Some Problems on ODE: (Homework):

(1) Verify that the following eqns are homogeneous and solve them:

(a) $(x^2 - 2y^2)\,dx + xy\,dy = 0$

(b) $x^2 y' - 3xy - 2y^2 = 0$, where $y' = \frac{dy}{dx}$

(c) $x^2 y' = 3(x^2 + y^2)\tan^{-1} y/x + xy$

(d) $x \sin y/x \frac{dy}{dx} = y \sin y/x + x$

(e) $xy' = y + 2xe^{-y/x}$

(f) $(x - y)\,dx - (x + y)\,dy = 0$

(2) Solve the following ODEs:

(a) $y' = (x + y)^2$ (b) $y' = \sin^2(x - y + 1)$

(c) $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$

(d) $\frac{dy}{dx} = \frac{x+y+4}{x+y-6}$

(3) Determine which of the following eq^{n}s are exact and solve it:

(a) $(x+\frac{y}{y})\,dy + y\,dx = 0$

(b) $(y-x^3)\,dx + (x+y^3)\,dy = 0$

(c) $(y+y\cos xy)\,dx + (x+x\cos xy)\,dy = 0$

(d) $(\sin x \sin y - xe^y)\,dy = (e^y + \cos x \cos y)\,dx$

(e) $-\frac{1}{y}\sin\frac{x}{y}\,dx + \frac{x}{y^2}\sin\frac{x}{y}\,dy = 0$

(f) $(2xy^3 + y\cos x)\,dx +$
$(3x^2y^2 + 8\sin x)\,dy = 0$

Answers :

(1) (a) $y^2 = x^2 + cx^4$

(b) $y = cx^2(x+y)$

(c) $y = x\tan cx^3$

(d) $\cos y/x + \log cx = 0$

(e) $y = x\log(\log cx^2)$

(f) $x^2 - 2xy - y^2 = c$

(2) (a) $x+y = \tan(x+c)$

(b) $\tan(x-y+1) = x+c$

(c) $\tan^{-1}\left(\frac{y+5}{x-1}\right) = \log\sqrt{(x-1)^2+(y+5)^2} + c$

(d) $y - x = 5\log(x+y-1) + c$

3 (a) $xy + \log y^2 = c$

(b) $4xy - x^4 + y^4 = c$

(c) $xy + \sin xy = c$

(d) $xe^y + \sin x \cos y = c$

(e) $\cos \frac{x}{y} = c$ or $\frac{x}{y} = c$

(f) $x^2y^3 + y\sin x = c$

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