

MA1140 ELEMENTARY LINEAR ALGEBRA

MARCH 28 TO MAY 02, 2022 (1-2 SEGMENT)

ASSIGNMENT 4 (DUE DATE: 29.04.2022, 11:59 PM)

Rules:

- Answer all questions.
- Provide complete answers with full justification and all steps worked out.
- The deadline is strict and even a one minute late submission cannot be accepted. Late submissions receive 0 marks.
- Only three grades are possible for each question: 0 for a wrong answer, 1.5 for a partially correct answer or 3 for a fully correct answer.

Questions:

1. Find the eigenvalues, eigenspaces, algebraic multiplicities and geometric multiplicities for the matrix $\begin{bmatrix} -1 & 2 \\ -6 & 6 \end{bmatrix}$.
2. For $A \in M_{n \times n}(\mathbb{C})$, show that the constant term in the characteristic polynomial of A is equal to its determinant.
3. If A and B are similar matrices, then show that A^m and B^m are also similar matrices for any $m \in \mathbb{Z}_{>0}$.
4. Check if the following matrix can be diagonalised:

$$\begin{bmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{bmatrix}.$$

If it is possible, diagonalise it.

5. Compute the exponential matrix e^{At} for $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$.
6. Let $u, v \in \mathbb{R}^3$ be linearly independent and $P = \text{span}\{u, v\}$.
 - (a) Let $v^\perp = v - \frac{(u,v)}{(u,u)}u$. Is $v^\perp \in P$?
 - (b) Find the angle between v^\perp and u .
 - (c) Find a third vector in \mathbb{R}^3 that is perpendicular to both v^\perp and u .
 - (d) Construct an orthonormal basis for \mathbb{R}^3 from u and v .
 - (e) Taking $u = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ repeat the above procedure.