ME3210

Control Systems

ME21BTECH11001

Abhishek Ghosh

Question 1

ME3120 Control Systems :-

Assignment - 1

ME21BTECH11001
Abhishek Ghosh

Question 1

(a) $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Ans : $\mathcal{L}[f(t)] = F(s)$

Also, $\mathcal{L}[f'(t)] = s\,\mathcal{L}[f(t)] - f(0)$

LHS $\rightarrow$ $\mathcal{L}[f'(t)] = \mathcal{L}\left[\frac{\partial}{\partial t}(f(t))\right] = \int_0^\infty e^{-st}\left(\frac{\partial}{\partial t} f(t)\right)\partial t$

$\Rightarrow \int_0^\infty e^{-st}\frac{\partial}{\partial t}(f(t))\,\partial t = s\,F(s) - f(0)$

$\rightarrow \lim_{s\to 0}\mathcal{L}\left[\frac{\partial}{\partial t} f(t)\right] = \lim_{s\to 0}\int_0^\infty \frac{1}{e^{st}}\frac{\partial}{\partial t}(f(t))\,\partial t$

$= \int_0^\infty 1 * \frac{\partial}{\partial t}(f(t))\,\partial t$

$= f(\infty) - f(0) = f(\infty)$

RHS $\lim_{s\to 0}[sF(s) - f(0)] = \lim_{s\to 0} sF(s)$

$\therefore \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \longrightarrow$ Prooved

(b) $\mathcal{L}[t f(t)] = -\frac{\partial F(s)}{\partial s}$

Ans: $\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)\,e^{-st}\,\partial t$

$\frac{\partial}{\partial s} F(s) = \frac{\partial}{\partial s}\left(\int_0^\infty f(t)\,e^{-st}\,\partial t\right)$

$$= \int_0^\infty \frac{\partial}{\partial s}\left(f(t)\, e^{-st}\right) \partial t$$

$$= -\int_0^\infty e^{-st}\ t\ f(t)\, \partial t$$

$$= -\mathcal{L}[t f(t)] \longrightarrow \text{Prooved}$$

(c) $\mathcal{L}\left[\int_0^t f(\tau)\,\partial\tau\right] = \frac{F(s)}{s}$

Considering $g(t) = \int_0^t f(\tau)\,\partial\tau$

$$g'(t) = f(t) \quad \& \quad g(0) = 0$$

$$\mathcal{L}[g'(t)] = s\,\mathcal{L}[g(t)] - g(0)$$

$$\Rightarrow \mathcal{L}[f(t)] = s\,\mathcal{L}[g(t)]$$

$$\rightarrow \mathcal{L}[g(t)] = \frac{\mathcal{L}[f(t)]}{s}$$

$$= \frac{F(s)}{s}$$

$$\Rightarrow \mathcal{L}\left[\int_0^t f(\tau)\,\partial\tau\right] = \frac{F(s)}{s} \longrightarrow \text{Prooved}$$

Question 2

Question 2 :-

Given, $\ddot{y} + 15\dot{y} + 56y = u(t)$

for unit step, $u(t) = 1$

unit impulse, $u(t) = \delta(t)$

unit ramp, $u(t) = t$

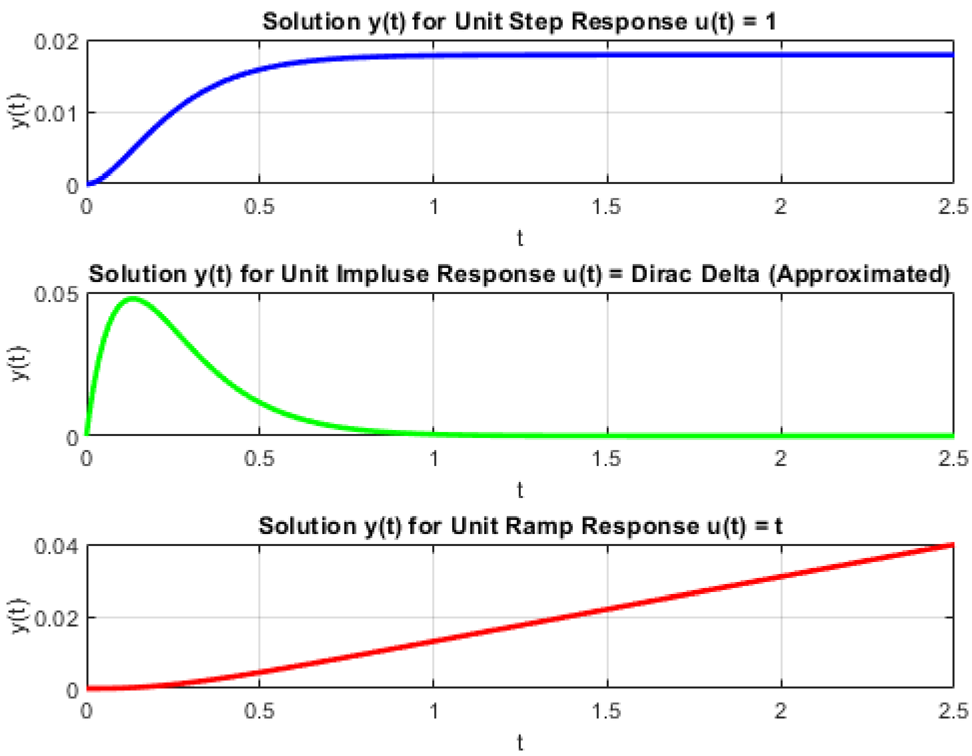
Numerical Solutn for second order solved using ode45 in matlab.

Code:

```
%Abhishek Ghosh
%ME21BTECH11001
%Question 2

% Parameters
tspan = [0 2.5];  % Time interval
y0 = [0; 0];  % Initial conditions: y(0) = 0, dy(0) = 0

% Case 1: u(t) = 1
u1 = @(t) 1;  % Define the function u(t) = 1
[t1, y1] = ode45(@(t, y) odefunc(t, y, u1), tspan, y0);

% Case 2: u(t) = t
u2 = @(t) t;  % Define the function u(t) = t
[t2, y2] = ode45(@(t, y) odefunc(t, y, u2), tspan, y0);

% Case 3: u(t) = Dirac Delta approximation
impulse_magnitude = 100000;  % Large magnitude to approximate Dirac delta
impulse_duration = 0.00001;  % Very short duration for impulse
u_dirac = @(t) (t >= 0 & t <= impulse_duration) * impulse_magnitude ;

[t_dirac, y_dirac] = ode45(@(t, y) odefunc(t, y, u_dirac), tspan, y0);

% Plotting the results
figure;

% Plot for u(t) = 1
subplot(3, 1, 1);
plot(t1, y1(:, 1), 'b', 'LineWidth', 2);
title('Solution y(t) for Unit Step Response u(t) = 1');
xlabel('t');
ylabel('y(t)');
grid on;

% Plot for u(t) = Dirac Delta approximation
subplot(3, 1, 2);
plot(t_dirac, y_dirac(:, 1), 'g', 'LineWidth', 2);
title('Solution y(t) for Unit Impluse Response u(t) = Dirac Delta (Approximated)');
```

```
xlabel('t');
ylabel('y(t)');
grid on;

% Plot for u(t) = t
subplot(3, 1, 3);
plot(t2, y2(:, 1), 'r', 'LineWidth', 2);
title('Solution y(t) for Unit Ramp Response u(t) = t');
xlabel('t');
ylabel('y(t)');
grid on;


% Function to define the system of first-order ODEs
function dydt = odefunc(t, y, u)
    dydt = zeros(2, 1);
    dydt(1) = y(2);  % dy1/dt = y2
    dydt(2) = u(t) - 15 * y(2) - 56 * y(1);  % dy2/dt = u(t) - 15*y2 - 56*y1
end
```

Plots:



Published with MATLAB® R2023a

Question 3

Questn 3 :-

Given, $\ddot{y} + 15\dot{y} + 56y = u(t)$

Taking Laplace,

$$s^2 Y(s) - sY(0) - \dot{y}(0)$$
$$+15\,(sY(s) - y(0)) + 5C\,Y(s) = U(S)$$

→ Initial conditn $y(0) = 0$ & $\dot{y}(0) = 0$

→ $s^2 Y(s) + 15sY(s) + 56Y(s) = U(C)$

$$Y(S) = \frac{U(S)}{(s^2 + 15s + 56)}$$

Transfer functn,

$$P(S) = \frac{Y(S)}{U(S)} = \frac{1}{s^2 + 15s + 56}$$

(1) For unit step response,

$u(t) = 1$, $u(S) = 1/S$

$$Y(S) = P(S)\,U(S)$$
$$= \frac{1}{(s^2 + 15s + 56)} \times \frac{1}{S}$$
$$= \frac{1}{S(S+8)(S+7)} = \frac{A}{S} + \frac{B}{S+8} + \frac{C}{S+7}$$

Solving $A = 1/56$ $B = 1/8$ $C = -1/7$

$$Y(S) = \frac{1}{56S} + \frac{1}{8(S+8)} - \frac{1}{7(S+7)}$$

Taking Inverse Laplace

$$Y(t) = \frac{1}{56} + \frac{1}{8}e^{-8t} - \frac{1}{7}e^{-7t}$$ —— Ans

(iii) Unit Impulse Response

$u(t) = \delta(t)$ $\quad\quad$ $u(s) = 1$

direc delta

$$Y(S) = P(S) \cdot u(s)$$

$$= \frac{1}{s^2 + 15s + 56} \times 1$$

$$= \frac{1}{(s+8)(s+7)}$$

$$Y(S) = \frac{1}{s+7} - \frac{1}{s+8}$$

Inverse Laplace

$$y(t) = e^{-7t} - e^{-8t}$$ ——— Ans

(iii) Unit Ramp Response

$u(t) = t$ $\quad\quad$ $u(s) = 1/s^2$

$$Y(s) = P(s)\, u(s)$$

$$= \frac{1}{s^2 + 15s + 56} \times \frac{1}{s^2}$$

$$= \frac{1}{s^2(s+8)(s+7)}$$

$$= \frac{1}{56s^2} + \frac{1}{49(s+7)} - \frac{1}{64(s+8)} - \frac{15}{3136s}$$

Inverse Laplace

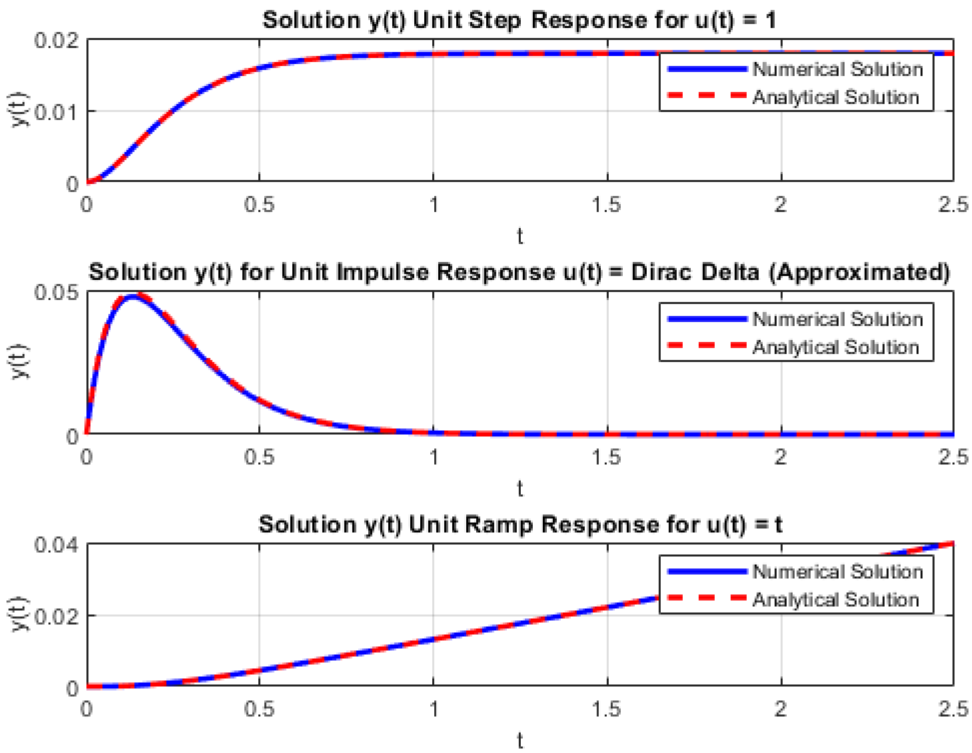
$$y(t) = \frac{1}{56}t + \frac{1}{49}e^{-7t} - \frac{1}{64}e^{-8t} - \frac{15}{3136}$$ ——— Ans

Code:

```
%Abhishek Ghosh
%ME21BTECH11001
%Question 3


% Parameters for the ODE
tspan = [0 2.5];  % Time interval
y0 = [0; 0];  % Initial conditions: y(0) = 0, dy(0) = 0
impulse_magnitude = 100000;  % Large magnitude to approximate Dirac delta
impulse_duration = 0.00001;  % Very short duration for impulse

% Numerical Solutions Using ode45

% Case 1: u(t) = 1
u1 = @(t) 1;  % Define the function u(t) = 1
[t1_num, y1_num] = ode45(@(t, y) odefunc(t, y, u1), tspan, y0);

% Case 2: u(t) = t
u2 = @(t) t;  % Define the function u(t) = t
[t2_num, y2_num] = ode45(@(t, y) odefunc(t, y, u2), tspan, y0);

% Case 3: u(t) = Dirac Delta approximation
u_dirac = @(t) (t >= 0 & t <= impulse_duration) * impulse_magnitude;
[t_dirac_num, y_dirac_num] = ode45(@(t, y) odefunc(t, y, u_dirac), tspan, y0);

% Analytical Solutions Using Laplace Transforms
% Analytical solution for u(t) = 1
y1_analytical = @(t) ((1/56) + (1/8) *(exp(-8*t)) - (1/7) * exp(-7*t));

% Analytical solution for u(t) = t
y2_analytical = @(t) ((1/56) * t + (1/49) *(exp(-7*t)) - (1/64) * exp(-8*t) -
15/3136);

% Analytical solution for u(t) = Dirac Delta
y_dirac_analytical = @(t) (exp(-7*t) - exp(-8*t));

% Time vector for plotting analytical solutions
t_analytical = linspace(0, 2.5, 1000);

% Plotting the comparison between numerical and analytical solutions
figure;

% Case 1: u(t) = 1
subplot(3, 1, 1);
plot(t1_num, y1_num(:, 1), 'b', 'LineWidth', 2); hold on;
plot(t_analytical, y1_analytical(t_analytical), 'r--', 'LineWidth', 2);
title('Solution y(t) Unit Step Response for u(t) = 1');
xlabel('t');
ylabel('y(t)');
legend('Numerical Solution', 'Analytical Solution');
grid on;

% Case 3: u(t) = Dirac Delta approximation
subplot(3, 1, 2);
plot(t_dirac_num, y_dirac_num(:, 1), 'b', 'LineWidth', 2); hold on;
plot(t_analytical, y_dirac_analytical(t_analytical), 'r--', 'LineWidth', 2);
title('Solution y(t) for Unit Impulse Response u(t) = Dirac Delta (Approximated)');
xlabel('t');
ylabel('y(t)');
legend('Numerical Solution', 'Analytical Solution');
grid on;

% Case 2: u(t) = t
subplot(3, 1, 3);
```

```
plot(t2_num, y2_num(:, 1), 'b', 'LineWidth', 2); hold on;
plot(t_analytical, y2_analytical(t_analytical), 'r--', 'LineWidth', 2);
title('Solution y(t) Unit Ramp Response for u(t) = t');
xlabel('t');
ylabel('y(t)');
legend('Numerical Solution', 'Analytical Solution');
grid on;



% Function to define the system of first-order ODEs
function dydt = odefunc(t, y, u)
    dydt = zeros(2, 1);
    dydt(1) = y(2);  % dy1/dt = y2
    dydt(2) = u(t) - 15 * y(2) - 56 * y(1);  % dy2/dt = u(t) - 15*y2 - 56*y1
end
```

Plots:



Published with MATLAB® R2023a

Advantages of Laplace Transform :—

i) Time domain eqⁿ can be converted into simple algebraic form

ii) for a known transfer functⁿ, o/p response is easy to determine for any i/p.

iii) Helps to determine important parameter like poles / zeros.

iv) Stability of system can be analysed.

Question 4

Questⁿ 4 :—

$\mathcal{L}[P(t)] = P(s) \longrightarrow$ Transfer functⁿ

$|P(t)| \longrightarrow 0$ as $t \longrightarrow \infty$

$Y(s) = P(s) \cdot u(s)$ $\quad$ $y(t) = P(t) * u(t)$

$y(t) = \int_0^t P(t-\tau)\, u(\tau)\, d\tau$ $\quad$ convolutⁿ

for BIBO stability $\quad u(t) \rightarrow$ bounded

$|u(t)| \leq m < \infty \quad \forall \quad t > 0$

$$y(t) = \left| \int_0^t P(t-\tau)\, u(\tau)\, d\tau \right|$$

$$\leq \int_0^t |P(t-\tau)|\,|u(\tau)|\, d\tau |$$

$$\leq m \int_0^t |P(t-\tau)|\, d\tau$$

BIBO $\longrightarrow$ $y(t)$ should be bounded when $t \to \infty$

$\Rightarrow \int_0^t |P(t-\tau)|\, d\tau$ should be bounded @ $t \to \infty$

Area of curve $|P(t)|$ should be constant in $t \in [0, \infty)$

$\Rightarrow \lim_{t \to \infty} |P(t)| = 0$

since $|y(t)| \leq m \int_0^t |P(t-\tau)|\, d\tau$

if $\lim_{t \to \infty} |P(t)| \neq 0$

$\lim_{t \to \infty} \int_0^t |P(t-\tau)|\, d\tau \longrightarrow \infty$

$y(t) \longrightarrow$ unbounded

$\therefore \lim_{t \to \infty} |P(t)| = 0 \longrightarrow$ necessary condit^n for BIBO stability

$\therefore$ If $\lim_{t \to \infty} |P(t)| = 0$ is given

then $|y(t)| \leq m \int_0^t |P(t-\tau)|\, d\tau$ is bounded

$\Rightarrow \lim_{t \to \infty} |P(t)| = 0$ is necessary & sufficient condit^n for BIBO stability

if $|P(t)| \longrightarrow \infty$

then $|y(t)| \leq m \int_0^t |P(t-\tau)|\, d\tau \longrightarrow \infty$

$\Rightarrow |y(t)|$ is unbounded

$\therefore$ This system is not BIBO stable since for bounded i/p $\longrightarrow$ unbounded o/p

$P(t) \longrightarrow \infty$

for a finite value of $t \longrightarrow$ no system

$P(s) = \frac{1}{s-a}$ $\quad P(t) = e^{at} \longrightarrow \;\to \infty$ when $t \to \infty$

Question 5

Question 5 :-

System :- An automatic fan control system.

This system manages the operatn of an electric fan based on temperature readings. This system can be used in computer cooling systems or industrial processes where we need to maintain stable temp.

Necessity :-

- To prevent overheating
- To reduce energy consumptn
- To extend lifespan of fan/device.

Constraints :-

- The system must maintain temp within a predefined range.
- Response Time : System should respond quick enough to change temp.
- Power Consumptⁿ : The system should minimize power usage and be cost effective

Assumptⁿs :-

- Assuming fan's speed is linear to its speed.
- Temp sensors and fan respond instantaneously to changes.
- External factors are constant or slow changing

Mathematical Model :-

$T(t)$: Temp of system at time t

T_{set} : Desired setpoint Temp.

T_{env} : Ambient Temp.

$u(t)$: Control i/p | fan speed

K_f : fan Constant.

Heat Balance Eqtⁿ : $C \frac{\partial T(t)}{\partial t} = -K_f u(t) + \dot{Q}_{env}$

where C : Thermal capacity of system

Control eqtⁿ : $\frac{\partial T(t)}{\partial t} = -\frac{K_f}{C} u(t) + \frac{K_f}{C} T_{set} + \frac{\dot{Q}_{env}}{C}$

Taking Laplace Transform

$$sT(s) = -\frac{K_b}{C} U(s) + \frac{K_b}{C}\frac{T_{set}}{S} + \frac{\dot{q}_{en}}{C}\frac{1}{S}$$

Isolating I/p & O/p relatn

$$T(s) = -\frac{K_b}{C}\frac{U(s)}{s}$$

Transfer functn : $\frac{T(S)}{U(S)} = \frac{-K_b}{CS}$

$$\Rightarrow H(S) = \frac{T(S)}{U(S)} = \frac{-K_b}{CS}$$