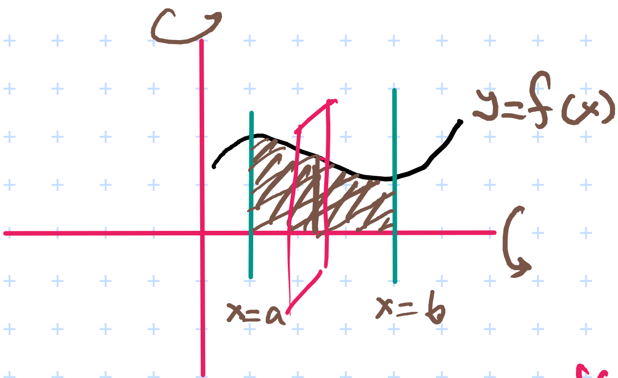
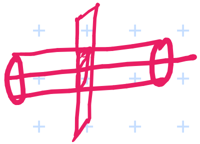
Volume of solid

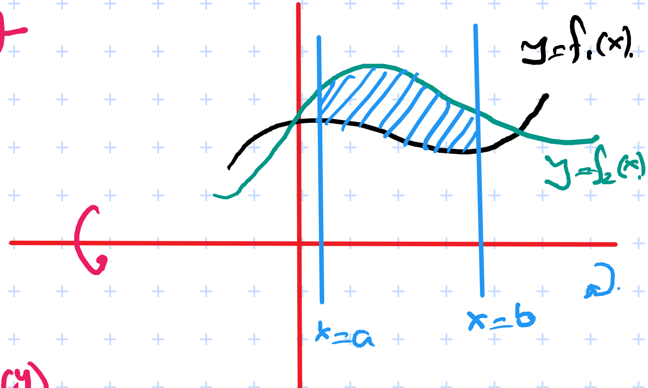
$$\int_a^b A(x)\,dx$$

Area of a slice.

Solids of revolutions:







$x=f(y)$ $x=g(y)$

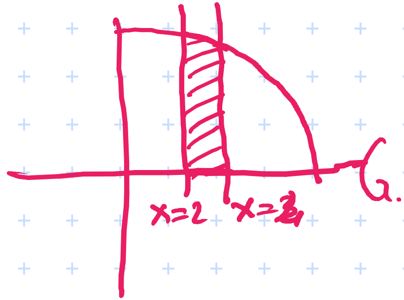
$$\int_a^b \pi f(x)^2\,dx$$

$$\int_a^b \pi\left(f_2(x)^2 - f_1(x)^2\right)dx.$$

Region: $y=\sqrt{25-x^2} = f(x)$ $\quad x=2 \quad$ about x-axis.

$y=0 = g(x)$ $\quad x=4.$

$y^2+x^2=25$



Volume: $$\int_2^4 \pi\left(\sqrt{25-x^2}\right)^2 dx$$

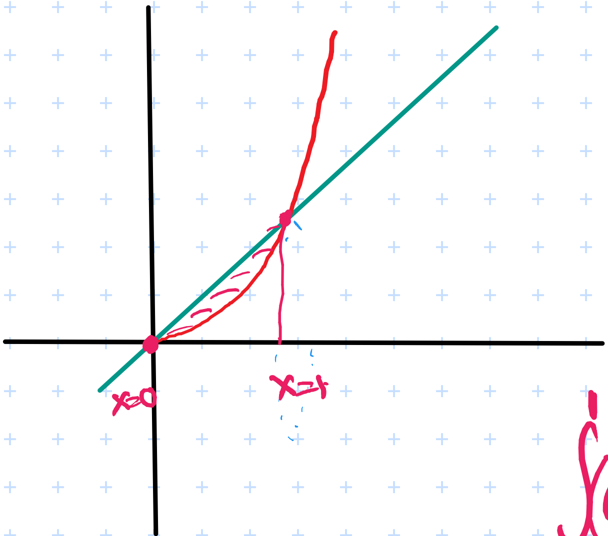
$$\pi\int_2^4 \left(f(x)^2 - g(x)^2\right)dx$$

$$= \int_2^4 (25-x^2)\pi \, dx$$

$$\overset{FTC②}{=} \frac{94}{3}\pi.$$

Region: $\boxed{\begin{array}{l} y = x^3 \\ y = x. \end{array}}$ $x \geq 0$ about x-axis.

Points of intersection when $x \geq 0$ are $(0,0)$ and $(1,1)$.



When $0 \leq x \leq 1$, $x \geq x^3$.

Volume $\pi \int_0^1 (x^2 - x^6)\,dx$

$$\int_0^1 \left((x)^2 - (x^3)^2\right) dx$$

$$= \frac{4}{21}\pi.$$

Region:

$$R = \{(x,y) \mid x \geq 1,\ 0 \leq y \leq 1/x\}$$

Volume.

$$= \int_1^\infty \pi \cdot \frac{1}{x^2}\, dx \quad \text{converges}$$

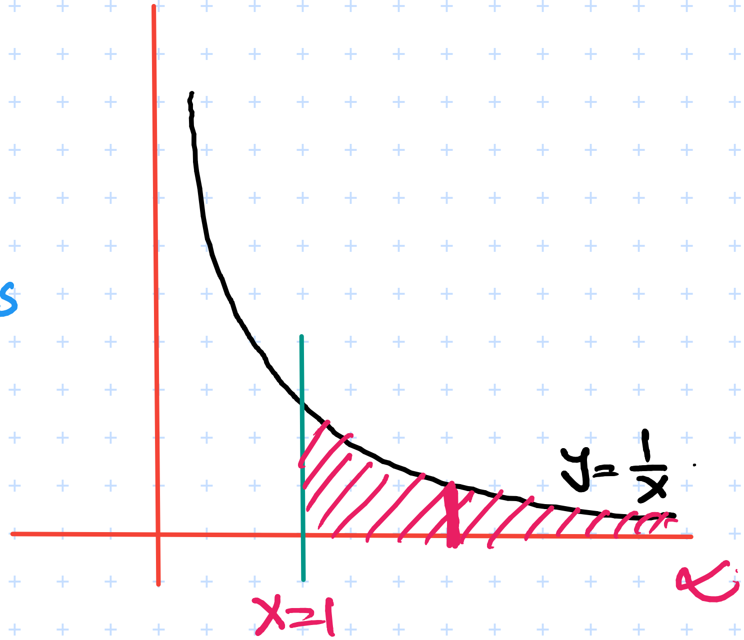
Surface area.

$$= \int_1^\infty 2\pi \cdot y \sqrt{1+f'(x)^2}\, dx$$

$$= \int_1^\infty 2\pi \cdot \frac{1}{x} \sqrt{1+\left(-\frac{1}{x^2}\right)^2}\, dx$$

$$= \boxed{2\pi \int_1^\infty \frac{1}{x^3} \sqrt{x^4+1}\, dx}$$

Diverges.



GABRIEL'S HORN.

Let $f(x) = \frac{1}{x^3}\sqrt{x^4+1}$.

$$g(x) = \frac{1}{x^3} \sqrt{x^4}$$

$$= \frac{1}{x}$$

$$f(x) > g(x)$$

$$\frac{\sqrt{x^4+1}}{x^3} > \frac{\sqrt{x^4}}{x^3}$$

$$= \frac{1}{x}$$

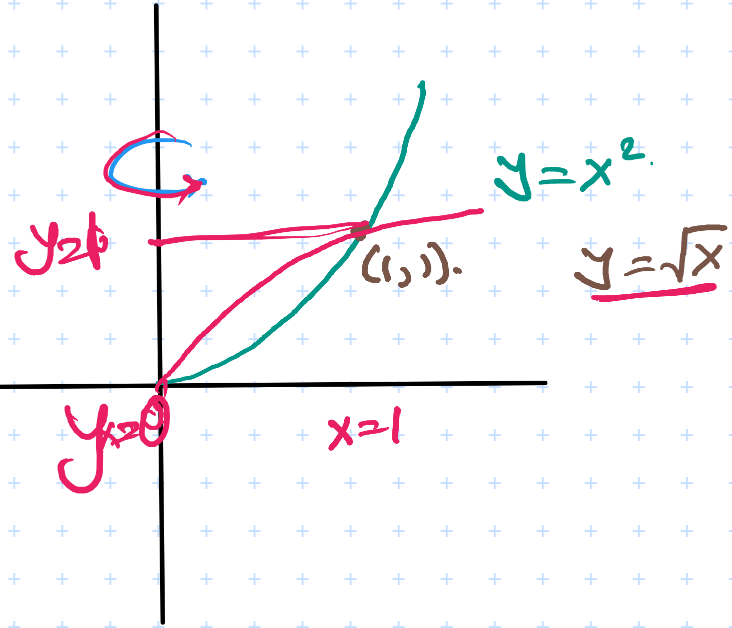
$$\int_1^\infty \frac{1}{x}\, dx$$

Region:

$y=\sqrt{x}$

$y=x^2$

y-axis



$\boxed{x=\sqrt{y}}$

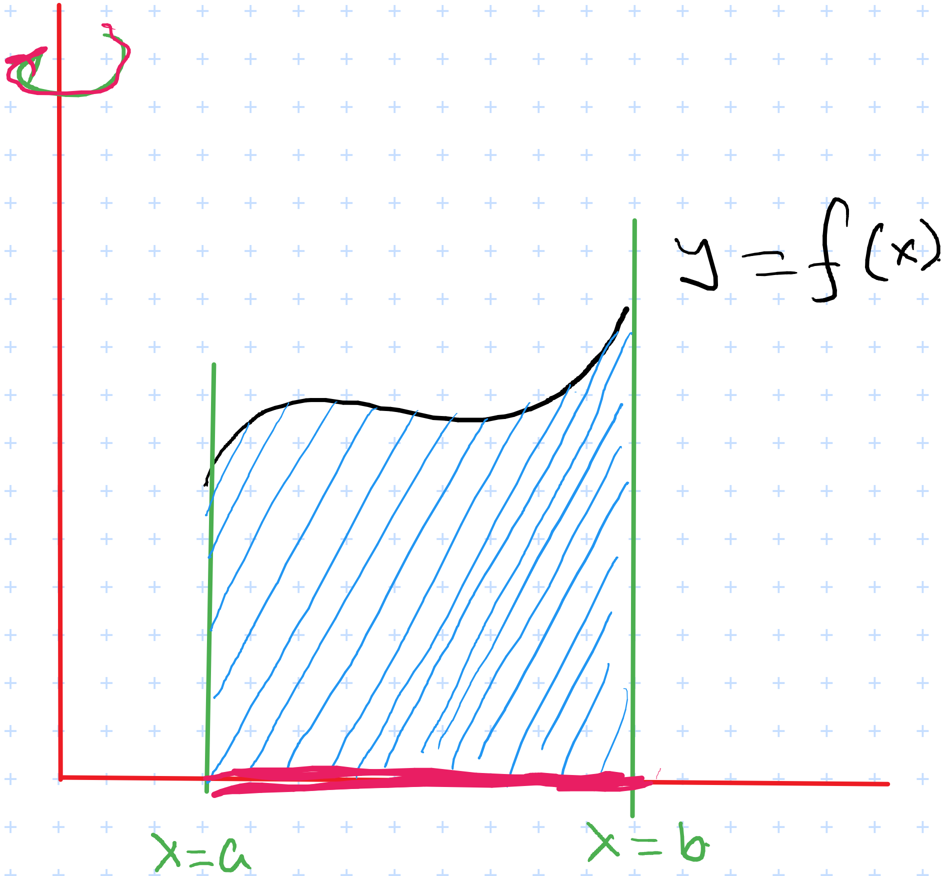
$x=y^2$

$x=y^2$

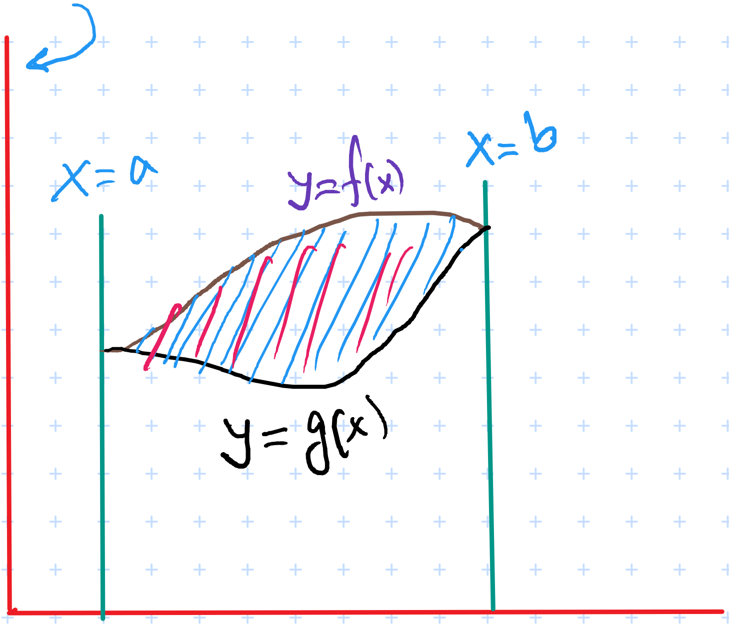
$x=\sqrt{y}$

$$\text{Volume} = \int_0^1 \pi\left(f_1(y)^2 - f_2(y)^2\right)dy$$

$$= \int_0^1 \pi\left(y - y^4\right)dy = \boxed{\frac{3}{10}\pi}$$



$$\text{Volume} = \int_a^b 2\pi x\, f(x)\, dx.$$



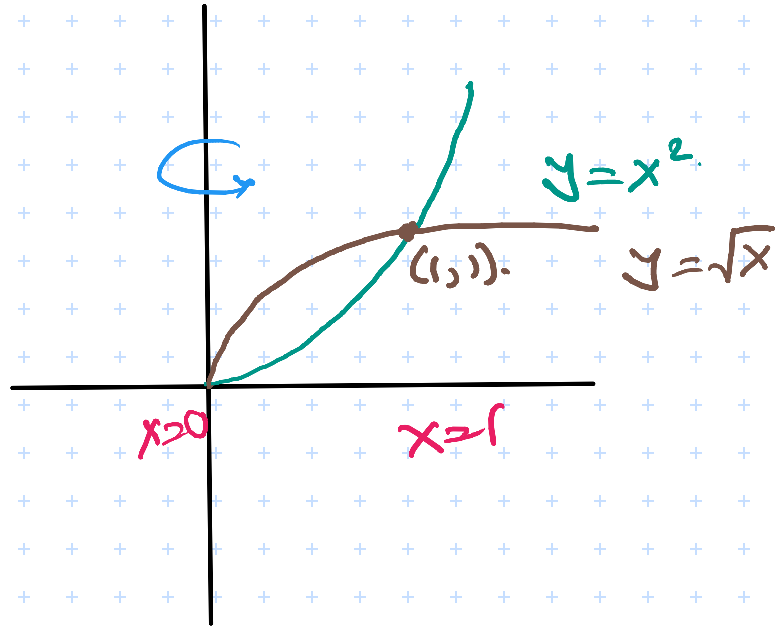
$$\boxed{\text{Volume} = \int_a^b 2\pi x \left(f(x) - g(x)\right) dx}$$

Region:

$y=\sqrt{x}$

$y=x^2$

y-axis



$x=y^2$

$x=\sqrt{y}$

$$\text{Volume} = 2\pi\int_0^1 x\left(\sqrt{x}-x^2\right)dx$$

$$= 2\pi\int_0^1 \left(x^{3/2}-x^3\right)dx$$

FTC 2

$$= \frac{3}{10}\pi$$