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Ordinary Differential Equation (MA-1150)

Answers to Assignment 2 (Second and Higher Order Linear ODEs)

1. Solutions of the given differential equations:

(a) $y = c_1 e^x + c_2 e^{\frac{x}{2}}$

(b) $y = (c_1 + c_2\, x)\, e^{-2\,x}$

(c) $y = c_1 e^{mx} + c_2 e^{-mx} + c_3 \cos mx + c_4 \sin mx$

(d) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-3x}$

(e) $y = e^{-\frac{1}{2}x}\left[(c_1 + c_2 x)\cos\frac{\sqrt{3}}{2}x + (c_3 + c_4 x)\sin\frac{\sqrt{3}}{2}x\right]$

(f) $x = 0$

(g) $y = 2(\cos x - \sin x)$

(i) $Q = Q_0\, e^{-\frac{Rt}{2L}}\left(\cos nt + \left(\frac{R}{2Ln}\right)\sin nt\right)$, where $n = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$.

(j) $\theta = c_1 e^{-\frac{k}{2m}t}\cos\left(\frac{a}{2m}t + c_2\right)$, where $a = \sqrt{4mc - k^2}$.

and 2nd solution: $\theta = c_1 e^{-\frac{k}{2m}t}\cos\left(\sqrt{\frac{c}{m}}\ t + c_2\right)$.

2. Solutions of the given differential equations:

(a) $y = A\cos ax + B\sin ax - \frac{x}{a}\cos ax + \frac{1}{a^2}\sin ax \log\sin ax$

(b) $y = A\cos ax + B\sin ax - \frac{1}{a^2}\cos ax \log\ |\sec ax + \tan ax|$

(c) $y = Ae^x + Be^{-2x} + C\,x\,e^{-2x} - \frac{1}{18}(x^3 + x^2)\,e^{-2x}$

(d) $y = c_1 e^{3x} + c_2 e^{-2x} - 4xe^{-2x}$

(e) $y = c_1 e^{-3x} + c_2\, x\, e^{-3x} + 12x^2 e^{-3x}$

(f) $y = c_1 e^{-x} + c_2 \cos x + c_3 \sin x + \frac{2\cos 2x - \sin 2x}{15}$

(g) $y = c_1\, e^{-2x} + c_2\, e^{-3x} - \frac{1}{10}e^{-2x}(\cos 2x + 2\sin 2x)$

(h) $y = c_1\, e^{-x} + (c_2 \cos 2x + c_3 \sin 2x)\, e^{x} - \frac{1}{65} e^{x}(2 \cos 3x + 3 \sin 3x)$

(i) $y = c_1\, e^{x} + c_2\, e^{-x} + \dfrac{x^2}{4} \cosh x - \dfrac{x}{4} \sinh x$

(j) $y = c_1 \cos \sqrt{2}\, x + c_2 \sin \sqrt{2}\, x + \frac{e^{3x}}{11}\left[x^2 - \frac{12x}{11} + \frac{50}{121}\right] - \frac{e^{x}}{17}(-4 \sin 2x + \cos 2x)$

(k) $y = c_1 \cos x + c_2 \sin x + \frac{3}{2} - \frac{1}{2} \cos 2x - \frac{3}{4} x \cos x + \frac{1}{16} \sin 3x$

(l) $y = (c_1 + c_2\, x) + (c_3 + c_4\, x)\, e^{x} + \frac{x^5}{20} + \frac{x^4}{2} + 3x^3 + 12x^2$

(m) $y = c_1 e^{-x} + e^{\frac{x}{2}} \left(c_2 \cos \frac{x\sqrt{3}}{2} + c_3 \sin \frac{x\sqrt{3}}{2}\right) + \frac{e^{2x}}{130}(3 \sin x - 11 \cos x)$

$$-\frac{1}{6}\, x\, e^{\frac{x}{2}} \left(\sin \frac{x\sqrt{3}}{2} + \sqrt{3} \cos \frac{x\sqrt{3}}{2}\right)$$

(n) $r = c_1 e^{\omega t} + c_2 e^{-\omega t} + \frac{g}{2\omega^2} \sin \omega t.$

3. Solutions of the given differential equations:

(a) $y = c_1 + c_2 e^{2x} - \frac{e^{x}}{2} \cos x$

(b) $y = e^{-x}(c_1 + c_2 x) - e^{-x}(1 + \log x)$

(c) $y = c_1 e^{x} + c_2 e^{2x} + (e^{x} + e^{2x})\, \log(1 + e^{x}) - (x + 1)\, e^{x} - x\, e^{2x}$

(d) $y = c_1 + c_2 \cos x + c_3 \sin x + \log(\operatorname{cosec} x - \cot x) - \cos x \log \sin x - x \sin x$

(e) $y = c_1 x + \frac{c_2}{x+1} + \frac{x^2}{6} \frac{(4x+3)}{x+1}$

(f) $y = x(c_1 + c_2 e^{x}) - x(x + 1)$

(g) $y = c_1 x + \frac{c_2}{x} + x^2 \left(\frac{1}{3} \log x - \frac{4}{9}\right)$

(h) $y = \frac{c_1}{x} + c_2 + c_3 x - \log x$

(i) $y = c_1 e^{x} + \frac{c_2}{x} - \frac{1}{3}\, x^2 - x - 1.$

4. Solutions of the given differential equations:

(a) $y = \frac{c_1}{x} + x\{c_2 \cos(\log x) + c_3 \sin \log x\} + 5x + \frac{2}{x} \log x$

(b) $y = c_1(x+a)^3 + c_2(x+a)^2 + \frac{1}{2}(x+a) - \frac{1}{6}a$

(c) $y = c_1 \cos(2\log(2+x)) + c_2 \sin(2\log(2+x)) - \frac{1}{2}\log(2+x)\cos(2\log(2+x))$

(d) $y = c_1 x^2 + \frac{c_2}{x^2} + c_3 \cos(\log x) + c_4 \sin(\log x) + \frac{1}{20}x^2 \log x - \frac{1}{5}\log x \sin(\log x)$

(e) $y = \frac{1}{x}(c_1 + c_2 \log x) + \frac{1}{x}\log\frac{x}{x-1}$

(f) $y = z^2\left(c_1 z^{\sqrt{3}} + c_2 z^{-\sqrt{3}}\right) + \frac{1}{6z} + \frac{\log z}{61z}(5 \sin \log z + 6 \cos \log z)$

$$+\frac{2}{3721}.\frac{1}{z}(27 \sin \log z + 191 \cos \log z)$$

(g) $y = c_1 + c_2 \log(x+1) + \{\log(x+1)\}^2 + x^2 + 8x$

(h) $y = x^m\,(c_1 \sin \log x^n + c_2 \cos \log x^n + \log x)$

(i) $V = (c_1 + c_2 \log r) + \pi\rho r^2$