Lab 11 (2nd Mar 2022)

Problem 1

Implement the dynamic programming solution to the matrix minimum-cost path problem covered in yesterday's lecture.

- Take the dimensions of the cost matrix M as input, followed by the entries of M.
- Take an additional input (x, y).
- Output the cost of a minimum cost path P from any of the possible starting cells to the cell $M[x, y]$.
- Output the path P starting from a cell in the bottom row, and ending in (x, y).

Problem 2

- Analyse the running time of Euclid's GCD algorithm. Compare against the running time of the naïve algorithm which iterates from 2 upwards till finding the largest common divisor. In both cases, express running times in terms of the input length.

Problem 3

- Take input an array A with n non-zero integers.
- Design an algorithm that modifies A such that all negative integers in A appear at the beginning, followed by all the positive integers.
- Your algorithm should run in time $O(n)$.

  Hint: Use two markers i and j such that at any point in the algorithm, all values in $A[0 \ldots, i-1]$ are negative, and all values from $A[j, \ldots, n]$ are positive. Then based on $A[i]$, take appropriate action and either grow the submatrix $A[0, \ldots, i]$ or grow $A[j, \ldots, n]$.

Problem 4

Consider a rectanglular area R of length n and width 2. We would like to cover this area with pretty tiles, each of length 2 and width 1. Take n as input, and output the number of ways in which we can cover R with such tiles. For example, for $n = 4$, we can cover it in the 5 different ways. See figure.