Lecture 10

Dimensionless Coefficients for Axial Flow Machines

Problem

Dimensionless Coefficients for Wind Turbines

Axial Flow Turbomachines

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For axial flow machines there are four dimensionless coefficients that are important

- *Flow coefficient*
- *Stage loading*
- *Efficiency*
- *Reaction*

Flow Coefficient $\left(\phi = \frac{V_x}{U_m}\right)$:

- *It is the ratio of axial velocity and mean blade speed*
  - ***For a given blade speed higher flow coefficient means higher flow axial velocity (more flow through turbomachine***

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Stage Loading $\left(\psi = \frac{\omega_x}{U_m^2} = \frac{\Delta h_0}{U_m^2}\right)$:

- *It is the ratio of work interaction in the stage to square of the mean blade speed.*
- *Recall Euler Work equation for specific work stage loading can be written as*

$$\psi = \frac{\Delta(UV_\theta)}{U_m^2} = \frac{\Delta(V_\theta)}{U_m}$$

Table 7.1: Typical Values of Ψ and Φ

Case	Flow Coefficient	Stage Loading Coefficient
Aircraft Engine Compressor	0.4 to 0.70	0.35 to 0.50
Aircraft Engine HP Turbine	0.5 to 0.65	1.0 to 2.0
Aircraft Engine LP Turbine	0.9 to 1.0	1.0 to 2.0

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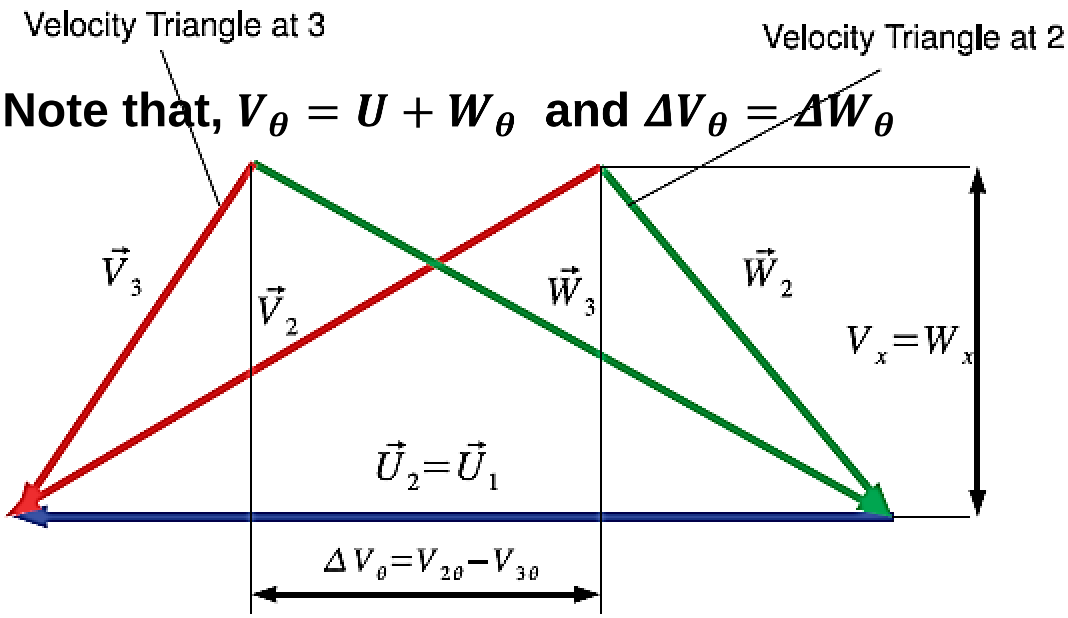
Combined inlet & exit flow velocity triangle

- *For a given blade speed, stage loading coeff. Gives the width of diagram*
- *For constant axial velocity, flow coeff. Gives the height of diagram*

Note that, $\boldsymbol{V_\theta = U + W_\theta}$ **and** $\boldsymbol{\Delta V_\theta = \Delta W_\theta}$



Example Given a turbine blade row with constant axial velocity of $150 m/s$ at $5000 rpm$ on a mean radius of $0.7\ m$ and an absolute flow angle at exit from the stator of $70°$. The turbine operates with axial leaving flow and is a repeating stage. Calculate the flow coefficient, stage loading coefficient and reaction.

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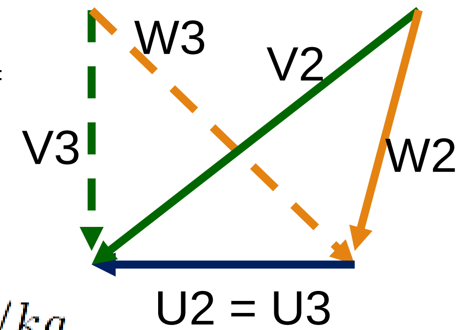
Given: $V_x = W_x = 150$ m/s; N = 5000 rpm, $r_m = 0.7$ m; $\alpha 2 = 70°$;

Repeating Stage Turbine (same inlet to stator & exit of rotor velocity & flow angles). Also, axial leaving flow

To Find: flow coefficient ϕ = ??; Stage Loading Coefficient ψ = ??; Reaction R = ??

$$U_m = \omega r = 5000 \times \frac{2\pi}{60} \times 0.7 = 366.5\, m/s \qquad V_{2\theta} = V_x \tan \alpha_2 = 150 \tan 70 = 412.12\, m/s$$

$$\phi = \frac{V_x}{U_m} = \frac{150}{366.5} = 0.409 \qquad \psi = \frac{412.12}{366.5} = 1.124$$



$$\psi = \frac{w}{U_m^2} = \frac{U_m(V_{3\theta} - V_{2\theta})}{U_m^2} = \frac{V_{3\theta} - V_{2\theta}}{U_m}$$

$$\Delta h_{STAGE} = \Delta h_0 = w = U_m(\Delta V_\theta) = 366.5(412.12) = 151.04\, kJ/kg$$

$$\Delta h_{ROTOR} = \frac{W_3^2 - W_2^2}{2} = \frac{W_{3\theta}^2 - W_{2\theta}^2}{2} = \frac{412.12^2 + 0^2}{2} = 84.92\, kJ/kg$$ **<= How??**

$$R = \Delta h_{ROTOR}/\Delta h_{STAGE} = 84.92/151.04 = 0.56$$

Coefficients for Wind Turbines

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For wind turbines two dimensionless coefficients are usually referred

- *Power Coefficient*
- *Tip speed ratio*

Power Coefficient

$$C_P = \frac{\text{Power from Turbine}}{\text{Power in Wind}} = \frac{P}{\frac{1}{2}\rho V^3 A}$$

- *It is the ratio of power produced by turbine to total wind power (kinetic energy). V is wind vel. 'A' is area swept by blades.*

Tip Speed Ratio

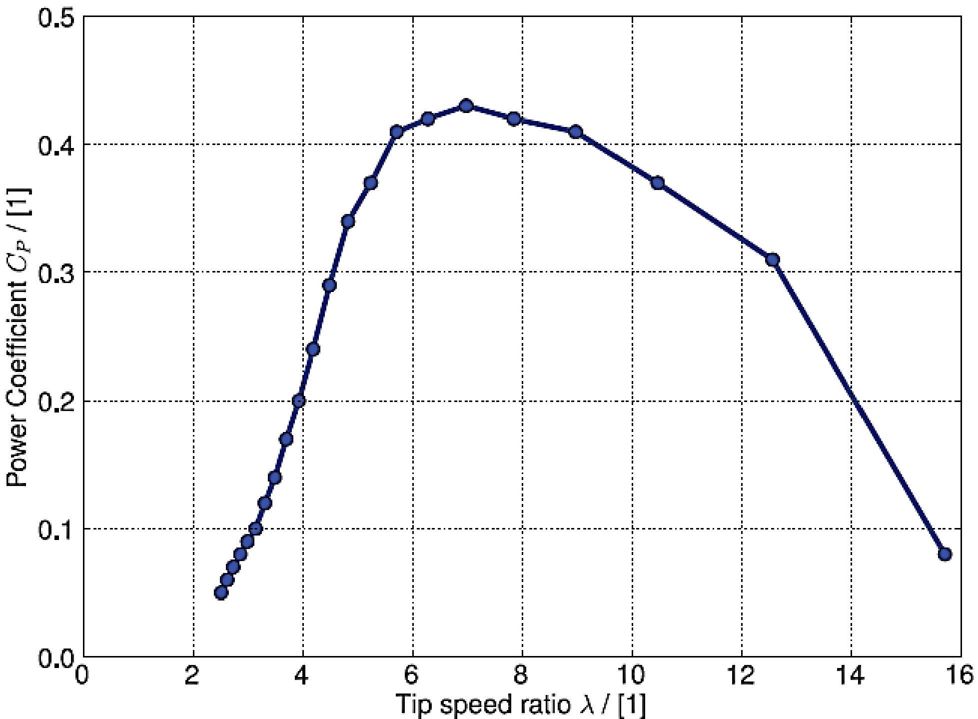
$$\lambda = \frac{\omega r_{tip}}{V}$$

- *It is the ratio of blade speed at tip to the velocity of wind*

Example Given the power curve shown in Figure 7.2. Calculate the power output of the device at 10 rpm in a 10 m/s wind given that the turbine is 80m in diameter and the density of air can be taken as $1.15 kg/m^3$.

$$\lambda = \frac{\omega r_{tip}}{V} = \frac{\frac{10\times 2\pi}{60} \times \frac{80}{2}}{4} = 10.47$$

$$C_P = \frac{P}{\frac{1}{2}\rho V^3 A} \implies P = C_P \frac{1}{2}\rho V^3 A = 0.37 \times \frac{1}{2} \times 1.15 \times 10^3 \times 5026 = 1.069\ MW$$



Exercise Problem for Students

1. An axial flow gas turbine has three stages of compression and one stage for the turbine. The compressor operates with an axial inlet flow velocity of 250 m/s at 3000 rpm. The rotor relative exit angle is set at $-34°$ at a mean radius of 0.75 m. Draw the velocity triangle for the first stage rotor exit and hence estimate the specific work output. *Answer:* 15.8 kJ/kg

2. The turbine from the previous questions operates with an axial inlet velocity of 300 m/s. The axial velocity is kept constant through the stage. The stator blades have an absolute exit angle of 60° and the rotor blades have a relative exit angle of $-30°$. Calculate the specific work output and calculate the stage loading coefficient for the turbine at a mean radius of 0.75 m. *Answers:* $w = 107.7\ kJ/kg$, $\psi_{turbine} = 1.94$

3. Compare the stage loading coefficient of the turbine and the compressor in the previous two questions. Why is the turbine stage loading coefficient higher than that of the compressor? What limits compressor loading? *Answer:* $\psi_{comp} = 0.28$

Lecture 11

Coefficients for Hydraulic Turbomachines

Specific Speeds for Turbines

Specific Speeds for Pumps

Problem

Dimensionless Coefficients for Hydraulic Turbomachines

Mostly incompressible water is used as working fluid in hydraulic turbomachines.

Three dimensionless parameters and efficiency are widely used for performance prediction/mapping.

- *Flow coefficient*
- *Head coefficient*
- *Power coefficient*
- *Efficiency*

$$\eta_t = \frac{\textbf{Actual Work}}{\textbf{Ideal Work}}; \quad \frac{\Pi_3}{\Pi_1 \Pi_2} = \frac{P}{\rho Q g H} = \eta$$

$$\eta_{pump} = \frac{\textbf{Ideal Work}}{\textbf{Actual Work}} \quad \frac{\Pi_1 \Pi_2}{\Pi_3} = \frac{\rho Q g H}{P} = \eta$$

Flow Coefficient	Head Coefficient	Power Coefficient
$\Pi_1 = \frac{Q}{ND^3}$	$\Pi_2 = \frac{gH}{N^2 D^2}$	$\Pi_3 = \frac{P}{\rho N^3 D^5}$

Utility of Dimensionless Coefficients

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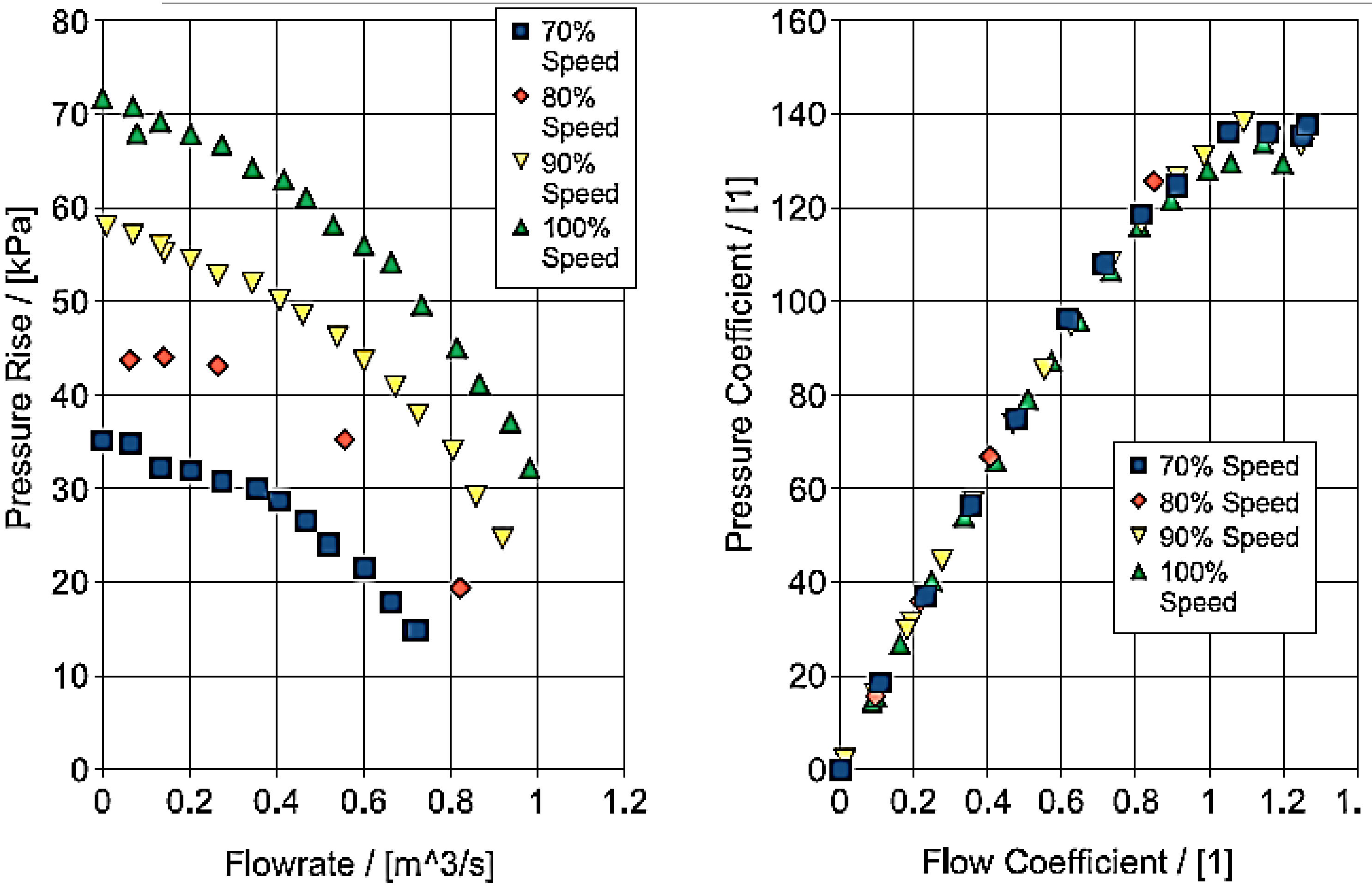
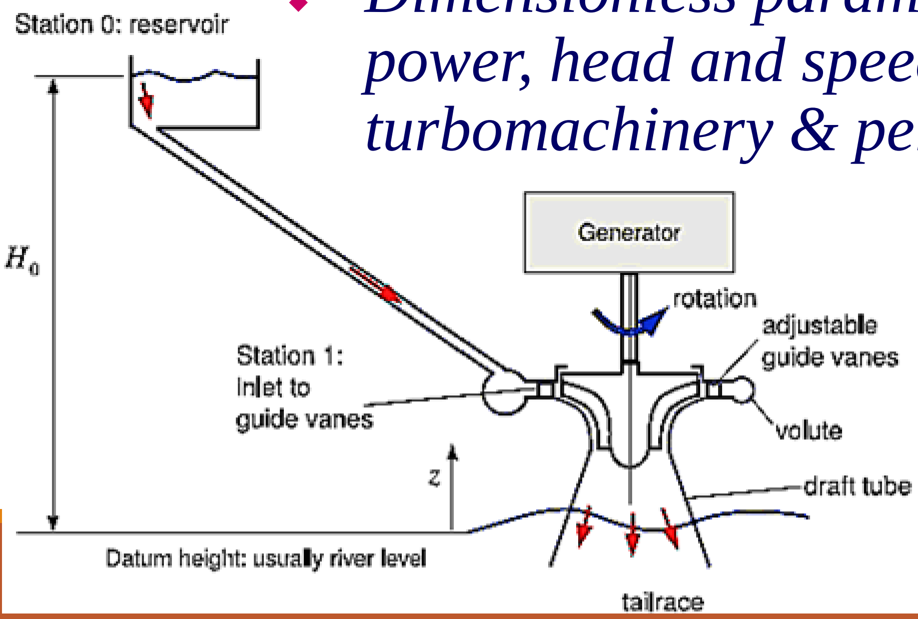


Figure : Collapsing Pump Data onto Non-dimensional Curves

Need for Specific Speeds for Turbine

Other than the physical size of a turbine; **power produced** by a turbine in hydroelectric power plant **depends on the head available and rotational speed**.

- *N fixed by frequency of electricity supply, P and H are set by flow rate and height of location*
- *Dimensionless parameter not involving size and relating power, head and speed would be useful for scaling of turbomachinery & performance prediction.*



$$K_s = \frac{\Pi_3^{1/2}}{\Pi_2^{5/4}} = \frac{N P^{1/2}}{\rho^{1/2}(gH)^{5/4}}$$

Specific Speeds for Turbine

Why head & power coefficient only are involved in K_s?

- *Power production is of interest in case of turbines*

$$K_s = \frac{\Pi_3^{1/2}}{\Pi_2^{5/4}} = \frac{NP^{1/2}}{\rho^{1/2}(gH)^{5/4}}$$

K_s is the shape parameter independent of size 'D'

Normally, hydraulic turbines use water and remain on earth surface hence 'ρ' & 'g' can be eliminated to obtain dimensional specific speed given by

$$N_S = \frac{NP^{1/2}}{H^{5/4}}$$

- *Where N is in rpm, P is in kW & H is in meter.*
  - **Then N_s = 1042 K_s**

Specific Speed for Pumps

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Head developed and discharge that pumps can provide are important characteristics for pump.

- *Using flow and heat coefficients, another dimensionless parameter not involving physical size variables can be estimated as follows.*

$$\Pi_5 = \frac{\Pi_1^{1/3}}{\Pi_2^{1/2}} = \frac{Q^{1/3} N^{2/3}}{(gH)^{1/2}} \longrightarrow K_{s'} = \frac{N Q^{1/2}}{(gH)^{3/4}}$$

Dimensional specific speed is given by

$$N_S = \frac{N Q^{1/2}}{H^{3/4}}$$

- *Where N is in rpm, Q in m^3/s and H in meter.*
  - **$N_s = 333\ k_{s'}$**

Specific Speeds for Different Turbomachines

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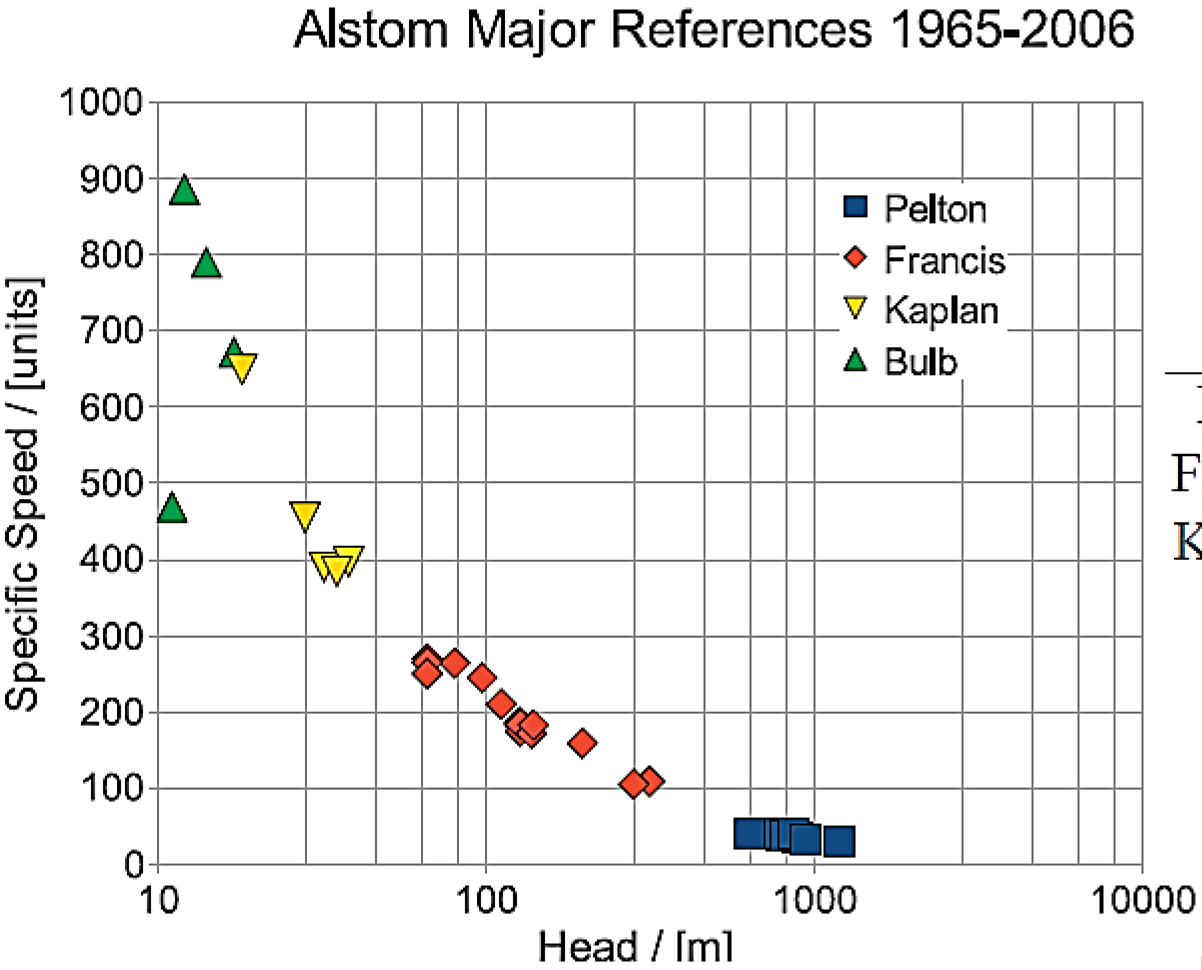

Case	Specific Speed
Pelton Wheel	50 and below
Francis Turbine	100 to 300
Kaplan Turbine	400 and above

Figure: Specific Speed for a Number of Hydraulic Turbines

Example A turbine is to be designed for a site with 400 m of head an an expected power of 1 MW, the turbine will feed electricity into a 50 Hz electrical grid. Using specific speed estimate which sort of turbine design should be investigated further.

Solution: Turbine speed in RPM is given by

- *$N = 50 \times 60 = 3000$ rpm.*

$$N_S = \frac{NP^{1/2}}{H^{5/4}} = \frac{3000 \times 1000^{1/2}}{400^{5/4}} = 53.03$$

Case	Specific Speed
Pelton Wheel	50 and below
Francis Turbine	100 to 300
Kaplan Turbine	400 and above

- *For specific speed range upto 53, Pelton wheel is suitable*

4. The Sellrain-Silz power station in Austria has two turbines each delivering 260 MW of power. The operating head is 1233 m and the rotational speed is 500 rev/min. Show that a multiple nozzle Pelton turbine is best suited for this application.

Steps to solution is to find the specific speed and see where it falls.

$$N_S = \frac{NP^{1/2}}{H^{5/4}}$$

Case	Specific Speed
Pelton Wheel	50 and below
Francis Turbine	100 to 300
Kaplan Turbine	400 and above

Specific Speed $\Leftrightarrow N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{500\times\sqrt{260\times10^3}}{1233^{1.25}} = 34.8942$

Hence, multiple jet Pelton turbine is justified.

Derive the flow, head & power coefficient expressions using Buckingham Pi theorem

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Steps:

- *List the ‘k’ parameters along with their units and decide ‘r’ fundamental dimensions to describe them*

Q_1	Rotational Speed	N	T^{-1}
Q_2	Diameter	D	L
Q_3	Density	ρ	$M \cdot L^{-3}$
Q_4	Volumetric Flow Rate	Q	$L^3 \cdot T^{-1}$
Q_5	Head	H	$L^2 \cdot T^{-2}$
Q_6	Power	P	$M \cdot L^2 \cdot T^{-3}$

6 Q's - 3 Dimensions = 3 Π's

- *Choice of repeating Variables ρ, N & D*

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$$\Pi_1 = N^a \cdot D^b \cdot \rho^c \cdot Q$$

$$\left(T^{-1}\right)^a \cdot (L)^b \cdot \left(M \cdot L^{-3}\right)^c \left(L^3 \cdot T^{-1}\right) = M^0 \cdot T^0 \cdot L^0$$

$$\Pi_1 = \frac{Q}{ND^3} = \phi = Flow coefficient$$

$$\Pi_2 = N^a \cdot D^b \cdot \rho^c \cdot H$$

$$\left(T^{-1}\right)^a \cdot (L)^b \cdot \left(M \cdot L^{-3}\right)^c \left(L^2 \cdot T^{-2}\right) = M^0 \cdot T^0 \cdot L^0$$

$$\Pi_2 = \frac{gH}{N^2 D^2} = \psi = Energy\ transfer\ or\ head\ coefficient$$

$$\Pi_3 = N^a \cdot D^b \cdot \rho^c \cdot P$$

$$\left(T^{-1}\right)^a \cdot (L)^b \cdot \left(M \cdot L^{-3}\right)^c \left(M \cdot L^2 \cdot T^{-3}\right) = M^0 \cdot T^0 \cdot L^0$$

$$\Pi_3 = \frac{P}{\rho N^3 D^5} = \xi = Power\ coefficient$$