Few useful parameters:
Plank's constant = h = 6.6261×10^{-34} J s
Velocity of light = c = 2.9979×10^{8} ms^{-1}
Rest mass of an electron = 9.11×10^{-31} kg
Rest mass of a proton = 1.67×10^{-27} kg
hc = 1239.8 eV nm = 1239.8 keV pm = 1239.8 MeV fm
1 eV =1.6022×10^{-19} J
##
Q1. A free electron has wave function: 5+5

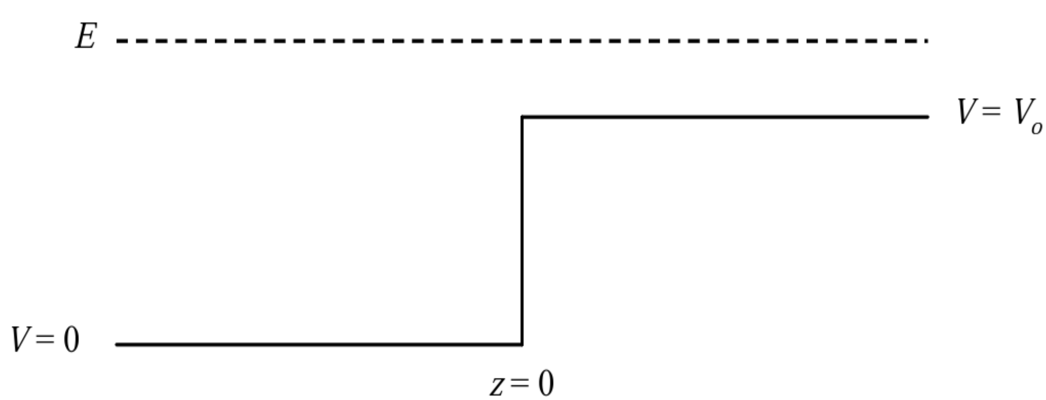
$$\Psi(x,t) = \sin(kx - \omega t)$$

1a. Determine the electron's de Broglie wavelength, momentum, kinetic energy and speed when $k = 50\ nm^{-1}$.

1b. Determine the electron's de Broglie wavelength, momentum, total energy, kinetic energy and speed when $k = 50\ pm^{-1}$.

Q2: An electron is in a potential well of thickness 1 nm, with infinitely high potential barriers on either side. It is in the lowest possible energy state in this well. What would be the probability of finding the electron between 0.1 and 0.2 nm from one side of the well? 10

Q3: Consider the one-dimensional problem, in the z direction, of an infinitely thick barrier of height Vo = 1 eV, at z = 0, beside an infinitely thick region with potential V = 0. We are interested in the behavior of an electron wave with electron energy E = 1.5 eV (i) For the case where the barrier is to the right, i.e., the barrier is for z > 0, as shown below:



and the electron wave is incident from the left, (a) solve for the wavefunction everywhere, within one arbitrary constant for the overall wave function amplitude (b) sketch the resulting probability density, giving explicit expressions for any key distances in your sketch, and being explicit about the phase of any standing wave patterns you find. 5+5

Q4: The electron scattering experiment gives a value of 2 x 10^{15} m for the radius of a nucleus.
4a. Estimate the order of kinetic energies of electrons used for the experiment. Use relativistic expressions.
[Hint: In an electron scattering experiment, the de Broglie wavelength of an electron is of the order of the diameter of a nucleus and total relativistic energy, $E^2 = p^2c^2 + m_0^2c^4$, where m_0 is the rest mass of an electron] 7+3
4b. Proton beam is used to obtain information about the size and shape of atomic nuclei. In the above experiment, if we replace the electron with a proton, what would be its kinetic energy?

Q5a: The average lifetime of an excited atomic state is 10^{-9} s. If the spectral line associated with the decay of this state is 600 nm, estimate the width of the line [Hint: think of Heisenberg Uncertainty Principle]. 5+5

Q5b: Normalize the wave function $\psi(x) = Ae^{(-ax^2)}$, A and a are constants, over the domain $-\infty < x < +\infty$.

Q6: The mathematical representation of a spherical wave travelling outwards from a point is given by

$$\psi(r) = A/r \, . \, e^{ikr}$$

Find out the probability current density $j(r)$ and plot the result 8+2

Q7: The time-independent wave function of a particle of mass m moving in a potential
$v(x) = \alpha x^2$ is
$\psi(x) = exp\,(-\,(\sqrt{m\alpha^2}/\sqrt{2\hbar^2}).\, x^2)$, α is being a constant.

Find out the total energy of the system. 10

Q8: A relativistic electron has a de Broglie wavelength of 1.5 x 10^{-12} m. Find its (i) kinetic energy (in eV or KeV unit), (ii) group and (iii) phase velocities (in terms of c, the velocity of light) of its matter waves. 4+3+3

Q9: What is the Compton wavelength ($\lambda_C = h/m_0c$) of an electron?
Find the condition (in terms of c) at which de Broglie wavelength of an equals the Compton wavelength 3+7

Q10: An operator O is said to be linear if $O[cf(x) + dg(x)] = cOf(x) + dOg(x)$
A and B are two operators defined by $A\psi(x) = \psi(x) + x$ and $B\psi(x) = (d\psi/dx) + 2\psi(x)$
Check for their linearity. 5+5