EP1108 Nuclear Physics

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References

These slides are mainly based upon the following two books

- Elements of Modern Physics S.H. Patil
- Concepts of Modern Physics by Arthur Beiser

For more details (beyond the scope of this course) and additional information one can refer to

- Introduction to Nuclear Physics by S.B. Patel
- Introductory Nuclear Physics by K.S. Krane

Introduction

- Rutherford's experiment demonstrated the existence of a heavy positively charged nucleus near the center of an atom of Ze.
- When distance of closest approach was $< 10^{-14}$ m, scattering showed deviations from Coulomb scattering indicating the distance of a nuclear over 10^{-14} m. (This requires the existence of a new short range interaction known as strong interactions.

This lecture will discuss constituents of nucleus, stability criteria, nuclear reactions, nuclear decays and basics of nuclear force.

Properties of the Nucleus

- Nucleus made up of protons and neutron. (Hydrogen atom consists of only protons). Properties of proton are as follows:

- Proton rest mass is equal to 938.256 MeV/c^2 or a mass of 1.0072766 amu (1 amu =1/12 th mass of Carbon atom)
- It has a postive charge equal to e and spin (angular momentum) equal to $\hbar/2$.
- Proton is a fermion and obeys Fermi-Dirac statistics.
- Proton (in stable nuclei) is absolutely stable (Some Grand unified theories predict that proton has a finite lifetime ($> 10^{30}$) years. However, these decays have not been observed.)

Properties of the neutron

- Neutron has a rest mass of 939.55 MeV/c^2 or 1.0086654 amu.
- Zero net charge and spin (angular momentum) equal to $\hbar/2$.
- Neutron is a fermion and obeys Fermi-Dirac statistics.
- A free neutron has a lifetime of about 10 minutes. $n \rightarrow p + e + \bar{\nu}$, where ν is known as anti-neutrino

Magnetic Moment of Proton and Neutron

Both proton and neutron have a magnetic moment proportional to their spins

$$\mu_p = g_p \frac{e}{m_p} S \tag{1}$$

$$\mu_n = g_n \frac{e}{m_n} S \tag{2}$$

$$\tag{3}$$

where $g_p = 2.793$ and $g_n = -1.913$. For a Dirac spin-1/2 particle, one would expect $g_p = 1$ and $g_n = 0$. However, this is different from what is observed. The observed values indicate a complicated internal structure for proton and neutron.

Binding Energies

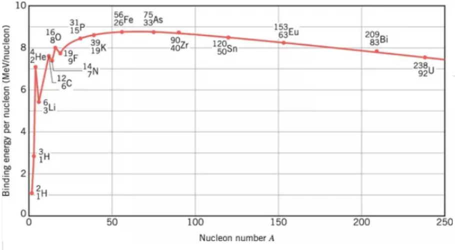
The binding energy E_b of a nucleus of mass M_A containing Z protons and $(A - Z)$ neutrons is given by

$$E_b = c^2[Zm_p + (A - Z)m_n - m_A] \tag{4}$$

This is the minimum energy required to separate a nucleus into its constituent nucleons (equivalent to ionization potential for an atom)
A nucleus (X) with Z protons and mass number equal to A is denoted by ${}^A_Z X$

- Isotopes -Same no of protons but different neutrons
- Isotones -Same no of neutrons but different no of protons
- Isobars -Nuclei with same A but different Z
- Isomers -Excited states of nuclei with very long lifetime

Binding Energy per nuclei



Binding energy per nucleon as a function of A. (Source: wikipedia)

- Rises from a low value of E_b/A of 1.1 MeV for Deuteron to 7.1 MeV for α particle.
- It reaches a peak value of 8.7 MeV for $A \approx 56$ corresponding to Iron (Fe) nucleus.
- For nuclei with larger A, E_b/A decreases (due to Coulomb repulsion) to about 7.5 MeV for heavier nuclei.

Nuclei with higher A undergo fission and those with lower A under fusion to increase the binding energy per nucleon.

Unstable Nuclei

If a lower energy state is available it will decay to that state with emission of other ancillary particles, assuming all conservation laws (energy, momentum, charge, etc) are satisfied. The decays are characterized by lifetime τ.
Important decay reactions are:

- α **decay** : Characterized by ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He}\ (\alpha)$
  Example : ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$
- γ **decay** : Excited nucleus undergoes transition to lower energy state by emission of a photon.. This can be represented as: $X^* \rightarrow X + \gamma$, where X^* is the nuclear excited state. Photon energies in nuclear transitions are of the order of MeV, compared to eV in atomic transitions. The corresponding lifetimes are 10^{-14} seconds, except isomers which have lifetimes of several hours.

Unstable nuclei

- β **decay** : Two Types of β decays
  - β^- decay in which a n is converted to a p, e ,and $\bar{\nu_e}$.
    Example: ${}^{14}_{6}C \rightarrow^{14}_{7} N + e + \bar{\nu_e}$
  - β^+ decay in which a proton decays to n, e^+ and ν_e. This is also called positron emission.
    Example: ${}^{23}_{12}Mg \rightarrow^{23}_{11} Na + e^+ + \nu_e$
- **Inverse beta decay:**
  $\bar{\nu_e} + p \rightarrow e^+ + n$. This reaction is used to detect neutrinos
- **Electron capture** The nucleus absorbs one of its innermost orbital electrons and gets converted to a neutron.
  $p + e \rightarrow n + \nu$
  Example: ${}^{26}_{13}Al + e \rightarrow^{26}_{12} Mg + \nu_e$

Nuclear Radius

Nuclear forces dominate over a distance of 10^{-15}m. The nuclear radius is estimated from the following methods:

- Scattering of electrons and neutrons (of energy about 100 MeV) by the nucleus
- Analyzing the effect of finite size of nucleus on nuclear and atomic binding energies.

From the first method, the fraction of neutrons scattered at various angles can be used to deduce the nuclear size

$$R \approx r_0 A^{1/3}, \tag{5}$$

where $r_0 \approx$ 1.3-1.4 fm The volume per nucleon is same for all nuclei:

$$\begin{aligned} V &= \frac{4\pi r^3}{3} / A \qquad (6) \\ &= \frac{4\pi r_0^3}{3} \qquad (7) \end{aligned}$$

Thus nuclear density is the same for all nuclei.

Nuclear Magnetic Moment and Angular Momentum

The total angular momentum of nuclei is made up of spins and orbital angular momentum of constituent nucleons and associated with angular momentum is magnetic moment. Nuclear angular momentum can be obtained from hyperfine interactions between magnetic moment of nuclei and electrons.

$$H = AI.J$$

where I and J are the angular momentum of the nucleus and total electrons respectively. The total angular momentum is given by $F = I + J$ and the corresponding quantum numbers take on the value F=J+I, J+I-1,...... |J-I| The shift in energy of these states is given by

$$\Delta E = \frac{1}{2} A\hbar^2 \left[F(F+1) - I(I+1) - J(J+1) \right]$$

Nuclear magnetic moments (contd)

This leads to a separation of
$E_F - E_{F-1} = A\hbar^2(I+J), A\hbar^2(I+J-1), ...A\hbar^2(|I-J|-1)$
I can be determined from measurements of ΔE. Magnetic moment can be determined from angular momentum using

$$\mu = \frac{e}{m_p} g I$$

where g is the gyromagnetic ratio and can be obtained from nuclear magnetic resonance experiments using atomic beams .
Results are as follows:

- Even A and even Z nuclei have zero angular momentum and zero magnetic moment
- Even A and odd Z nuclei have integral angular momentum
- Odd A nuclei have half integral angular momentum

Electric Quadrupole moment

A nucleus is generally non-spherically . The distortion along the axes of rotation is expressed in terms of electric quadrupole moment.

$$Q = \frac{1}{e}\int (3z^2 - r^2)\rho(r)dV$$

For a spherically symmetric $\rho(r)$, Q is zero. For an ellipsoidal distribution one can show that

$$Q = \frac{2}{5}\frac{q}{e}(a^2 - b^2)$$

where q is the total charge and a is the semi-major axes along the axis of rotation.

- $Q > 0$ implies an elongated or prolate nucleus.
- $Q < 0$ implies a flattened or oblate nucleus

Q can be determined from hyperfine structure of the atomic spectra. The values of Q range from $Q = 10^{-28}m^2$ for ^{123}Sb and $8 \times 10^{-28}m^2$ for ^{176}Lu. Deuteron has a value of Q of about $2.74 \times 10^{-31}m^2$

Nuclear Forces

Some general properties of the nuclear force are as follows:

1. Nuclear forces are strong and about 100 times stronger than electromagnetic forces. This follows from the large nuclear binding energies.
2. Nuclear forces have a very short range. They dominate over a distance of 1 fm but vanish rapidly for distance > few femtometers.
3. Nuclear forces are independent of nuclear charges. One can assign an isospin to the nucleons. Isospin described by two numbers :(I), which is the total isospin and I_3, which is the third component of the spin vector in a given direction. Proton and neutron have $I = 1/2$ and $I_3 = 1/2$ (proton) and $-1/2$ (neutron).
4. Charge independence of nuclear forces is equivalent to statement that nuclear forces are independent of orientation of isospin.

Yukawa theory

Nuclear force arise from exchange of virtual particles (called π-mesons or pions). One can derive the potential from the following arguments.

$$\frac{1}{(\hbar c)^2}(E^2 - c^2 p^2 - m^2 c^4) = 0$$

From plugging in the quantum mechanical operators of E and p (cf. quantum mechanics lectures) and including a point source, the equation for the potential becomes

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2 c^2}{\hbar^2}\right)\phi_m = g\delta(r)$$

The static solution to this equation is found to be

$$\phi_m = -\frac{g}{4\pi r}\exp[-rmc/\hbar]$$

This is known as Yukawa potential. The approximate range of r_0 is given by $r_0 = \hbar/mc$

Yukawa theory (Contd)

- Yukawa argued that since nuclear forces have a range $r_0 = 10^{-15}$m arise from exchange of a particle of mass $m \approx h/r_o c \approx 200 MeV$
- The π meson with a mass of 140 MeV and discovered in cosmic rays would be a good candidate for the quantum whose exchange gives rise to nuclear forces.
- The short range nature of the nuclear force is due to rapidly decreasing exponential function in the potential.