Spacecraft Dynamics End Sem

ME21BTECH11001

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For code see the attached Code file (.ipynb)

Question 1 and 2:

Spacecraft Dynamics & Control (End-Sem)

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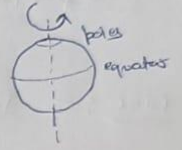
Questn 1 :-

For equitorial or low inclination orbit, chose pad near equator. Because the Earth rotating and launching from near the equator gives it extra push (tangenttally) due to rotating, which is not in the case when launching from pole. This would save fuel & carry extra load



@ equator $\longrightarrow$ $R_{equator} = 6378$ kms

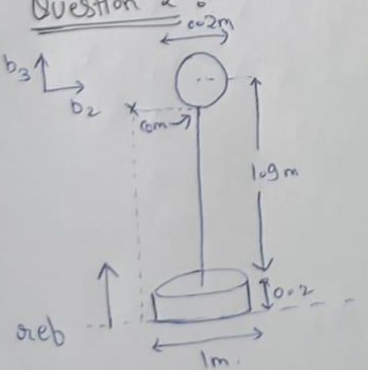
$$T = 1\,\text{day} = 24 \times 60 \times 60 = 86400\,s$$

$$v_{extra} = \frac{2\pi R_{equator}}{T} = \frac{2\pi \times 6378 \times 10^3}{86400} = 463.58$$

$$\approx \underline{464\ m/s} \rightarrow \text{extra boost at equator}$$

But if its polar or sun-synchronous orbit, pick pad near poles. Because these orbits go over Earth & launching them from higher latitude makes it easier when compared to equator. So depending on choice of orbit, we can select where to launch from.

Question 2 :-



$m_s = 19$ kg

$m_d = 1$ kg

Given massless rod,

finding com (Z)

$Z_s = 2.1$ m $\quad Z_d = 0.1$ m

$$Z = \frac{m_s Z_s + m_d Z_d}{m_s + m_d} = 2\,m \quad \text{(from ref)}$$

Calculating $\underline{I}$ at this com :-

Sphere :- $I_{b_3} = \frac{2}{5} m_s r_s^2 = \frac{2}{5} \times 19 \times (0.1)^2 = 0.076 \ kg m^2$

$I_{b_1} = I_{b_2} = \frac{2}{5} m_s r_s^2 + m_s (z_{com} - z_s)^2$ [Parallel axis theorem]

$= 0.076 + 1.9$

$= \cancel{1.976}\ 0.266 \ kg\, m^2$

$$I_{sphere, com} = \cancel{\begin{bmatrix} 1.976 & 0 & 0 \\ 0 & 1.976 & 0 \\ 0 & 0 & 0.076 \end{bmatrix}} \begin{bmatrix} 0.266 & 0 & 0 \\ 0 & 0.266 & 0 \\ 0 & 0 & 0.076 \end{bmatrix} kg\, m^2$$

Disc :- $I_{b_3} = \frac{1}{2} m_d r_d^2 = 0.125 \ kg m^2$ (Axis passes through z_{com})

$I_{b_1} = I_{b_2} = \frac{1}{4} m_d r_d^2 + m_d (z_{com} - z_d)^2$ [Parallel axis theorem]

$= 0.0625 + 3.61$

$= 3.6725 \ kg m^2$

$$I_{disc, com} = \begin{bmatrix} 3.6725 & 0 & 0 \\ 0 & 3.6725 & 0 \\ 0 & 0 & 0.125 \end{bmatrix} kg\, m^2$$

$$I_{com} = I_{sphere, com} + I_{disc, com} = \cancel{\begin{bmatrix} 5.6485 & 0 & 0 \\ 0 & 5.6485 & 0 \\ 0 & 0 & 0.201 \end{bmatrix}} kg m^2$$

$$= \begin{bmatrix} 3.9385 & 0 & 0 \\ 0 & 3.9385 & 0 \\ 0 & 0 & 0.201 \end{bmatrix} kg m^2 \text{ in } (b_1, b_2, b_3) \text{ axis}$$

Question 3

Questn 3 :-

Euler equations for a Torque-free case

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3$$
$$I_2\dot{\omega}_2 = (I_3 - I_1)\omega_3\omega_1$$
$$I_3\dot{\omega}_3 = (I_1 - I_2)\omega_1\omega_2$$

when initial angular momentum is not aligned with b_1, b_2 or b_3 the angular velocity ω precesses around fixed angular momentum vector $\vec{L}$. (The body wobbles)

In our case of spinning $I_1 = I_2 > I_3$

Euler equations →

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3 \quad \text{— (i)}$$
$$I_2\dot{\omega}_2 = (I_3 - I_1)\omega_3\omega_1 \quad \text{— (ii)}$$
$$I_3\dot{\omega}_3 = 0 \Rightarrow \omega_3 = \text{constant} \quad \text{— (iii)}$$

eq (i) & (ii)

$$\dot{\omega}_1 = \left(\frac{I_1 - I_3}{I_1}\right)\omega_2\omega_3 \qquad (\text{using } I_1 = I_2)$$

$$\dot{\omega}_2 = -\left(\frac{I_1 - I_3}{I_1}\right)\omega_3\omega_1$$

$$\ddot{\omega}_1 = \left(\frac{I_1 - I_3}{I_1}\right)\omega_3\dot{\omega}_2 = -\left(\frac{I_1 - I_3}{I_1}\right)^2\omega_3^2\omega_1$$

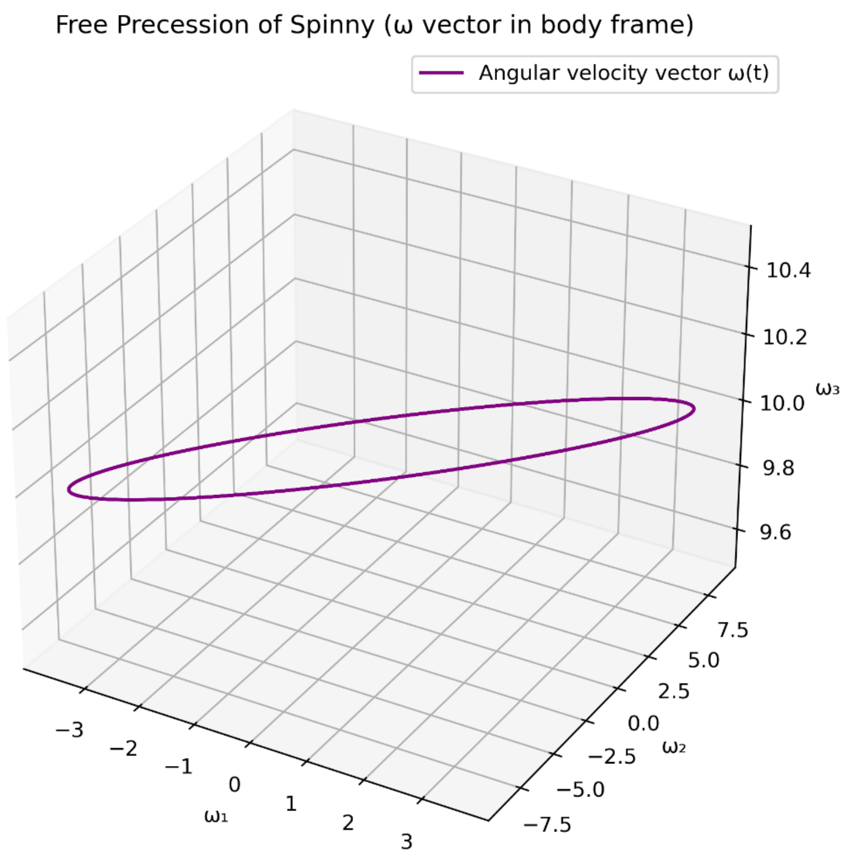
Let $$\Omega = \left(\frac{I_1 - I_3}{I_1}\right)\omega_3$$

$$\Rightarrow \ddot{\omega}_1 + \Omega^2\omega_1 = 0 \longrightarrow \text{Simple harmonic Oscillator}$$

General soltn:-

$$\omega_1(t) = A\cos(\Omega t) + B\sin(\Omega t)$$
$$\omega_2(t) = C\cos(\Omega t) + D\sin(\Omega t)$$
$$\omega_3(t) = \text{const}$$

⟶ Equations describe an elliptical path [See Plot & Code]



Free Precession of Spinny (ω vector in body frame)
Angular velocity vector ω(t)
10.4
10.2
10.0
9.8
9.6
ω₃
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
ω₂
−3
−2
−1
0
1
2
3
ω₁

Question 4

Question 4 :-

When initial angular momentum is along the minimum inertia axis, the system is unstable, small perturbations can cause large variations due to instability around this axis.

When energy dissipatn is introduced, system can be modelled as damping,

Euler equatn become :-

$$I_1 \dot{\omega}_1 = (I_2 - I_1)\omega_2\omega_3 - k\omega_1$$
$$I_2 \dot{\omega}_2 = (I_3 - I_1)\omega_3\omega_1 - k\omega_2$$
$$I_3 \dot{\omega}_3 = (I_1 - I_2)\omega_1\omega_2 - k\omega_3$$

where k is damping co-efficient

Case I : Circular Disc, $(I_1 = I_2 > I_3)$

Due to unstability & losses, system tries to stablize along more stable axis → one with higher moment of Inertia (Along this axis system has minimum energy).

When currently along b_3 axis, the system would try to rotate about b_1 or b_2 ⇒ Either axis would be stable because $(I_1 = I_2)$

Case II : Elliptical Disc, $(I_1 > I_2 > I_3)$

Here when system tries to move away from low MoI to higher MoI axis, then it can either go to b_1 or b_2. Going around b_2 (intermediate axis) would cause it to tumble → Dzhanibekov Effect makes it oscillate unpredictably

Going around the maximum MoI axis b_1 would make it stable.

Energy Ellipsoid :- $E = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2)$

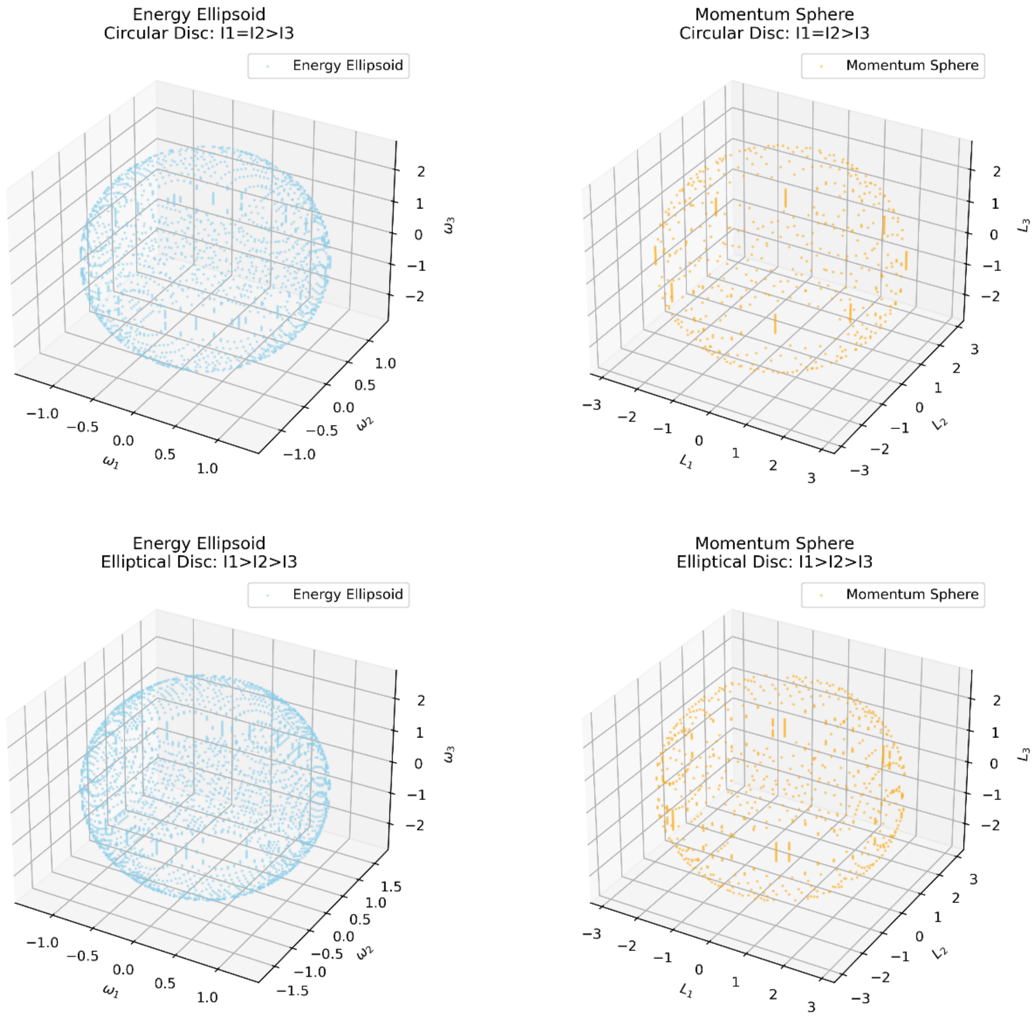
For circular disc, ellipsoid is symmetric.

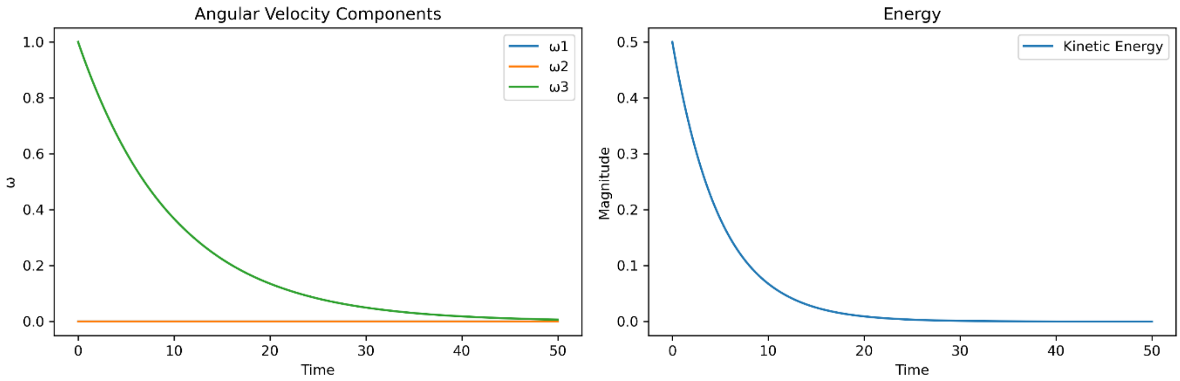
For elliptical, the ellipsoid is stretched along I_1 & compressed along I_3

Momentum Sphere :- $L = I_1\omega_1 \hat{e}_1 + I_2\omega_2 \hat{e}_2 + I_3\omega_3 \hat{e}_3$

momentum sphere is deformed due to varying moI.

Angular momentum vector traces momentum sphere & angular velocity vector traces the energy ellipsoid.




Angular Velocity Components
ω
Time
ω1
ω2
ω3
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
Energy
Magnitude
Time
Kinetic Energy
0.5
0.4
0.3
0.2
0.1
0.0
0
10
20
30
40
50

Question 5

Question 5 :-

let I_1, I_2 & I_3 be principal with ~~I_3 → maximum MOI~~
$(I_1 = I_2 > I_3)$

orbital frame {O} defined by :-

- $\hat{x}_o$ → along velocity vector
- $\hat{z}_o$ → Nadir (towards Earth center)
- $\hat{y}_o$ → $\hat{z}_o \times \hat{x}_o$

Gravity gradient torque tends to align the axis of maximum MOI with the nadir direction to minimize potential energy

Stable configuration :-

- $\cancel{z_b} \parallel z_o$
- ~~$x_b \parallel x_o$~~
- ~~$y_b \parallel y_o$~~

} Body frame aligned with orbital frame & or it can be $y_b \parallel z_o$ with z_b lies in orbital plane

3-2-1 Euler angle sequence

(ϕ, θ, ψ) can be $(0°, 90°, 0°)$ or $(90°, 0°, 0°)$ (not unique)

Energy $U = \frac{3\mu}{2r^3} (I_1 r_1^2 + I_2 r_2^2 + I_3 r_3^2)$

gravity gradient tries to minimize U under

constraint $r_1^2 + r_2^2 + r_3^2 = 1$

Solvinging

$$\mathcal{L}(r_1, r_2, r_3, \lambda) = I_1 r_1^2 + I_2 r_2^2 + I_3 r_3^2 - \lambda(r_1^2 + r_2^2 + r_3^2 - 1)$$

$$\frac{\partial L}{\partial r_i} = 0 \longrightarrow \lambda = I_i \quad 2 r_i \neq 0$$

minimum of U when, $U = I_1$ when $r_1 = 1$, $r_2 = r_3 = 0$

$U = I_2$ when $r_2 = 1$ $r_1 = r_3 = 0$

$U = I_3$ when $r_3 = 1$ $r_2 = r_1 = 0$

minimum U when $r_1 = 1$ or $r_b = \hat{x}_b$ (x axis point towa Earth)

OR $r_2 = 1$ or $r_b = \hat{y}_b$ (Y " " " ")

where max MOI is along X / Y axis ($I_1 = I_2 > I_3$)

Question 6

<u>Question 6 :—</u>

for our spinny, $I_1 = I_2 > I_3$

ⓐ the equa^ns of motion are, where spin axis is b_3

$$I_1 \dot{\omega}_1 = (I_2 - I_3)\,\omega_2 \omega_3 + I_{\omega s}\, \omega_2\, \Omega$$

$$I_2 \dot{\omega}_2 = (I_3 - I_1)\,\omega_1 \omega_3 - I_{\omega s}\, \omega_1\, \Omega$$

$$I_3 \dot{\omega}_3 = (I_1 - I_2)\,\omega_1 \omega_2 - I_{\omega s}\, \dot{\Omega}$$

$$(\dot{\Omega} = 0)$$

putting $I_1 = I_2 = I$

$$I\dot{\omega}_1 = (I - I_3)\,\omega_2\omega_3 + I_{ws}\,\omega_2\,\Omega \quad \text{--- (i)}$$

$$I\dot{\omega}_2 = -(I - I_3)\,\omega_1\omega_3 - I_{ws}\,\omega_1\,\Omega \quad \text{--- (ii)}$$

$$I_3\dot{\omega}_3 = 0 \implies \omega_3 = \text{const} \quad \text{--- (iii)}$$

(b) $$\dot{\omega}_1 = \left(\frac{I - I_3}{I}\right)\omega_2\omega_3 + \frac{I_{ws}}{I}\,\omega_2\,\Omega$$

$$\ddot{\omega}_1 = \left(\frac{I - I_3}{I}\right)\dot{\omega}_2\,\omega_3 + \frac{I_{ws}}{I}\,\Omega\,\dot{\omega}_2$$

from (ii)

$$\ddot{\omega}_1 = -\left(\frac{I - I_3}{I}\right)\omega_3\left(\frac{I - I_3}{I}\,\omega_1\omega_3 + \frac{I_{ws}}{I}\,\omega_1\,\Omega\right)$$
$$+ \frac{I_{ws}}{I}\,\Omega\left(\frac{I - I_3}{I}\,\omega_1\omega_3 + \frac{I_{ws}}{I}\,\omega_1\,\Omega\right)$$

$$= -\left(\frac{I - I_3}{I}\,\omega_3 + \frac{I_{ws}}{I}\,\Omega\right)\left(\frac{I - I_3}{I}\,\omega_3 + \frac{I_{ws}}{I}\,\Omega\right)\omega_1$$

$$= -k^2\,\omega_1$$

where $k = \left(\frac{I - I_3}{I}\,\omega_3 + \frac{I_{ws}}{I}\,\Omega\right)$

such that $k > 0$

with $\Omega = 0$ $\quad I > I_3 \Rightarrow I_1 = I_2 > I_3 \Rightarrow$ $\boxed{\text{Spin about axis of maximum MoI}}$

(c) Non dimensional spin rate, $\hat{\Omega} = \frac{\Omega}{\omega_{e_3}}$

$$\Rightarrow k = \omega_{e_3}\left(\frac{I - I_3}{I} + \frac{I_{ws}}{I}\,\hat{\Omega}\right) = 0$$

$$\hat{\Omega}_1 = \frac{I_3 - I}{I_{ws}}$$

Dual Spin stability conditions

1) $\hat{\Omega} > \hat{\Omega}_1$

or 2) $\hat{\Omega} < \hat{\Omega}_1$

for our case $\hat{\Omega}_1 = \dfrac{I_3 - I_1}{I_{ws}} = \dfrac{0.201 - 3.9385}{5.19}$

$= -29.9$

Range $\rightarrow \Omega <$ -600 rpm & $\Omega >$ 600 rpm

(d)

- without flywheel, satellite spinning is unstable
- with flywheel, Total angular momentum can be shifted to make satellite spin about maximum MOI axis
- Stability can be achieved with $\underline{\Omega}$.