Mid Semester Exam

Transform Techniques (MA2120)

Full Marks: 25
Time- 50 Minutes

Name: **Roll No:**

Solve all the questions: (Here L stands for Laplace Transform and L^{-1} stands for Inverse Laplace Transform)

(1) If $L[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)\, dt$, then which of the following statement (s) is/are always true?

(i) $f(t)$ is continuous or piecewise continuous and of exponential order [2 Marks]

(ii) $\lim\limits_{s\to\infty} F(s) = 0$ (iii) $\int_0^\infty e^{-st} f(t)\, dt$ converges (iv) $\lim\limits_{s\to\infty} s\,F(s)$ is finite

(a) (i), (ii), (iii) (b) (ii), (iii) (c) (ii), (iii), (iv) (d) All of the above

2. $L\left\{\sqrt{t}\cos\sqrt{7t}\right\}$ equals to [3 Marks]

(a) $\frac{\sqrt{\pi}}{4s^{\frac{3}{2}}}\, e^{-\frac{7}{4s}}$ (b) $\sqrt{\frac{\pi}{s}}\, e^{-\frac{7}{4s}}$ (c) $\frac{\sqrt{\pi}\,(2s-7)}{4s^{\frac{5}{2}}}\, e^{-\frac{7}{4s}}$ (d) None of these

3. $L\{f(t)\}$ equals to where $f(t) = \begin{cases} t, & 0 \le t < \frac{1}{2} \\ t-1, & \frac{1}{2} \le t < 1 \\ 0, & t \ge 1 \end{cases}$ [2 Marks]

(a) $-\frac{1}{s}\, e^{-\frac{s}{2}} + \frac{1-e^{-s}}{s^2}$ (b) $-\frac{1}{s}\, e^{-\frac{s}{2}} - \frac{1-e^{-s}}{s^2}$ (c) $-\frac{1}{s}\, e^{-\frac{s}{2}} + \frac{e^{-s}}{s^2}$ (d) None of these

4. $L\{\sinh at \cos at\}$ equals to [3 Marks]

(a) $\frac{2a^2 s}{s^4+4a^4}$ (b) $\frac{a(s^2+a^2)}{s^4+4a^4}$ (c) $\frac{a(s^2-2a^2)}{s^4+4a^4}$ (d) None of these

5. The value of $L\{(\cos at)/t\}$ [2 Marks]

(a) $-\frac{1}{2}\log\,(s^2+a^2)$ (b) $\cot^{-1}\left(\frac{s}{a}\right)$ (c) $\tan^{-1}\left(\frac{s}{a}\right)$ (d) does not exist

6. The value of the integral $\int_0^\infty e^{-tx^2} dx$ is [3 Marks]

(a) $\frac{1}{2}\sqrt{\frac{\pi}{t}}$ (b) $\frac{1}{2}\sqrt{\pi t}$ (c) $\frac{2}{\sqrt{\pi}\, t}$ (d) None of these (P.T.O.)

7. $L^{-1}\left[\frac{5}{s^2}+\left(\frac{(\sqrt{s}-1)^2}{s^2}\right)-\frac{7}{3s+2}\right]$ is [3 Marks]

(a) $6t+1-\sqrt{\left(\frac{t}{\pi}\right)}-\left(\frac{7}{3}\right)e^{-\frac{2t}{3}}$ (b) $6t-4\sqrt{\left(\frac{t}{\pi}\right)}-\left(\frac{7}{3}\right)e^{-\frac{2t}{3}}$ (c) $6t+1-4\sqrt{\left(\frac{t}{\pi}\right)}-\left(\frac{7}{3}\right)e^{-\frac{2t}{3}}$

(d) None of these

8. $L^{-1}\left[\frac{s}{s^4+s^2+1}\right]$ is [3 Marks]

(a) $\frac{2}{\sqrt{3}}\cosh\left(\frac{t}{2}\right)\sin\left(\frac{\sqrt{3}t}{2}\right)$ (b) $\frac{2}{\sqrt{3}}\sinh\left(\frac{t}{2}\right)\cos\left(\frac{\sqrt{3}t}{2}\right)$ (c) $\frac{2}{\sqrt{3}}\sinh\left(\frac{t}{2}\right)\sin\left(\frac{\sqrt{3}t}{2}\right)$ (d) None of these

9. $L\{f(t)\}$ equals to where $f(t)=\begin{cases}\sin\omega t, & \frac{2n\pi}{\omega}<t<\frac{(2n+1)\pi}{\omega}\\ -\sin\omega t, & \frac{(2n+1)\pi}{\omega}<t<\frac{(2n+2)\pi}{\omega}\end{cases}$, $n=0,1,2\ldots$ [3 Marks]

(a) $\frac{\omega}{s^2+\omega^2}\coth\frac{\pi s}{2\omega}$ (b) $\frac{\omega}{s^2+\omega^2}\tanh\frac{\pi s}{2\omega}$ (c) $\frac{\omega}{s^2+\omega^2}\sinh\frac{\pi s}{2\omega}$ (d) None of these

10. When $L[f(t)]=\frac{2s}{s^2-2s+5}$, the value of $f'(0)$ is [1 Mark]

(a) 4 (b) 2 (c) 0 (d) None of these

***********************Best of Luck*************************