EP1108 - Stern-Gerlach experiment and spin

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References

Most of this material has been obtained from the following sources

- Bransden and Joachain book on quantum Mechanics
- MIT open course ware lectures https://www.youtube.com/watch?v=AX9769eQV24&t=2737s by Barton Zweibech
- http://physics.mq.edu.au/~jcresser/Phys301/Chapters/Chapter6.pdf Lecture notes (somewhat advanced)

For a historical perspective and antecedents of this experiment check out the Physics today article https://physicstoday.scitation.org/doi/10.1063/1.1650229

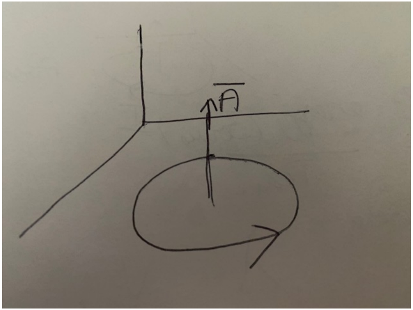
Stern-Gerlach experiment done in Frankfurt in 1922. The results of this experiment were puzzling and things were completely understood only by 1927.

- Pauli thought electron has two degrees of freedom
- Kronig thought it has to do with rotation of electron (criticized by Pauli as electron would rotate faster)
- 1925 Uhlenbeck and Goudsmith -same idea. Advisor (Ehrenfest) advised them to publish
- 1927 Results of Stern-Gerlach experiment finally fully understood

According to Bohr, the electron has angular momentum as it revolves around the nucleus. So they wanted to detect it. You never see spins directly, but only the spin-induced magnetic moment.

Primer on magnetic moments

Magnetic moment (μ) = magnetic analog of electic dipole. $\bar{\mu} = i\bar{A}$



$$\bar{\mu} = i\bar{A}$$

μB has dimensions of energy. So μ has units of Joules/Tesla. Note that Bransden and Jochain book uses the notation $\mathcal{M}$ for magnetic dipole moment.

Dipole moment for Ring of charge

Consider a ring of charge of some radius R with a total charge Q with a linear charge density λ. It has a mass M. Calculate the magnetic moment.

$$\begin{aligned} I &= \lambda v \\ &= \frac{Q}{2\pi r} v \\ \mu &= IA \\ &= \frac{Q}{2\pi r} v \pi r^2 \\ &= \frac{1}{2} Qvr \end{aligned}$$

$$L = Mvr$$

Combining the two, we get $\boxed{\mu = \frac{Q}{2M} L}$

Same is true for a particle that is rotating.

Relation between spin and magnetic moment for electron

Can we write the following for electron ?

$$\mu = \frac{e}{2m_e} S??$$

We can rewrite RHS as

$$\frac{e\hbar}{2m_e}\left(\frac{S}{\hbar}\right)$$

First term has magnetic moment units. Last term is dimensionless. For electron you need an additional fudge factor called g, called Lande factor or spin gyromagnetic ratio

$\mu = g\frac{e\hbar}{2m_e}\left(\frac{S}{\hbar}\right)$ For electron, $g = 2$ (predicted by Dirac's equation).

$\mu_b \equiv \frac{e\hbar}{2m_e}$ known as Bohr magneton $= 9.3 \times 10^{-24}$ J/T

For electron $\boxed{\bar{\mu} = -g\mu_b\frac{S}{\hbar}}$ (Minus sign is because electron has negative charge)

In case of both spin and orbital angular momentum, we have

$$\bar{\mu} = -\mu_b\frac{(L + gS)}{\hbar}$$

Interaction of magnetic dipole with a magnetic field

If an atom with magnetic moment μ is placed in a magnetic field B, the energy of interaction is given by

$$W = -\mu . B$$

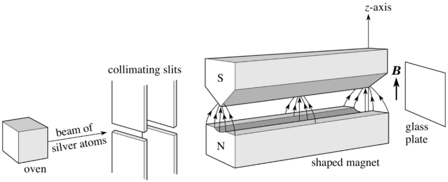
. System experiences a torque given by

$$T = \mu \times B$$

It experiences a net force given by $F = -\nabla W$ The cartesian components of F are given by:
$F_x = \mu . \frac{\partial B}{\partial x}$; $F_y = \mu . \frac{\partial B}{\partial y}$; $F_z = \mu . \frac{\partial B}{\partial z}$
If magnetic field is uniform , no net force experienced by magnetic dipole.
In an inhomogenous magnetic field, atom produces a net force proportional to the magnitude of the dipole moment.

Stern-Gerlach experiment

In 1921 Stern suggested that the magnetic moment of atom could be measured by detecting the deflection of an atomic beam in an inhomogeneous magnetic field. First set of experiments done with silver atom.



Stern-Gerlach expt setup
quantum-mechanics.gatech.edu/resources/resources.html

A sample of silver atoms is vaporized in an oven and a fraction of atoms emerging from a small hole is collimated by a system of slits so that it enters the magnetic field region as a narrow and nearly parallel atomic beam.

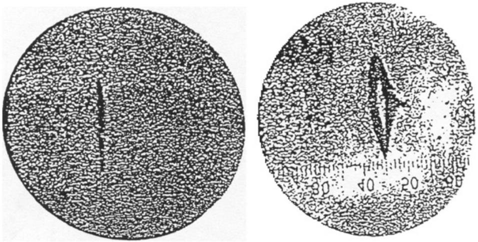
Stern-Gerlach expt (contd)

The beam is then passed between the poles of a magnet shaped to produce an inhomogeneous magnetic field. The beam is finally detected by allowing it to fall on a glass plate. If the magnetic field is only in the z-direction, Force on each atom is given by:
$F_x = \mu_z \frac{\partial B_z}{\partial x}$; $F_y = \mu_z \frac{\partial B_z}{\partial y}$; $F_z = \mu_z \frac{\partial B_z}{\partial z}$;
The magnet is symmetric about the xz plane and the beam is confined to the plane. Therefore, it follows that $\frac{\partial B_z}{\partial y} = 0$. Apart from edge effects $\frac{\partial B_z}{\partial x} = 0$. so only only force on atoms in the beam is in the Z-direction.

Expected results

In the incident beam, direction of the magnetic moment (M) of the atoms is completely at random and in the z-direction every value of M_z given by $-M \leq \mu_z \leq M$ would occur such that the deposit on the collecting plate would be spread continuously over a region symmetrically disposed about the point of no deflection.

Observed results



Result of Stern-Gerlach experiment (from original experiment). https://plato.stanford.edu/entries/physics-experiment/app5.html

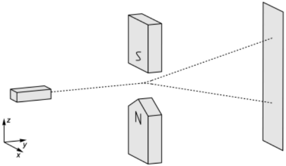


Figure 1: In the Stern-Gerlach experiment, electrons are shot from an emitter (on the left) through an inhomogeneous magnetic field produced by two magnets (in the center), after which their final locations are recorded when they hit a detector screen (on the right). The dotted lines show the paths of z-spin up and z-spin down electrons.

Interpretation of Results

In Stern-Gerlach original experiment, B=0.1 T and spacing between the two spots was 1/5 mm.
What they found that the magnetic moment is quantized.
$\implies$ that the component of angular momentum along Z-direction is quantized and can only take certain values. They refered to this as "space quantization".
They tried to explain this by postulating electron has quantized orbital angular momentum (as in Bohr's model). However, this does not agree since the Ag atom has one unpaired electron with $l = 0$, where l is the orbital angular momentum quantum number. The expected multiplicity is equal to $(2l + 1)$ and hence no splitting expected. Stern and Gerlach had assumed that the atoms are in $l = 1$ state. The expected multiplicity would be equal to three. But the observed multiplicity was equal to two.

Electron Spin

- Goudsmith and Uhlenbeck showed that the splitting can be explained if electrons possess an intrinsic spin orbital angular momentum or "spin" S given by $\mu = -g_s \mu_b \frac{S}{\hbar}$
- If we introduce a *spin quantum number* s, multiplicity of spin in a given direction is equal to $(2s+1) \implies$ electron must have a $s = 1/2$.
- Possible values of S_z of the electron spin S in the z-direction is equal to $\pm\hbar/2$
- Magnitude of electron spin is given by $\sqrt{(s)(s+1)}\hbar = \sqrt{3/4}\hbar$
- In case electron has both spin and orbital angular momentum we have $\mu = -\mu_b(L + gS)/\hbar$