ME 1020

Lecture -4

Mechanics

1) Kinematics : motion of the body
2) Kinetics → Statics → $\Sigma F = 0$
   → Dynamics → $\Sigma F =$ Inertial forces ma

↓ Action of forces

Force is a **vector** : 3 components

$$\underline{F} = F_1 \underline{e}_1 + F_2 \underline{e}_2 + F_3 \underline{e}_3$$

(e_1, e_2, e_3) are orthonormal

$e_1 \cdot e_2 = e_2 \cdot e_3 = e_3 \cdot e_1 = 0$

$e_1 \cdot e_1 = e_2 \cdot e_2 = e_3 \cdot e_3 = 1$

$$\begin{Bmatrix} F \cdot e_1 \\ F \cdot e_2 \\ F \cdot e_3 \end{Bmatrix} = \begin{bmatrix} e_1 \cdot e_1 & e_2 \cdot e_1 & e_3 \cdot e_1 \\ e_1 \cdot e_2 & e_2 \cdot e_2 & e_3 \cdot e_2 \\ e_1 \cdot e_3 & e_2 \cdot e_3 & e_3 \cdot e_3 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\underline{F} = F_1' \underline{a} + F_2' \underline{b} + F_3' \underline{c}$$

$[a, b, c] \neq 0$

$F \cdot a = F_1' a \cdot a + F_2' b \cdot a + F_3' c \cdot a$

$F \cdot b = F_1' a \cdot b + F_2' b \cdot b + F_3' c \cdot b$

$F \cdot c = F_1' a \cdot c + F_2' b \cdot c + F_3' c \cdot c$

$$\begin{Bmatrix} F \cdot a \\ F \cdot b \\ F \cdot c \end{Bmatrix} = \begin{bmatrix} a \cdot a & b \cdot a & c \cdot a \\ a \cdot b & b \cdot b & c \cdot b \\ a \cdot c & b \cdot c & c \cdot c \end{bmatrix} \begin{Bmatrix} F_1' \\ F_2' \\ F_3' \end{Bmatrix}$$

$\{R\} = [A]_{3\times 3}$

$\{x\} = [A]^{-1} \{R\}$

$F \cdot e_i = F_i$

$F \cdot e_j = F_j$

$$F = F_1 e_1 + F_2 e_2 + F_3 e_3$$

$$= \sum_{i=1}^{3} F_i e_i$$

$$= \sum F_i e_i$$

$$= F_i e_i$$

$$\boxed{\underline{F} = (F \cdot e_i) e_i}$$

Stephen Timoshenko