THEO - JANSEN'S MECHANISM

ME2220

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Dimensional Synthesis of Theo - Jansen Mechanism

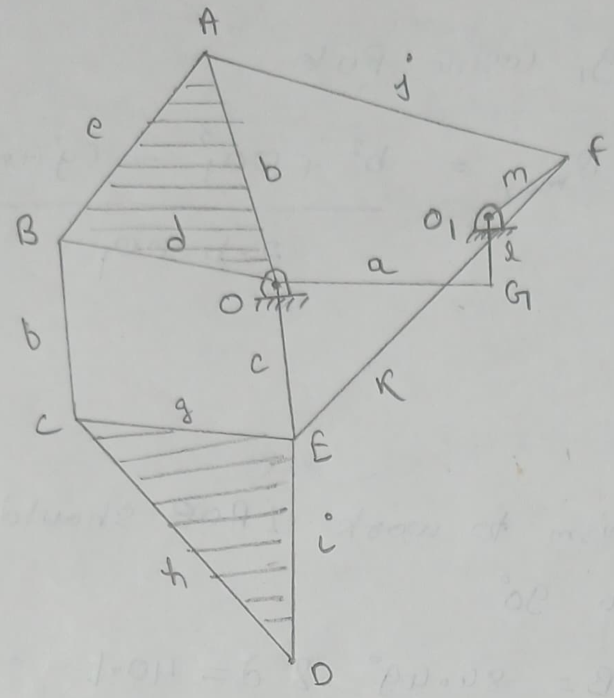


fig: Theo - Jansen model

The first step is to design of two four- bar subcomponents and the second is the design of a parallel four-bar mechanism.

for the first 4-bar-mech OO_1fA is a crank rocker. (m is crank, b is rocker)

i) OO_1fA synthesis

Taking $a = 38$ and $l = 7.8$

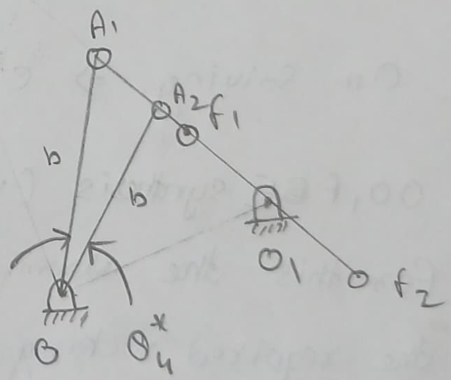
$OO_1 = \sqrt{a^2 + l^2} = 38.79$

The required rocking angle

$\theta_4^* = 42.37^\circ$ (Constraint)

Also, $\frac{m}{b} = \sin\left(\frac{\theta_4}{2}\right)$

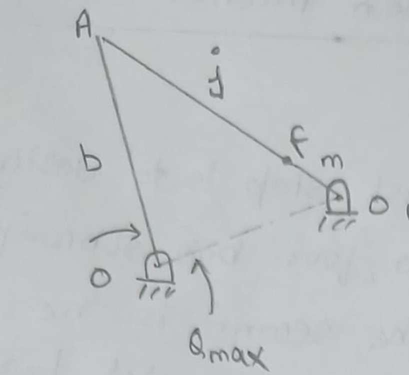
$\frac{m}{b} = 0.361$



$A_1A_2 = 2m$

Taking $m = 15 \Rightarrow b = \frac{15}{0.361}$

$= 41.5$



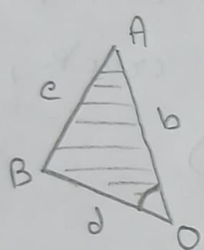
Assuming the max angle (θ_{max}) b/w rocker and OO_1 to be $108.06°$

Using, Cosine Rule

$$\cos \theta_{max} = \frac{b^2 + OO_1^2 - (j + m)^2}{2 \cdot b \cdot OO_1}$$

On solving, $\Rightarrow j = 50$

ii) ΔOAB



for the mechanism to work $\angle AOB$ should be b/w $0°$ to $90°$

Assuming $\angle AOB = 84.49°$ & $d = 40.1$

Using cosine Rule

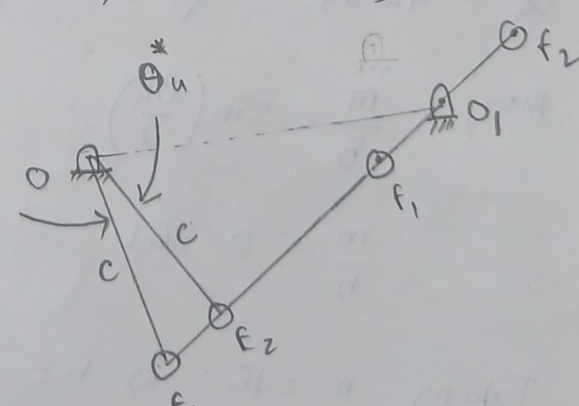
$$\cos 84.49° = \frac{b^2 + d^2 - e^2}{2 \cdot b \cdot d}$$

On Solving $\Rightarrow e = 55.8$

iii) $OO_1 f E$ synthesis (m is crank, c is rocker)

For this the assuming the required rocking angle

$\theta_4^* = 44.87°$ (Constraint)



from $\Delta O E_1 E_2$

$$\frac{m}{c} = \sin\left(\frac{\theta_4^*}{2}\right)$$

$E_1 E_2 = 2m$

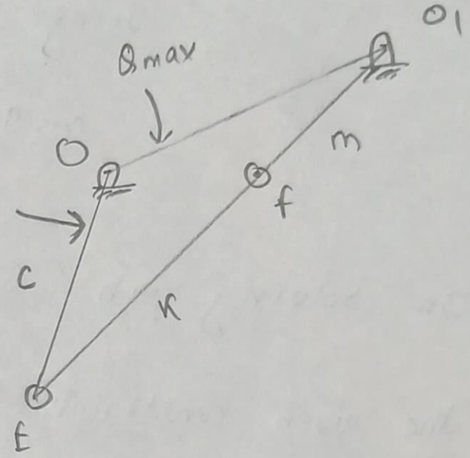
$O_1 f = m$

$$\frac{m}{c} = 0.381$$

$$\Rightarrow \quad c = \frac{m}{0.381} = \frac{15}{0.381}$$

$$\Rightarrow \quad c = 39.3$$

Assuming max angle θ_{max} b/w rocker & OO_1 to be 160°



Using cosine Rule

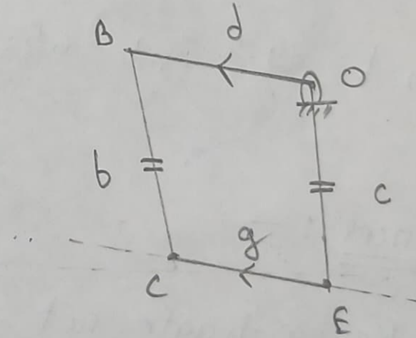
$$\cos 160^\circ = \frac{c^2 + OO_1^2 - (K+m)^2}{2 \cdot c \cdot OO_1}$$

On Solving $\Rightarrow K = 61.9$

iv) OBCE synthesis

Since this is a parallel four bar mech with $OB \parallel CE$ & $BC = OE$

$$b = c = 39.3$$

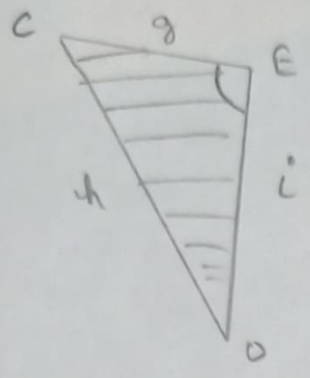


With OE known, draw $CE \parallel BO$

Now construct BC of length 39.3

This gives length of $CE = g = 36.7$.

v) ΔECD



for given mechanism $\angle CEO$ should range from 0° to 180°

Assuming $\angle CEO = 99.1^\circ$ & $i = 49$

Using cosine Rule

$$\cos 99.1^\circ = \frac{g^2 + i^2 - h^2}{2 \cdot g \cdot i}$$

On Solving $\Rightarrow h = 65.7$

With the given constraints of Rocking angle and assuming max angle ($\angle AOO_1 = 108.06^\circ$ & $\angle CEO = 99.1)^\circ$ & lengths $a = 38$, $d = 7.8$, $d = 40.1$, $m = 15$, $i = 49$ the following are found out to be

$b = 41.5$, $f = 50$, $e = 55.8$, $c = 39.3$, $k = 61.9$, $g = 36.7$, $l = 39.3$, $h = 65.7$

To prove Trajectory of point T only depends on input angle :-

Theory :-

Theo - Jansen schematic diagram is shown below. It consists of two stiff triangles AON, FBT and rods $KA = a$, $KB = z$, $NF = g$, $OB = l$, $O_1K = r$.

Hinges → A, O, N, B, F, K, O_1

with O and O_1 being fixed firmly.

The mechanism is set to motion by moving the rod O_1K (r). The vertex T of triangle FBT follows a certain trajectory.

Let $AN = w$, $OA = v$, $ON = u$, $FB = b$, $BT = p$, $FT = s$,

$\angle AON = \alpha$, $\angle FBT = \beta$ & $OO_1 = d$.

In order for T to follow optimal trajectory, we need to find numerical values of the parameters →

$a, z, d, g, l, v, u, w, b, p, s, r$.

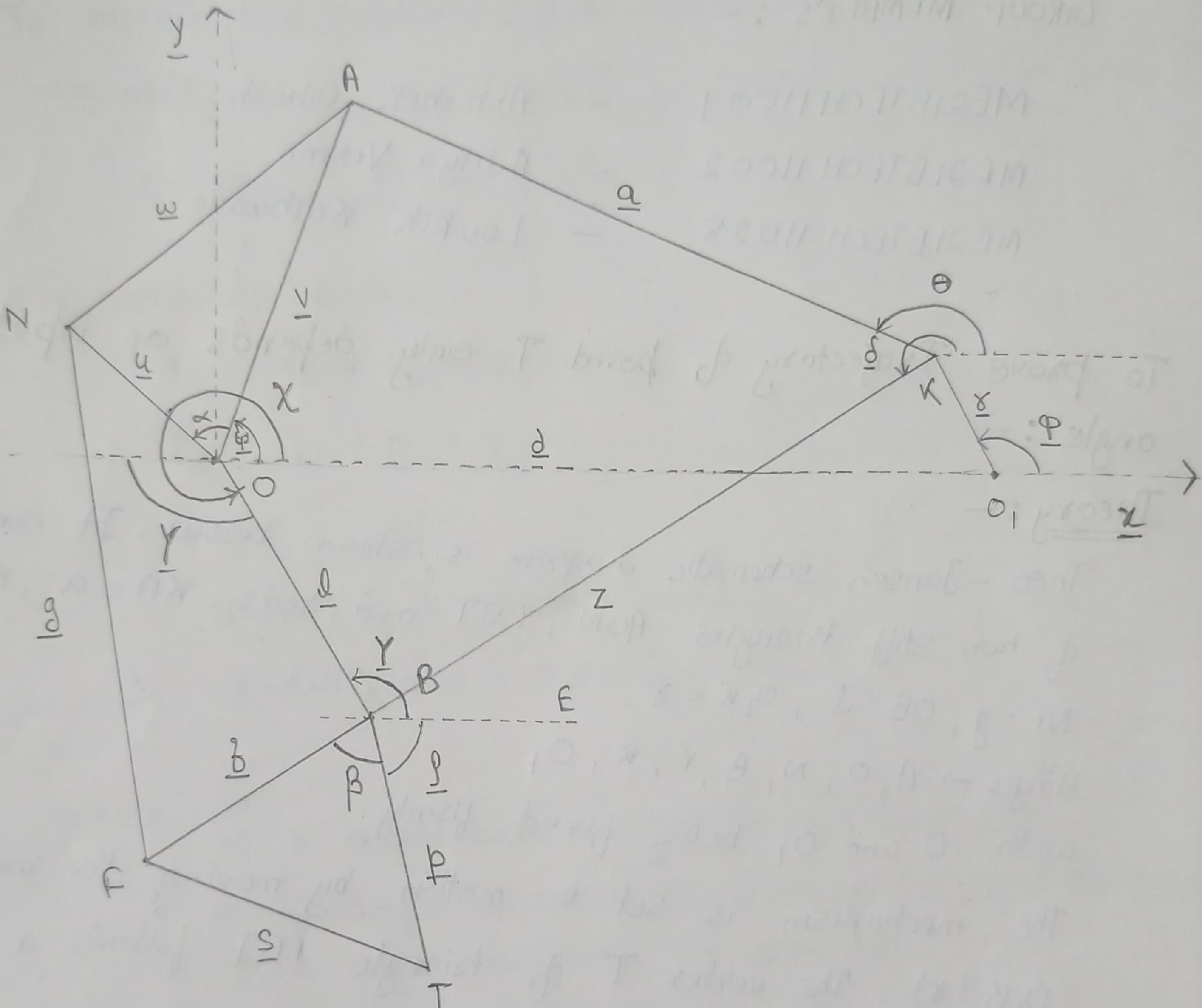


Fig1: Schematic Diagram of Theo Jansen Mechanism

The coordinates of point T =:

$$x_T = x_B + p\cos\rho$$
$$y_T = y_B + p\sin\rho \quad -(1)$$

where x_B, y_B are the coordinates of point B.

ρ = angle between BE and BT.

To obtain the coordinates of point B, we used the vector equality. $\vec{OB} = \vec{OK} + \vec{KB}$ —(2) (where, $\vec{OK} = \vec{OO_1} + \vec{O_1K}$)

or say, $\vec{l} = (\vec{d} + \vec{r}) + \vec{z}$ —(3)

In coordinate form:

$$x_B = x_d + x_r + x_z$$
$$y_B = y_d + y_r + y_z \quad —(4)$$

or

$$l\cdot\cos\chi = d + r\cos\phi + z\cos\delta$$
$$l\cdot\sin\chi = 0 + r\sin\phi + z\sin\delta$$

where,
φ = angle between horizontal and O_1K.
δ = angle between the horizontal and KB (figure)
χ = angle between Ox and OB.

from eq -(4), it follows

$$z\cdot\sin\delta = l\sin\chi - r\sin\phi$$
$$z\cdot\cos\delta = l\cos\chi - r\cdot\cos\phi - d \quad -(5)$$

by ratio;

$$(z\sin\delta)^2 + (z\cos\delta)^2 = z^2 \quad -(6)$$

from eq^n - (6) we found -:

$$A \sin x + B \cdot \cos x = C \quad - (7)$$

$$A = 2lr \sin\phi$$
$$B = 2lr \cos\phi + 2dl \quad - (8)$$
$$C = l^2 + r^2 + d^2 + 2rd\cos\phi - z^2$$

let, $\sin x = \dfrac{2t}{1+t^2}$, $\cos x = \dfrac{1-t^2}{1+t^2}$ $\quad - (9)$

substituting the values of $\sin x$ and $\cos x$ in eq^n - (7)

$$(B+C)t^2 - 2At + (C-B) = 0 \quad - (10)$$

the roots of above equation will be real ~~and~~ at a positive discriminant.

ie. $A^2 + B^2 - C^2 \geq 0$

roots of equation - (10) $\Rightarrow$

$$t_{1,2} = \frac{2 \cdot A \pm \sqrt{4 \cdot A^2 - 4(B+C)(C-B)}}{2(B+C)} = \frac{A \pm \sqrt{A^2+B^2-C^2}}{B+C} \quad - (12)$$

$$x_{1,2} = 2\tan^{-1}(t) \quad - (13)$$

Since X_2 depends on angle φ. so,

The coordinates of point B :

$$x_B = l \cdot \cos(X_2(\varphi))$$
$$y_B = l \sin(X_2(\varphi)) \quad - (14)$$

Similarly, as we did for coordinates of point B. The coordinate of point A are -:

$$\vec{OA} = \vec{OK} + \vec{KA} \quad - (15)$$
$$\vec{v} = \vec{d} + \vec{r} + \vec{a} \quad - (16)$$

in coordinate form,

$$x_A = x_d + x_r + x_a \qquad v\cos\varphi = d + r\cos\varphi + a\cos\theta$$
$$y_a = y_d + y_r + y_a \qquad \text{or} \qquad v\sin\varphi = 0 + r\sin\varphi + a\sin\theta$$

$$\left(\begin{aligned} a\sin\theta &= v\sin\varphi - r\sin\phi \\ a\cos\theta &= v\cos\varphi - d - r - \cos\phi \end{aligned} \right) \quad - (17)$$

from ratio :

$$(v\sin\varphi - r\sin\phi)^2 + (v\cos\varphi - (r\cos\varphi + d))^2 = a^2 \quad - (18)$$

$$A_1 \sin\varphi + B_1 \cos\varphi = C_1 \quad - (19)$$

where

$$A_1 = 2vr\sin\phi$$
$$B_1 = 2vr\cos\phi + 2dv \quad - (20)$$
$$C_1 = v^2 + r^2 + d^2 + 2rd\cos\varphi - a^2$$

let say, $\sin\varphi = \dfrac{2\tau}{1+\tau^2}$, $\cos\varphi = \dfrac{1-\tau^2}{1+\tau^2}$ $\quad - (21)$

Substituting the value of $\sin\varphi$ and $\cos\varphi$ in equation-(19)

$$(B_1+C_1)\tau^2 - 2A_1\tau + (C_1-B_1) = 0 \quad -(23)$$

roots of above quadratic equation $\Rightarrow$

$$\tau_{1,2} = \frac{2A_1 \pm \sqrt{4A_1^2 - 4(B_1+C_1)(B_1-B_1)}}{2(B_1+C_1)}$$

$$= \frac{A_1 \pm \sqrt{A_1^2 + B_1^2 - C_1^2}}{B_1+C_1} \quad -(24)$$

Hence,

$$\psi_{1,2} = 2\tan^{-1}(\tau_{1,2}) \quad -(25)$$

Since ψ_1 depends on φ

Hence, the coordinates of point A -:

$$x_A = v\cos(\psi_1(\varphi))$$
$$y_B = v\sin(\psi_1(\varphi)) \quad -(26)$$

From quad BFNO, angle ρ is determined

$$2\pi - \rho(\phi) = \eta(\phi) + \nu(\phi) + \beta + \gamma(\phi)$$

ν & η are determined by cosine rule :-

$$b(\phi) = \sqrt{u^2 + d^2 - 2u \cdot d \cos(\gamma(\phi))}$$

where

$$\gamma(\phi) = 2\pi - \psi_1(\phi) - \chi_2(\phi) - \alpha$$

where from ΔAON

$$\alpha = \cos^{-1}\left(\frac{v^2 + u^2 - w^2}{2 \cdot u \cdot v}\right)$$

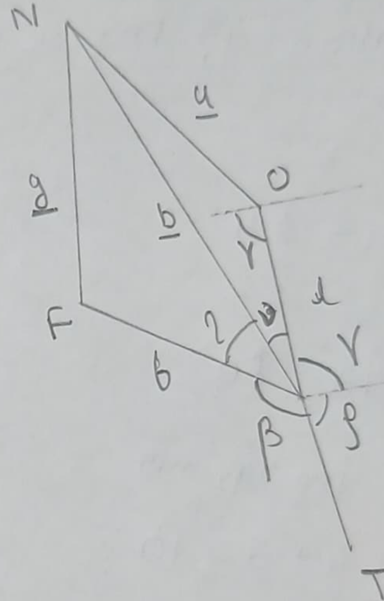


Fig: Quad BFNO

Also,

$$\nu(\phi) = \cos^{-1}\left(\frac{b(\phi)^2 + d^2 - u^2}{2 \cdot b(\phi) \cdot d}\right)$$

& $$\eta(\phi) = \cos^{-1}\left(\frac{b(\phi)^2 + f^2 - g^2}{2 \cdot b(\phi) \cdot f}\right)$$

Also from ΔfBT,

$$\beta = \cos^{-1}\left(\frac{f^2 + p^2 - s^2}{2 \cdot f \cdot p}\right)$$

After knowing $\eta(\phi)$, $\nu(\phi)$, β & $\gamma(\phi)$ we find $\rho(\phi)$

Knowing $\rho(\phi)$ we know the trajectory of T.

⇒ Movement of Point T only depends on initial input angle.

Calculation of Degree of freedom :-

$n = 8$

$j_1 = \cancel{3}\ 4$

$j_2 = \cancel{3}\ 3$

$j_3 = \cancel{1}\ 0$

$j = j_1 + 2j_2 + 0$

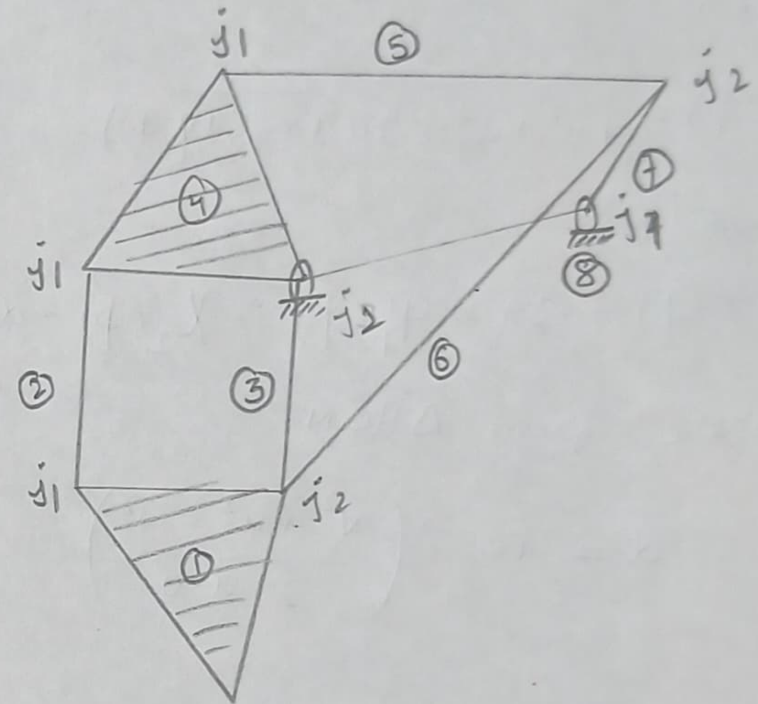
$= 4 + 6 = 10$

$F = 3(n-1) - 2j$

$= 3(8-1) - 2(10)$

$= 21 - 20$

$= \underline{\underline{1}}$



References :-

1) Researchgate. net

2) sciencedirect. com

Note :- All assumptions have been made with help of Geogebra model.

LINK: https://www.geogebra.org/classic/wvxbctek