EP1108 Black Body Radiation

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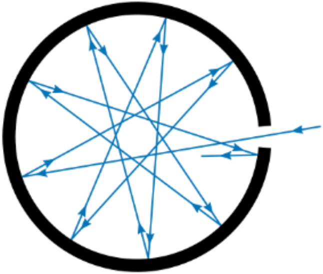
Introduction

- Surface of a hot body emits electromagnetic radiation. When radiation falls on the surface of a body, some of it is absorbed and some is reflected.
- At low temperatures, most of the emitted energy is concentrated at long wavelengths and vice-versa.
- Absorption coefficient is the fraction of radiant energy incident on a surface which is absorbed at that wavelength.
- If a body is in thermal equilibrium with its surroundings it must emit and absorb the same amount of radiant energy per unit time. This radiation emitted and absorbed under these circumstances is known as *thermal radiation*.

Black Body

A black body has an absorption coefficient of one at all wavelengths. Thermal radiation emitted or absorbed by a black body is called *black body radiation*.



A cavity at constant temperature is a good approximation to a black body. Credit:wikiwand.com

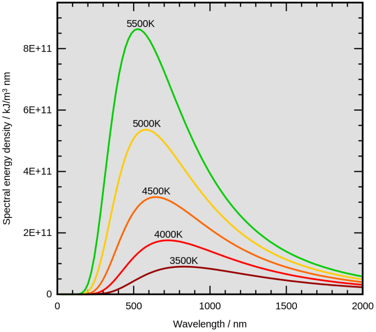
Any radiation incident upon the body is absorbed after multiple reflections inside the cavity, so that the hole has an effective absorption coefficient close to unity.

Kirchoff's law

Thermal radiation emitted or absorbed by a black body is called black body radiation.
- Emissive Power (or Spectral Emittance) = Power emitted per unit area at a given wavelength and can be denoted as $R(\lambda, T)$
Kirchoff proved that the ratio of emissive power to absorption coefficient is same for all bodies at the same temperatures and is equal to the emissive power of a black body at that temperature. This is known as *Kirchoff's law*.
Therefore, since a black body has the maximum absorption coefficient of one, this shows that a black hole is not only the perfect absorber, but also the most efficient emitter of electromagnetic energy.

Wien's Law

The first measurements of $R(\lambda, T)$ were made by Lummer and Pringsheim. $R(\lambda, T)$ is shown as a function of λ for different temperatures.



Source:wikipedia

$\boxed{\lambda_{max} T = b}$, where $b = 2.898 \times 10^{-3} mK$.
This is known as Wien's displacement law and b is known as Wien displacement constant

Stefan-Boltzmann Law

Define *Total emissive power* is the total power emitted per unit area from a black body at the absolute temperature T

$$R(T) = \int_0^\infty R(\lambda, T) d\lambda$$

In 1879, Stefan found that $\boxed{R(T) = \sigma T^4}$
In 1884, Boltzmann deduced this law from thermodynamics and this is known as Stefan-Boltzmann law. where $\sigma = 5.67 \times 10^{-8} W/m^2/K^4$ and is known as Stefan-Boltzmann constant.

Spectral Distribution Function

Kirchoff showed that for a cavity (used to approximate the black body) the radiation is homogeneous and isotropic. More convenient to define the *spectral distribution function* or *monochromatic energy density*

$\rho(\lambda, T)d\lambda \equiv$ Energy per unit volume of the radiation in the wavelength interval between $(\lambda,\lambda + d\lambda)$ at absolute temperature T. *(Note that some books also define a corresponding quantity per frequency interval)*

$$\rho(\lambda, T) = \frac{4}{c}R(\lambda, T)$$

(See Ref. [2] for derivation)

Rayleigh-Jeans Law

From thermodynamics and laws of classical Physics, Rayleigh and Jeans showed that:

$$\rho(\lambda, T) = \frac{8\pi}{\lambda^4}\bar{\epsilon}$$

(See Bransden and Jochain for derivation), where $\bar{\epsilon}$ is the mean energy at wavelength λ, equal to kT

$$\implies \rho(\lambda, T) = \frac{8\pi}{\lambda^4}kT \tag{1}$$

2-3 problems with this law

- It only agrees with experiment at large wavelengths.
- At low wavelengths it does not exhibit the observed maximum (shown earlier).
- It diverges as $\lambda \to 0$
- The total energy per unit volume $\rho_{tot}(T) = \int_0^\infty \rho(\lambda, T)d\lambda$ is equal to infinity, which is a problem.

Planck's quantum theory

No solution to the problems were found using classical Physics. Planck postulated the following hypothesis

- The energy of an oscillator at a given frequency ν cannot take arbitrary values between zero and infinity.
- It can only take discrete values, given by $n\epsilon_0$, where ϵ_0 is a finite amount called *quantum* of energy, which could be a function of frequency, ν and n is a positive integer or zero.

Therefore

$$\bar{\epsilon} = \frac{\sum_{n=0}^{\infty} n\epsilon_0 \exp(-\beta n\epsilon_0)}{\sum_{n=0}^{\infty} \exp(-\beta n\epsilon_0)} = -\frac{d}{d\beta}\left[\log \sum_{n=0}^{\infty} \exp(-\beta n\epsilon_0)\right]$$

$$\implies \bar{\epsilon} = -\frac{d}{d\beta}\left[\log\left(\frac{1}{1-\exp(-\beta\epsilon_0)}\right)\right] = \frac{\epsilon_0}{\exp(\beta\epsilon_0)-1}$$

where $\beta = \frac{1}{kT}$

Planck spectral distribution law

Substituting the value of $\bar{\epsilon}$ from Planck's theory into Rayleigh-Jeans formula

$$\rho(\lambda, T) = \frac{8\pi}{\lambda^4}\frac{\epsilon_0}{\exp(\epsilon_0/kT) - 1}$$

In order to satisfy the observed empirical behaviour of $R(\lambda, T)$, $\epsilon \propto \nu$ or

$$\boxed{\epsilon_0 = h\nu = \frac{hc}{\lambda}}$$

where h is known as Planck's constant. Often one uses reduced Planck constant $\hbar \equiv h/2\pi$

$$\boxed{\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp(hc/\lambda kT) - 1}} \tag{2}$$

Connection with Rayleigh-Jeans Law and short λ behavior

- At long wavelengths $\rho(\lambda, T) \rightarrow \frac{8\pi kT}{\lambda^4}$ in agreement with Rayleigh-Jeans law.
- For short wavelengths, the presence of $\exp(hc/\lambda kT)$ in the denominator ensures that $\rho \rightarrow 0$ as $\lambda \rightarrow 0$

At short wavelengths the available quantum steps are widely separated in energy in comparison to the thermal energies.
At long wavelengths,$\epsilon_0 << kT$ and quantum steps are small with respect to thermal energy and hence the quantum steps are almost continuously distributed.

Connection with Wein's Law

Maximum value of $\rho(\lambda, T)$ (Eq. 2) can be evaluated from $\frac{d\rho(\lambda,T)}{d\lambda} = 0$ and one can find that

$$\lambda_{max} T = \frac{hc}{4.965K}$$

$\implies b = \frac{hc}{4.965K}$

Total Radiated Power

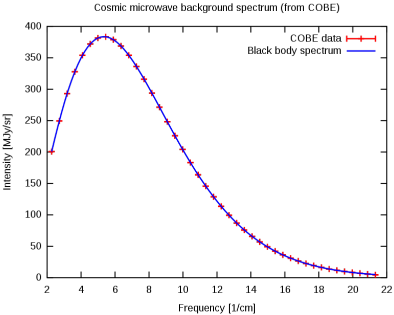
The total energy per unit volume

$$\rho_{tot}(T) = \int_0^\infty \rho(\lambda, T) d\lambda = aT^4$$

where $a = \frac{2\pi^5}{15} \frac{k^4}{h^3c^2}$
Since $\rho_{tot} = 4R/c$, Stefan constant σ is given by $\sigma = \frac{2\pi^4}{15} \frac{k^4}{h^3c^2}$
From the observed values of ρ_{tot} and Wien's law, one has three unknowns, h, c, and k.
By 1900, c was known. So h and k could be estimated. So we get $h = 6.62618 \times 10^{-34} Js$ and $k = 1.38 \times 10^{-23} JK^{-1}$

Cosmic Microwave Background Radiation

The cosmic microwave background (CMB) is the most perfect black body, seen in nature.



Cosmic Microwave Background Black body spectrum as measured by COBE satellite (source :wikipedia). Original reference ApJ 354L, 37 (1990)

For more details on history of CMB, one can look at
http://www.astro.ucla.edu/~wright/CMB.html

References for this lecture

The material in these slides has been distilled from *Bransden and Jochain : Introduction to Quantum Mechanics (Chapter 1)* Other references for this material:

- Any book on Modern Physics
- https://mcgreevy.physics.ucsd.edu/s12/lecture-notes/chapter07.pdf [2]
- http://www.iucaa.in/~dipankar/ph217/contrib/blackbody.pdf