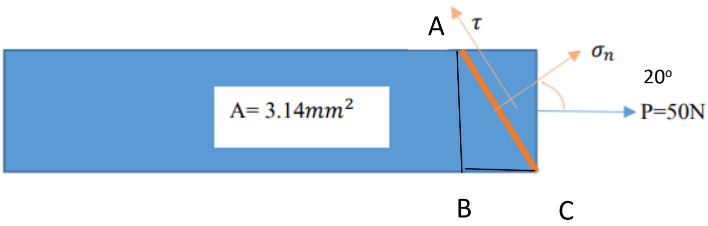
<u>ME1020 Homework 5</u>

Abhishek Ghosh me21btech11001

<u>Question 1:-</u>



$Angle\ BAC = 20^0$

$Given\ A = 3.14mm^2\ ; P = 50N$

$Area\ along\ AC\ , A_{AC} = \dfrac{A}{cos20^o}$

$Force\ perpendicular\ to\ AC\ is\ given\ by\ , F_{\perp} = P\cos 20^o$

$Force\ parale\ to\ AC\ is\ given\ by\ , F' = P\sin 20^o$

$The\ normal\ stress\ along\ the\ cross\ section\ AC\ is\ given\ by\ \sigma_n;$

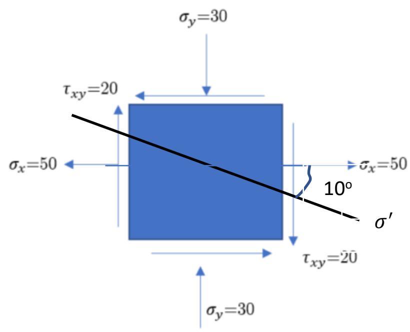
$$Where\ \sigma_n = \frac{F_{\perp}}{A_{AC}} = \frac{P\cos 20^o}{\frac{A}{\cos 20^o}} = \frac{P}{A}\,(\cos 20^o)^2$$

$$= \frac{50}{3.14\ X\ 10^{-6}}(0.881) = \mathbf{14.04}\boldsymbol{M\ Pa}$$

$$The\ shear\ stress\ is\ given\ by\ \tau = \frac{F'}{A_{AC}} = \frac{P\sin 20^o}{\frac{A}{cos20^o}} = \frac{P}{A}\sin 20^o\cos 20^o$$

$$= \frac{50}{3.14\ X\ 10^{-6}}0.321 = \mathbf{5.11}\ \boldsymbol{M\ Pa}$$

Question 2:-



+x axis is + σ

-x axis is - σ

+y axis is τ_{cw}

-y axis is τ_{ccw}

a) $Aloong\ y\ axis\ the\ force\ is\ compressive\ , \sigma_y = -30\ M\ Pa;$

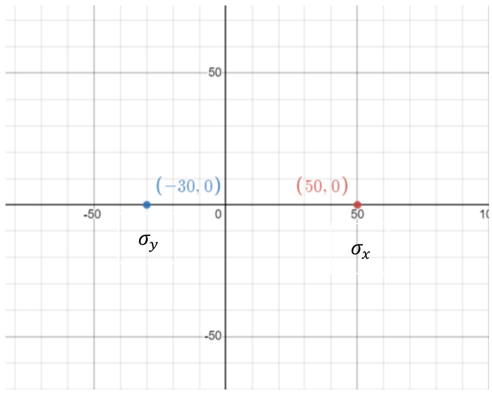
$Along\ x\ axis\ the\ force\ is\ tensile\ , \sigma_x = 50\ M\ Pa;$ (cw is +ve and ccw is -ve)

$Given\ \tau_{xy} = 20; Along\ y\ this\ shear\ is - ve\ and\ along\ x\ this\ is + ve$

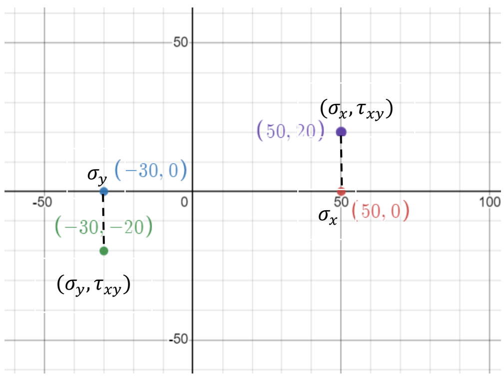
$$Centre\ of\ Mohr's\ circle\ , C = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 50}{2} = 10\ M\ Pa$$

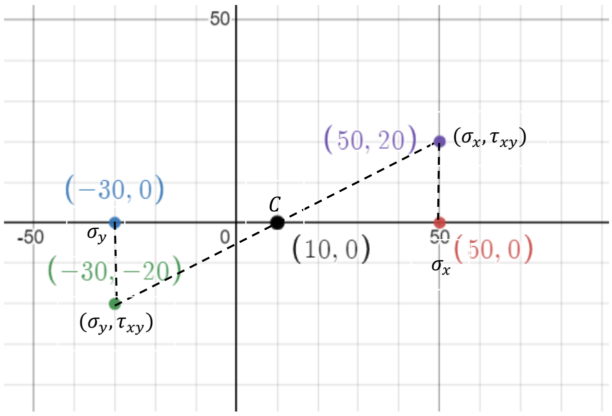
$$Radius\ of\ Mohr's\ circle\ is\ , R = \tau_n = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + 20^2}$$

$$= 20\sqrt{5} = 44.72 M\ Pa$$

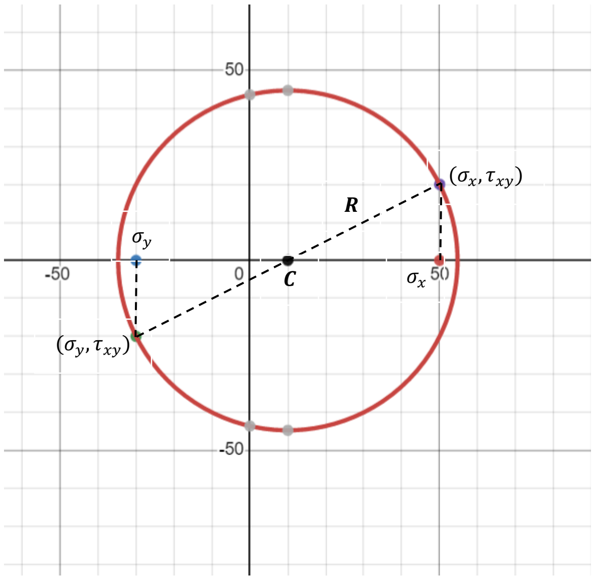


Denoting shear stress





Drawing mohr's circle with C as centre and R as radius



b) $Maximum\ shear\ principal\ stress\ , \sigma_a = \sigma_{avg} + R = 54.72\ M\ Pa$

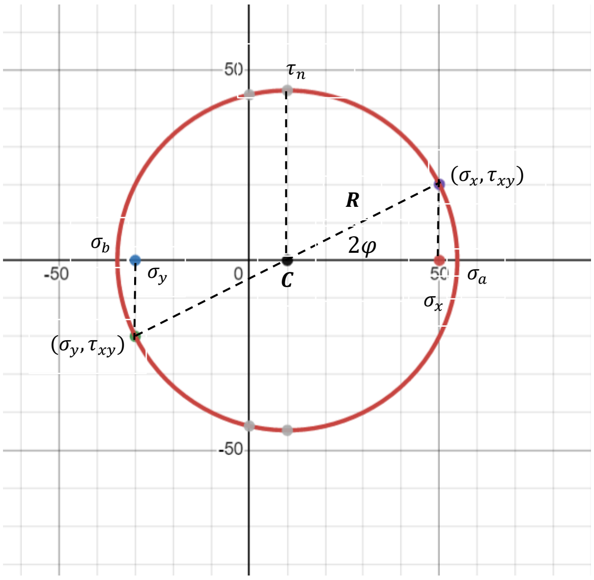
$Minimum\ shear\ principal\ stress\ , \sigma_b = \sigma_{avg} - R = -34.72\ M\ Pa$

$Maximum\ shear\ stress\ , \tau_n = R = 44.72\ M\ Pa$

$Angle\ of\ maximum\ principal\ stress\ \varphi\ on\ the\ body\ and\ 2\varphi\ on\ the\ mohr's\ circle$

$is\ given\ by\ \tan(2\varphi) = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{2(20)}{50 - (-30)} = \dfrac{1}{2} = 0.5$

$\Rightarrow \varphi = \dfrac{1}{2}\tan^{-1} 0.5 = \mathbf{13.28^o}$



c) *At angle* $\varphi = 10^o$ *with* σ_x *of the material*

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\varphi + \tau_{xy}\sin 2\varphi$$

$$= 10 + 40\cos 20^o + 20\sin 20^o = \mathbf{54.42}\ M\ Pa$$

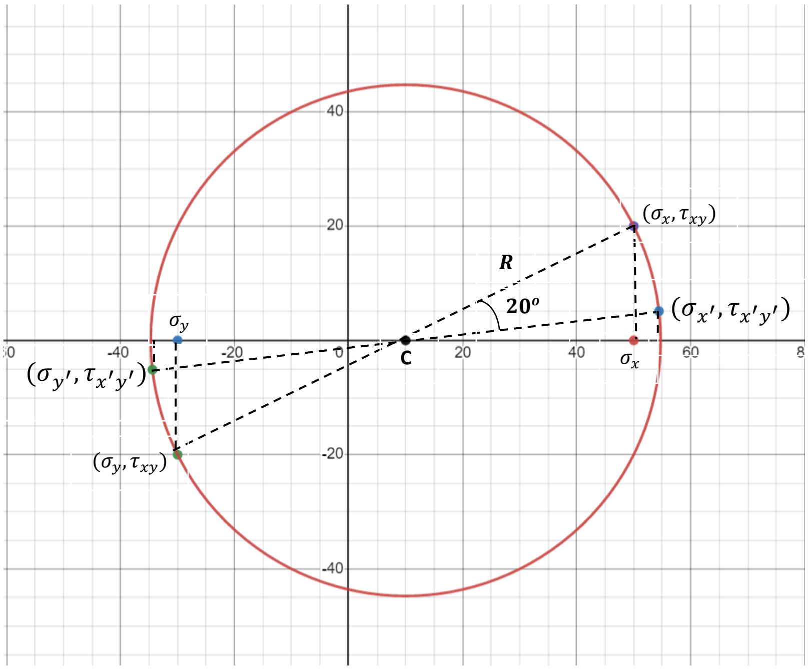
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\varphi - \tau_{xy}\sin 2\varphi$$

$$= 10 - 40\cos 20^o - 20\sin 20^o = -\mathbf{34.42}\ M\ Pa$$

For angle on labelled figure see 1st diagram of this ques.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\varphi + \tau_{xy}\cos 2\varphi$$

$$= -40\sin 20^o + 20\cos 20^o = \mathbf{5.11}\ M\ Pa$$



Question 3:-

Given $\sigma = \begin{bmatrix} 10 & 20 & -50 \\ -30 & 44 & 0 \\ 72 & 28.8 & -5 \end{bmatrix}$

Also $\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} = \sigma_{hyd} + \sigma_{dev};$

Where $\sigma_{hyd} = \frac{\sigma_{xx}+\sigma_{yy}+\sigma_{zz}}{3} = \frac{10+44+(-5)}{3} = \frac{49}{3} = 16.33$

$\Rightarrow \boldsymbol{\sigma_{hyd}} = \begin{bmatrix} \mathbf{16.33} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{16.33} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{16.33} \end{bmatrix};$

$\Rightarrow \sigma_{dev} = \sigma - \sigma_{hyd} = \begin{bmatrix} 10 & 20 & -50 \\ -30 & 44 & 0 \\ 72 & 28.8 & -5 \end{bmatrix} - \begin{bmatrix} \frac{49}{3} & 0 & 0 \\ 0 & \frac{49}{3} & 0 \\ 0 & 0 & \frac{49}{3} \end{bmatrix} = \begin{bmatrix} \frac{-19}{3} & 20 & -50 \\ -30 & \frac{83}{3} & 0 \\ 72 & 28.8 & \frac{34}{3} \end{bmatrix};$

$\Rightarrow \boldsymbol{\sigma_{dev}} = \begin{bmatrix} \mathbf{-6.33} & \mathbf{20} & \mathbf{-50} \\ \mathbf{-30} & \mathbf{27.67} & \mathbf{0} \\ \mathbf{72} & \mathbf{28.8} & \mathbf{-21.34} \end{bmatrix};$

Question 4:-

In generalized hooke's law $\sigma_{ij} = C_{ijkl}\epsilon_{kl}$

C_{ijkl} *is the stiffness tensor(* 4^{th} *order)having* 81 *components because stress and strain have* 9 *components each.*

Stiffness tensor gives the relation between stresses nad the strains and use to represent solid'stiffness.

4^{th} *order tensor is tensor product of* 4 *linear elastic components*

Symmetry influences the C matrix in following ways:—

i) In generic anisotropic C matrix has 36 components.
ii) If the anisotropic C matrix has strain symmetry, then the number of independent quantities are 21.
iii) In orthotropic materials having 3 planes of symmetry the independent quantities are 9.

iv) Isotropic materials have 2 independent quantities.

C Matrix for anisotropic material:-

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix}$$

C matrix for orthotropic material:-

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

C matrix for isotropic material is:-

$$\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix}$$

$$Where\ C_{44} = \frac{C_{11} - C_{12}}{2}$$

Question 5:-

a) Total weight w=80*9.8=784 N

b) Weight on one leg $w' = \frac{w}{2} = 392\ N$

c) 

d) Young's Modulus Y for rubber slipper is = $0.1\ G\ Pa = 10^8\ Pa$

e) Area of slipper (using solid edge software)

$$A_{slipper} = 23913.87\ mm^2 = 2.4\ X\ 10^{-2}\ m^2$$

Stress on slipper $\sigma = \frac{w'}{A_{slipper}} = \frac{392}{2.4\ X\ 10^{-2}} = \ 1.64\ X\ 10^4 N\ m^{-2}$