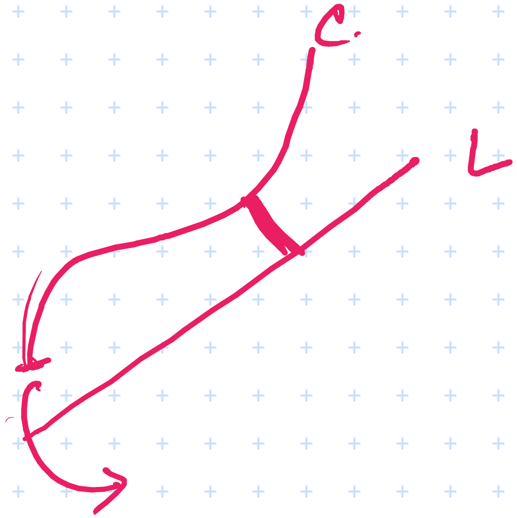
Surface area

C is a smooth curve given by $C=(x(t),y(t))$ $t\in[\alpha,\beta]$.



L: $ax+by+c=0$

C does not cross L.

Area of surface of revolution obtained by rotating C around L is.

$$\int_\alpha^\beta 2\pi \frac{|ax+by+c|}{\sqrt{a^2+b^2}} \sqrt{x'(t)^2+y'(t)^2}\,dt.$$

L $=$ x axis, C: $y=f(x)$. $\rightsquigarrow$ $\displaystyle\int_\alpha^\beta 2\pi\cdot|y|\sqrt{1+f'(x)^2}\,dx.$

L $=$ y axis, C: $x=f(y)$. $\rightsquigarrow$ $\displaystyle\int_\alpha^\beta 2\pi\cdot|\cancel{x}|\sqrt{1+g'(y)^2}\,dy.$

(1) $C: y = x^3$ $\quad L = x$ axis $\quad 0 \le x \le 2.$

(*) Note: L and C intersect at $(0,0)$ but C does not cross L.

Area: $\displaystyle\int_0^2 2\pi f(x)\sqrt{1+f'(x)^2}\,dx.$ $\qquad f'(x) = 3x^2.$

$$= \int_0^2 2\pi x^3\sqrt{1+9x^4}\,dx$$

$$\underset{\text{FTC (2)}}{\overset{\text{Substitution}}{=}} \frac{\pi}{27}\left(145^{3/2}-1\right).$$

2. $C: y = \frac{x^3}{6} + \frac{1}{2x}$ $\frac{1}{2} \le x \le 1.$

$L:$ x. axis.

$$\text{Area}(S) = \int_{1/2}^{1} 2\pi \cdot f(x) \cdot \sqrt{1 + f'(x)^2}\, dx.$$

$$= \int_{1/2}^{1} 2\pi \left(\frac{x^3}{6} + \frac{1}{2x}\right) \sqrt{1 + \left(\frac{3x^2}{6} - \frac{1}{2x^2}\right)^2}\, dx$$

$$= \int_{1/2}^{1} 2\pi \left(x^3/6 + 1/2x\right) \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}\, dx$$

$$\overset{\text{FTC}(2)}{=} \frac{263}{256}\pi.$$

3. C: $x = \sqrt{a^2 - y^2}$, $\quad 0 \le y \le \frac{a}{2}$

L: y axis.

$$\text{Area} = \int_0^{a/2} 2\pi\, g(y) \sqrt{1 + g'(y)^2}\, dy$$

$$g'(y) = \frac{1}{2} \cdot \frac{-2y}{\sqrt{a^2 - y^2}} = -\frac{y}{\sqrt{a^2 - y^2}}$$

$$= \int_0^{a/2} 2\pi \sqrt{a^2 - y^2} \sqrt{1 + \frac{y^2}{a^2 - y^2}}\, dy$$

$$= \int_0^{a/2} 2\pi \sqrt{a^2 - y^2}\, \frac{a}{\sqrt{a^2 - y^2}}\, dy$$

$$= \int_0^{a/2} 2\pi a\, dy$$

$$= 2\pi \frac{a^2}{2} = \pi a^2.$$