Engineering Mechanics Notes

- Body consists of small particles or molecules between which forces are acting
- Molecular forces tend to resist the change in body shape or size, when an external force is acted on the body
- If the particles are displaced, it will continue until the equilibrium occurs between the internal forces and applied external force.
- If the external force displaces the particles, then the body is said to be under a "*state of strain.*"
- As a response to the strain applied, the material will experience stress. Therefore, the strain is the fundamental responsible for the material's stress.
- During the loading, the material will undergo deformation; if the material reaches the initial/original shape after removal of the load, then the loading is said to be within the limit of Elasticity.
- The property of the material regaining its shape after removal of the load is said to be Elasticity.
- If the material deformed permanently even after removing the load, it is said to be in the plastic region.

Hooke's Law-2D

Consider a material is subjected to a load of F, as shown in the below figure

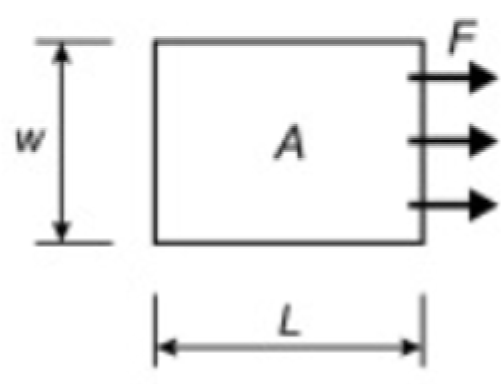


Figure 1 Load applied on a body

It has been established for many structural materials that the elongation of the bar is proportional to the tensile force within certain limits. This simple linear relationship between the force the elongation, which Robert Hooke first coined it in 1678.

$$\delta = \frac{FL}{AE}$$

Here δ = total elongation of the bar

F = Total applied load

A = Cross-sectional area perpendicular to applied to the load

E = Elastic modulus/Young's modulus of the material/ Modulus of Elasticity

$$\delta = \frac{L}{E}\left(\frac{F}{A}\right)$$

Stress:

The force per unit area is known as the stress in the body. $\sigma = \left(\frac{F}{A}\right)$. The above equation can be written as

$$\delta = \frac{L}{E}\sigma$$

$$E\,\frac{\delta}{L} = \sigma$$

Strain:

The elongation per unit length of the material is known as strain.$\epsilon = \frac{\delta}{L}$

$$E\,\epsilon = \sigma$$

The above expression is widely known as Hooks law.

Example Problems:

1. Determine the total elongation of a steel bar 25 in. long, if the tensile stress is equal to 15×10^3 lbs. per sq. in.

Answer.

$$\delta = \epsilon \times l = \frac{25}{2,000} = \frac{1}{80} \text{ in.}$$

2. Determine the tensile force on a cylindrical steel bar of one inch diameter, if the unit elongation is equal to $.7 \times 10^{-3}$.

Solution. The tensile stress in the bar, from eq. (4), is

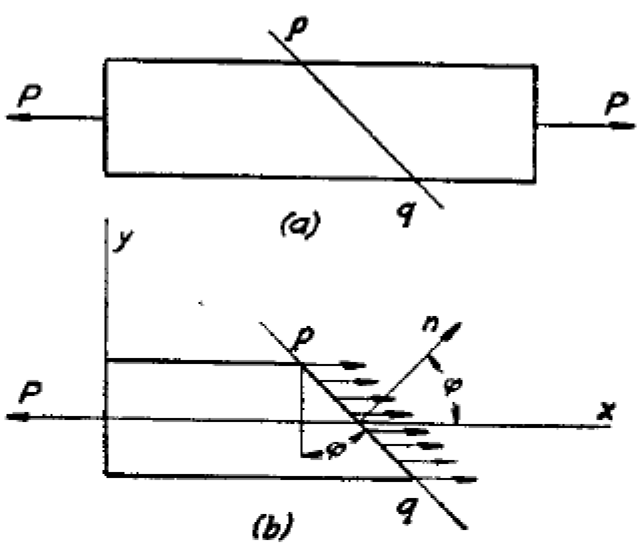
$$\sigma = \epsilon \cdot E = 21 \times 10^3 \text{ lbs. per sq. in.}$$

The tensile force, from eq. (2), is

$$P = \sigma \cdot A = 21 \times 10^3 \times \frac{\pi}{4} = 16,500 \text{ lbs.}$$

Above examples are taken from the strength of materials by Timoshenko

Let's try to understand the variation of stress on different sections (normal stress and shear stress). Consider the loading P on the body as shown in below image



The normal stress on the cross-section(pq) is given by,

$$\sigma = \frac{Force}{Area} = \frac{Pcos\phi}{A/cos\phi} = \sigma_x(cos\phi)^2$$

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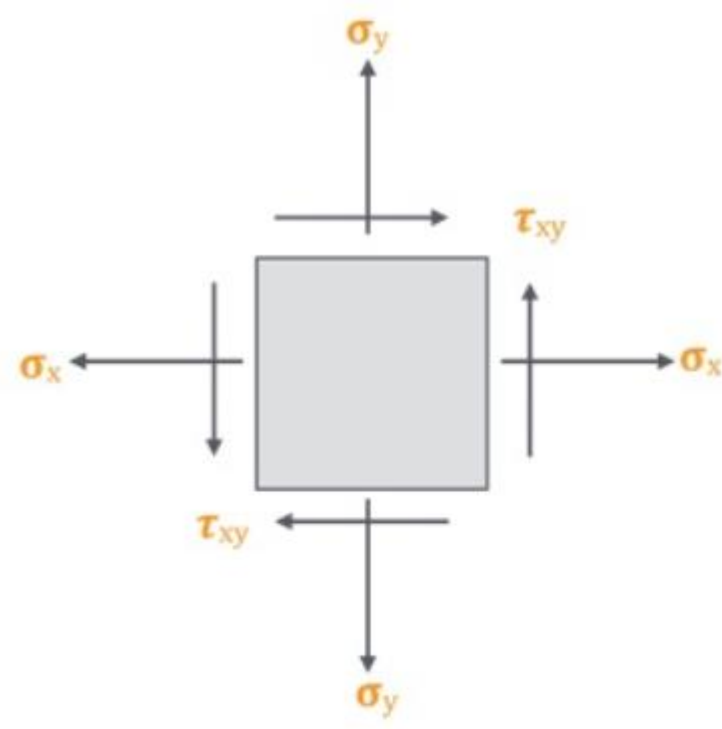
The area of the cross-section pq is given by $A_{pq} = \frac{A}{cos\phi}$. Here A is the area of the cross-section normal to the loading. Here, ϕ is the angle between the x-axis and normal to the cross-section pq.

The stress parallel to the cross-section pq is known as shear stress and is given by the following. The force acting parallel to the surface pq will be $P\ sin\phi$, and the cross-section area of the surface pq is $A_{pq} = \frac{A}{cos\phi}$. There the shear stress is given by

$$\tau = \frac{Psin\phi}{A/cos\phi} = \sigma_x sin\phi cos\phi$$

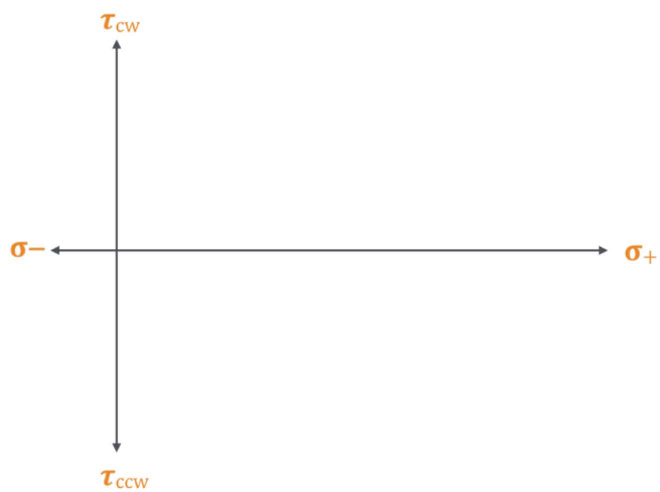
Mohr's circle

Mohr's circle is a graphical tool commonly used to analyze the principal and maximum shear stresses on any body plane. Consider an element of the body having the normal and shear forces applied on the surface, as shown in the below figure.

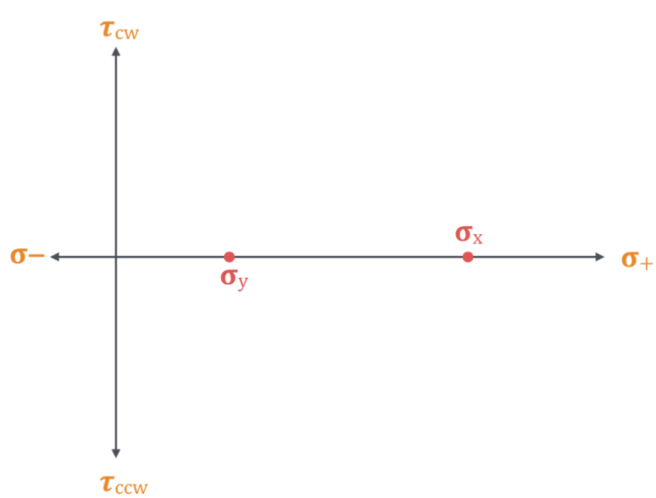


Procedure:

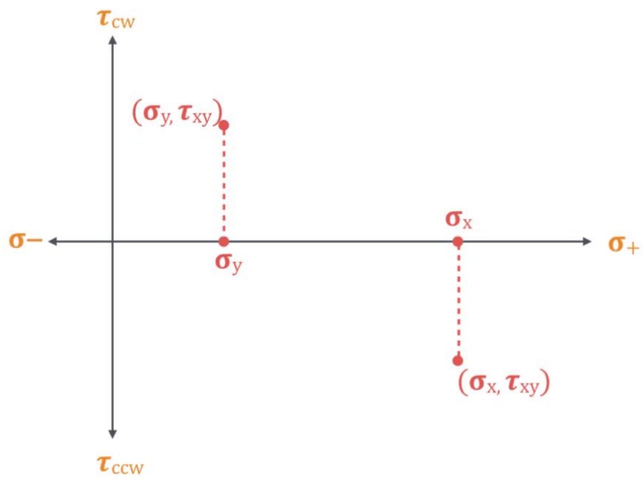
1. Define the coordinate system; usually, the X-axis is the normal stress, and Y-axis is the shear stress
2. +ve X denotes the tensile stress, -ve X denotes the compressive normal stress
3. +ve Y denotes the clockwise shear, -ve Y counter-clockwise shear. As shown in the below figure



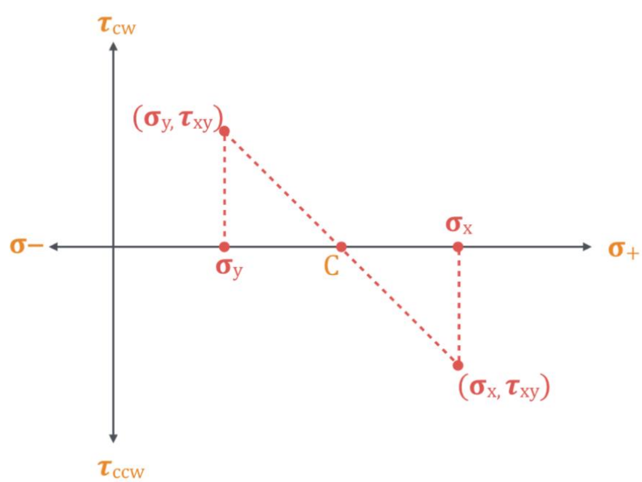
4. Denote the points σ_x,σ_y applied on the material. Here, in this case, both are +ve and $\sigma_x > \sigma_y$



5. Denote the shear stress τ_{xy} on the plane as shown below.



6. Obtain the
   center of Mohr's circle $\sigma_{avg} = \frac{\sigma_x+\sigma_y}{2}$ and
   the radius of the circle $R = \tau_n = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$



7. Draw the circle

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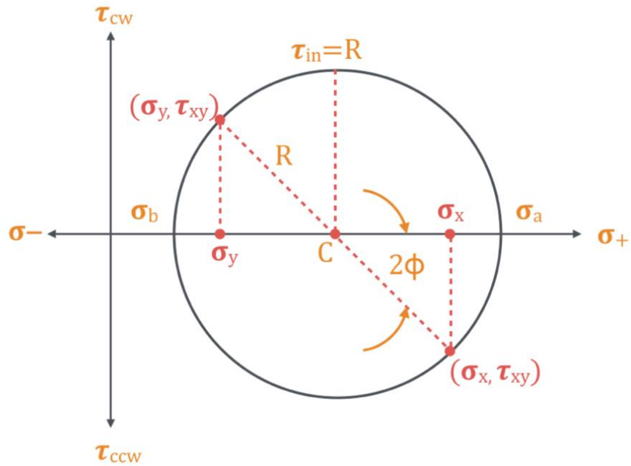


From the circle, we can find the maximum and minimum principal stresses

$$\sigma_{a=}\sigma_{avg} + R$$

$$\sigma_{b=}\sigma_{avg} - R$$

$$\sigma_{a=}\frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{b=}\frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

With this circle, we can find the stress state on any plane; if ϕ is the plane angle in the body, it will be 2ϕ in the Mohr's circle.

$$\sigma_{avg} = \frac{\sigma_a + \sigma_b}{2};\ \tan(2\phi) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Stresses on any plane can be calculated given the $\sigma_x, \sigma_y, \tau_{xy}$ using

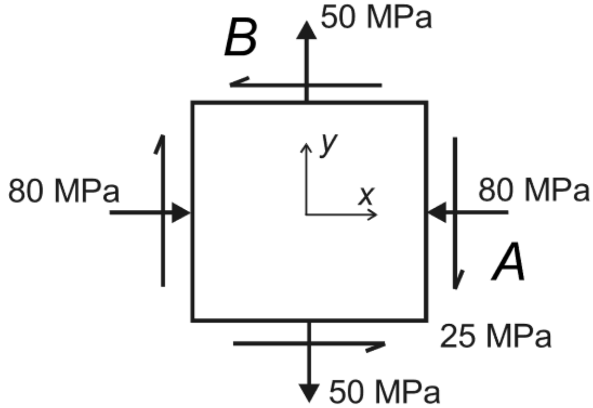
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \sigma_x + \sigma_y - \sigma_{x'}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

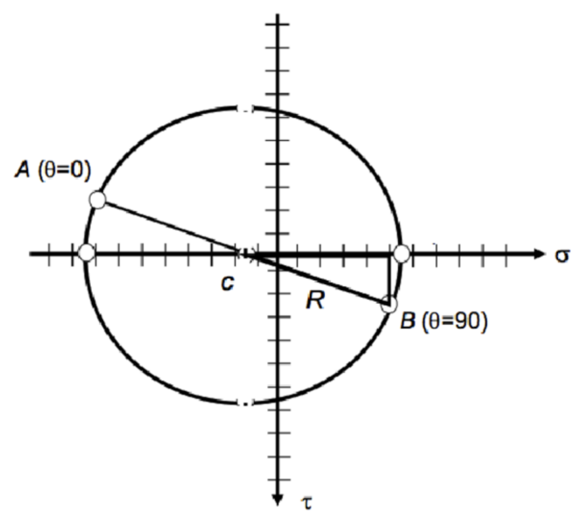
Example:



$$c = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-80+50}{2} = -15$$

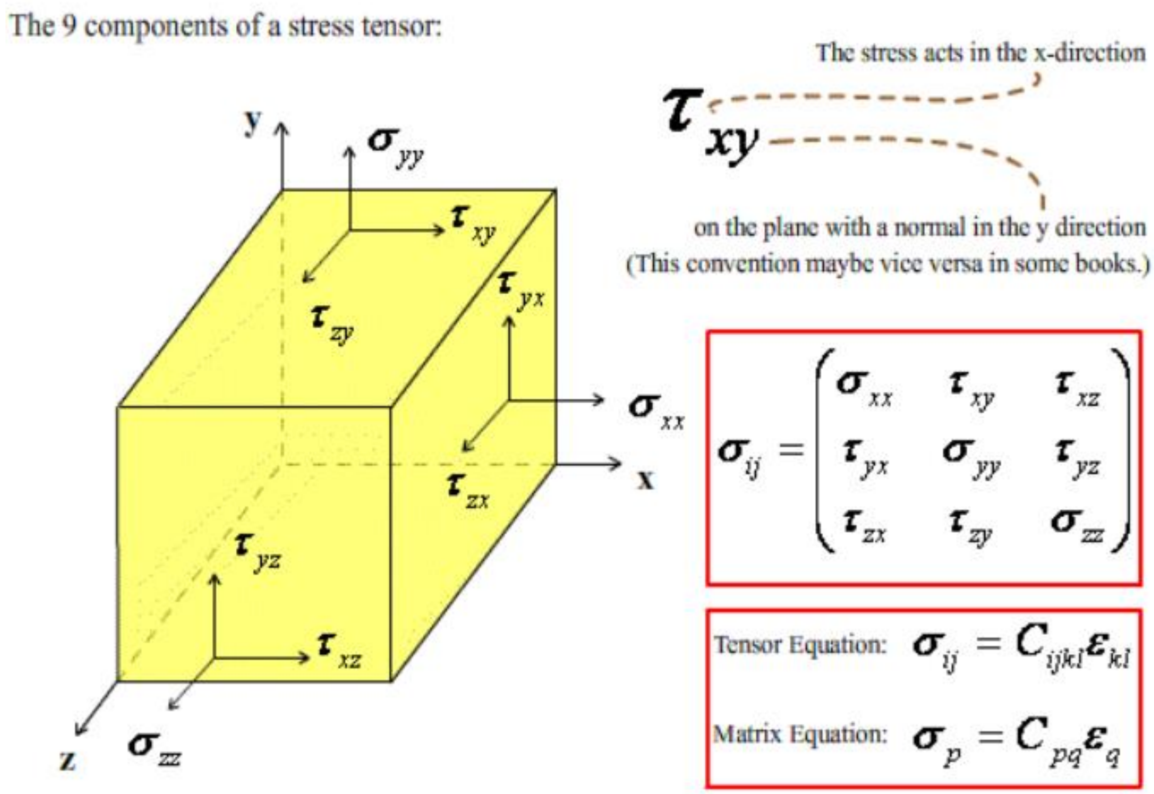
$$R = \sqrt{(50-(-15))^2 + (25)^2}$$

$$R = \sqrt{65^2 + 25^2} = 69.6$$



Hooke's Law-3D

Consider the 3D body, as shown in the figure below



The σ_{ij} is known as stress tensor. Scalar will have only magnitude; Vector will have magnitude and one direction, whereas tensor will have magnitude and more than one direction. It can be written in the matrix form, as shown in the above figure.

Generalized Hook's law

$$\boldsymbol{\sigma_{ij} = C_{ijkl}\epsilon_{kl}}$$

Here C_{ijkl} is known as stiffness tensor having 81 components.

$$\boldsymbol{\epsilon_{ij} = S_{ijkl}\sigma_{kl}}$$

Here S_{ijkl} is known as a compliance tensor having 81 components.

These tensors can be written in terms of matrices. The stress can be divided into two parts 1) Hydrostatic part 2) Deviatoric part

$$\sigma_{ij} = \sigma_{hyd} + \sigma_{\text{dev}}$$

$$\sigma_{hyd} = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$$

Example:

Given the following stress tensor

$$\boldsymbol{\sigma} = \begin{bmatrix} 50 & 30 & 20 \\ 30 & -20 & -10 \\ 20 & -10 & 10 \end{bmatrix}$$

The hydrostatic stress is

$$\sigma_{\text{Hyd}} = \frac{50 + (-20) + 10}{3} = 13.3$$

which can be written as

$$\boldsymbol{\sigma}_{\text{Hyd}} = \begin{bmatrix} 13.3 & 0 & 0 \\ 0 & 13.3 & 0 \\ 0 & 0 & 13.3 \end{bmatrix}$$

Subtracting the hydrostatic stress tensor from the total stress gives

$$\boldsymbol{\sigma}' = \begin{bmatrix} 50 & 30 & 20 \\ 30 & -20 & -10 \\ 20 & -10 & 10 \end{bmatrix} - \begin{bmatrix} 13.3 & 0 & 0 \\ 0 & 13.3 & 0 \\ 0 & 0 & 13.3 \end{bmatrix} = \begin{bmatrix} 36.7 & 30.0 & 20.0 \\ 30.0 & -33.3 & -10.0 \\ 20.0 & -10.0 & -3.3 \end{bmatrix}$$