FEM and CFD Theory (ME3180)
Department of Mechanical and Aerospace Engineering
Assignment 3
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Due Date: 25/11/2023 (11:59 PM)

2D Unsteady Heat Conduction

Consider a 2D rectangular plate with thermal diffusivity ($\alpha = 9.7 \times 10^{-5}$ m^2/s) and dimensions as shown in Figure 1. Initially, the plate is at 30°C and then suddenly, at a later instant, its surfaces are maintained at temperatures as shown in Figure 1. Solve the unsteady 2D heat conduction equation numerically and plot the temperature distribution with time within the plate.

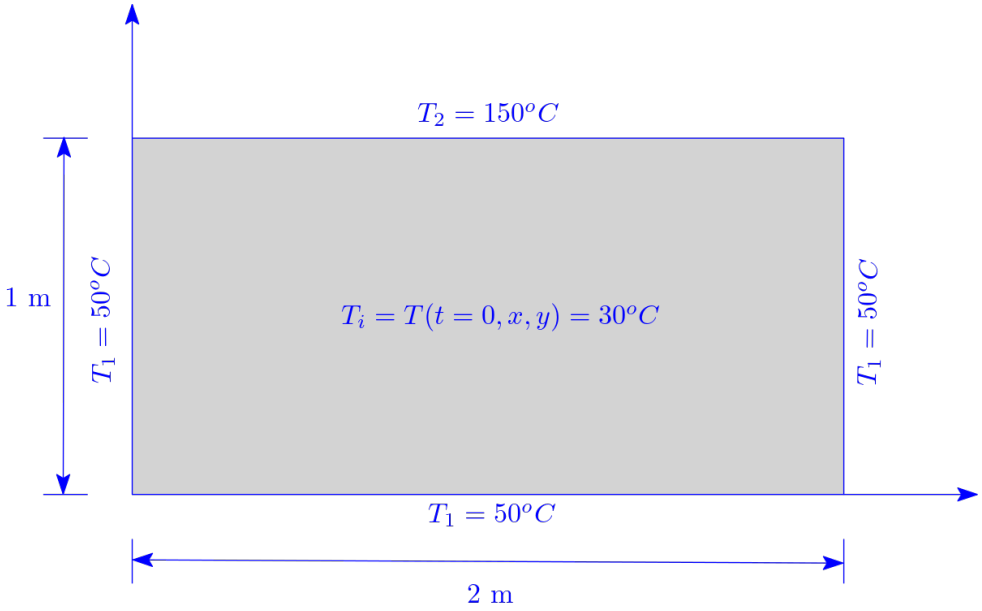

Figure 1: Physical domain of the plate with rectangular cross-section.

Make a note of the following points:

- Use Explicit, Implicit, Crank-Nicholson, and Alternating Direction Implicit Method (ADI) method to solve the governing PDE.
- Perform a grid independence test to show that your numerical solution is not dependent on your grid size. You may consider only a uniform grid for your computations.
- For the grid independence study you may consider any one of the methods of your choice.

- If the analytical solution at the steady state is given by

$$\frac{T - T_1}{T_2 - T_1} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} + 1}{n} \sin(\frac{n\pi x}{L})(\frac{\sinh \frac{n\pi y}{L}}{\sinh \frac{n\pi}{L}})$$

  then compare the center line temperature distribution along the x and y directions at steady state against the analytical solution.

- To obtain the steady state solution from the unsteady formulation you need to run your unsteady code till the steady state is reached.

*** Please prepare a clean report and submit it on the due date. The report should contain the Governing equation, the complete discretization of the governing equation, and all the results (as asked). Please append your code at the end of your report. Upload your report and the code as one Zip file in the Google Classroom. Kindly note that the submission date will not be extended, and no late submission will be accepted.

**********Best Wishes**********