ME2240: Fluid Mechanics
Assignment - 4

Instructor: Harish N Dixit
Department of Mechanical & Aerospace Engineering,
IIT Hyderabad.

Due date: 11th October 2022, before the class begins.

Note: Proxy submissions of assignments will not be allowed unless you have taken prior approval from me. Attendance will be taken during the submission. Students submitting assignment of their absent friends will be risking getting a zero in the assignment.

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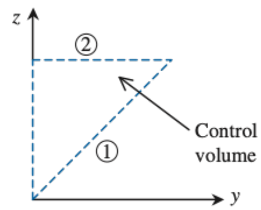
1. The velocity field in the region shown is given by $\vec{V} = (a\hat{\boldsymbol{j}} + by\hat{\boldsymbol{k}})$ where $a = 10 m/s$ and $b = 5 s^{-1}$. For the $1m \times 1m$ triangular control volume (depth $w = 1m$ perpendicular to the diagram), an element of area ① may be represented by $d\vec{A}_1 = wdz\hat{\boldsymbol{j}} - wdy\hat{\boldsymbol{k}}$ and an element of area ② by $d\vec{A}_2 = -wdy\hat{\boldsymbol{k}}$.



   (a) Find an expression for $\vec{V} \cdot dA_1$.

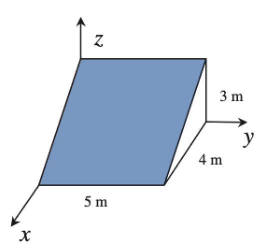
   (b) Evaluate $\int_{A_1} \vec{V} \cdot dA_1$.

   (c) Find an expression for $\vec{V} \cdot dA_2$.

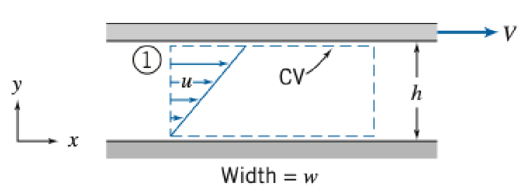
   (d) Find an expression for $\vec{V}\left(\vec{V} \cdot dA_2\right)$.

   (e) Evaluate $\int_{A_2} \vec{V}\left(\vec{V} \cdot dA_2\right)$.

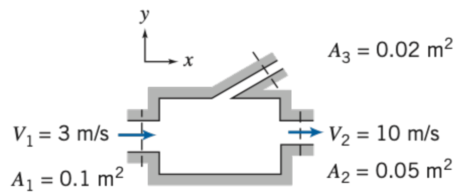
2. The shaded area shown is in a flow where the velocity field is given by $\vec{V} = ax\hat{\boldsymbol{i}} + by\hat{\boldsymbol{j}}; a = b = 1s^{-1}$, and the coordinates are measured in meters. Evaluate the volume flow rate and the momentum flux through the shaded area ($\rho = 1 kg/m^3$).



3. Obtain expressions for the volume flow rate and the momentum flux through cross section ① of the control volume shown in the diagram.

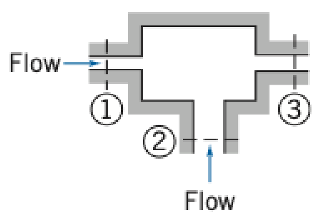


4. Consider steady, incompressible flow through the device shown. Determine the magnitude and direction of the volume flow rate through port 3.



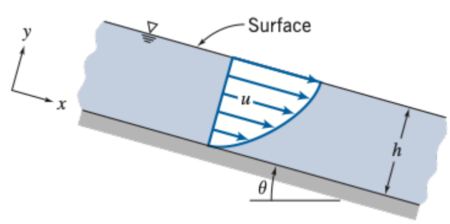
5. Ventilation air specifications for classrooms require that at least 8.0 L/s of fresh air be supplied for each person in the room (students and instructor). A system needs to be designed that will supply ventilation air to 6 classrooms, each with a capacity of 20 students. Air enters through a central duct, with short branches successively leaving for each classroom. Branch registers are 200 mm high and 500 mm wide. Calculate the volume flow rate and air velocity entering each room. Ventilation noise increases with air velocity. Given a supply duct 500 mm high, find the narrowest supply duct that will limit air velocity to a maximum of 1.75 m/s.

6. In the incompressible flow through the device shown, velocities may be considered uniform over the inlet and outlet sections. The following conditions are known: $A_1 = 0.1m^2$, $A_2 = 0.2m^2$, $A_3 = 0.15m^2$, $V_1 = 10e^{-t/2}m/s$, and $V_2 = 2\cos(2\pi t)m/s$ (t in seconds). Obtain an expression for the velocity at section ③, and plot V_3 as a function of time. At what instant does V_3 first become zero? What is the total mean volumetric flow at section ③?



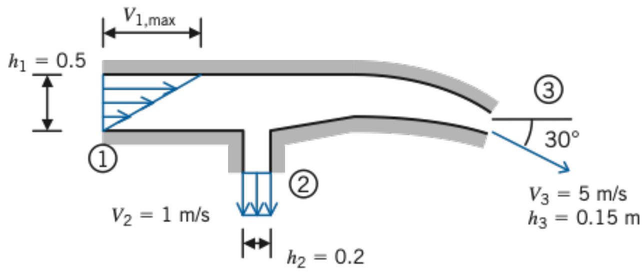
7. Oil flows steadily in a thin layer down an incline plane. The velocity profile is

$$u = \frac{\rho g \sin\theta}{\mu}\left[hy - \frac{y^2}{2}\right].$$



Express the mass flow rate per unit width in terms of $\rho, \mu, g, \theta,$ and h.

8. A two-dimensional reducing bend has a linear velocity profile at section ①. The flow is uniform at sections ② and ③. The fluid is incompressible and the flow is steady. Find the maximum velocity, $V_{1,\max}$, at section ①.



9. Water enters a two-dimensional, square channel of constant width, $h = 75.5mm$, with uniform velocity, U. The channel makes a 90° bend that distorts the flow to produce the linear velocity profile shown at the exit, with $V_{\max} = 2V_{\min}$. Evaluate $V_{\min}$, if $U = 7.5m/s$.