Assignment 1 on First Order ODE

Differential Equation (MA-1150)

1. **Solve any three problems of the following differential equations: (Rest of the problems for your practice only)**

   (a) $x^2\,du - u\,dx = 2\sin\frac{1}{x}\,dx$

   (b) $x^2\,dy + xy\,dx = \sqrt{1 - x^2y^2}\,dx$

   (c) $(y^3 - 2xy^2)\,dx + (2xy^2 - x^3)\,dy = 0$

   (d) $(xy\sin xy + \cos xy)\,y\,dx + (xy\sin xy - \cos xy)\,x\,dy = 0$

   (e) $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)dx + \frac{1}{4}(x + xy^2)\,dy = 0$

   (f) $(y^2 + 2x^2y)\,dx + (2x^3 - xy)\,dy = 0$

   (g) $\sin x\,\frac{dy}{dx} + 3\,y = \cos x$

   (h) $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$

   (i) $x^3\,\frac{dy}{dx} - x^2y + y^4\cos x = 0$

   (j) $\frac{du}{dx} + xu = e^{-x^2}, u(1) = 0$

   (k) Show that $M(x,y)\,dx + N(x,y)\,dy = 0$ is exact iff $[M(x,y) + g(x)]\,dx + [N(x,y) + h(y)]\,dy = 0$ is exact.

   (l) Verify that $e^{\int p_0(x)\,dx}\,[p_0(x)\,y - q(x)]\,dx + e^{\int p_0(x)\,dx}\,dy = 0$ is exact and hence solve it.

2. **Solve any three problems of the following differential equations: (Rest of the problems for your practice only)**

   (a) $3\,e^x\tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$

   (b) $(x - y^2\,x)\,dx + (y - x^2y)\,dy = 0$

   (c) $\sec^2\theta\tan\varphi\,d\varphi + \sec^2\varphi\tan\theta\,d\theta = 0$

   (d) $xy^2\,dy = (x^3 + y^3)\,dx$

   (e) $x\cos\frac{y}{x}\,(y\,dx + x\,dy) = y\sin\frac{y}{x}\,(x\,dy - y\,dx)$

(f) $(x + 2\,y + 1)\,dx - (2x + 4y + 3)\,dy = 0$

(g) $(y^3 - x)\,dy = y\,dx$

(h) $2(3xy^2 + 2x^3)\,dx + 3(2x^2y + y^2)\,dy = 0$

(i) $\frac{1}{x^2} + \frac{3\,y^2}{x^4} = \frac{2y}{x^3}\,\frac{dy}{dx}$

(j) $x^{-1}\cosh y\,dx + \sinh y\,dy = 0$

3. Solve any three problems of the following differential equations: (Rest of the problems for your practice only)

(a) $x^3\frac{dy}{dx} + 3x^2y = \frac{1}{x}$

(b) $\frac{1}{3}\,(1 - 2x)\,y^4\,dx = dy + \frac{1}{3}\,y\,dx$

(c) $x\,dy + 4y\,dx = 8x^4\,dx, \quad y(1) = 2$

(d) $dy\,(\sinh 3y - 2xy) = y^2\,dx$

(e) $2xy\,dy + (x - 1)\,y^2\,dx = x^2\,e^x\,dx$

(f) $dy + y\sin x\,dx = e^{\cos x}\,dx$

(g) $(x - 1)\,dy = \tan y\,dx$

(h) $x\,dy + \cot y\,dx = 0, \quad y(\sqrt{2}) = \frac{\pi}{4}$

(i) $\frac{x+u-a}{x+u-b}\,du = \frac{x+u+a}{x+u+b}\,dx$

(j) $\cos(x + y)\,dy = dx$

(k) $x + y\,\frac{dy}{dx} = a\left(\frac{dy}{dx}\right)^2$

(l) $y = x\,\frac{dy}{dx} + a\,\frac{dx}{dy}$

(m) $x^2\left(y - x\frac{dy}{dx}\right) = y\left(\frac{dy}{dx}\right)^2$

(n) $\left(\frac{dy}{dx}\right)^3 - 4xy\frac{dy}{dx} + 8y^2 = 0$

(o) $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$

(p) $(4x^2y - 6)\,dx + x^3\,dy = 0$