}$, the demand for bank deposits is 0. If $\psi < \frac{\mu_m}{1+i_m}$, the demand for CBDC is 0.

As before, (11)–(13) define the demand function of D and Z_E as the following:

$$D = \begin{cases} \Psi^{-1}(\psi) & \psi < \frac{\mu_m}{1+i_m} \\ [0, \Psi^{-1}(\psi)] & \psi = \frac{\mu_m}{1+i_m} \\ 0 & \psi > \frac{\mu_m}{1+i_m} \end{cases} \quad \text{and} \quad Z_E = \begin{cases} 0 & \psi < \frac{\mu_m}{1+i_m} \\ \Psi^{-1}(\frac{\mu}{1+i_m}) - D & \psi = \frac{\mu_m}{1+i_m} \\ \Psi^{-1}(\frac{\mu}{1+i_m}) & \psi > \frac{\mu_m}{1+i_m} \end{cases},$$

where Ψ is the same as the one defined in Section 2 for some $d \in [0, \Psi^{-1}(\frac{\mu_m}{1+i_m})]$.

This defines a new inverse demand function for deposits:

$$\hat{\Psi}(D) = \begin{cases} [\frac{\mu_m}{1+i_m}, \infty) & D = 0 \\ \frac{\mu_m}{1+i_m} & D \in [0, \Psi^{-1}(\frac{\mu_m}{1+i_m})] \\ \Psi(D) & D \geq \Psi^{-1}(\frac{\mu_m}{1+i_m}) \end{cases}.$$

Figure 4 illustrates $\hat{\Psi}$ and Ψ . Introducing CBDC truncates the original inverse demand function for deposits. With a CBDC, bankers can no longer drive the deposit rate below $\frac{\mu_m}{\beta(1+i_m)} - 1$ by restricting the supply of deposits because buyers then will choose to hold the CBDC.

Following (10) and assuming that $1/\mu - 1 \leq \rho < \bar{\rho}$, we can now write the bank's problem as follows:

$$\max_{\ell_j, d_j} \left\{ \left[(1 + \rho) (1 - \chi) - c + \frac{\chi}{\mu} \right] \hat{\Psi} (d_{-j} + d_j) d_j - d_j \right\}.$$

Denote by $\tilde{D}(\rho)$ the total deposit supply by the banks under ρ and $\hat{\Psi}$. Let $\bar{\psi}(\rho) \equiv \Psi(D(\rho))$ be the price of deposits given by Cournot competition if there is no CBDC and the lending rate is ρ . In addition, define $\hat{\psi}(\rho) = \left[(1 + \rho)(1 - \chi) - c + \frac{\chi}{\mu} \right]^{-1}$, which is the outcome of marginal cost pricing. Notice that $\hat{\psi}(\rho)$ and $\bar{\psi}(\rho)$ are both decreasing in ρ if $D(\rho)$ is strictly increasing. In addition, $\hat{\psi}(\rho) < \bar{\psi}(\rho)$ because bankers just break even at $\hat{\psi}(\rho)$ but earn a positive profit at $\bar{\psi}(\rho)$. Then the following lemma holds.

Lemma 2 *Suppose Assumption 1 holds. If $\max(1/\mu, 1 + \tilde{\rho}) < 1 + \rho < 1 + \bar{\rho}$, then we have the following:*

1. if $\frac{\mu_m}{1+i_m} \in (\bar{\psi}(\rho), \infty)$, then $\tilde{D}(\rho) = D(\rho)$;
2. if $\frac{\mu_m}{1+i_m} \in (\hat{\psi}(\rho), \bar{\psi}(\rho))$, then in the symmetric pure strategy equilibrium $\tilde{D}(\rho) = \Psi^{-1}(\frac{\mu_m}{1+i_m})$;
3. if $\frac{\mu_m}{1+i_m} \in (0, \hat{\psi}(\rho))$, then $\tilde{D}(\rho) = 0$.

Proof. We only prove 2. The other two are obvious. If $d_j < \Psi^{-1}(\frac{\mu_m}{1+i_m}) - d_{-j}$, increasing d_j does not change the price of the deposit, which is fixed at $\mu_m / (1 + i_m)$. Then the FOC of bank j is

$$\xi \frac{\mu_m}{1+i_m} - 1 > \xi \hat{\psi}(\rho) - 1 = 0,$$

where $\xi = (1 + \rho)(1 - \chi) - c + \chi/\mu$. Therefore, bank j can always increase its profit by increasing d_j .

Now suppose $\tilde{D}(\rho) > \Psi^{-1}(\frac{\mu_m}{1+i_m})$. Because $\frac{\mu_m}{1+i_m} < \bar{\psi}(\rho)$, $\tilde{D}(\rho)$ is larger than the Cournot outcome without CBDC. Therefore, by Assumption 1, a banker's marginal profit is

$$\xi \left[\Psi(\tilde{D}(\rho)) + \frac{\tilde{D}(\rho)}{N} \Psi'(\tilde{D}(\rho)) \right] - 1 < 0.$$

This shows that it is profitable for a banker to reduce its supply of deposit. Hence, $\tilde{D}(\rho) > \Psi^{-1}(\frac{\mu_m}{1+i_m})$ cannot happen. ■

Lemma 2 describes how the amount issued by the banks changes if we introduce an interest-bearing CBDC. Interestingly, if the interest rate of the CBDC is intermediate, then, for a given ρ , the deposits created by banks increase and the CBDC is not used in transactions. Intuitively, banks restrict their supply to get a higher price if there is no CBDC. If the CBDC is introduced and has an intermediate interest, the interest rate on deposits has to equal the interest on the CBDC. Then banks lose the ability to lower the rates on deposit by restricting supply. So they would increase the supply of deposits. As long as the interest rate is not further lowered, banks' profit is increasing in their deposits. Therefore, they supply enough deposits to satisfy all the transaction needs and drive out the CBDC from transactions.

This result on the quantity of deposits translates to the following result on loan supply, which is helpful for determining the equilibrium. Define

$$\begin{aligned}\hat{\rho} &= \max \left\{ \frac{1}{\mu} - 1, \left(\frac{1+i_m}{\mu_m} + c - \frac{\chi}{\mu} \right) \frac{1}{1-\chi} - 1 \right\}, \\ \check{\rho} &= \bar{\psi}^{-1} \left(\frac{\mu_m}{1+i_m} \right) \text{ if } \Psi(Nd^*(\bar{\rho})) \leq \frac{\mu_m}{1+i_m} < \lim_{\rho \downarrow 1/\mu-1} \bar{\psi}(\rho).\end{aligned}$$

Here, $\hat{\rho}$ is the maximum of two terms. The first term is the return from holding money. If the lending rate is less than the return from money, bankers hold money rather than lending. Therefore, the loan supply is 0. The second term is the marginal cost of lending given that a banker prefers lending rather than holding money and that the deposit rate equals the CBDC rate. It equals the real rate paid to the depositors plus the deposit handling cost minus the return to the money holding necessary for the reserve requirement, all scaled by $1/(1-\chi)$. The scale is needed because loans are at most $1-\chi$ fraction of total deposits. In other words, the second term is the lowest lending rate at which bankers are willing to operate if they are forced to lend up to the reserve requirement. As a result, if $\rho < \hat{\rho}$, there is no

lending because bankers just hold money or do not operate. If $\rho = \hat{\rho}$, bankers break even if the deposit rate matches the CBDC rate. While $\check{\rho}$ is the real lending rate at which the equilibrium deposit rate from the Cournot competition without CBDC equals the current CBDC rate, $\check{\rho}$ is only well defined for a range of CBDC rates. For example, if the CBDC rate is too high, the deposit rate without CBDC is always below the CBDC rate for any ρ . Similarly, if the real CBDC rate is too low, the deposit rate without CBDC is always above the CBDC rate for any ρ . By definition, $\check{\rho}$ is larger than $1/\mu - 1$, and at $\check{\rho}$ bankers have a positive profit if the deposit rate matches the real rate of CBDC. But $\hat{\rho}$ is equal to $1/\mu - 1$, or at $\hat{\rho}$ bankers break even if the deposit rate matches the real rate of CBDC. Therefore, $\check{\rho}$ is bigger than $\hat{\rho}$ if the former is well defined. Also notice that both $\check{\rho}$ and $\hat{\rho}$ are increasing in i_m .

Lemma 3 *Suppose Assumption 1 holds. Suppose that the CBDC pays an interest i_m such that $\Psi(0) > \frac{\mu_m}{1+i_m}$ and $\frac{\mu_m}{1+i_m} \geq \beta$. Then, the loan supply with CBDC \tilde{L}^s satisfies the following:*

1. If $\frac{\mu_m}{1+i_m} \geq \lim_{\rho \downarrow 1/\mu-1} \bar{\psi}(\rho)$, $\tilde{L}^s = L^s$.
2. If $\Psi(Nd^*(\bar{\rho})) \leq \frac{\mu_m}{1+i_m} < \lim_{\rho \downarrow 1/\mu-1} \bar{\psi}(\rho)$,

$$\tilde{L}^s(\rho) = \begin{cases} 0 & \text{if } \rho < \hat{\rho} \\ \left[0, (1-\chi) \frac{\mu_m}{1+i_m} \Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right)\right] & \text{if } \rho = \hat{\rho} \\ (1-\chi) \frac{\mu_m}{1+i_m} \Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right) & \text{if } \rho \in (\hat{\rho}, \check{\rho}) \\ L^s(\rho) & \text{if } \rho > \check{\rho} \end{cases}.$$

3. If $\Psi(Nd^*(\bar{\rho})) > \frac{\mu_m}{1+i_m}$,

$$\tilde{L}^s(\rho) = \begin{cases} 0 & \text{if } \rho < \hat{\rho} \\ \left[0, (1-\chi) \frac{\mu_m}{1+i_m} \Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right)\right] & \text{if } \rho = \hat{\rho} \\ (1-\chi) \frac{\mu_m}{1+i_m} \Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right) & \text{if } \rho \in (\hat{\rho}, \bar{\rho}) \\ \left\{ (1-\chi) \frac{\mu_m}{1+i_m} \Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right) \right\} \cup \left[\frac{(1-\chi)N\beta y^*}{N-1}, \infty \right] & \text{if } \rho = \bar{\rho} \end{cases}.$$

We omit the proof because it only involves checking the conditions in Lemma 2. This lemma suggests that if the real CBDC rate is too low, it does not affect the loan supply curve. In this case, a CBDC is not an attractive alternative to bank deposits and hence no one is willing to hold it in the economy. If i_m becomes higher, it starts to reshape the loan supply curve. If ρ is not too high, bankers are then forced to offer deposits at a rate that matches the CBDC rate. But if ρ is sufficiently high, the deposit rate without the CBDC is higher than the real rate of the CBDC; then the loan supply curve is not affected. This happens if $\rho > \check{\rho}$ if i_m is not too high. However, if i_m is sufficiently high, $\tilde{L}^s(\rho) \neq L^s(\rho)$ for all $\rho > \hat{\rho}$.

To investigate the effects of CBDC, we can plot the loan demand and loan supply curves before and after introducing a CBDC. As in previous sections, we focus on the case where $D\Psi(D)$ is increasing. The results are shown in Figure 5. In any of these graphs, the solid red curve is the loan demand curve and the solid blue curve is the loan supply curve without the CBDC. They intersect at point a, which is the equilibrium without the CBDC. After the CBDC is introduced, the loan demand curve is not affected, whereas the loan supply curve is changed to the dashed black line. As shown by Lemma 3, the loan supply curve has the following properties. If $\rho < \hat{\rho}$, the loan supply is 0. This is because the loan rate is too low and the rate on deposits cannot fall below i_m . Then it is not profitable for banks to issue deposits. If $\rho = \hat{\rho}$, the loan rate is sufficiently high to cover the cost of issuing deposits. Hence, banks are indifferent between a range of deposits to offer. But total deposits issued do not go above $\Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right)$. Otherwise, the deposit rate increases, which reduces the profits of banks. As a result, the total loan that can be provided can be any value between 0 and $(1-\chi)\frac{\mu_m}{1+i_m}\Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right)$. If $\rho \in (\hat{\rho}, \check{\rho})$, then the loan rate is high enough that issuing more deposits always increases the profit of the banks as long as the deposit rate is not affected. This is because with more deposits, banks can make more loans. Then the loan supply is at $(1-\chi)\frac{\mu_m}{1+i_m}\Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right)$. If the

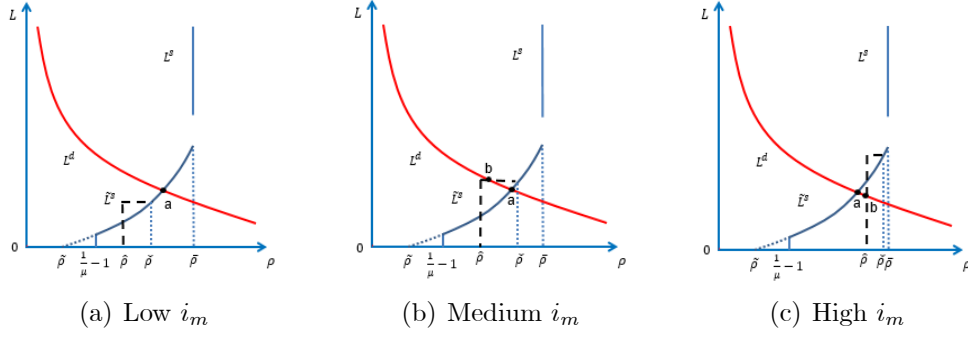


Figure 5: Effects of CBDC

dashed black curve joins the solid blue curve at $\check{\rho}$ and if $\rho > \check{\rho}$, these two curves coincide. This is because ρ is sufficiently high and the deposit rate without CBDC is above i_m . Hence, introducing a CBDC does not affect the loan supply, and the supply curve coincides with the one without CBDC.

Now if i_m is low, as shown in Figure 5(a), the dashed curve joins the solid blue curve before point a . Then with CBDC, the equilibrium stays at a . As i_m increases, both $\hat{\rho}$ and $\check{\rho}$ shift to the right, and also the horizontal part of the dashed curve increases because $(1 - \chi) \frac{\mu_m}{1 + i_m} \Psi^{-1} \left(\frac{\mu_m}{1 + i_m} \right)$ increases with i_m . If i_m is sufficiently high, as in Figure 5(b), the dashed black curve joins the solid blue curve at a point to the right of point a . Now the equilibrium with CBDC is at point b . In this equilibrium, the amount of loans is higher and the loan rate is lower. This is because now banks have less incentive to restrict the deposit supply because they have less ability to affect the equilibrium deposit rate. They then take more deposits, which translates into more loan supply. In this case, further increasing i_m increases the loans and decreases the loan rate. Hence, a higher interest rate on the CBDC increases the intermediation in the economy. If i_m further increases, $\hat{\rho}$ increases further to lie to the right of point a , as shown in Figure 5(c). Now with the CBDC, the equilibrium has a higher lending rate and a lower amount of loans. This is the case where introducing a CBDC crowds out bank lending and private investment (Keister and Sanches, 2018). The following proposition summarizes these

discussions. The effects of the CBDC interest rate are depicted schematically in Figure 6 for the case where $c = \chi = 0$.

Proposition 4 *Suppose bankers cannot hold CBDC. If Assumptions 1 and 2 hold and $\iota < \bar{\iota}$, there exists at least one monetary equilibrium. Moreover, if $D\Psi(D)$ is increasing, there exists a unique monetary equilibrium and*

1. *if i_m is set such that $\rho^* \in (\hat{\rho}, \check{\rho})$, then CBDC increases lending and a higher i_m induces more lending;*
2. *if i_m is set such that $\rho^* = \hat{\rho}$, then CBDC decreases lending and a higher i_m induces less lending.*

Proof. In the text above. ■

This proposition delivers an important message, i.e., introducing a CBDC need not cause disintermediation by reducing loans and deposits. On the contrary, it may increase lending and deposits by introducing more competition to the banking sector if its rate is appropriately designed. Unlike Andolfatto (2018), our result does not require that the central bank lend to private banks. This is because bankers make their deposits more attractive in order to compete with the CBDC. Because their deposits are more attractive, they are able to make more loans in the form of deposits. In addition, they have incentives to do so because their impact on deposit rates is contained by the CBDC, i.e., they cannot reduce the deposit rate too much by restricting the supply of deposits. Also notice that because $\hat{\rho}$ and $\check{\rho}$ are both increasing in i_m , there exists $\underline{i}_m < \bar{i}_m$ such that $\rho^* \in (\hat{\rho}, \check{\rho})$ iff $i_m \in (\underline{i}_m, \bar{i}_m)$. Therefore, there is a range of i_m under which introducing a CBDC increases lending.

It is also worth noticing that in Figure 5(a), no CBDC is used for transactions. Buyers continue to hold only deposits. However, this has a real effect. Intuitively, the existence of the CBDC disciplines the off-equilibrium outcome. If the bankers

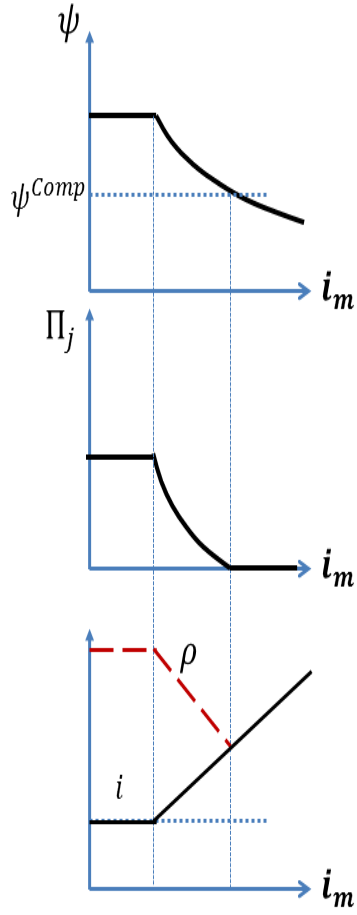


Figure 6: Effects of Change in CBDC Interest Rate with No Reserve Requirements and No Handling Cost of Deposits

reduce their deposit rates, they know buyers would switch to hold a CBDC. In this case, the CBDC acts like a *potential entrant*.

Note the difference between introducing the CBDC and introducing another bank. Although in both scenarios the lending and deposit rates both get closer to their perfect competition level, introducing the CBDC sets the floor for the deposit rate at any target level that the central bank wants. In contrast, increasing the number of banks affects the deposit rates only indirectly and may not increase the rate to the target level. Moreover, although the central bank/government can affect banks' entry through regulation and taxation in the long run, these tools are unlikely to be useful for influencing the rates in the short run, which is the focus of monetary policy.

If i_m increases a little bit more, the L^d intersects \tilde{L}^s on its vertical region. In this case, the CBDC and bank deposits are both used as means of payment. An important implication of this finding is that the usefulness of the CBDC should not be judged on how frequently people use it, but rather on how much it affects the deposit rates. Indeed, the CBDC increases lending most if i_m is set such that at the equilibrium $\rho = \hat{\rho}$. In this case, the CBDC is never used as a means of payment.

4.2 CBDC Accessible by Bankers

Now suppose bankers have access to a CBDC but the CBDC does not serve as reserves. They now can choose whether to lend to banks, hold cash reserves or hold the CBDC. If the CBDC gives higher rates than lending, they choose to hold the CBDC. Under this scenario, the CBDC has the most negative impact on bank lending given the CBDC is designed as a perfect substitute for bank deposits. First, bankers cannot hold the CBDC as reserves, and hence interest on the CBDC does not reduce the cost of holding reserves. Second, because the CBDC pays interest, the bankers may want to choose holding the CBDC instead of making loans. We show that even within this scenario, introducing the CBDC can help lending if the

interest rate on the CBDC is set properly.

Their problem changes to

$$\begin{aligned} \max_{z_j^E, z_j, \ell_j, d_j} & \left\{ (1 + \rho) \ell_j + \frac{z_j}{\mu} + \frac{(1 + i^m) z_j^E}{\mu_m} - \left[d_j + \hat{\Psi} (d_{-j} + d_j) d_j c \right] \right\} \\ \text{st} \quad & \ell_j + z_j + z_j^E = \hat{\Psi} (d_{-j} + d_j) d_j \\ & z_j \geq \chi \hat{\Psi} (d_{-j} + d_j) d_j. \end{aligned} \quad (14)$$

If $1/\mu > (1 + i^m)/\mu_m$, then the bankers continue to hold only money if the lending opportunity is not good because fiat money has a higher return compared with the CBDC. If $1/\mu \leq (1 + i^m)/\mu_m$, then the bankers would instead hold the CBDC because it has a higher return. In both cases, the bankers operate if ρ is sufficiently high for them to at least break even. Recall that without the CBDC, the break-even $\rho = \hat{\rho}$. Now with the CBDC, if $1/\mu > (1 + i^m)/\mu_m$, the break-even point remains $\hat{\rho}$ because bankers never hold the CBDC. If $1/\mu \leq (1 + i^m)/\mu_m$, then $\hat{\rho} > (1 + i^m)/\mu_m - 1$. Intuitively, bankers can never break even if $\hat{\rho} \leq (1 + i^m)/\mu_m - 1$ because bankers need to pay $(1 + i^m)/\mu_m - 1$ to depositors and incur the cost of holding reserves and handling deposits given $\chi > 0$ or $c > 0$. If $\chi = 0$ and $c = 0$, then $\hat{\rho} = (1 + i^m)/\mu_m - 1$. The above analysis means that without loss of generality, we can assume that they *do not* hold the CBDC as a saving instrument.

Proposition 5 *Suppose bankers can hold a CBDC as a saving instrument but not as reserves. If Assumptions 1 and 2 hold and $\iota < \bar{\iota}$, there exists at least one monetary equilibrium. Moreover, if $D\Psi(D)$ is increasing, there exists a unique monetary equilibrium. The CBDC increases lending if i_m is set such that $\rho^* \in (\hat{\rho}, \check{\rho})$. Furthermore, a higher i_m induces more lending if the equilibrium lending rate $\rho_E^* \in (\hat{\rho}, \check{\rho})$.*

Interestingly, not only does Proposition 5 hold, but the equilibria coincide with the case where bankers cannot hold the CBDC. This is because if bankers operate,

$\hat{\rho}$ must be above the CBDC rate, and as a result, bankers do not hold the CBDC in equilibrium.

4.3 CBDC as Reserves

Lastly, we consider the case where bankers can hold a CBDC as reserves. Then the banker's problem becomes

$$\max_{z_j^E, z_j, \ell_j, d_j} \left\{ (1 + \rho) \ell_j + \frac{z_j}{\mu} + \frac{(1 + i^m) z_j^E}{\mu_m} - \left[d_j + \hat{\Psi} (d_{-j} + d_j) d_j c \right] \right\} \quad (15)$$

$$\begin{aligned} \text{st} \quad & \ell_j + z_j + z_j^E = \hat{\Psi} (d_{-j} + d_j) d_j \\ & z_j^E + z_j \geq \chi \hat{\Psi} (d_{-j} + d_j) d_j. \end{aligned} \quad (16)$$

If $\max \{1/\mu, (1 + i^m) / \mu^m\} - 1 < \rho < \bar{\rho}$, then the banker's problem changes to

$$\max_{\ell_j, d_j} \left\{ \left[(1 + \rho) (1 - \chi) - c + \chi \max \left(\frac{1}{\mu}, \frac{1 + i^m}{\mu^m} \right) \right] \hat{\Psi} (d_{-j} + d_j) d_j - d_j \right\}.$$

To solve for the equilibrium, we first solve for the equilibrium where households cannot hold a CBDC. In this case, all the analysis in Section 3 goes through if we replace $1/\mu$ by $\max \{1/\mu, (1 + i^m) / \mu^m\}$ everywhere. More specifically, let

$$\begin{aligned} \tilde{\rho}_R &= \left[\frac{1}{\Psi(0)} + c - \max \left(\frac{1}{\mu}, \frac{1 + i_m}{\mu_m} \right) \right] (1 - \chi)^{-1} + \max \left(\frac{1}{\mu}, \frac{1 + i^m}{\mu^m} \right) - 1, \\ \bar{\rho}_R &= \frac{c + 1/\beta + \chi \left[1 - \max \left(\frac{1}{\mu}, \frac{1 + i_m}{\mu_m} \right) \right]}{1 - \chi}, \\ \xi_R &= \max \left\{ \left[1 + \rho - \max \left(\frac{1}{\mu}, \frac{1 + i_m}{\mu_m} \right) \right] (1 - \chi), 0 \right\} + \max \left(\frac{1}{\mu}, \frac{1 + i^m}{\mu^m} \right) - c, \\ \xi_R^{-1} &= \Psi' (N d_R^* (\rho)) d_R^* (\rho) + \Psi (N d_R^* (\rho)). \end{aligned}$$

Now, $\tilde{\rho}_R$, $\bar{\rho}_R$, and ξ_R are the counterparts of $\tilde{\rho}$, $\bar{\rho}$, and ξ . The difference is that now these rates take into account that the CBDC serves as reserves and pays interest.

Then we can define

$$D_R(\rho) = \begin{cases} 0 & \text{if } \mu > \bar{\mu} \text{ and } \rho < \tilde{\rho}_R \\ Nd_R^*(\rho) & \text{if } \rho < \bar{\rho}_R \text{ and } \beta < \mu < \bar{\mu} \\ & \text{or } \rho > \tilde{\rho}_R \text{ and } \mu > \bar{\mu} \\ [\frac{N}{N-1}y^*, \infty] \cup \{Nd_R^*(\rho)\} & \text{if } \rho = \bar{\rho}_R \\ \infty & \text{if } \rho > \bar{\rho}_R \end{cases}$$

to be the new deposit function. Notice that now $D_R(\rho)$ is increasing in i^m under Assumption 1 because higher i^m increases ξ_R . Then we can obtain the loan supply curve,

$$\tilde{L}_R^s(\rho) = \begin{cases} 0 & \text{if } \rho < \hat{\rho}_R \\ \left[0, (1-\chi) \frac{\mu_m}{1+i_m} \Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right)\right] & \text{if } \rho = \hat{\rho}_R \\ (1-\chi) \frac{\mu_m}{1+i_m} \Psi^{-1}\left(\frac{\mu_m}{1+i_m}\right) & \text{if } \rho \in (\hat{\rho}_R, \check{\rho}_R) \\ L_R^s(\rho) & \text{if } \rho > \check{\rho}_R \end{cases},$$

where $L_R^s(\rho)$ is the loan supply if the households cannot hold the CBDC,

$$L_R^s(\rho) = \begin{cases} 0 & \text{if } \rho < \max\left(\frac{1}{\mu}, \frac{1+i^m}{\mu^m}\right) - 1 \\ [0, (1-\chi) D_R(\rho) \Psi(D_R(\rho))] & \text{if } \rho = \max\left(\frac{1}{\mu}, \frac{1+i^m}{\mu^m}\right) - 1 \\ (1-\chi) D_R(\rho) \Psi(D_R(\rho)) & \text{if } \rho \in \left(\max\left(\frac{1}{\mu}, \frac{1+i^m}{\mu^m}\right) - 1, \bar{\rho}_R\right) \\ \{(1-\chi) Nd_R^*(\rho) \Psi(Nd_R^*(\rho))\} \cup \left[\frac{(1-\chi)N\beta y^*}{N-1}, \infty\right] & \text{if } \rho = \bar{\rho}_R \\ \infty & \text{if } \rho > \bar{\rho}_R \end{cases},$$

$\check{\rho}_R$ solves $\Psi(D_R(\rho)) = \mu_m / (1 + i_m)$, and

$$\hat{\rho}_R = \max\left\{\frac{1}{\mu}, \frac{\frac{1+i_m}{\mu_m} - \chi \max\left(\frac{1}{\mu}, \frac{1+i_m}{\mu_m}\right) + c}{1-\chi}\right\} - 1.$$

Notice that under Assumption 1, $\tilde{L}_R^s(\rho) \geq L_R^s(\rho)$ for all $\rho > \hat{\rho}_R$. This is because bankers can earn interest on their reserves by holding the CBDC. Also notice that $1/\mu \geq (1 + i_m) / \mu_m$, and $\hat{\rho}_R = \hat{\rho}$. Otherwise, $\hat{\rho}_R < \hat{\rho}$, which suggests that bankers are willing to lend at a lower ρ .

Proposition 6 *Suppose bankers can hold a CBDC as reserves. If Assumptions 1 and 2 hold and $\iota < \bar{\iota}$, there exists at least one monetary equilibrium. Moreover, if $D\Psi(D)$ is increasing, there exists a unique monetary equilibrium. The CBDC*

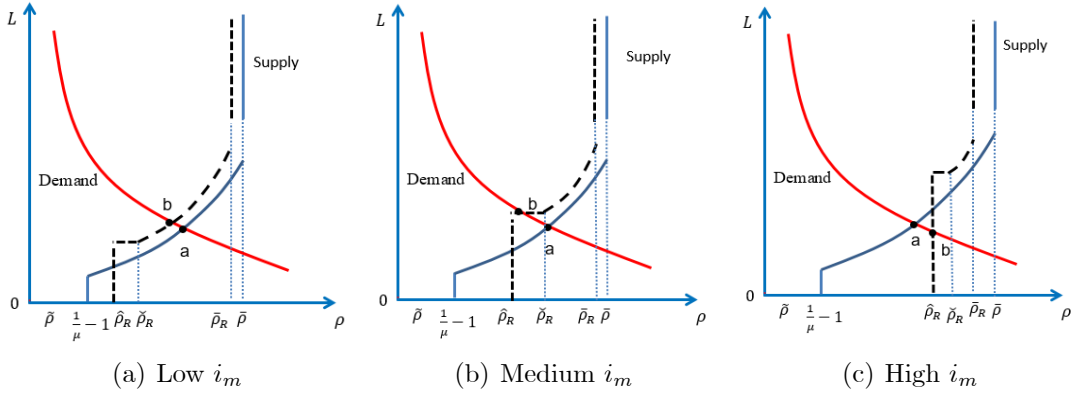


Figure 7: CBDC as Reserves

increases lending if i_m is set such that $\rho^* > \hat{\rho}_R$ and $i_m > \mu_m/\mu - 1$. Furthermore, higher i_m induces more lending if the equilibrium lending rate $\rho_R^* > \hat{\rho}_R$.

Figure 7 illustrates the result. The solid blue lines are the loan supply before introducing the CBDC and the dashed black lines are that after introducing the CBDC. As in the previous two subsections, the loan supply curve is flat if $\rho \in (\hat{\rho}_R, \check{\rho}_R)$. Unlike in the previous two subsections, the CBDC increases loan supply for all $\rho > \hat{\rho}_R$. In particular, the $\tilde{L}_R^s(\rho)$ does not overlap with $L^s(\rho)$ for ρ that is sufficiently high. This is because interest on reserves lowers the cost of lending, which would increase lending even if there is no competition effect.

Figure 7(a) shows the case where the CBDC rate is higher than $\mu_m/\mu - 1$ but still lower than the deposit rate in the equilibrium without the CBDC. The loan demand curve intersects the loan supply curve with CBDC in its increasing region. In this case, buyers strictly prefer bank deposits over the CBDC. As a result, the CBDC does not introduce additional competition. However, the equilibrium with CBDC (point b) still features higher bank lending compared with the equilibrium without (point a). This additional lending is induced by the fact that CBDC has a higher return than money and hence reduces the cost of holding reserves for lending.

Figure 7(b) shows the case where the CBDC rate is higher than the deposit rate when there is no CBDC but still not so high that $\rho^* > \hat{\rho}_R$. Now the red curve intersects the dashed black curve in its flat region. CBDC induces more lending through two effects. First, it reduces the cost of holding reserves. Second, it introduces more competition to the deposit market. In Figure 7(a) and Figure 7(b), we have $\rho_R^* > \hat{\rho}_R$, and a higher CBDC rate promotes lending. Moreover, CBDC is not used in the equilibrium.

If the CBDC rate is sufficiently high, then $\rho^* < \hat{\rho}_R$. The equilibrium is shown in Figure 7(c). The red curve intersects the black dashed line in its vertical region. Now the CBDC is used by households for payment. And the equilibrium lending rate equals $\hat{\rho}_R$. In this case, the CBDC is too attractive, and bankers hold the CBDC instead of lending to entrepreneurs. As a result, lending shrinks. In addition, higher i_m increases $\hat{\rho}_R$ and decreases bank lending.

Notice that all the analysis above assumes that except for the CBDC, bankers can hold only money as reserves. If bankers can hold other types of central bank reserves that pay interest, all the above analysis stays valid if we replace $1/\mu$ everywhere with $(1 + i_r)/\mu$, where i_r is the interest rate on central bank reserves. In other words, the above analysis can be viewed as a special case where $i_r = 0$.

4.4 Implications of Other Designs of CBDC

Suppose a CBDC is still interest bearing, but is a perfect substitute for cash, i.e., it can be used in type 1 and type 3 meetings. Again, we focus on the cases in Figure 1 where L^d intersects L^s in its strictly increasing region. If the nominal interest rate that the CBDC pays is above 0, then it dominates cash for households, so households do not hold cash.

If the CBDC serves as reserves, a higher CBDC rate can potentially have two effects: a cost reduction effect and a substitution effect. The cost reduction effect is present because a higher CBDC rate reduces the bankers' cost of holding reserves.

This effect increases lending. The substitution effect can be present because a higher CBDC rate can reduce the demand for deposits by making the CBDC more attractive. This reduces bank deposits and lending. The cost reduction effect is always present, while the substitution effect is active if and only if the CBDC and deposits are substitutes, which happens if and only if households are constrained in type 3 meetings. The net effect of a higher CBDC rate on lending depends on which of the two effects dominates and hence is in general ambiguous. But if households are not constrained in type 3 meetings, a higher CBDC rate increases lending.

If the CBDC does not serve as reserves, only the substitution effect can be present. Then a higher CBDC rate weakly reduces lending.

5 Quantitative Analysis

In the previous section, we establish theoretically that an interest-bearing CBDC can increase bank lending if the interest rate is in a certain range. However, it remains an empirical question how large the range is. This is crucial for policy decisions. To answer this question, we calibrate our model to the US data and then conduct a counterfactual analysis to evaluate (1) how large the relevant range of interest is; (2) how much additional lending can be created by the CBDC; and (3) what the effect on output and welfare is.

We consider an annual model. First, we parametrize the model such that $U(x) = B \log x$, $u(y) = y^{1-\sigma}/(1-\sigma)$, and $f(k) = Ak^\eta$. We first set $\beta = 0.96$ and $\mu = 1.02$. We do not have a good measure of the deposit handling cost c . As a benchmark, we choose $c = 0$, under which CBDC has the maximum benefit. The reserve requirement ratio χ is set to be 0.1 to match the regulation in the US. We also set $\eta = 0.66$ to match the elasticity of commercial loans with respect to the prime rate. Then the remaining unknowns are $(A, B, N, \alpha_1, \alpha_2, \alpha_3, \sigma)$. To proceed, we first set the sum of all α s to be 0.5 because it is difficult to separately identify the

total DM trading probability and σ (Lagos and Wright 2005). Second, according to the Survey of Consumer Payment Choice, the fraction of online transactions is around 25.37% of total transactions.⁷ Therefore, we set $\alpha_2 = 0.1268$. Lastly, we choose $(A, B, N, \alpha_1, \sigma)$ to jointly match the money demand curve and the ratio of cash holding to checking account balances. The latter is calculated from the Survey of Consumer Payment Choice in 2016. Table 1 summarizes all the results. In particular, we obtain that around 4.4% of transactions accept only cash. Figure 8 shows the money demand curve from the model and the data. In the data, the money demand becomes close to constant if the interest rate is sufficiently high. Our model is able to capture this feature because we have three types of meetings. Intuitively, when inflation is sufficiently high, households become constrained in type 3 meetings. As a result, the money demand becomes less elastic. For more details on data and calibration, see Appendix C.

5.1 Effects of CBDC Lending

Now we consider introducing a CBDC that is a perfect substitute. We consider both the case where the CBDC does not serve as reserves and the case where the CBDC serves as reserves. The growth rate of the CBDC is set to be the same as the fiat money. We then focus on how changes in the CBDC rate affect the real economy as well as the deposit and lending rates. In particular, we are interested in whether the CBDC increases lending and output.

Figure 9 shows the result. The first column displays real variables divided by corresponding equilibrium values without CBDC, as the CBDC rate changes. The second column shows the changes in deposit and loan rates and their difference, i.e., the spread. The blue curve is constructed assuming that the CBDC does not

⁷This is calculated using data from 2016. See Greene and Stavins (2018). We sum up the total number of online transactions, online bill payments, and automatic bill payments. Then we divide this number by the total number of transactions. We have also experimented with data from 2015 and 2017. Results are very similar.

Parameters	Notation	Value	Notes
Calibrated externally			
Discount factor	β	0.96	Standard in literature
Curvature of production	η	0.66	Elasticity of commercial loans
Money growth rate	μ	1.02	2% inflation
Prob. of type 2 meeting	α_2	0.1268	Frac. of online purchase (SCPC)
Cost of handling deposits	c	0.00	Bank expenses/deposits 0%
Reserve requirement	χ	0.1	US regulation
Calibrated internally			
Prob. of type 1 meeting	α_1	0.0222	Cash/Checking ratio
Prob. of type 3 meeting	α_3	0.3509	
Coeff. on CM consumption	B	1.2043	Money demand curve
Curv. of DM consumption	σ	0.1595	Money demand curve
TFP	A	1.1290	Deposit rate 0.05%
Number of banks	N	4	NIM 2.98% (Dempsey 2018)

Table 1: Calibration Results

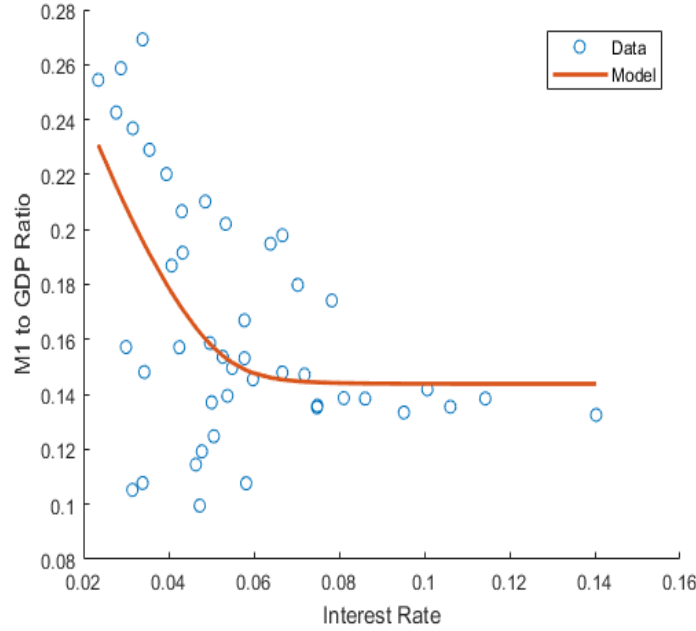


Figure 8: M1 to GDP Ratio versus Interest Rate

serve as reserves and the red curve is under the scenario where the CBDC serves as reserves. Notice all the rates are nominal rates and are in percentages.

Let us first focus on the total amount of loans, which is shown in the second graph in the first column. If i_m is lower than 0, the real rate of the CBDC is below that of fiat money. Therefore, it is not used and the economy behaves as if there were no CBDC. In this region, the blue curve and the red curve overlap. Although it is not clear in the graph, the red curve starts to increase as i_m increases to 0, while the blue curve stays put before i_m goes above some positive value. In this region, the deposit rate without CBDC is still above the CBDC rate. Therefore, the CBDC is not a direct competitor to the deposit. If it does not serve as reserves, it does not change the equilibrium. But if the CBDC serves as reserves, interest on the CBDC reduces the lending cost and increases lending. This is because by the reserve requirement regulation, bankers have to hold reserves for deposits. Without the CBDC, holding the only reserve, which is fiat money, is costly because there is inflation. The CBDC reduces this cost because it pays positive interest. But this effect is very small and therefore not obvious in the graph.

If i_m further increases, the CBDC becomes an attractive competitor to the bank deposits, which forces the bankers to increase their deposit rates, which increases the demand for deposits and bank lending. Notice that in this region, the blue curve again overlaps with the red curve. This is consistent with our theoretic results in the previous sections. In this region, the equilibrium ρ is in $(\hat{\rho}, \check{\rho})$ when the CBDC does not serve as reserves. As a result, the CBDC serving as reserves does not change the equilibrium.

However, if i_m is sufficiently high, then both curves start to decrease. In this case, bankers act as if they were in a perfectly competitive deposit market, and their profit is driven to 0. To compensate for the higher deposit rate, they have to charge a higher lending rate, which reduces lending. In this region, lending increases if the

CBDC serves as reserves. Therefore, the red curve is always above the blue curve. This happens because the CBDC pays positive interest and reduces the lending cost for bankers.

If we set i_m to maximize lending, we can increase lending by 6.82% if the CBDC does not serve as reserves and by 6.99% if the CBDC serves as reserves. Lending increases in i_m if i_m is below around 0.5%. And introducing a CBDC increases lending if its rate is between 0.05% and 2.55% when it does not serve as reserves. When it serves as reserves, introducing a CBDC increases lending if its rate is between 0% and 2.84%.

To summarize, the region of i_m in which CBDC increases bank lending is more than 2.5%. As a result, this lessens the concern that introducing CBDC would reduce lending. In addition, an appropriate i_m can increase bank lending by around 7.0%, which is substantial.

5.2 Effects on Total Output

Now we move to total output, which is shown in the third graph in the first column. The pattern is similar to the loan: as i_m increases, total output first increases and then decreases. Quantitatively, there are two differences. First, the increase in output is much smaller. The highest increase is 1.10% if the CBDC does not serve as reserves and 1.13% if the CBDC serves as reserves; both are achieved at $i_m = 0.5\%$. Second, introducing a CBDC increases total output iff $i_m \in (0.05\%, 1.91\%)$ when it does not serve as reserves and iff $i_m \in (0, 2.09\%)$ when it serves as reserves. These ranges are much smaller compared with the regions where lending increases but these ranges are still more than 1.5%.

The effects on total output are smaller than the effect on lending because there is an off-setting effect of CBDC. If the CBDC becomes more valuable, households substitute out of money and hold more CBDC. As a result, they choose lower money holdings and hence can consume less in type 1 meetings, which reduces total output.

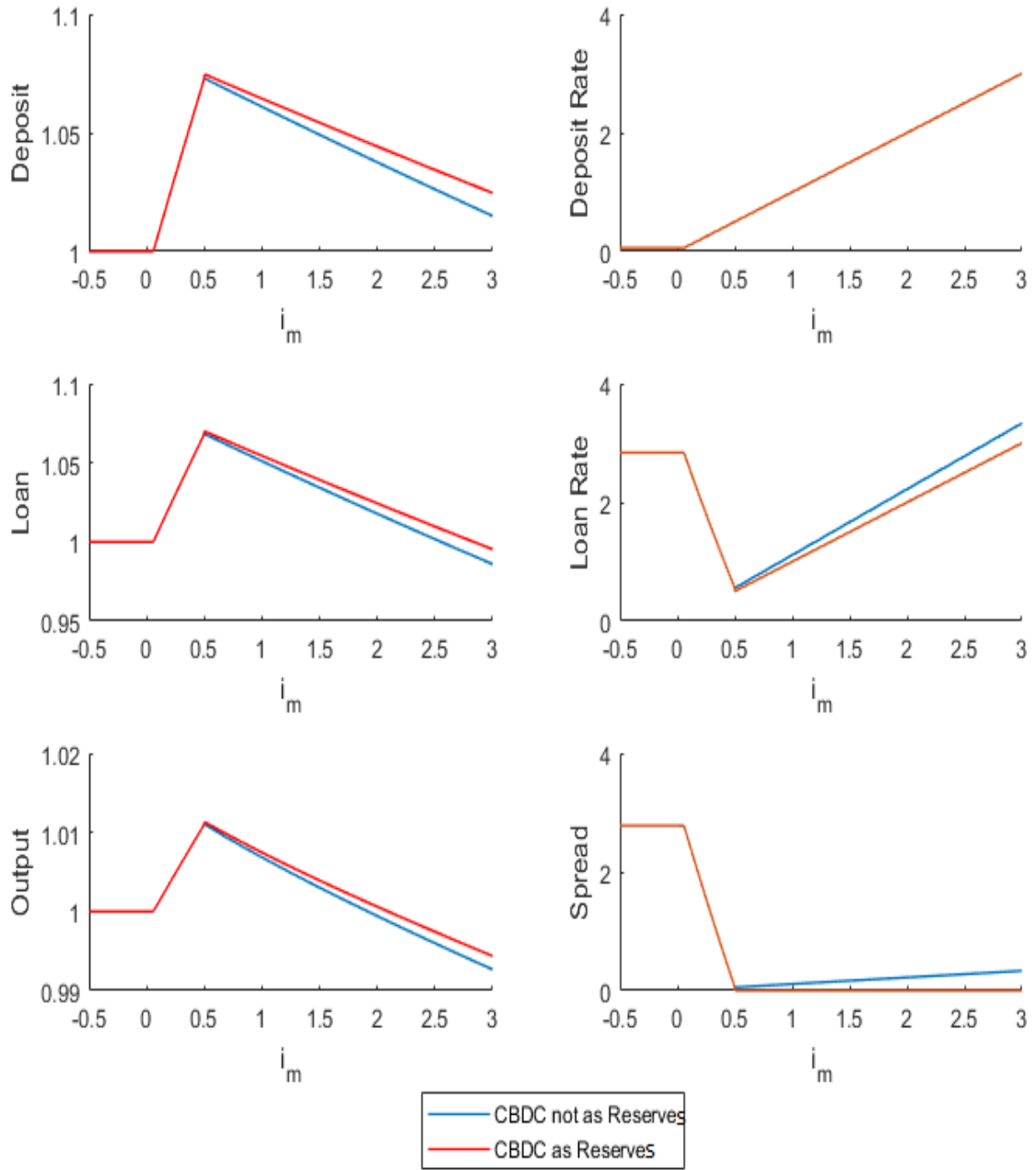


Figure 9: Effects of CBDC Rate

Under the calibrated parameters, this off-setting effect turns out to be large, and as a result total output falls.

5.3 Effects on Interest Rates

The second column of Figure 9 shows how deposits, lending rates, and spread change with the CBDC rate. All these rates are nominal.

The deposit rate is shown in the first panel in the second column. It is constant if i_m is low and then it coincides with the 45°-line. This reflects that the CBDC rate serves as a floor of the deposit rate. It is also worth noting that if i_m is positive but small, the deposit rate increases if the CBDC can be used as reserves. But the effect is too small to discern in the graph.

The loan rate has the inverse pattern of loans, as shown in the second panel. If i_m is set appropriately, the loan rate reduces to less than .56% from around 3% when there is no CBDC. But if i_m is too high, the loan rate can be higher compared with the no CBDC equilibrium.

The spreads, which are the difference between the nominal lending rate and the nominal deposit rate, are shown in the third panel in the second column. The spread is increasing in bankers' market power. CBDC reduces spreads by introducing more competition into the deposit market. If i_m is sufficiently high, bankers act as if the market is perfectly competitive. Then the lending rate equals the marginal cost of lending, which is the cost of maintaining deposits and holding reserves. Interestingly, even if i_m is sufficiently high, the spread increases if the CBDC does not serve as reserves. However, this does not mean that bankers have higher market power as i_m increases. Intuitively, under perfect competition, bankers need the spread to be high enough to compensate for the difference between the deposits rate, which is equal to i_m , and the rate on reserve. If the CBDC does not serve as reserves, this difference increases as the rate on reserve keeps constant at 0. Bankers need to have a higher spread. However, if the CBDC serves as reserves, this difference is constant

and, therefore, the spread keeps constant, as shown by the red curve.

5.4 Effects on Welfare

We end this section with a discussion on welfare. Figure 10 shows changes in welfare for buyers, entrepreneurs, and bankers. We do not show the effect on sellers because their welfare is not affected by CBDC.⁸ Welfare is measured as the percentage change in the equilibrium consumption without CBDC that makes an agent indifferent between no CBDC and the CBDC with interest rate i_m . If it is positive, CBDC increases the welfare of the agent. Otherwise, CBDC reduces the welfare. All the y-axes are in percentages.

CBDC can have a positive effect on buyers that increases in i_m on the range in concern. Without hurting lending, buyers' surplus can be raised by around 1%. Entrepreneurs highly benefit from CBDC. Their maximum welfare gain is about 4.5%. CBDC improves their welfare as long as i_m does not exceed 2.55% if the CBDC does not serve as reserves and does not exceed 2.84% if it serves as reserves. Intuitively, entrepreneurs benefit directly from a lower lending rate. The buyers benefit from a higher deposit rate but suffer from less valuable money. Therefore, the welfare gain for the buyers is much smaller.

Bankers lose because CBDC introduces more competition into the deposit market. If i_m is sufficiently high, bankers behave as if the market is perfectly competitive. Their profit and hence consumption is reduced to 0, which is a 100% reduction compared with the equilibrium without CBDC.

5.5 Robustness

The quantitative results depend crucially on the value of α_1 , α_2 , and α_3 . In this subsection, we consider an alternative method to calibrate these three parameters

⁸This is because we assume that buyers make take-it-or-leave-it offers in the DM. Therefore, sellers do not get a surplus from the DM transactions.

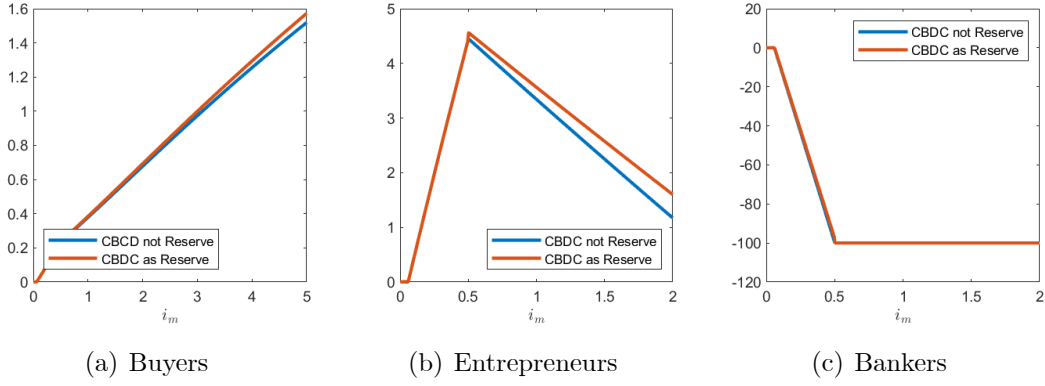


Figure 10: Welfare Change for Each Type

Parameters	Notation	Value	Notes
Calibrated externally			
Discount factor	β	0.96	Standard in literature
Curvature of production	η	0.66	Elasticity of commercial loans
Money growth rate	μ	1.02	2% inflation
Prob. of type 1 meeting	α_1	0.075	Diary data
Prob. of type 2 meeting	α_2	0.01	Diary data
Prob. of type 3 meeting	α_3	0.415	Diary data
Cost of handling deposits	c	0.03	Bank expenses/deposits 3%
Reserve requirement	χ	0.1	US regulation
Calibrated internally			
Coeff. on CM consumption	B	1.73	Money demand curve
Curv. of DM consumption	σ	0.2147	Money demand curve
TFP	A	0.4855	Deposit rate 0.05%
Number of banks	N	1	NIM 2.98% (Dempsey 2018)

Table 2: Alternative Calibration Results

and assess the robustness of our results. For this calibration, we rely on the Diary of Consumer Payment Choice data from the Federal Reserve Bank of Atlanta website. In the diary, respondents are asked whether cash and credit/debit card would have been accepted in each transaction. From 2016 data, people report that around 15% of transactions do not accept credit/debit card and around 2% of transactions do not accept cash. Assuming that every transaction accepts at least cash or cards, the above number implies that $\alpha_1 = 0.075$, $\alpha_2 = 0.01$, and $\alpha_3 = 0.415$. Notice that we maintain the assumption that the sum of α s equals 0.5.⁹ The parameters are shown in Table 2. Notice that this result is very different from the benchmark calibration. In particular, 15% of transactions accept only cash compared with 4.4% in the benchmark case. And 2% of transactions accept only deposits compared with 25.37% in the benchmark case. Therefore, the market power of banks is much lower in this case because deposits are not as useful. Given that the choices of α s are very different from the benchmark, it is not surprising that other parameters are very different. This shows that the parameters can be sensitive to the value of α s. However, it is worth noting that the benchmark calibration fits the money demand curve better.

Given that the parameters are very different from the benchmark case, the effects on most equilibrium objects, which is shown in Figure 11, are different from the benchmark case. However, it is interesting to point out that it remains valid that a wide range of i_m increases lending. More specifically, if the CBDC does not serve as reserves, any $i_m \in (0.05\%, 1.5\%)$ increases lending. If the CBDC serves as reserves, $i_m \in (0.05\%, 1.73\%)$ increases lending. And the maximum increase can be around 5%. This suggests that the conclusion that there is a wide range of i_m at which CBDC can increase lending is relatively robust to the calibration method.

⁹See Consumer Payment Research Centre (2018). These numbers are slightly different for 2017. See Foster (2018).

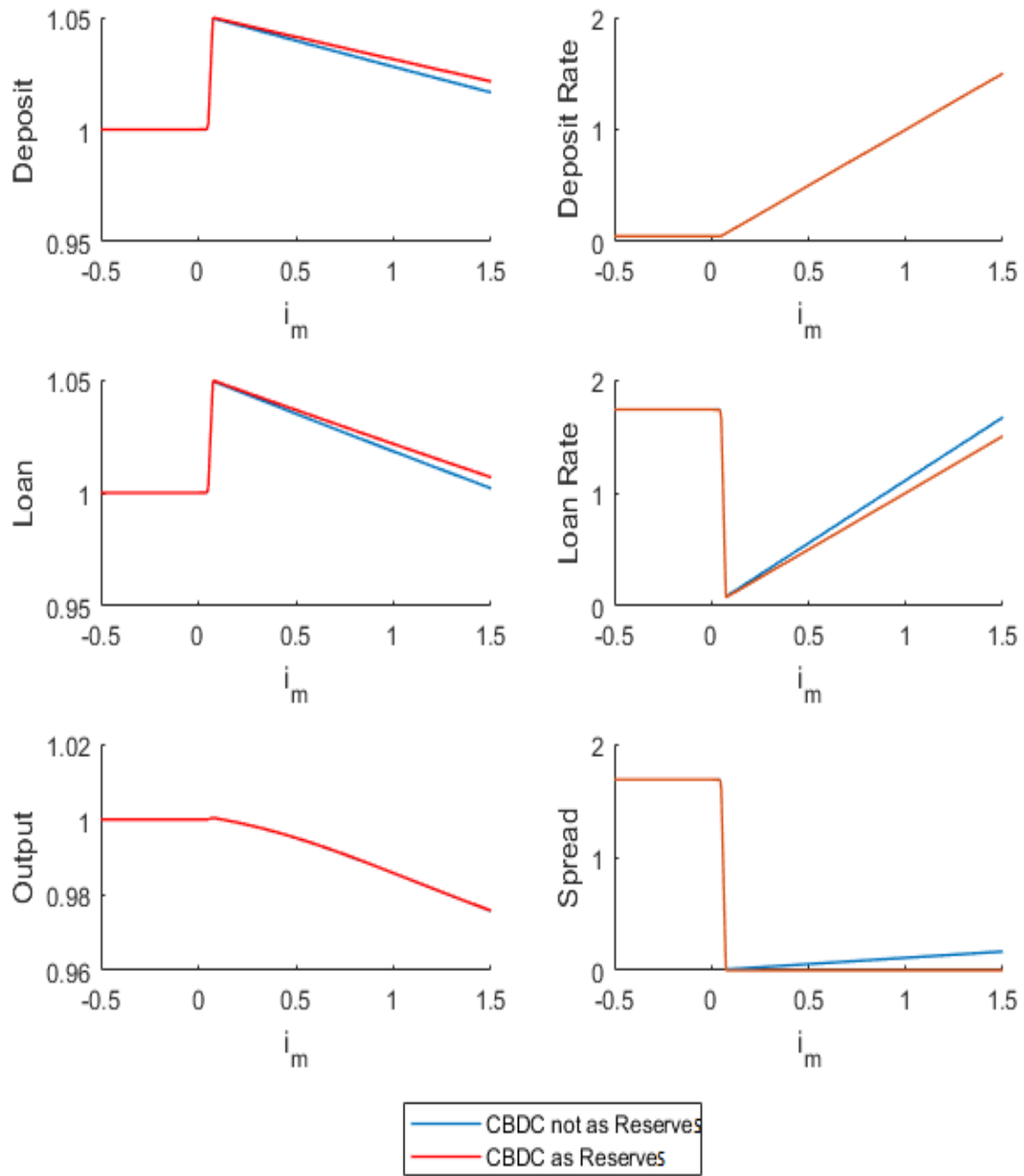


Figure 11: Effects of CBDC Rate: Alternative Calibration

6 Conclusion

Our paper develops a model with imperfect competition in the deposit market. We use the model to analyze whether introducing a CBDC causes disintermediation of banks. Contrary to common wisdom, we show that an interest-bearing CBDC can improve bank intermediation. Intuitively, if banks have market power, they would restrict the deposit supply to lower the deposit rate. An interest-bearing CBDC reduces their market powers by setting a floor for the deposit rate, which leads to the creation of more deposits. Banks then lend more because they have more funding. This can raise total lending and output. However, more intermediation happens only if the interest rate on the CBDC is set properly. In particular, if the CBDC rate is too low, the CBDC does not affect the equilibrium. If the rate is too high, the CBDC causes disintermediation. The CBDC only helps if its interest rate is in some intermediate range.

We abstract from many important issues, such as endogenous decisions of banks on the composition of both the asset side and the liability side of their balance sheets in terms of risk and maturity, and how these endogenous decisions may have macroeconomic and financial stability implications. More specifically, introducing an interest-bearing CBDC would increase banks' funding costs. On the asset side of their balance sheets, it may induce banks to take more risk to make up for their lower profit margin. This can increase the total risk to the financial system, leading to a less stable system. On the liability side, it can induce banks to switch to other sources, such as wholesale funding. These funding sources are generally considered less stable than deposits. Therefore, more reliance of banks on wholesale funding may increase the likelihood of runs in the wholesale market. These issues are all left for future research.

Appendices

A Multiplicity under Perfect Competition

Now we show that this model can have multiple equilibria even under perfect competition in both the deposit and loan markets. Set $\alpha_3 = 0$, which implies money and deposit dichotomize. In addition, normalize the measure of firms to be 1. Assuming that ι is sufficiently small, in the equilibrium, real balances are determined by

$$\iota = \alpha_1 \lambda(Z). \quad (17)$$

Given ψ , the demand for deposit is determined by

$$\psi = \alpha_2 \beta \lambda(D) + \beta. \quad (18)$$

Notice that ψ can never go below β ; if $\psi = \beta$, then the demand for D can be any value lying between D^* and ∞ ; if $\psi < \beta$, then $D = \infty$.

Suppose there is a continuum of banks with measure 1 and they are price takers. Given ψ and ρ , they solve

$$\max_{\ell_j, d_j} \left\{ \left(1 + \rho - \frac{1}{\mu}\right) \ell_j - \left(1 + \psi c - \frac{\psi}{\mu}\right) d_j \right\} \quad (19)$$

$$\text{st} \quad \ell_j \leq (1 - \chi) \psi d_j. \quad (20)$$

As long as $\psi < (1/\mu - c)^{-1}$, the constraint is binding and the problem reduces to

$$\max_{d_j} \left\{ \left[(1 + \rho)(1 - \chi) + \frac{\chi}{\mu} - c \right] \psi d_j - d_j \right\}. \quad (21)$$

Because $\rho = \varrho(L) \equiv f'(L) - 1$, the optimization problem along with the constraint imply

$$\{1 + \varrho[(1 - \chi) \psi D]\} (1 - \chi) + \frac{\chi}{\mu} - c = 1/\psi. \quad (22)$$

Given ψ , this equation defines D as a function of ψ : $D = \Delta(\psi)$, which can be non-monotone depending on the curvature of the production function. If $\psi \geq$

$(1/\mu - c)^{-1}$, then the constraint is not binding, which means $\rho = 1/\mu - 1$ and $L = \varrho^{-1}(1/\mu - 1)$. If $\psi > (1/\mu - c)^{-1}$, then $D = \infty$. If $\psi = (1/\mu - c)^{-1}$, then banks are indifferent between any amount of capital. To be consistent with market clearing, $D \geq \varrho^{-1}(1/\mu - 1)(1 - \chi)$. One can show that $\varrho^{-1}(1/\mu - 1)(1 - \chi) = \Delta(\psi)$ if $\psi = (1/\mu - c)^{-1}$. To summarize, the supply for deposit is the following:

$$D = \begin{cases} \Delta(\psi) & \psi < (1/\mu - c)^{-1} \\ \infty & \psi > (1/\mu - c)^{-1} \\ [\Delta(\psi), \infty) & \psi = (1/\mu - c)^{-1} \end{cases}. \quad (23)$$

Any intersection between (18) and (23) determines an equilibrium.

Proposition 7 *Monetary equilibrium exists iff $\iota < \alpha_1 \lambda(0)$ and $\beta(1/\mu - c) \leq 1$.*

Proof. From (17), one can see that $Z > 0$ iff $\iota < \alpha_1 \lambda(0)$. If ψ is sufficiently small, (18) defines $D = \infty$ and if ψ is sufficiently large, D that is sufficiently small. On the other hand, (23) implies that D is finite if ψ is low and $D = \infty$ for ψ that is sufficiently large. Then by continuity, these two curves have at least one intersection. Hence, at least one equilibrium exists. ■

In general, the equilibrium is not unique. We next use numerical examples to illustrate this. To this end, parametrize $f'(l) = A(l + \varepsilon)^{-\xi} \mathbf{1}\{l > \bar{l}\} + Bl^{-\omega} \mathbf{1}\{l \leq \bar{l}\}$. Here, $A, \varepsilon, \bar{l} > 0$ and $\xi > 1, 0 < \omega < 1$ are the parameters to choose. Then B is chosen such that f' is continuous. One can integrate this function and impose $f(0) = 0$ to obtain f . Since f' is positive and strictly decreasing, f is strictly increasing and concave.

Results are shown in ψ - D space in Figure 12. In all graphs, the blue curve is the deposit demand curve and the red curve is the deposit supply curve. The demand curve is monotonically decreasing while the supply curve can be monotonically increasing or non-monotone, depending on the curvature of f . If $f(l) = Al^\omega$ with $\omega < 1$, the supply curve is increasing as in Figures 12(a) and 12(b). In this case, we have a unique equilibrium. If, however, $f(l) = A \left[(l + \varepsilon)^{1-\xi} - \varepsilon^{1-\xi} \right] / (1 - \xi)$ with

$\xi > 1$ and ε that is sufficiently small, the supply curve is decreasing for the most part if $\psi < (1/\mu - c)^{-1}$. This is shown in 12(c). In this case, we have three equilibria. One has $\psi = \beta$, i.e., the deposit does not carry a liquidity premium. One has $\rho = 1/\mu - 1$ and $\psi = (1/\mu - c)^{-1}$. In this case, the price for deposit is sufficiently high that the banks are willing to take any amount of deposits and then hold them in cash reserves. Notice at this intersection, $D > \Delta(\psi)$. Consequently, the reserve requirement constraint is not binding, i.e., banks hold voluntary reserves. There is another equilibrium where the reserve requirement constraint is binding.

If f is a combination of the previous two cases, the supply curve can be non-monotone even if $\psi < (1/\mu - c)^{-1}$, as shown in Figure 12(d). In this case, there are two equilibria where the reserve requirement constraint is binding.

B Imperfect Competition in Lending Market

Now consider the extension that there is a competitive interbank market and the lending market features imperfect competition. We consider two cases: a Cournot lending market, and a search and matching market. In both cases, the deposit market has Cournot competition as in the previous section.

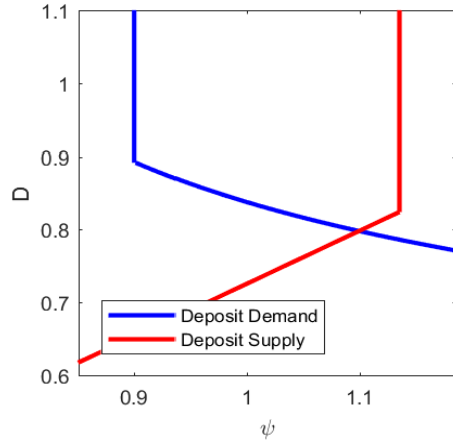
B.1 Cournot Lending Market

Now let ρ^B be the real rate in the interbank market. Then the loan supply function L^s is the same as before except that now it depends on the interbank rate ρ^B . The loan makers solve

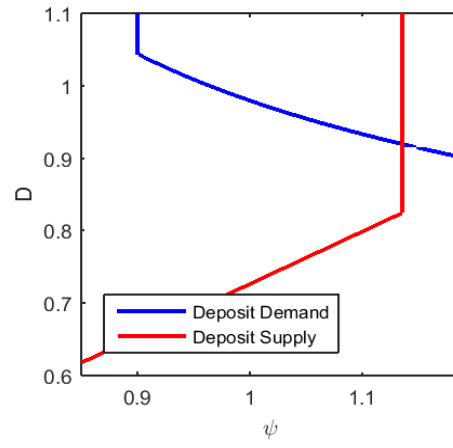
$$\max_{\ell_j} f' \left(\frac{\ell_{-j} + \ell_j}{N^E} \right) \ell_j - (1 + \rho^B) \ell_j.$$

To guarantee the existence of a pure strategy equilibrium given ρ^B on the loan side, we require the following condition:

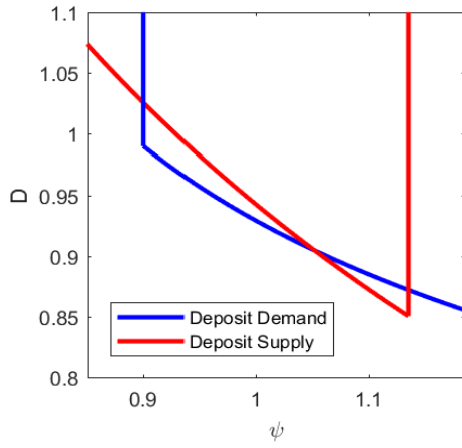
Condition 2 $f''(L) + f'''(L) L \leq 0$.



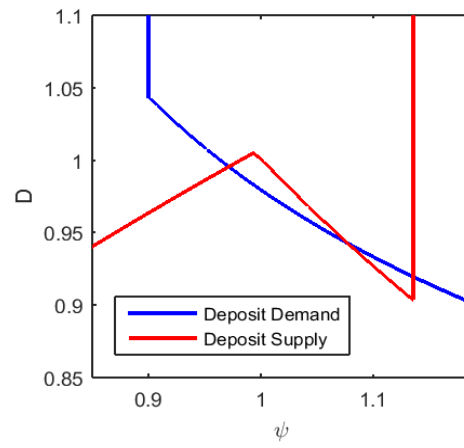
(a) Case 1



(b) Case 2



(c) Case 3



(d) Case 4

Figure 12: Equilibrium

Then the equilibrium satisfies

$$f''\left(\frac{L}{N^E}\right) \frac{L}{N^E N} + f'\left(\frac{L}{N^E}\right) = 1 + \rho^B.$$

This defines $L^d(\rho^B)$, which is decreasing and continuous in ρ^B . Now the equilibrium interbank market rate is determined by $L^s(\rho^B) = L^d(\rho^B)$. Then the loan rate is given by $\rho = f'(L^s(\rho^B)) - 1$.

Proposition 8 *If $\iota < \bar{\iota}$ and Assumptions 1 and 2 hold, there exist one or three monetary equilibria. If, in addition, $\Psi(D)$ is increasing, the monetary equilibrium is unique.*

Then comparative statics can be analyzed as before.

B.2 Search for Loans

Now suppose that each bank has a continuum of loan officers with measure N_l . They have access to a competitive interbank market and randomly search and match with firms. The matching probability is $\alpha(\lambda)$, where $\lambda = N_f/N_l N_b$. Upon a meeting, the loan officer bargains with the firm on the terms of loans given the interbank market rate ρ . The surplus is split with Kalai's bargaining solution, where the firm has a bargaining power η :

$$\begin{aligned} & \max_{\ell, p} f(\ell) - p \\ \text{st} \quad & f(\ell) - p = \eta [f(\ell) - (1 + \rho^B) \ell]. \end{aligned}$$

In this case, $f'(\ell) = \ell + \rho^B$ and $p = (1 - \eta) f(\ell) + (1 + \rho^B) \ell$. This implies the loan rate,

$$\rho^E = \eta \rho^B + (1 - \eta) \left[\frac{f(\ell)}{\ell} - 1 \right],$$

which is a weighted sum of the interbank market rate ρ^B and the average investment return $f(\ell)/\ell - 1$. Then the loan demand curve is

$$L^d(\rho^B) = \alpha(\lambda) N_l N_b f'^{-1}(1 + \rho^B).$$

Then the equilibrium ρ^B is determined in the same fashion as in the competitive loan market case. Consequently, we have the following proposition:

Proposition 9 *If $\iota < \bar{\iota}$ and Assumption 1 holds, there exist one or three monetary equilibria. If, in addition, $\Psi(D)$ is increasing, the monetary equilibrium is unique.*

Notice that in this case, the total supply of loans is efficient given the interbank market rate ρ^B . However, the bankers get a positive surplus from lending. We can then calculate the spread between the interbank rate and lending rate as $\rho^E - \rho^B = (1 - \eta) \left[\frac{f(\ell)}{\ell} - 1 - \rho^B \right]$.

C Calibration Method and Data

The data we use are from FRED on the website of the Federal Reserve Bank of St. Louis. For money demand, we use the data series of M1 to GDP ratio. For interest rate, we use the annualized three-month T-bill rate. For both, we use annual data from 1959 to 2006. We exclude three years after 9/11 and all the data after the financial crisis because the nominal rates are almost 0 in these years. FRED lists nominal interest on non-jumbo deposits after the year 2009. The average annual rate on checking accounts is less than 0.05% and that on saving accounts is around 0.09%. This is consistent with the Survey of Consumer Payment Choice data, where more than 80% of respondents report checking account rates below or at 0.05%. (See Consumer Payment Research Center (2018) in the reference list.) We then use 0.05% in our benchmark calibration. Data on cash holding, checking account balances and the fraction of online payments are all based on the 2016 Survey of Consumer Payment Choices from the Federal Reserve Bank of Atlanta.

One straightforward way to calibrate the model is to solve the whole equilibrium given the parameter and then fit the money demand curve and the deposit rates. This, however, can be computationally cumbersome because one needs to solve the

model for each data point used for the money demand and then optimize over a five-dimensional parameter. One key insight is that the money demand can be solved independent of the banking sector given the deposit rate. This leads to the following algorithm that greatly simplifies the calibration.

1. Fit the money demand and cash to the checking account balance ratio by choosing B , α_1 , and σ given A . More specifically, take the following steps:

- (a) Given α_1 , for each interest rate, calculate the steady state equilibrium using the nominal interest rate and the deposit rate. Then solve a non-linear least squares problem over (B, σ) . The M1 to GDP ratio in the model can be calculated by $(Z + D) / \mathbf{Y}$, where

$$\mathbf{Y} = \sum_{j=1}^3 \alpha_j y_j + 2B + A [(1 - \chi) D \Psi(D)]^\eta - D + (1 - \chi) D \Psi(D)$$

is the output. Here \mathbf{Y} is the sum of the consumption of households in DM and CM, the consumption of bankers and entrepreneurs, and the investment.

- (b) Set α_1 to match the cash-checking ratio 5.53%.
 - (c) Update α_1 and iterate (a) and (b) until convergence.
2. Given B , α_1 , and σ , find A and N such that the solution of the Cournot competition leads to a 0.05% rate on deposit and the net interest margin is 2.98%.
 3. Update A and repeat 1 and 2 until convergence.

This greatly simplifies the calibration because we reduce a problem of five parameters into two problems of lower dimensions that are linked by A . It is also worth noting that if we assume that bankers and entrepreneurs transfer all the profit back to the households, \mathbf{Y} can be computed independently of A . In that case, we only need to do 1 and 2 once to recover all the unknown parameters.

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Do Bank Runs Exist in the Diamond-Dybvig Model?

by

TEH-MING HUO* and MIN-TEH YU*

This paper explores the condition for the existence of a bank-run equilibrium in the Diamond-Dybvig banking model. (JEL: G 21, G 33)

1. Introduction

The seminal work of DIAMOND and DYBVIG [1983] has become a standard reference in the literature of banking. They have shown that, in an environment characterized by incomplete information, bank demand deposit contracts could be used to implement the social optimal allocation attained only under full information. Given such contracts, they also argued that there is another (undesirable) equilibrium in the form of a bank run. This equilibrium occurs because agents expect the bank is going to fail, and henceforth it fails.

The modeling approach of Diamond and Dybvig has been subject to numerous extensions, yet, as pointed out by POSTLEWAITE and VIVES [1987, 485]:

“... a potential problem with this approach is that bank runs should not be observed in equilibrium since no one would deposit anticipating a run. One way around this, as suggested by Diamond and Dybvig, is to suppose that the equilibrium is selected depending on a publicly observable random variable *such as sunspots*. Then agents may still deposit in the bank provided the probability of a run is small enough.” (*italic added*)

The purpose of this paper is to point out an inherent difficulty in the original Diamond-Dybvig model in which a bank-run equilibrium will not emerge as a second (pure strategy Nash) equilibrium. We will also propose a modification of their original model so that such an equilibrium will exist and the sunspot interpretation is meaningful.

* We thank Douglas W. Diamond, J. Huston McCulloch and an anonymous referee for comments. Any remaining errors are our own responsibility. Please send all correspondence to Prof. Min-Teh Yu, Department of Finance, National Central University, Chung-Li, Taiwan.

2. The Diamond-Dybvig Model

The model consists of three periods: a planning period 0, and two subsequent consumption periods 1 and 2. A single homogeneous consumption good exists at each date. All agents are endowed with one unit of the good at the planning period.

A. Production

The consumption good can be invested in a production process that yields $R > 1$ units of output in period 2 for each unit in period 0. If production is interrupted in period 1, each unit invested may be salvaged on a one-for-one basis. Once interrupted, the production process cannot be resumed. R is a constant known to all at period 0. Across periods, individuals may store the consumption good at zero cost.

B. Preferences

All agents are identical at period 0, but may differ thereafter. At period 1, a fraction t of agents discover themselves to be of one type (type-1) and the remaining fraction $1 - t$ find themselves to be of a second type (type-2). Type-1 agents (early diers) wish to consume only in period 1, and type-2 agents (late diers) wish to consume in periods 1 and 2. Each agent j has a state-dependent utility function of the form

$$U(c_1, c_2; \Theta) = \begin{cases} u(c_1) & \text{if } j \text{ is of type-1 in state } \Theta \\ \rho u(c_1 + c_2) & \text{if } j \text{ is of type-2 in state } \Theta, \end{cases}$$

where $u: R_+ \rightarrow R^1$ is twice continuously differentiable, increasing, strictly concave, and satisfies Inada conditions. Also, it is assumed that

$$(1) \quad 1 \geq \rho > \frac{1}{R} \quad \text{and} \quad -\frac{cu''(c)}{u'(c)} > 1, \quad c \geq 0.$$

C. Full Information

When types are publicly observable as of period 1, it would be possible to write an optimal insurance contract. Denoting by c_k^i the consumption in period k of an agent who is of type- i , contract gives rise to the optimal consumption $\{c_k^i\}^*$ satisfying

$$(2) \quad c_1^{2*} = c_2^{1*} = 0,$$

$$(3) \quad u'(c_1^{1*}) = \rho R u'(c_2^{2*}),$$

¹ R_+ denotes the nonnegative real line.

$$(4) \quad tc_1^{1*} + \frac{(1-t)c_2^{2*}}{R} = 1.$$

From (1)–(4), it is well known that the optimal consumption satisfies $R > c_2^{2*} > c_1^{1*} > 1$.

D. Incomplete Information

When types are private information and there are no credible ways of revealing one's type, two possible scenarios may arise. One scenario is that each agent produces only for his own consumption with the consumption pattern $c_1^1 = 1$, $c_2^1 = c_1^2 = 0$, and $c_2^2 = R$. Clearly, this is dominated by the optimal consumption as characterized by (2)–(4).

Another scenario is characterized by the existence of a bank. The bank offers a demand deposit contract that requires an initial deposit at period 0 in exchange for the right to withdraw per unit of investment (at the discretion of the depositors and conditional on the solvency of the bank) either r_1 in period 1 or $R(1 - r_1f)/(1 - f)$ in period 2, where f is the fraction of agents who withdraw in period 1. By setting $r_1 = c_1^{1*}$ and $f = t$, Diamond and Dybvig have shown that the optimal consumption in (2)–(4) can be implemented by this demand deposit contract. However, given the demand deposit contract, the bank does not have sufficient funds to meet the withdrawal demands of all depositors in period 1. This gives type-2 agents an incentive to withdraw early in period 1. If, as claimed by Diamond and Dybvig, there are enough withdrawals by type-2 agents, the bank fails and hence a bank run occurs.

3. The Possibility of Bank Runs

We now consider whether the bank run can indeed occur as a pure strategy Nash equilibrium. Before we do so, it is noted that the class of utility functions $u(\cdot)$ satisfying (1) implies that $cu'(c)$ is strictly decreasing in c . It follows that $\lim_{c \rightarrow 0} cu'(c) > 0$ and $\lim_{c \rightarrow 0} u(c) = -\infty$, i.e., the utility functions are unbounded below at $c = 0$.² If a bank run occurs, some of the agents will be forced to consume nothing. But, agents' preferences must be well defined for consumption allocations in such an equilibrium. That is, the von Neumann-Morgenstern expected utility functions representing underlying preferences must take values in real numbers. Yet the class of utility functions in (1) does not satisfy this

² For a formal proof of the last assertion, see OBSTFELD and ROGOFF [1983] and HUO [1992]. An example of such preferences is the constant-relative-risk-aversion utility function

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1, c \geq 0.$$

requirement at $c = 0$. Therefore, a bank run equilibrium cannot exist.³ All these can be explained in a more intuitive manner.

Assuming the sequential service constraint and the deposit payoff structure, i.e., $r_1 = c_1^*$ and $f = t$, offered by a bank at date 0, those who withdraw at date 1 may face two possibilities. On the one hand, all late diers (type-2 agents) may refrain from withdrawing, the bank distributes promised payoffs at dates 1 and 2. This is a non-run equilibrium. On the other hand, if some late diers pretend to be early diers (type-1 agents) and withdraw at date 1, then a positive fraction of all depositors withdrawing at date 1 will not be able to do so, and will be forced to consume at zero level. If those who consume nothing at date 1 include early diers, then all agents, after making an expected utility calculation at date 0, will not join the bank. Hence, no banks will be formed at date 0 and bank runs cannot happen. If those who consume nothing at date 1 include late diers, then those late diers will again consume nothing at date 2, since the bank has already exhausted all its resources. Again, this implies that no one will join the bank at date 0. Therefore, if we take the position that outcomes must match anticipations, bank runs cannot happen under the preference structure assumed in (1).

3. A Modification

The foregoing conclusion of the impossibility of bank runs predisposes the following query: under what condition(s) will a demand deposit contract be able to implement the full information social optimal allocation, as characterized by equations (2)–(4), and allow for possible bank runs? Fortunately, such a query could be readily answered without altering much of the original construct of Diamond and Dybvig. Let the preferences be represented by

$$U(c_1, c_2; \Theta) = \begin{cases} u(c_1) & \text{if } j \text{ is of type-1 in state } \Theta \\ \rho u(c_1 + c_2) & \text{if } j \text{ is of type-2 in state } \Theta, \end{cases}$$

where

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad 0 < \gamma < 1, \quad c \geq 0,$$

and

$$1 > \frac{R^\gamma}{R} > \rho > \frac{1}{R}.$$

Then the demand deposit contract (with $r_1 = c_1^*$, $f = t$ under the sequential service constraint) has the potential of implementing the full information social

³ Among the fast-growing literature on banking, the following papers have employed the similar type of preferences as Diamond and Dybvig did: BHATTACHARYA and GALE [1987], CHARI [1989], ENGINEER [1989], FREEMAN [1988], LOEWY [1991], SMITH [1984], [1991], WALLACE [1988], [1990].

optimal allocation as a pure strategy Nash equilibrium,⁴ while another equilibrium of bank runs is still possible.

Finally, one cannot overemphasize the importance of requiring the demand deposit contract implementing the (unique) social optimal allocation.⁵ One can easily construct a demand deposit contract in their original model that is subject to runs, and all resulting consumption allocations are not Pareto optimal.⁶ Suppose that all agents deposit $0 < \alpha < 1$ of their wealth in the bank and retain $1 - \alpha$, then the floor on date 1 consumption is $1 - \alpha$, and on date 2 consumption is $R(1 - \alpha)$. This allocation is clearly nonoptimal, but will keep consumptions above zero with certainty, and will allow for a chance of a bank run even if $-cu''(c)/u'(c) > 1$.⁷ One can also alter the environment slightly, as WALLACE [1988] did, in allowing for the suspension of payments as part of the demand deposit contract, then it is possible to achieve the social optimal allocation with positive autarkic investment.

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⁴ Proof: given that t is known, it is sufficient to show $1 < c_1^*(t) < c_2^*(t) < R$ under the new assumption. The optimal consumption plan $[c^* = c^*(t)]$ should satisfy (2)–(4). Note that, $\rho R > 1$ is sufficient to ensure that $c_1^*(t) < c_2^*(t)$. It suffices to show that $c_1^*(t) > 1$. This follows as $\rho Ru'(R) < u'(1)$. For the utility functions and the number γ in the text, this is true since $\rho R^{1-\gamma} < 1$.

⁵ This was also the perspective of Diamond and Dybvig. After all, as a minimum requirement, why would anyone be interested in a contract that cannot implement the social optimal allocation?

⁶ The following example is due to D. Diamond.

⁷ Given the demand deposit contract specified earlier, not only the social optimal allocation cannot be implemented, but there is a possibility of disintermediation, i.e., no banks formed at date 0, as noted in MCCULLOCH and YU [1990].

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