MA1140 Elementary linear algebra

March 28 to May 02, 2022 (1-2 segment)

Assignment 2 (Due date: 15.04.2022, 11:59 PM)

Rules:

- Answer all questions.
- Provide complete answers with full justification and all steps worked out.
- The deadline is strict and even a one minute late submission cannot be accepted. Late submissions receive 0 marks.
- Only three grades are possible for each question: 0 for a wrong answer, 1.5 for a partially correct answer or 3 for a fully correct answer.

Questions:

1. Determine whether all matrices of the form

$$(a)\begin{pmatrix} \star & \star & 1 \\ \star & 1 & \star \\ 1 & \star & \star \end{pmatrix} \quad \text{or} \quad (b)\begin{pmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$$

   where $\star$ represents an arbitrary real number, form a vector space over $\mathbb{R}$ under the usual operations of matrix addition and scalar multiplication.

2. Suppose that V is a vector space over some field F, and $u, v, w \in V$. Using the properties of a vector space, show that if $w + u = w + v$, then $u = v$.

3. Are the elements of the subset S of $M_{2\times 2}(\mathbb{R})$ defined by

$$S = \left\{ \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} -3 & 1 \\ 2 & 1 \end{pmatrix} \right\}$$

   linearly independent?

4. Consider the subspace of $M_{2\times 2}(\mathbb{R})$ defined by the span

$$S = \left\langle \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} -3 & 1 \\ 2 & 1 \end{pmatrix} \right\rangle$$

   and also the matrix $A = \begin{pmatrix} -3 & 3 \\ 6 & -4 \end{pmatrix}$. Does $A \in S$?

5. Consider the subspace S of $M_{2\times 2}(\mathbb{R})$ defined by the set

$$S := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + b = c, b + c = d, c + d = a \right\}.$$

   Find the dimension of S.

6. Calculate the dimension of the null space of the matrix

$$\begin{pmatrix} 2 & -1 & -3 & 11 & 9 \\ 1 & 2 & 1 & -7 & -3 \\ 3 & 1 & -3 & 6 & 8 \\ 2 & 1 & 2 & -5 & -3 \end{pmatrix}$$

by computing its reduced row echelon form.