

CBA: Practice Problem Set 5
Topics: Confidence Intervals for Proportions

1. The Java computer language, developed by Sun Microsystems, has the advantage that its programs can run on types of hardware ranging from mainframe computers all the way down to handheld computing devices or even smart phones. A test of 100 randomly selected programmers revealed that 71 preferred Java to their other most used computer languages. Construct a 95% confidence interval for the proportion of all programmers in the population from which the sample was selected who prefer Java.

Ans: confidence interval for a population proportion.

$n=100$ (Sample size, Number of trials)

$x=71$ (Number of success).

$1-\alpha=0.95$ (Confidence level).

If n is large, the distribution of $z = \frac{x - np}{\sqrt{np(1-p)}} = \frac{p^{\wedge} - p}{\frac{\sqrt{p^{\wedge}(1-p)}}{n}}$ is approximately standard normal

$$CI = P^{\wedge} \pm z_{\alpha/2} * \sqrt{\frac{P^{\wedge} * (1 - P^{\wedge})}{n}}$$

$$CI = 0.71 \pm 1.96 * \sqrt{\frac{0.71 * (1 - 0.71)}{100}}$$

$$CI = 0.71 \pm 1.96 * \sqrt{\frac{0.71 * 0.29}{100}}$$

$$CI = 0.71 \pm 1.96 * \sqrt{\frac{0.2059}{100}}$$

$$CI = 0.71 \pm 1.96 * \sqrt{0.002059}$$

$$CI = 0.71 \pm 1.96 * 0.0454$$

$$CI = 0.71 \pm 0.089$$

$$CI = (0.71 - 0.089, 0.71 + 0.089)$$

$$CI = (0.621, 0.799)$$

Conclusion: We are 95% sure that interval [0.621, 0.799] contains the true population proportion of all programmers in the population from which sample was selected who prefer Java.

2. A small British computer-game firm, Eidos Interactive PLC, stunned the U.S.- and Japan-dominated market for computer games when it introduced Lara Croft, an Indiana Jones-like adventuress. The successful product took two years to develop. One problem was whether Lara should have a swinging ponytail, which was decided after taking a poll. If in a random sample of 200 computer-game enthusiasts, 161 thought she should have a swinging ponytail (a computer programmer's nightmare to design), construct a 95% confidence interval for the proportion of enthusiasts who would like here to have a swinging ponytail, in this market. If the decision to incur the high additional programming cost was to be made if $p > 0.90$, was the right decision made (when Eidos went ahead with the ponytail)?

Ans: Sample size $N = 200$

$$\text{Sample proportion} = p^{\wedge} x/N = 161/200 = 0.805$$

$$\text{As here } Np^{\wedge} = 200(0.805) = 161 > 10$$

$$N(1-p^{\wedge}) = 200(1 - 0.805) = 39 > 10$$

So conditions of normal distribution are met here hence we can use Z test to find required CI.

$$\text{CI for population proportion } p = p^{\wedge} \pm (Z_{\alpha/2}) \sqrt{\frac{P^{\wedge}(1-P^{\wedge})}{N}}$$

$$\text{For 95\% CI } Z_{\alpha/2} = 1.96$$

$$\text{So 95 \% Ci for population proportion } p = 0.805 \pm (1.96) \sqrt{\frac{(0.805) (1-0.805)}{200}}$$

$$= 0.805 \pm 0.055$$

$$= 0.805 - 0.055, 0.805 + 0.055$$

$$= (0.750, 0.860)$$

It means here 95% confidence population proportion lies between 0.750 – 0.860

Since 95% CI contain all values less than 0.90 so there is no sufficient evidence to conclude that population proportion $p > 0.90$ **Hence No it was not right Decision.**

3. According to a survey published in the *Financial Times*, 56% of executives at Britain's top 500 companies are less willing than they had been five years ago to sacrifice their family lifestyle for their career. If the survey consisted of a random sample of 40 executives, give a 95% confidence interval for the proportion of executives less willing to sacrifice their family lifestyle.

Ans: $p = 56\% = 0.56$

$$n = 40$$

$$95\% \text{ CI} \Rightarrow \alpha = 0.05$$

$$\text{Critical value, } Z_{\alpha/2} = 1.96$$

$$\text{CI} = p \pm Z_{\alpha/2} * \sqrt{\frac{P(1-P)}{n}}$$

$$\begin{aligned}
&= 0.56 \pm 1.96 * \sqrt{\frac{0.56(1-0.56)}{40}} \\
&= 0.56 \pm 0.15383 \\
&= (0.40617, 0.71383)
\end{aligned}$$

95% Confidence interval for the proportion of executives less willing to sacrifice their family lifestyle is (0.406, 0.713)

4. A survey of 5,250 business travelers worldwide conducted by OAG Business Travel Lifestyle indicated that 91% of business travelers consider legroom the most important in-flight feature. (Angle of seat recline and food service were second and third, respectively.) Give a 95% confidence interval for the proportion of all business travelers who consider legroom the most important feature.

Ans: Sample proportion $p^{\wedge} = 0.91$

Sample size $n = 5250$

Z critical for 95% confidence level = 1.96

$$\begin{aligned}
CI &= p^{\wedge} \pm z * \sqrt{\frac{(1-p^{\wedge}) * p^{\wedge}}{n}} \\
&= 0.91 \pm 1.96 * \sqrt{\frac{(1-0.91) * 0.91}{5250}} \\
&= 0.91 \pm 1.96 * 0.00395
\end{aligned}$$

95% Confidence Interval= (0.9023, 0.9177)

5. According to *Money*, 60% of men have significant balding by age 50.24 If this finding is based on a random sample of 1,000 men of age 50, give a 95% confidence interval for the proportion of men of 50 who show some balding.

Ans: Sample proportion $p^{\wedge} = 0.60$

Sample size $n = 1000$

Z critical for 95% confidence level = 1.96

$$\begin{aligned}
CI &= p^{\wedge} \pm z * \sqrt{\frac{(1-p^{\wedge}) * p^{\wedge}}{n}} \\
&= 0.60 \pm 1.96 * \sqrt{\frac{(1-0.60) * 0.60}{1000}} \\
&= 0.60 \pm 1.96 * 0.01549
\end{aligned}$$

95% Confidence Interval= (0.5696, 0.6304)