FEM and CFD Theory (ME3180)
Department of Mechanical and Aerospace Engineering
Assignment 1
Instructor: Sachidananda Behera
TA: Anirudh and Amulya
Due Date: 25/11/2023 (11:59 PM)

1D Steady State Heat Conduction Equation

Part 1

Consider a metal rod of length 1 meter with thermal conductivity ($k = 45$ W/m.K) shown in Fig 1. Both of its ends are maintained at a constant temperature of $T_a = 300$ K and $T_b = 400$ K, respectively. Assuming that the heat transfer is only taking place along the length of the rod, solve the steady state 1D heat conduction equation numerically and plot the steady state contour of the temperature distribution along the length of the rod. Use a second-order central difference scheme to discretize your governing partial differential equation.

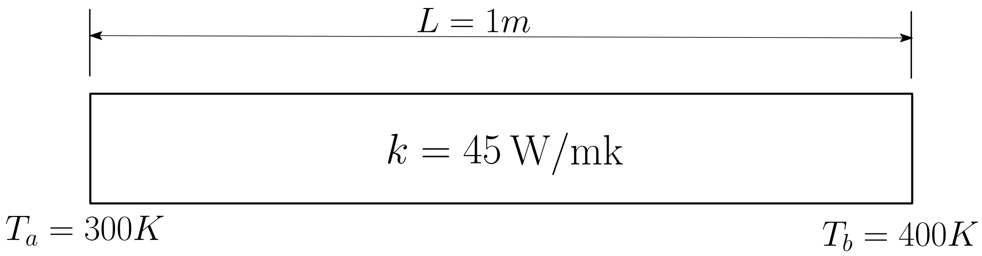

Figure 1: Schematic diagram of the rod

Make a note of the following points:

- Compare your numerical results against the analytical solution by plotting the temperature distribution along the rod in the same graph.
- Divide your computational domain using uniform grids. Use 10 different mesh sizes and show that the numerical error goes down at the same order as the order of accuracy of your discretization scheme.
- Solve the problem using a non-uniform grid and compare it against the results obtained using uniform grids. Give your comments on the results.
- Use Jacobi, Gauss-Seidel, and TDMA methods to solve the system of equations you obtained after discretizing the governing PDE.

Part 2

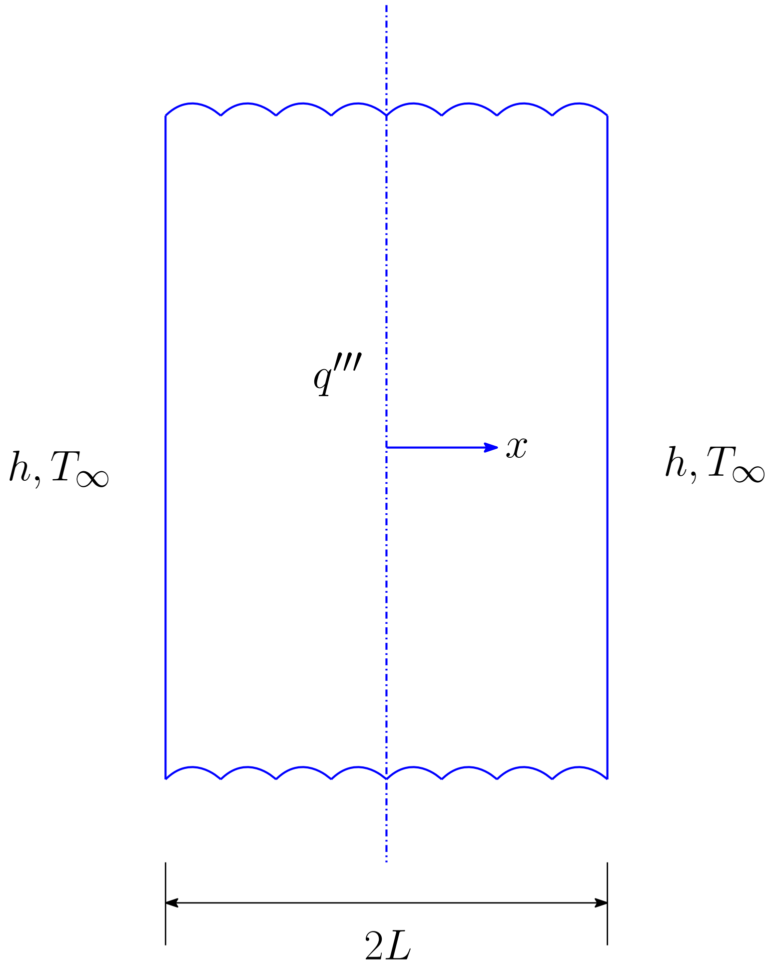

Figure 2: Schematic diagram of the slab

Consider a plane wall of thickness $2L$ (Fig. 2 with uniformly distributed heat source and, therefore, heat is generated at the rate $q''' = 5 \times 10^4$ W/m^3. On each exposed surface, the wall is bounded by a circulating fluid of temperature $T_\infty = 25$ °C. The convective heat transfer coefficient for both surfaces is $h = 22$ W/m^{2}K. The thermal conductivity of the wall is given as $k = 0.5$ W/mK. Assuming the thickness of the wall is 50 cm and heat transfer is one-dimensional, solve for the temperature distribution across the wall numerically.

- Compare your numerical results against the analytical solution by plotting the temperature distribution along the rod in the same graph.
- Use Jacobi, Gauss-Seidel, and TDMA methods to solve the system of equations you obtained after discretizing the governing PDE.
- Make sure that the numerical results are grid-independent. You can consider only uniform mesh distribution for this case.

*** Please prepare a clean report and submit it on the due date. The report should contain the Governing equation, the complete discretization of the governing equation, the derivation of the analytical solution, and all the results (as asked). Please append your code at the end of your report. Upload your report and the code as one Zip file in the Google Classroom. Kindly note that the submission date will not be extended, and no late submission will be accepted.

********************************Best Wishes********************************