Quiz-3

1. Question: Identify all the convergent series.

3 points

Check all that apply.

Thus
$$\frac{\sqrt{n}}{n\sqrt{n+1}} \ge \frac{\sqrt{n}}{2n\sqrt{n}} = \frac{1}{2n}$$

Thus $\frac{\sqrt{n}}{n\sqrt{n+1}} \ge \frac{\sqrt{n}}{2n\sqrt{n}} = \frac{1}{2n}$

hence diverges

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n\sqrt{n+1}}$$

Since $\sum_{n=1}^{\infty} \frac{1+n}{n^2}$

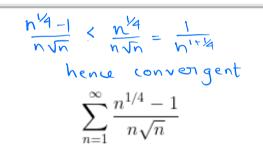
The series diverges

$$\frac{1+n}{n^2} > \frac{n}{n^2} = \frac{1}{n}$$
hence diverges
$$\sum_{n=1}^{\infty} \frac{1+n}{n^2}$$

Option 1

 $\frac{\sqrt{n+1}-\sqrt{n}}{n}=\frac{1}{n(\sqrt{n+1}+\sqrt{n})}<\frac{1}{2n\sqrt{n}}$ Since I Into converges, $\sum_{n=0}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}$ hence the series converges.

Option 2



Option 3

2. Question: Identify all the convergent series.

2 points

Check all that apply.

Convergent by Integral Test
$$D \circ it!$$

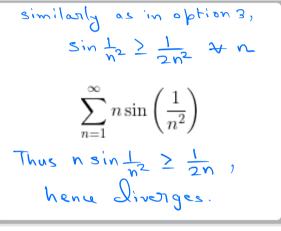
$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1.001}}$$

Option 1

It was proved that
$$\sin \frac{1}{n} > \frac{1}{2n} + n$$
Thus $n \sin \frac{1}{n} > 2 + n$

$$\sum_{n=1}^{\infty} n \sin \left(\frac{1}{n}\right)$$
hence the series diverges.

Option 2



Option 3

3. Question: Identify all the convergent series.

3 points

Check all that apply.

Convergent by Root Test
$$\lim_{N \to \infty} \left(1 - \frac{1}{n} \right)^n = \frac{1}{e} < 1$$

$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{n} \right)^{n^2}$$

$$\frac{1}{(1+n)^n} < \frac{1}{n^n} \le \frac{1}{n^2} \ \forall \ n \ge 2$$
hence convergent by
$$\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$$
Composison Test.

Option 1

Divergent by Root Test $\lim_{n \to \infty} \left(1 + \frac{1}{n} \right)^n = e > 1$ $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^{n^2}$

 $\frac{1}{n^{1+\Omega}nn} \leq \frac{1}{n^2} \quad \forall \quad n \geq 3.$ So convergent by $\sum_{n=1}^{\infty} \frac{1}{n^{1+\ln n}}$

Companison Test.

Option 3

Option 4

4. Question: Identify all the convergent series.

3 points

Check all that apply.

Apply Ratio Test
$$\frac{a_{n+1}}{a_n} = \frac{n+1}{e} \longrightarrow \infty$$

$$\sum_{n=1}^{\infty} \frac{n!}{e^n}$$
hence Divergent.

Apply Ratio Test
$$\frac{Q_{n+1}}{Q_n} = \frac{5^{n+1}((n+1)!)^2}{(2n+2)!} \cdot \frac{2n!}{5^n(n!)^2}$$

$$\sum_{n=1}^{\infty} \frac{5^n(n!)^2}{2n!}$$

$$= \frac{5(n+1)}{2(2n+1)} \rightarrow \frac{5}{4} > 1$$
hence divergent.

Option 1

Apply Ratio Test
$$\frac{2n+1}{dn} = \frac{2(n+1)}{2(2n+1)} \rightarrow \frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{2^n(n!)^2}{2n!}$$
hence convergent

Option 2

Apply Root Test
$$a_n^{1/n} = \frac{2^n}{n^{2/n}} \to \infty$$

$$\sum_{n=1}^{\infty} \frac{2^{n^2}}{n^2}$$
hence divergent.

Option 3

2 points

Check all that apply.

Sin
$$\left(\frac{2n+1}{2}\right) = \left(-1\right)^{n+1}$$

Thus, the series is $\sum_{n=1}^{\infty} \frac{-1}{n^2}$

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n+1)\pi}{2}\right)}{n^2}$$
Obviously absolutely carvey.

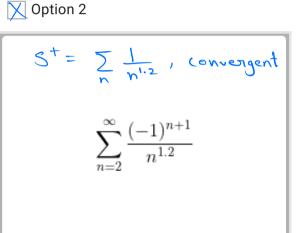
Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is conveg.

In this case
$$S^{+} = \sum_{n=1}^{\infty} \frac{1}{n}, \text{ not convg.}$$

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n+1)\pi}{2}\right)}{n}$$
 hence the series is not absolutely covg.

Option 1

Same as option 2 $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n}$



Option 3

6. Which among the following series is/are conditionally convergent?

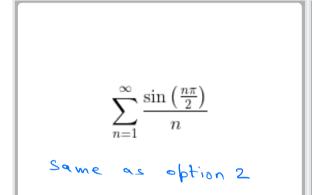
3 points

Check all that apply.

since absolutely converge hence not conditionally
$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n+1)\pi}{2}\right)}{n^2}$$
 converge.

Convergent by Afternating Series Test. $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n+1)\pi}{2}\right)}{n}$ Since not absolutely converge, hence conditionally converge.

Option 1



 $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^{1.2}}$ argument same as option 1.

Option 3

Option 4

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7. Question 3 points

Let $\{a_n\}_n$ be a sequence of positive terms such that $\sum a_n$ is convergent.

Which of the following series is always convergent? Check all that apply.

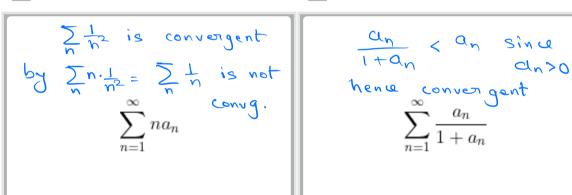
hence convg. by Comparison Test
$$\sum_{n=1}^{\infty} \frac{a_n}{n}$$

$$\sum_{n=1}^{\infty} \frac{a_n}{n}$$

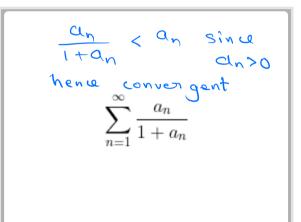
$$\sum_{n=1}^{\infty} \frac{a_n}{\ln(1+n)}$$

$$\frac{Cln}{\ln(1+n)} \leq Cln \quad \forall \quad n \geq 3$$
hence convergent
$$\sum_{n=1}^{\infty} \frac{a_n}{\ln(1+n)}$$

Option 1



Option 2



Option 3

Quiz-3

8. Question 2 points

Suppose $\sum a_n$ is a convergent series of positive terms. Pick the correct alternative(s).

Check all that apply.



$$\lim_{n\to\infty} a_n = 0$$

Van (& whenever an < & 2 and I happen after a stage $\overline{u_n}=0$

$$\lim_{n\to\infty} \sqrt{a_n} = 0$$

Thus Van< & + n>no hence Jan - 0

Option 1

 $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges but $\lim_{n\to\infty}\frac{1}{n^2n}=1$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = 0$$

Option 2

an < 1 after a stage no.

0 < an < an + n>no

 $\lim_{n\to\infty} a_n^n = 0$

Thus, an - 0 by

Sandwich Theorem.



Option 4

9. Question

If $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive terms. Then the series $\sum_{n=1}^{\infty} \sin a_n$ is absolutely convergent.

Mark only one oval.

True

Isinan / an

) False

hence Isinanl is convergent

=> Z sinan is absolutely

Convergent

2 points

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10. Question 2 points

If $\sum_{n=1}^{\infty} a_n$ is a	a convergent	series of positive	terms. Th	hen the series	$\sum_{n=1}^{\infty} \sin a_n$	is conditionally	convergent.
Mark only	ono oval						

Mark only one oval.

True

X False

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cannot	be co	moliti	onall	H
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