Conservation of Momentum

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Newton's second law of motion applied to rotating frame of reference gives expression for torque.

- *Rate of change in angular momentum produces torque.*

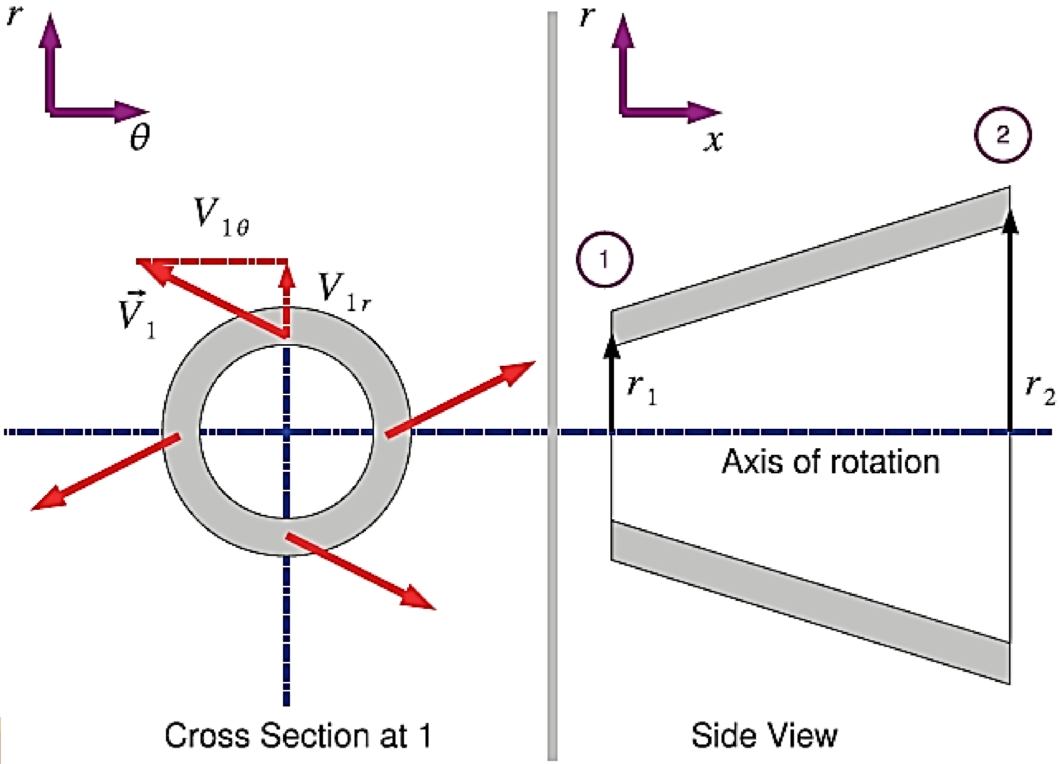

Figure 5.3: A Generic Turbomachinery Flow Passage

$$T = \frac{dL}{dt}$$

- *Where 'T' is torque, 'L' is angular momentum and 't' is time.*

Angular momentum is given by $L = mV_\theta r$

- *Where 'm' is mass, r is radius and V_θ is tangential component of velocity.*

Conservation of Momentum

Now applying time rate of change of momentum across inlet and exit for a turbomachinery, we obtain,

$$T = \frac{dL}{dt} = \frac{d}{dt}(m_2 V_{2\theta} r_2 - m_1 V_{1\theta} r_1)$$

$$= \frac{d}{dt} m(V_{2\theta} r_2 - V_{1\theta} r_1) \quad \text{Continuity}$$

$$= \dot{m}(V_{2\theta} r_2 - V_{1\theta} r_1)$$

Power produced is given by

$$\mathrm{P} = T\omega = \dot{m}\omega(V_{2\theta} r_2 - V_{1\theta} r_1) = \dot{m}(V_{2\theta} U_2 - V_{1\theta} U_1)$$

Power per unit mass flow rate is Euler turbomachinery eq:

$$\boldsymbol{\mathcal{W} = \omega(V_{2\theta} r_2 - V_{1\theta} r_1) = (V_{2\theta} U_2 - V_{1\theta} U_1) = \Delta(U V_\theta)}$$

Euler Work Equation

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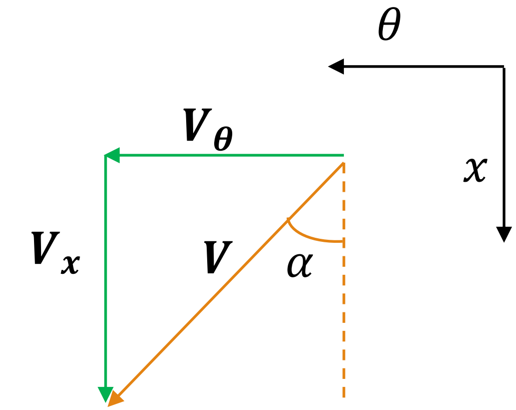
Note that for Euler Work Equation

- *Turbine work would be negative and compressor work would be positive*
- *Power input or power output is entirely given by change in angular momentum.*
  - **Essentially energy interaction is governed by flow turning.**
- *Change in radius affects the angular momentum (moment arm changes in moment of momentum); hence radial flow machine tend to have larger power production/absorption per stage compared to axial flow machine.*

Example A turbine stage with a rotational speed of 3000 rpm is to be designed with an absolute inlet angle of 60° and an absolute exit angle of −60° at a mean radius of 0.4 m. The machine is to be designed for a constant axial velocity of 450 m/s. Estimate the specific work from this stage.

Note that specific work is same as Euler work.

$$\mathcal{W} = \Delta(UV_\theta) = (V_{2\theta}U_2 - V_{1\theta}U_1) = U(V_{2\theta} - V_{1\theta})$$



$$U = 3000 \times \frac{2\pi}{60} \times 0.4 = 125.6\ m/s$$

$$V_\theta = V_x \tan\alpha$$

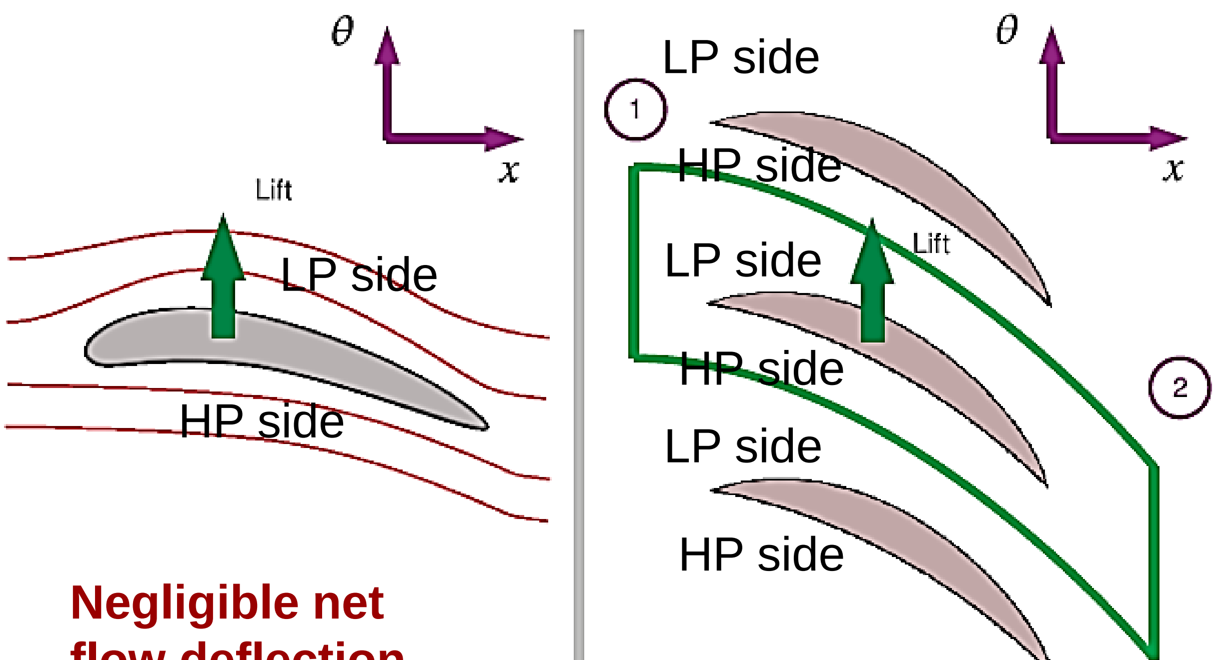
$$V_{1\theta} = 450 \tan 60 = 779\ m/s$$

$$V_{2\theta} = 450 \tan -60 = -779\ m/s$$

$$\mathcal{W} = U(V_{2\theta} - V_{1\theta}) = 125.6(-779 - 779) = -195.8\ kJ/kg$$

Cascade of Blades

Notice the difference in flow deflection for single and cascade of airfoils.



Negligible net flow deflection

Blade arrangement dictates net flow deflection

Figure 5.4: Isolated Aerofoil compared to a Cascade

- *In cascade of blades flow turning can be achieved more due to interference from adjacent blades with flow.*
- *Note that in Grant Ingram's book boundary layer effect is not included.*

Conservation of Energy & Rothalpy

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Recall steady flow energy equation form of first law of thermodynamics.

$$\dot{q} + \dot{w} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)\right]$$

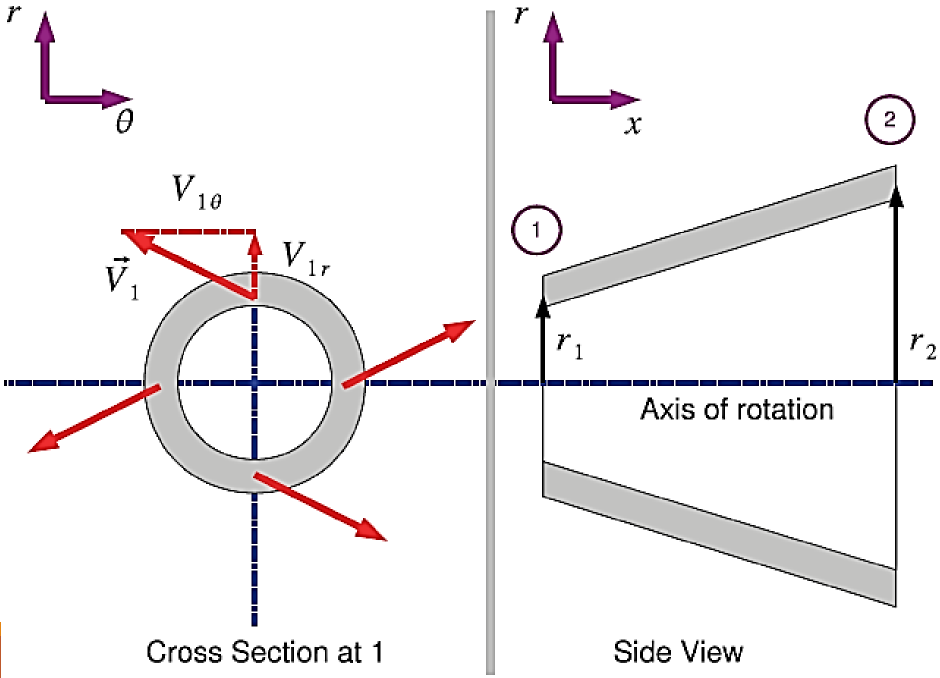


Figure 5.3: A Generic Turbomachinery Flow Passage

- *with negligible difference in datum head at inlet and exit of turbomachinery compare to change in velocity and enthalpy then work done is given by*

$$\dot{w} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2)\right]$$

$$\dot{w} = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right]$$

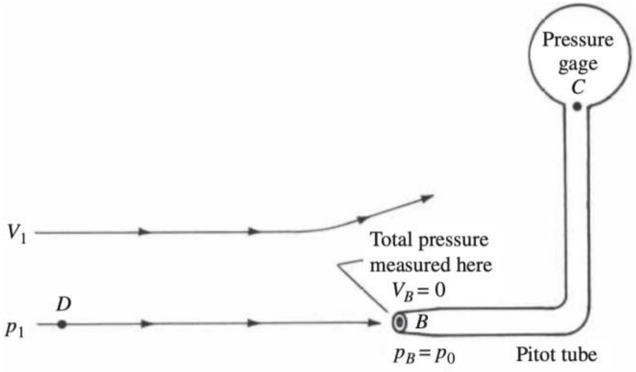
Conservation of Energy & Rothalpy

Recall static and stagnation/total properties of flow.

- *Rate of doing work can be written as*

$$h_0 = h + \frac{V^2}{2}$$

$$\dot{w} = \dot{m}\left[h_{02} - h_{01}\right]$$



- *Specific work expression can be written as*

$$\dot{w}/\dot{m} = w = h_{02} - h_{01}$$

- *Specific work is Euler's work derived previously.*

$$w = U_2V_{2\theta} - U_1V_{1\theta} = h_{02} - h_{01} \implies h_{01} - U_1V_{1\theta} = h_{02} - U_2V_{2\theta}$$

i.e. $h_0 - UV_\theta$ = Constant

Rothalpy

Defining Rothalpy as $I = h_0 - UV_\theta$

- *From rearrangement of steady flow energy equation we obtain expression for Rothalpy, which is conserved over the blade row*
  - Note that inlet station rothalpy is same as rothalpy at exit station.

For stators, blade speed U = 0; hence rothalpy conservation simply changes to

$$I_1 = I_2 \implies h_{01} = h_{02}$$

Rothalpy in Rotors

We know that, $V_x = W_x$ and $V_\theta = U + W_\theta$ & $V^2 = V_x^2 + V_\theta^2$

$$V^2 = V_x^2 + V_\theta^2 = W_x^2 + (W_\theta + U)^2 = W^2 + 2W_\theta U + U^2$$

Rothalpy conservation across rotor gives

$$I_1 = I_2 \implies h_{01} - UV_{1\theta} = h_{02} - UV_{2\theta}$$

i.e. $h_1 + \frac{V_1^2}{2} - UV_{1\theta} = h_2 + \frac{V_2^2}{2} - UV_{2\theta}$

Substituting expression

$$V_\theta = U + W_\theta$$

$$LHS = h_1 + \frac{V_1^2}{2} - UV_{1\theta} = h_1 + \frac{W_1^2}{2} - \frac{U_1^2}{2}$$

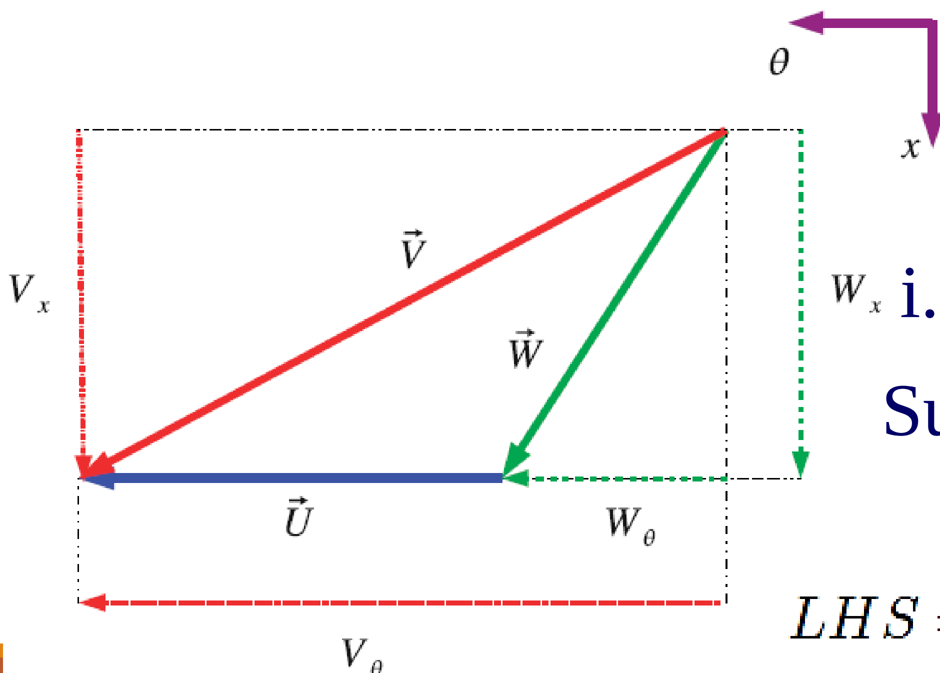


Figure 5.5: Generic Velocity Triangle

Rothalpy in Rotors

Defining relative stagnation enthalpy as follows

$$h_{0rel} = h + \frac{W^2}{2}$$

$$LHS = h_1 + \frac{V_1^2}{2} - UV_{1\theta} = h_1 + \frac{W_1^2}{2} - \frac{U_1^2}{2}$$

$$h_{0rel1} - \frac{U_1^2}{2} = h_{0rel2} - \frac{U_2^2}{2}$$

1. An industrial gas turbine operates at an 8.8:1 pressure ratio and a mass flow of $77\ kg/s$. The exhaust temperature is at $437°C$ and the inlet temperature to the machine is around $1000°C$. The machine is to be designed for a constant axial velocity of $200\ m/s$. (This data is based on the Rolls-Royce Avon, an engine that dates from the 1950s but is still used as low efficiency high reliability engine for stationary power such as pipeline pumping.)

   For this gas turbine, draw to scale the meridional view with:

   (a) a constant mean radius of $0.4\ m$.

   (b) a constant outer diameter (to minimise cross sectional area) of $1.05\ m$.

1. An industrial gas turbine operates at an 8.8:1 pressure ratio and a mass flow of 77 kg/s. The exhaust temperature is at $437°C$ and the inlet temperature to the machine is around $1000°C$. The machine is to be designed for a constant axial velocity of 200 m/s. (This data is based on the Rolls-Royce Avon, an engine that dates from the 1950s but is still used as low efficiency high reliability engine for stationary power such as pipeline pumping.)

   For this gas turbine, draw to scale the meridional view with:

   (a) a constant mean radius of 0.4 m.

   (b) a constant outer diameter (to minimise cross sectional area) of 1.05 m.

$\dot{m} = 77\ kg/s$

$T_2 = 437\ C$, $T_1 = 1000°C$

$V_x = 200\ m/s$

(a) $\rho_1 A_1 V_x = 77$

$\rho_1 A_1 = \frac{77}{200}$

$\rho_1 2\pi \overset{=0.4}{r_m} b = \frac{77}{200}$

$\rho_1 b = \left(\frac{77}{200 \times 2\pi \times 0.4}\right)$

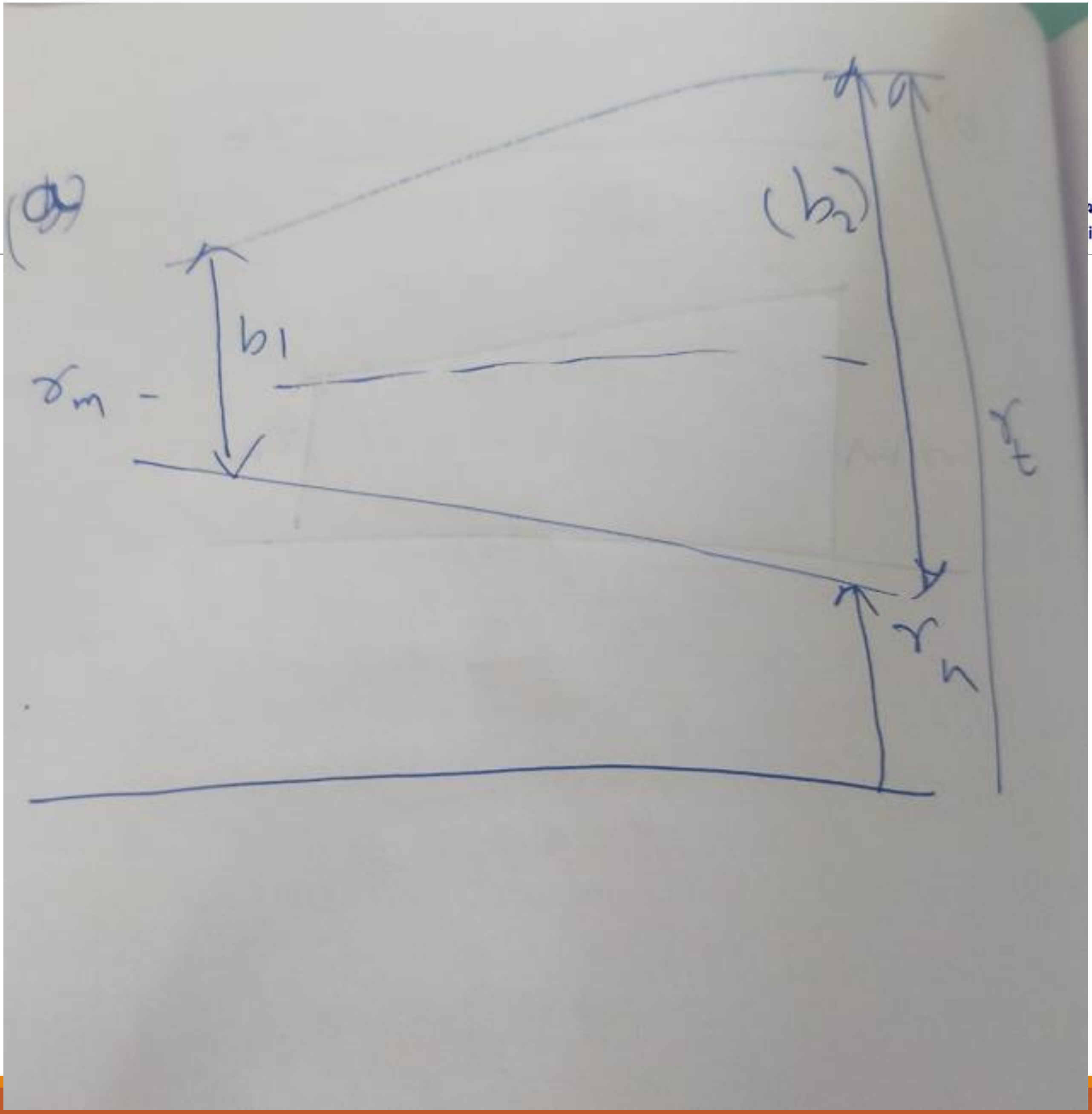
$\frac{\overset{8.8\times10^5 Pa}{P_1}}{\underset{287}{R}\underset{1000+273}{T_1}} b_1 = (\ )$

$b_1 =$ ✓

$b_2 =$ ✓

$P = \rho R T$

$\frac{P}{RT} = \rho$


(a)
b_1
r_m
(b_2)
r_t
r_h

1. An industrial gas turbine operates at an 8.8:1 pressure ratio and a mass flow of 77 kg/s. The exhaust temperature is at 437°C and the inlet temperature to the machine is around 1000°C. The machine is to be designed for a constant axial velocity of 200 m/s. (This data is based on the Rolls-Royce Avon, an engine that dates from the 1950s but is still used as low efficiency high reliability engine for stationary power such as pipeline pumping.)

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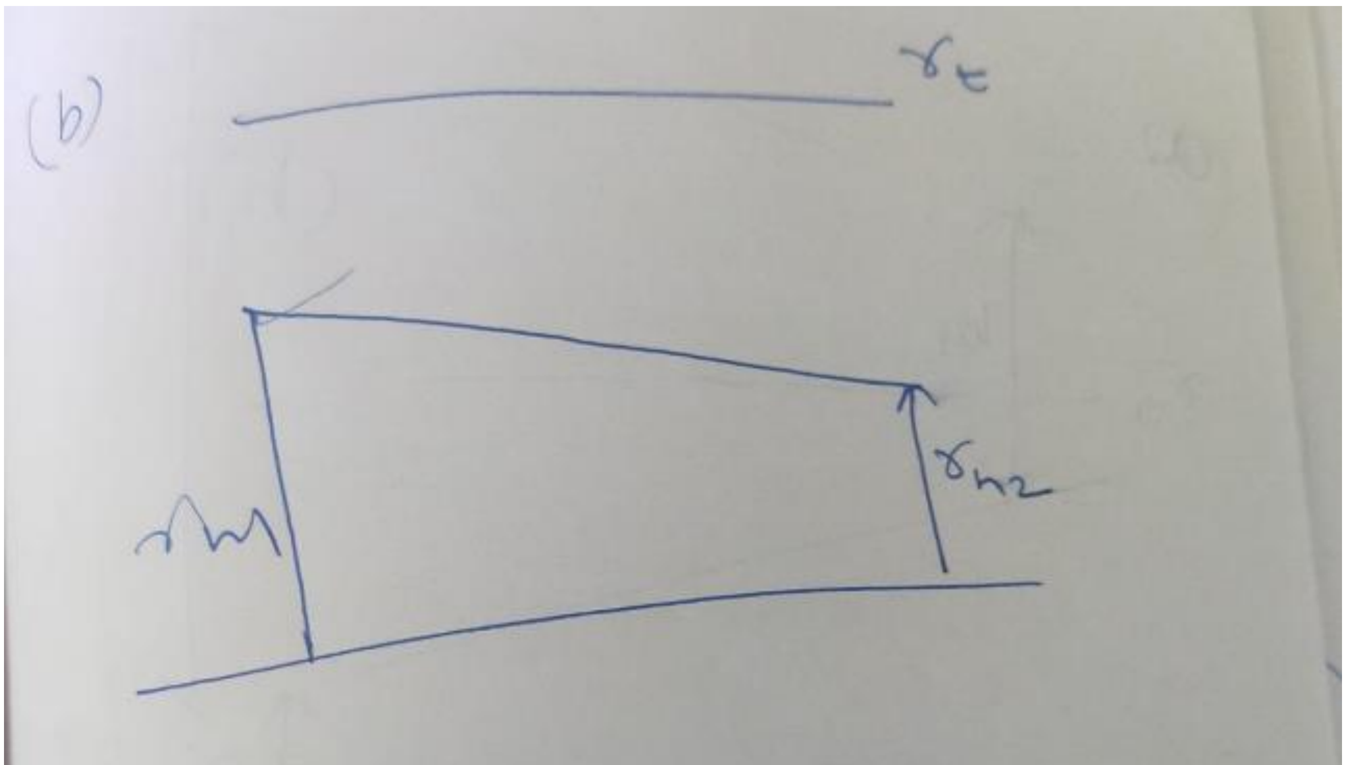

$r_t = 1.05\,m$

(b)

$$\rho_1 A_1 = \frac{\dot{m}}{V_1} = \frac{77}{200}$$

$$\rho_1 \times \pi\left(r_t^2 - r_h^2\right) = \frac{77}{200}$$

$$\frac{P_1}{RT_1} \times \pi\left((1.05)^2 - r_h^2\right) = \frac{77}{200}$$

Inlet $r_{h_1} = \checkmark$ (change $\frac{P_1}{RT_1}$)

Outlet $r_{h_2} = \checkmark$ $\left(\frac{P_2}{RT_2}\right)$

Consider an axial compressor rotor blade row. The inlet velocity is 150 m/s and is in the axial direction. The blade speed is 180 m/s and the relative outlet angle is $-30\circ$ to the axial. The axial velocity is constant across the row. Calculate the relative flow angle at inlet and the absolute swirl velocity at exit and the absolute flow angle. Calculate also the specific work output.

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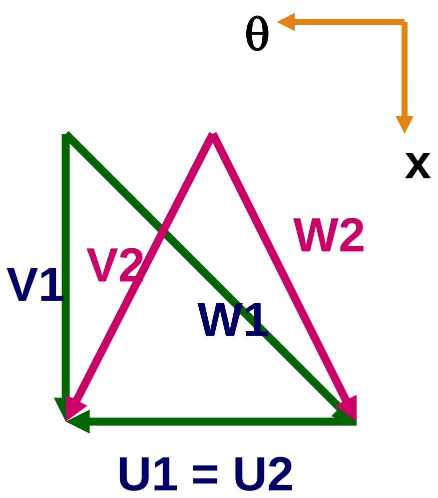
Given Data: $V_1 = V_{1x} = 150$ m/s $= V_{2x}$; $U = 180$ m/s; $\beta_2 = -30°$

To Find: $\beta_1 = ?$; $V_{2\theta} = ?$; $\alpha_2 = ?$; $\mathcal{W} = ?$



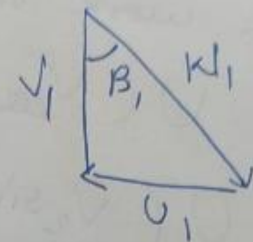
5.2)

$V_{1x} = 150\ m/s = V_{2x}$

$U = 180\ m/s$

$\beta_2 = -30^\circ c.$

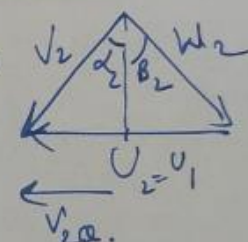
$\beta_1 = ?$



$$\tan\beta_1 = \frac{U_1}{V_1}$$

$$\boxed{\beta_1 = \tan^{-1}\frac{U_1}{V_1} = -50.2^\circ}$$

$$V_2\cos\alpha_2 = W_2\cos\beta_2 = V_{2x} = 150$$



$$W_2 = \frac{150}{\cos\beta_2}$$

$$W_2 = 173.2$$

$$V_2\sin\alpha_2 + W_2\sin\beta_2 = U = 180$$

$$V_2\sin\alpha_2 = 180 - 173.2\sin(30^\circ)$$

abs swirl velocity $\boxed{V_2\sin\alpha_2 = 93.4\ m/s}$

$$\tan\alpha_2 = \frac{V_2\sin\alpha_2}{V_2\cos\alpha_2} = \frac{93.4}{150}$$

$$\alpha = 31.9^\circ$$

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Specific work output:

$$U(V_{2a} - V_{1a})$$

$$U(V_2 \sin\alpha_2 - V_1 \cancel{\sin\alpha_1}^{0})$$

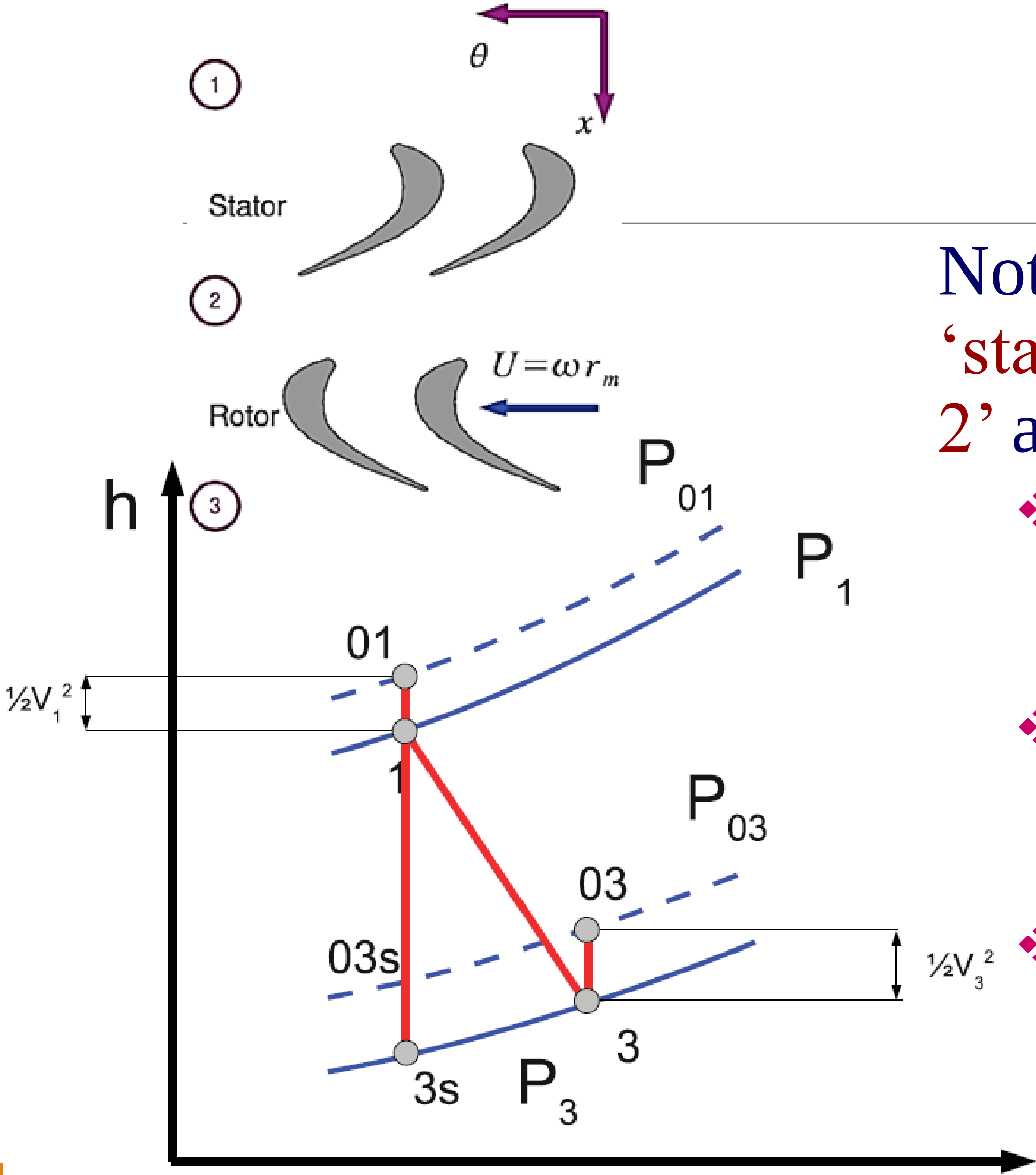
$$180 \times 93.4 = 16.811 \text{ KJ/kg}$$

Lecture 8

Efficiency and Reaction

Problems

Efficiency of a Turbine



Enthalpy entropy diagram of a turbine

Note that 'station 1' is inlet and 'station 3' is exit with 'station 2' as exit of stator/entry to rotor

- *Static pressure at entry to turbine is P_1 and Total pressure is P_{01}*
- *Similarly, static pressure at exit is P_3 and total pressure at exit is P_{03}*
- *Points 3s and 03s refer to static and total pressure at exit if expansion were isentropic in the turbine*

Efficiency of a Turbine

Efficiency can be defined in various ways as follows:

Total to Total efficiency for turbine is given by

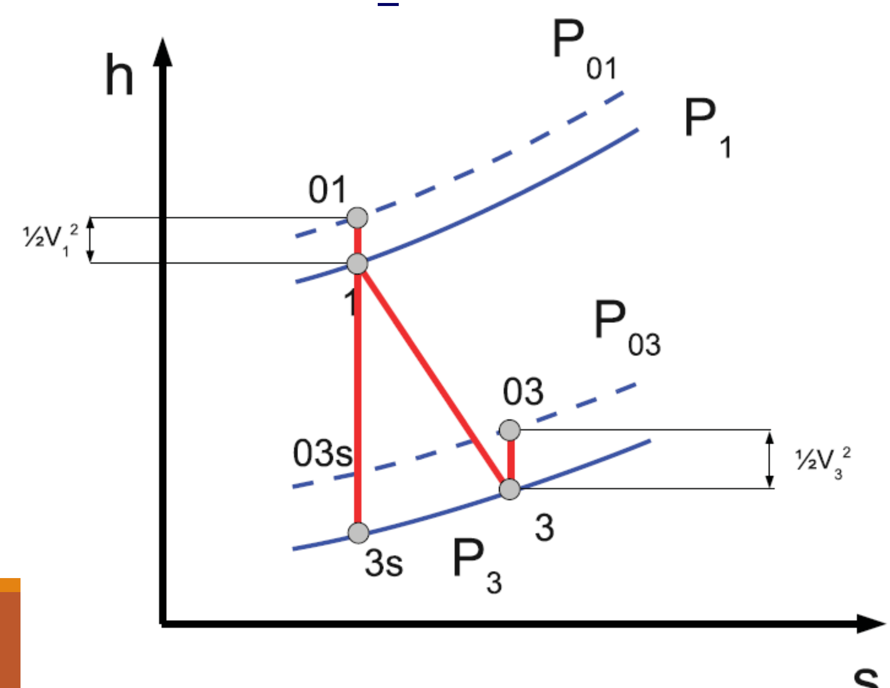
$$\eta_{it} = \frac{actual\ work\ done}{ideal\ work\ output} = \frac{(h_{03} - h_{01})}{(h_{03s} - h_{01})}$$

Total to Static efficiency

$$\eta_{it} = \frac{(h_{03} - h_{01})}{(h_{3s} - h_{1})}$$

Static to static efficiency

$$\eta_{it} = \frac{(h_{3} - h_{1})}{(h_{3s} - h_{1})}$$



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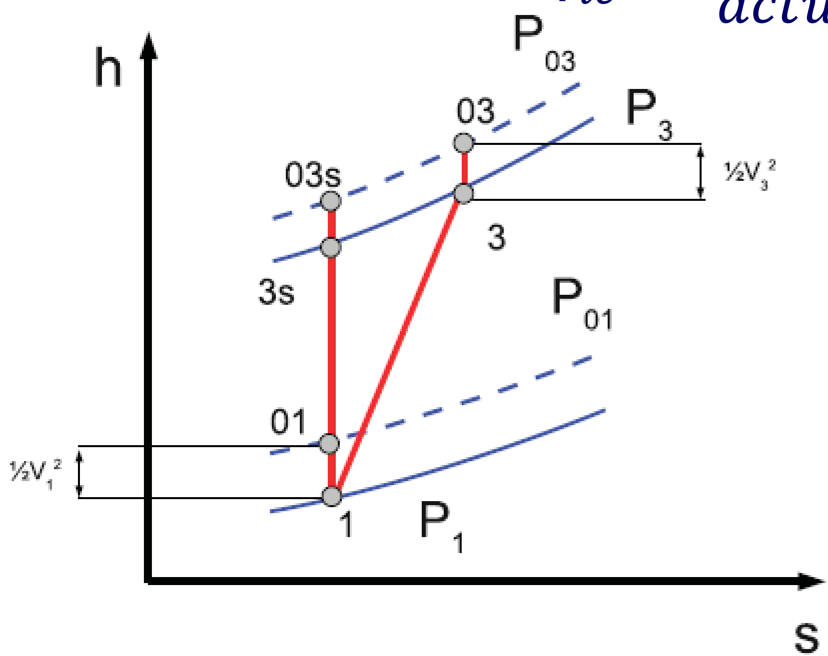
Efficiency of a Compressor

Compressor is work absorbing turbomachines and efficiency is defined exactly opposite of a turbine.

Total to Total efficiency for turbine is given by

$$\eta_{ic} = \frac{ideal\ work\ input}{actual\ work\ input} = \frac{(h_{03s} - h_{01})}{(h_{03} - h_{01})}$$



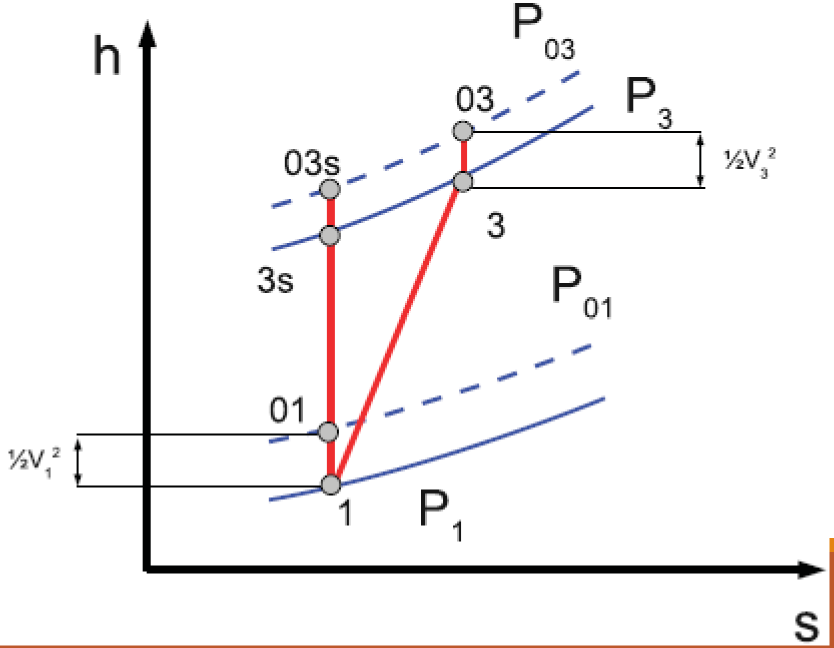
Example A compressor stage working on air at 1 bar and 25°C has a work input of 17 kJ/kg and an isentropic efficiency of 0.9. The velocity at inlet and exit is the same. Assuming that the air properties are unchanged over the stage calculate the pressure output of the stage.

Note that there is no change in KE at inlet and exit of compressor. Also, calorically perfect gas is to be assumed

$\Delta h = C_p \, \Delta T$

$P3/P1 = (T_{3s}/T_1)^{[\gamma/(\gamma-1)]}$

$$\eta_{it} = \frac{(h_{3s} - h_1)}{(h_3 - h_1)}$$



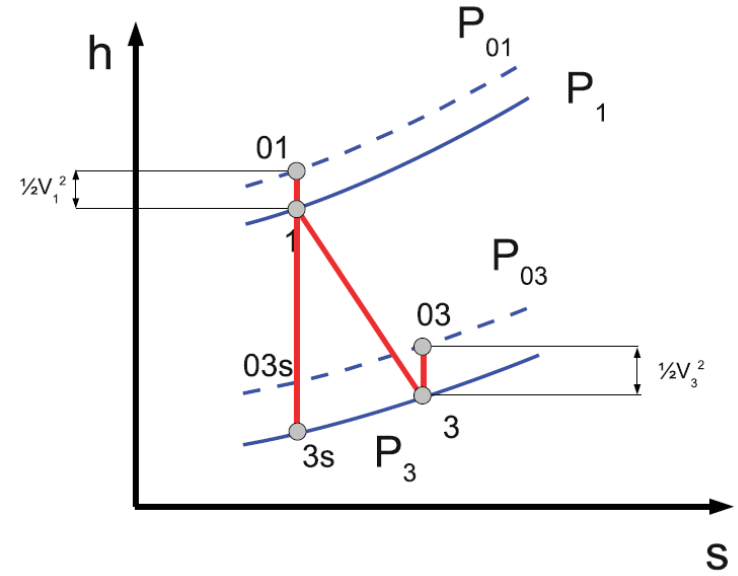
Example A steam turbine stage operates with inlet conditions of $120 bar$ and 500°, the exit pressure is 10 bar. If the efficiency of the stage is 90% calculate the specific work output.

Given: P_1 = 120 bar; P_3 = 10 bar; T1 = 500°C; η_{it} = 0.9;

To Find: specific work output (W_{actual})

h1:120 bar, 500C=3350KJ/Kg

h3s: 10 bar=2730KJ/kg



Either steam tables or Mollier chart

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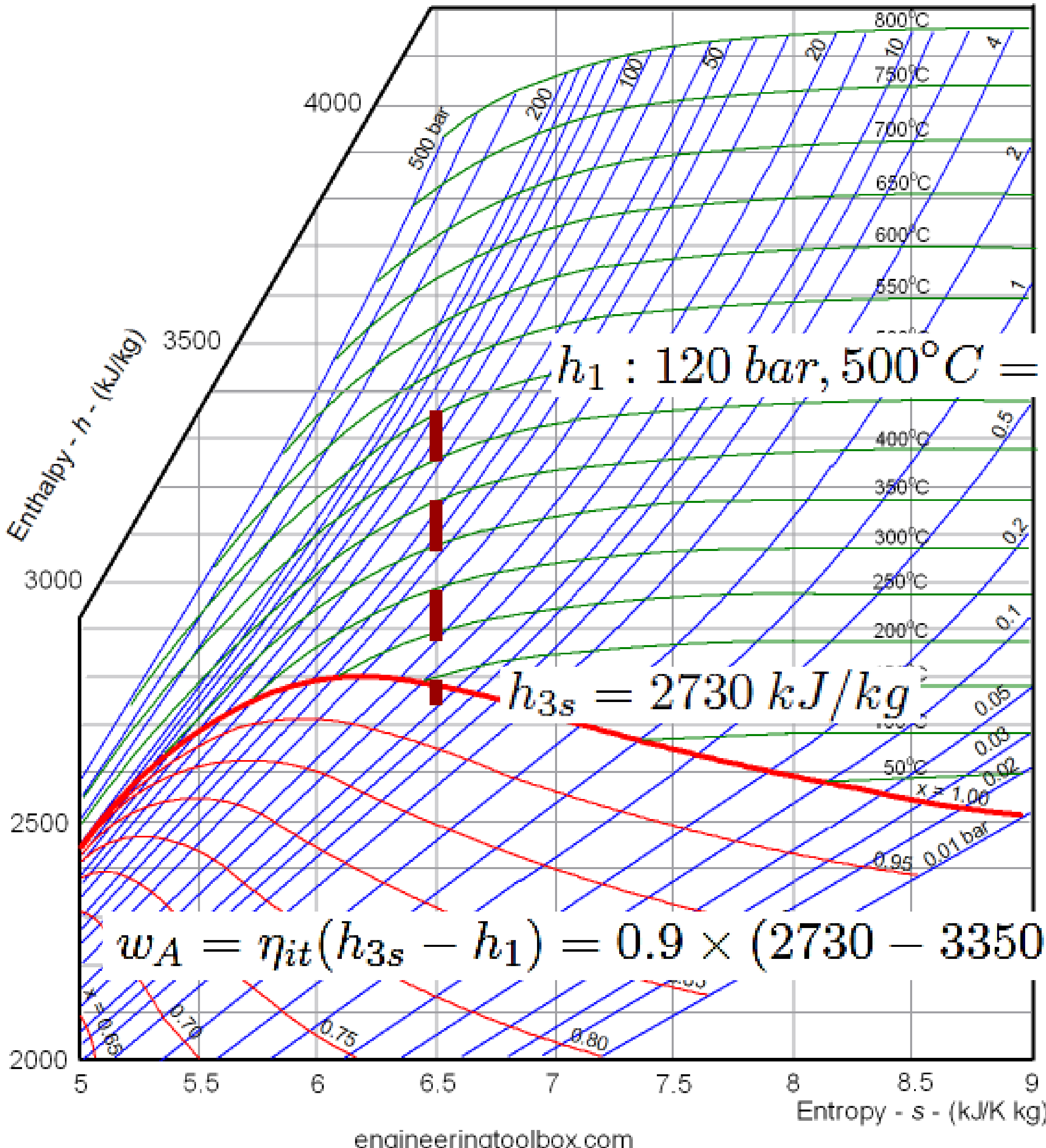
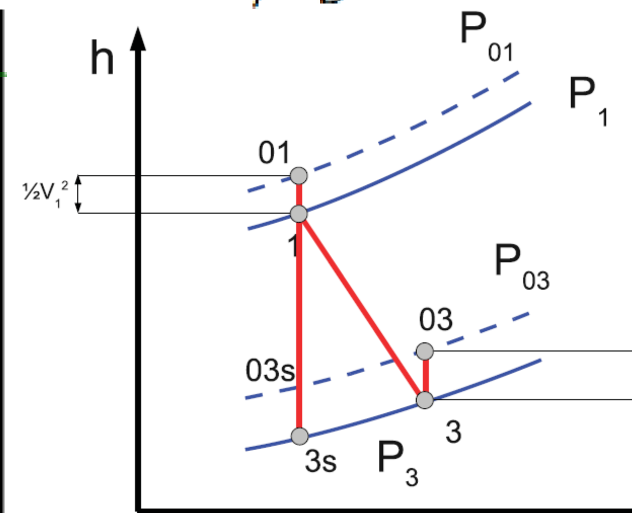


$h_1 : 120\ bar, 500^{\circ}C = 3350\ kJ/kg$

$h_{3s} = 2730\ kJ/kg$



$$w_A = \eta_{it}(h_{3s} - h_1) = 0.9 \times (2730 - 3350) = -558\ kJ/kg$$