ME1020 Homework 4

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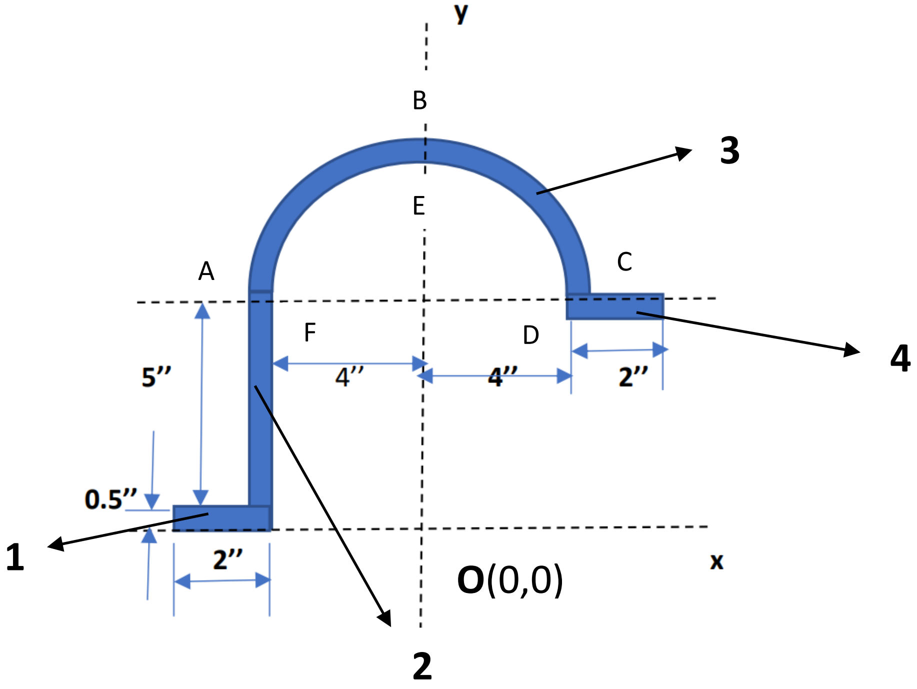
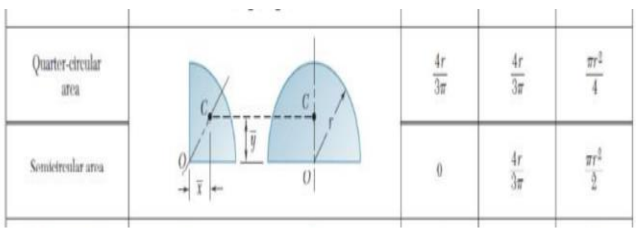
ME21BTECH11001

Question 1:-



Finding Centroids for respective labelled parts by considering O as origin

1) $A_1 = 0.5X2 = 1\ inch^2$
   $x_1 = -5\ ;\ y_1 = 0.25$
2) $A_2 = 0.5X5 = 2.5\ inch^2$
   $x_2 = -4.25\ ;\ y_2 = 3$
3) $A_3 = \frac{\pi}{2}(4.5^2 - 4^2) = 2.125\pi\ inch^2$
   $x_3 = 0$



Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

$Centroid\ for\ symmetric\ \ disc\ is\ \frac{4R}{3\pi}\ ;$

$\Rightarrow ABCDEF = Disc_{ABC} - Disc_{DEF}$

$y\ is\ y\ coordinate\ for\ centroid\ of\ disc$

$$\Rightarrow y_3 = \frac{A_{ABC}y_{ABC} - A_{DEF}y_{DEF}}{A_{ABC} - A_{DEF}} + 5.5 = \frac{\frac{\pi}{2}(4.5)^2\left(\frac{4}{3\pi}(4.5)\right) - \frac{\pi}{2}(4)^2\left(\frac{4}{3\pi}(4)\right)}{\frac{\pi}{2}(4.5^2 - 4^2)} + 5.5$$

$\Rightarrow y_3 = 8.21$

4) $A_4 = 0.5X2 = 1\ inch^2$
$x_4 = 5\ ; y_4 = 5.5$

For whole figure let centroid be $(\bar{x}, \bar{y})$
$\Sigma A_i = A_1 + A_2 + A_3 + A_4 = 4.5 + 2.125\pi = 11.1725\ inch^2$

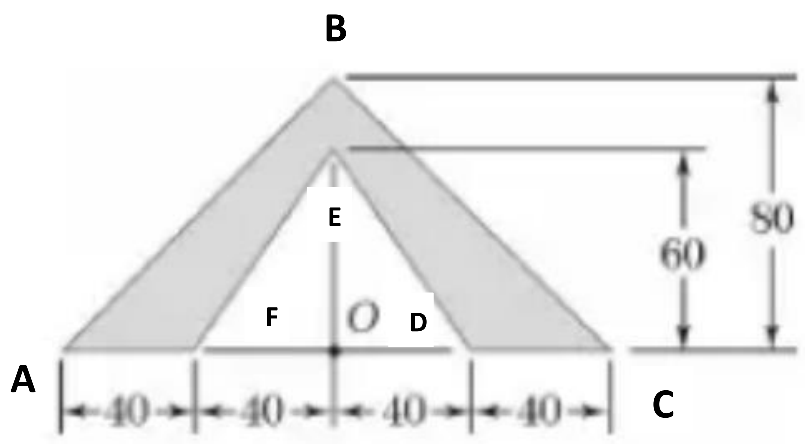
$$\Rightarrow \boldsymbol{\bar{x}} = \frac{\boldsymbol{\Sigma A_i x_i}}{\boldsymbol{\Sigma A_i}} = \frac{\boldsymbol{-5X1-4.25X2.5+0X2.125\pi+5X1}}{\boldsymbol{11.1725}} = \boldsymbol{-0.950\ inch}$$

Similarly,

$$\Rightarrow \boldsymbol{\bar{y}} = \frac{\boldsymbol{\Sigma A_i y_i}}{\boldsymbol{\Sigma A_i}} = \frac{\boldsymbol{1X0.25+2.5X3+8.21X2.125\pi+1X5.5}}{\boldsymbol{11.1725}} = \boldsymbol{6.08\ inch}$$

Centroid at $(\boldsymbol{-0.950, 6.08})\ inches$

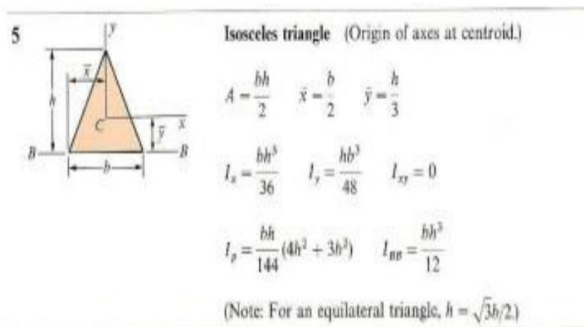
Question 2:-



a) Given figure $\mathrm{ABCDEF} = \mathrm{Traingle_{ABC}} - \mathrm{Triangle_{DEF}}$
For ABC:- (AC is x axis and OB is y axis)

For Triangle with base b and height h moment of inertia with centroid as origin of axes is

$$I_{xx} = \frac{bh^3}{36}$$



Using parallel axis theorem to find I_{xx} along base

$$\Rightarrow I_{xx} = \frac{bh^3}{36} + \frac{bh}{2}\left(\frac{h}{3}\right)^3 = \frac{1}{12}bh^3$$

$$\Rightarrow I_{xx} = \frac{1}{12}(160)(80)^3 = 6.8267\ X\ 10^6\ mm^4$$

About y axis ABC = AOB + COB =2 X MOI_{AOB}

And for a right Angled triangle moment of inertia about axis (origin of axes at vertex) is given by

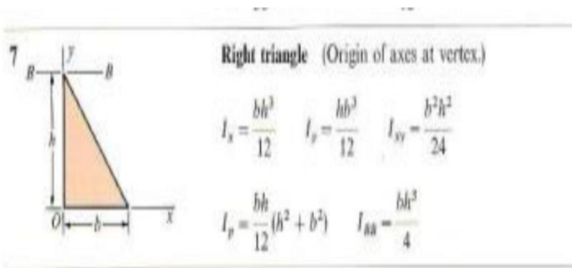
$$I_{yy} = \frac{hb^3}{12}$$

$$\Rightarrow I_{yy} = 2\left(\frac{1}{12}(80)(80)^3\right) = 6.8267\ X\ 10^6\ mm^4$$

Polar moment of inertia :-

$$\Rightarrow J_o = I_{xx} + I_{yy} = 2\ X\ 6.8267\ X\ 10^6\ mm^4 = 13.65\ X\ 10^6\ mm^4$$

Similarly for DEF :-



$$\Rightarrow I_{xx} = \frac{1}{12}(80)(60)^3 = 1.44\ X\ 10^6\ mm^4$$

Triangle DEF=DOE+FOE=2 X MOI_{DOE}

$$\Rightarrow I_{yy} = 2\left(\frac{1}{12}(60)(40)^3\right) = 0.64X\ 10^6\ mm^4$$

Polar moment of inertia :-

$$\Rightarrow J_o = I_{xx} + I_{yy} = (1.44 + 0.64)X\ 10^6\ mm^4 = 2.08\ X\ 10^6\ mm^4$$

For entire $\mathrm{ABCDEF} = \mathrm{Traingle_{ABC}} - \mathrm{Triangle_{DEF}}$

$$\Rightarrow (J_o)_{ABCDEF} = (J_o)_{ABC} - (J_o)_{DEF}$$

$$\Rightarrow \boldsymbol{(J_o)_{ABCDEF} = (13.65 - 2.08)\ X\ 10^6 = 11.57\ X\ 10^6\ mm^4}$$

b) Centroid of $\mathrm{ABCDEF} = \mathrm{Traingle_{ABC}} - \mathrm{Triangle_{DEF}}$
For triangle centre of mass is at h/3 from base
By symmetry $x_{ABC} = x_{DEF} = 0$

For ABC

$$A_{ABC} = \frac{1}{2}(160)(80) = 6400\,mm^2$$

$$y_{ABC} = \frac{80}{3}$$

For DEF

$$A_{DEF} = -\frac{1}{2}(80)(60) = -2400\,mm^2$$

$$y_{ABC} = \frac{60}{3} = 20$$

For whole figure let centroid be at $(\bar{x}, \bar{y})$

$$\Sigma A_i = A_{ABC} + A_{DEF} = 6400 - 2400 = 4000\,mm^4$$

$$\Rightarrow \bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{6400\left(\frac{80}{3}\right) - 2400(20)}{4000} = 30.67$$

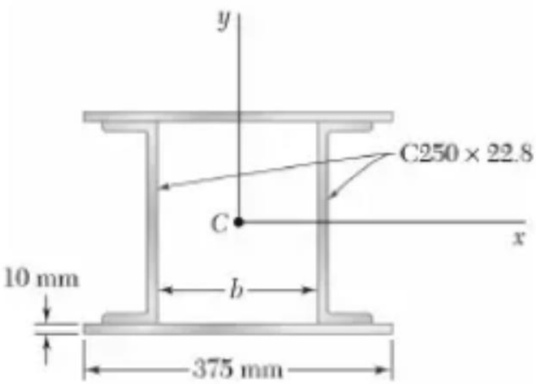
For finding MOI about centroid we use parallel axis theorem

$\Rightarrow J_o = J_{Centroid} + A_{ABCDEF}\, y^2$ (y= distance between o and centroid)

$$\Rightarrow J_{Centroid} = J_o - A_{ABCDEF}\, y^2 = 11.57\ X\ 10^6 - (4000)(30.67)^2$$

$$\Rightarrow \boldsymbol{J_{Centroid} = 7.807\ X\ 10^6\ mm^4}$$

<u>Question 3:-</u>



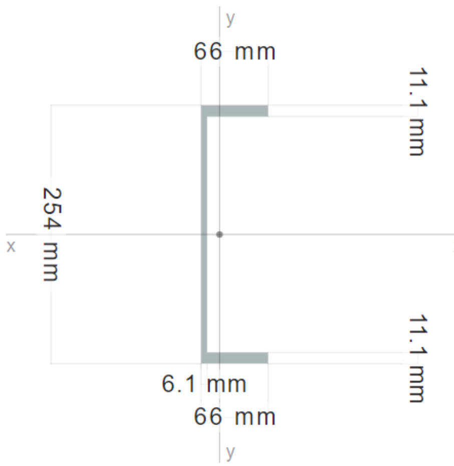


TABLE B-4 *Properties of Channel Sections: SI Units*

Designation	Mass (kg/m)	Area (mm^2)	Depth (mm)	Flange: Width (mm)	Flange: Thickness (mm)	Web thickness (mm)	Axis X-X: I (10^6 mm^4)	Axis X-X: $S = I/c$ (10^3 mm^3)	Axis X-X: $r = \sqrt{I/A}$ (mm)	Axis Y-Y: I (10^6 mm^4)	Axis Y-Y: $S = I/c$ (10^3 mm^3)	Axis Y-Y: $r = \sqrt{I/A}$ (mm)	x (mm)
C380 × 74	74.0	9480	381	94.5	16.5	18.2	168	882	133	4.58	61.8	22.0	20.3
× 60	60.0	7610	381	89.4	16.5	13.2	145	762	138	3.82	54.7	22.4	19.8
× 50.4	50.4	6450	381	86.4	16.5	10.2	131	688	143	3.36	50.6	22.9	20.0
C310 × 45	45.0	5680	305	80.5	12.7	13.0	67.4	442	109	2.13	33.6	19.4	17.1
× 37	37.0	4740	305	77.5	12.7	9.83	59.9	393	113	1.85	30.6	19.8	17.1
× 30.8	30.8	3920	305	74.7	12.7	7.16	53.7	352	117	1.61	28.2	20.2	17.7
C250 × 45	45.0	5680	254	77.0	11.1	17.1	42.9	339	86.9	1.64	27.0	17.0	16.5
× 37	37.0	4740	254	73.4	11.1	13.4	37.9	298	89.4	1.39	24.1	17.1	15.7
× 30	30.0	3790	254	69.6	11.1	9.63	32.8	259	93.0	1.17	21.5	17.5	15.4
→ × 22.8	22.8	2890	254	66.0	11.1	6.10	28.0	221	98.3	0.945	18.8	18.1	16.1

a)

From ASTM data for C250 X 22.8 we obtain

$A = 2890\ mm^2; I_{xx} = 28\ X\ 10^6\ mm^4; I_{yy} = 0.945\ X\ 10^6\ mm^4;$

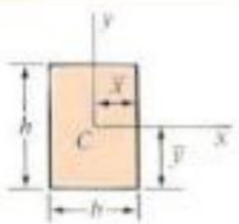
$Distance\ between\ edge\ to\ centroidal\ axis\ , x_1 = 16.10\ mm$

$Total\ Area\ , A_{tot} = 2\big(2890 + (10)(375)\big)\ mm^2 = 13.28\ X\ 10^3\ mm^2$

$Distance\ between\ centre\ of\ plate\ to\ C, d = 127 + 5 = 132\ mm$

$For\ a\ rectangular\ plate\ MOI\ with\ centre\ of\ axis\ about\ centroid\ is\ given\ \ by$

$$I_{xx} = \frac{1}{12}bh^3; I_{yy} = \frac{1}{12}hb^3$$

1 

Rectangle (Origin of axes at centroid)

$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$

$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$

$I_{xy} = 0 \qquad I_P = \frac{bh}{12}(h^2 + b^2)$

Also base $b = 250\ mm$; finding MOI about C :-

$$I_{xx} = 2(I_{xx})_{channel} + 2\big((I_{xx})_{plate} + A_{plate}d^2\big) \quad \text{\{parallel axis theorem\}}$$

$$= 2(28\ X\ 10^6\ ) + 2\left(\tfrac{1}{12}(375)(10) + (375)(10)(132)^2\right)$$

$\boldsymbol{I_{xx} = 186.74\ X\ 10^6 mm^4}$

$I_{yy} = 2((I_{yy})_{channel} + A_{plate}(\frac{250}{2} + x_1)^2\ ) + 2(I_{yy})_{plate}$ {parallel axis theorem}

$$= 2\left(\ 0.945\ X\ 10^6 + (2890)\left(\frac{250}{2} + 16.10\right)^2\right) + 2\left(\frac{1}{12}(10)(375)^3\right)$$

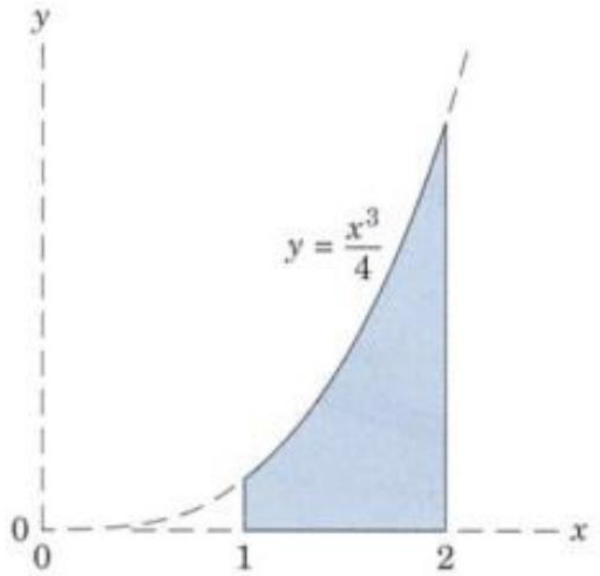
$$\boldsymbol{I_{yy} = 204.84\ X\ 10^6\ mm^4}$$

b)

$$k_x = \sqrt{\frac{I_{xx}}{A_{tot}}} = \sqrt{14.06\ X\ 10^3} = \mathbf{118.6}\ \boldsymbol{mm}$$

$$k_y = \sqrt{\frac{I_{yy}}{A_{tot}}} = \sqrt{15.42\ X\ 10^3} = \mathbf{124.19}\ \boldsymbol{mm}$$

Question 4:-



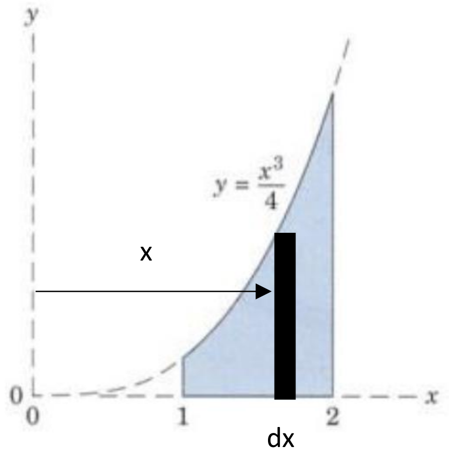
About y axis

$$dI_{yy} = x^2 dA$$

$$\Rightarrow I_{yy} = \int x^2 dA$$

$$\Rightarrow I_{yy} = \int_1^2 x^2 dA = \int_1^2 x^2 \frac{x^3}{4} dx = \int_1^2 \frac{x^5}{4} dx$$

$$\Rightarrow I_{yy} = \frac{[x^6]_1^2}{24} = \frac{2^6 - 1}{24} = \frac{63}{24} = 2.625\, mm$$



$$dA = ydx \Rightarrow dA = \frac{x^3}{4} dx$$

About x axis

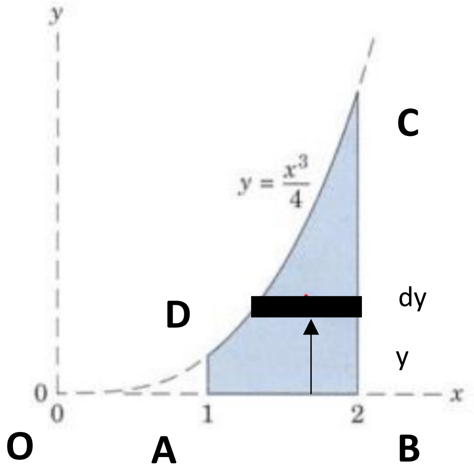
$$dI_{xx} = y^2 dA$$

For given figure

$$(I_{xx})_{ABCD} = (I_{xx})_{OBC} - (I_{xx})_{OAD}$$

For OBC $dA_1 = xdy \Rightarrow dA = \left(2 - (4y)^{\frac{1}{3}}\right) dy$

For OAD $dA_2 = xdy \Rightarrow dA = \left(1 - (4y)^{\frac{1}{3}}\right) dy$



$$I_{xx} = \int y^2 dA_1 - \int y^2 dA_2$$

$$\Rightarrow I_{xx} = \int_0^2 y^2 \left(2 - (4y)^{\frac{1}{3}}\right) dy \ - \ \int_0^{\frac{1}{4}} y^2 \left(1 - (4y)^{\frac{1}{3}}\right) dy$$

$$= \left[\frac{2y^3}{3} - 4^{\frac{1}{3}} \frac{y^{\frac{10}{3}}}{\frac{10}{3}}\right]_0^2 \ - \ \left[\frac{y^3}{3} - 4^{\frac{1}{3}} \frac{y^{\frac{10}{3}}}{\frac{10}{3}}\right]_0^{\frac{1}{4}}$$

$$= \left(\frac{16}{3} - \frac{(2^3)\,3}{5}\right) - \left(\frac{1}{3(2^6)} - \frac{3}{(2^6)10}\right) = \frac{2^{10}-1}{15(2^7)} = 0.533$$

$$Area\ , A = \int_1^2 dA = \int_1^2 ydx = \int_1^2 \frac{x^3}{4} dx = \frac{[x^4]_1^2}{16} = \frac{15}{16} = 0.9375$$

Radius of gyration:-

$$I_{yy} = k_y^2 A \Rightarrow \boldsymbol{k_y} = \sqrt{\frac{\boldsymbol{I_{yy}}}{\boldsymbol{A}}} = \sqrt{\mathbf{2.8}} = \mathbf{1.673}\ \boldsymbol{mm}$$

$$I_{xx} = k_x^2 A \Rightarrow \boldsymbol{k_x} = \sqrt{\frac{\boldsymbol{I_{xx}}}{\boldsymbol{A}}} = \sqrt{\mathbf{0.568}} = \mathbf{0.753}\ \boldsymbol{mm}$$

$$J_o = I_{xx} + I_{yy} = (k_x^2 + k_y^2)A = k_o^2 A$$

$$\Rightarrow k_o = \sqrt{\frac{I_{xx} + I_{yy}}{A}} = \sqrt{3.368} = 1.835\, mm$$