

Answers to Assignment 1 on First Order ODE

1. Solutions of the given differential equations:

(a) $u = \left(\cos \frac{1}{x} - \sin \frac{1}{x}\right) + ce^{-\frac{1}{x}}$

(b) $\sin(\log |cx|) = xy$

(c) $\frac{7}{12} \log |(x+3y)| - \log |y| - \frac{1}{4} \log |(y-x)| = \frac{1}{3} \log |x| + c,$

(d) $\left| \frac{x}{y} \sec xy \right| = c$

(e) $3x^4y + x^4y^3 + x^6 = c$

(f) $-\frac{2}{3} x^{-\frac{3}{2}} y^{\frac{3}{2}} + 4\sqrt{xy} = c$

(g) $y (\operatorname{cosec} x - \cot x)^3 = -x + \frac{7}{3} \tan \frac{x}{2} - \frac{1}{3} \tan \frac{x}{2} \sec^2 \frac{x}{2} + c$

(h) $x = \tan^{-1} y - 1 + ce^{-\tan^{-1} y}$

(i) $\frac{x^3}{y^3} - 3 \sin x + c = 0$

(j) $ue^{\frac{x^2}{2}} = \sqrt{\frac{\pi}{2}} \left(\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) - \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right),$ where $\operatorname{erf}(x)$ is the error function $= \frac{2}{\sqrt{\pi}} \int e^{-x^2} dx$

2. Solutions of the given differential equations:

(a) $|(e^x - 1)|^3 = c \tan y$

(b) $x^2 + y^2 = x^2y^2 + c$

(c) $\sin^2 \varphi + \sin^2 \theta = c$

(d) $y^3 = x^3 \log |cx^3|$

(e) $|xy \cos \frac{y}{x}| = c$

$$(f) \log |4x + 8y + 5| + 8y - 4x = c$$

$$(g) y^4 = 4xy + c$$

$$(h) 3(xy)^2 + x^4 + y^3 = c$$

$$(i) x^2 + y^2 = c x^3$$

$$(j) x \cosh y = c$$

3. Solutions of given differential equations:

$$(a) e^{x^3 y} = c x$$

$$(b) \frac{1}{y^3} = -2x - 1 + c e^x$$

$$(c) y x^4 = x^8 + 1$$

$$(d) x y^2 - \frac{1}{3} \cosh 3y = c$$

$$(e) 2y^2 e^x = x e^{2x} + c x$$

$$(f) y = e^{\cos x} (x + c)$$

$$(g) |\sin y| = c |(x - 1)|$$

$$(h) x \sec y = 2$$

$$(i) u + \frac{b-a}{2} \log |(x + u)^2 - ab| = x + c$$

$$(j) (x + y) - \tan\left(\frac{x+y}{2}\right) = x + c$$

$$(k) y = -\frac{x}{p} + a p \quad \text{and} \quad \frac{x\sqrt{1+p^2}}{p} = c + a \log |(p + \sqrt{1+p^2})|, \text{ These equations will provide the general solution.}$$

$$(l) \text{ The general solution is } y = cx + \frac{a}{c} \quad \text{and} \quad \text{the singular solution is } y^2 = 4ax.$$

$$(m) \text{ The general solution is } y^2 = cx^2 + c^2 \quad \text{and} \quad \text{the singular solution is } x^4 + 4y^2 = 0.$$

$$(n) y = c (x - c)^2$$

$$(o) |x|^2 = c |y| e^{\frac{1}{xy}}$$

$$(p) x^4 y = 3x^2 + c$$