ME 3180

FEM & CFD Theory

Assignment 4

Abhishek Ghosh

ME21BTECH11001

Question 1

a)

Abhishek Ghosh
ME21BTECH11001

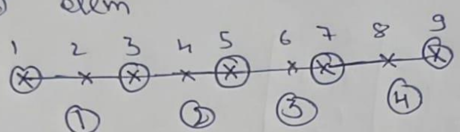
Question 1 :-

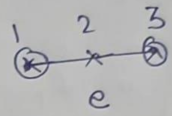
(a) Roll no = 01

$p = 01 \% 2 = 01$

case $p+1 = 2$

3 noded elem





Connectivity matrix

$$
CM = \begin{array}{cccc} e & \#1 & \#2 & \#3 \end{array}
\begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 3 & 5 & 6 & 7 \\ 4 & 7 & 8 & 9 \end{bmatrix}
$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$i \quad 2i-1 \quad 2i \quad 2i+1$$

(b) Code

(c) Code.

b)

```
% Abhishek Ghosh ME21BTECH11001
% ME3180 Assignment 4 Question 1
%

% n = input("number of elements: ");
cm = connectivity_matrix(10)

function mat = connectivity_matrix(n)
    mat = zeros(n, 4);

    for i=1 : n
        mat(i,1) = i;
        mat(i,2) = 2*i-1;
        mat(i,3) = 2*i;
        mat(i,4) = 2*i+1;
   end
end
```

```
Taking n = 10


cm =


     1     1     2     3

     2     3     4     5

     3     5     6     7

     4     7     8     9

     5     9    10    11

     6    11    12    13

     7    13    14    15

     8    15    16    17

     9    17    18    19

    10    19    20    21
```

c)

```
Verifying for n = 4
```

```
cm =

     1     1     2     3
     2     3     4     5
     3     5     6     7
     4     7     8     9
```

Question 2

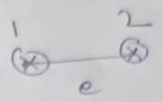
Assuming 0.2 at the x=0 for tapered end

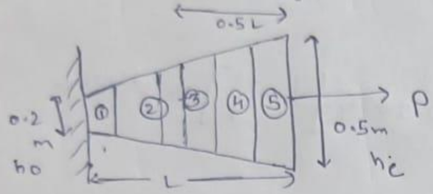
Question 2 :-

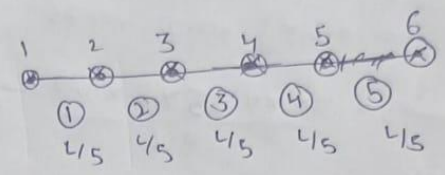
Roll no.= 01

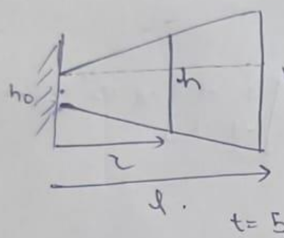
$p = 1 \% 4 = 1$

no. of elem $= 1 + 4 = 5$









$t = 5 \times 10^{-3}$ m

$$\frac{(h-h_0)/2}{x} = \frac{(h_e-h_0)/2}{L}$$

$$h = (h_e - h_0)\frac{x}{L} + h_0$$

$$= 0.3\frac{x}{L} + 0.2$$

(a) Connectivity matrix,

$$cm = \begin{array}{c} \\ \left[\begin{array}{ccc} e & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 3 \\ 3 & 3 & 4 \\ 4 & 4 & 5 \\ 5 & 5 & 6 \end{array}\right] \end{array}$$

(b) $E = 30$ MPa

Element stiffness matrix $k_e = \frac{E_e A_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(Taking mean h for each elem)

for e=1

$$h = (h_e - h_0)\frac{L/10}{L} + h_0 = 0.23 \Rightarrow A_1 = 0.23 \times 5 \times 10^{-3} \text{ m}^2$$

$$l_e = 0.20 \text{ m}$$

$$k_1 = \frac{30 \times 0.23 \times 5 \times 10^{-3}}{0.2} = 0.172,5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \times 10^6 \text{ N/m}$$

for e=2

$$h = 0.3 \times \frac{3}{10} + 0.2 = 0.29$$

$$k_2 = \frac{30 \times 0.29 \times 5 \times 10^{-3}}{0.2} = 0.2175 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \times 10^6 \, N/m$$

for e=3

$$h = 0.3 \times \frac{1}{2} + 0.2 = 0.35$$

$$k_3 = \frac{30 \times 0.35 \times 5 \times 10^{-3}}{0.2} = 0.2625 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \times 10^6 \, N/m$$

for e=4

$$h = 0.3 \times \frac{7}{10} + 0.2 = 0.41$$

$$k_4 = \frac{30 \times 0.41 \times 5 \times 10^{-3}}{0.2} = 0.3075 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \times 10^6 \, N/m$$

for e=5

$$h = 0.3 \times \frac{9}{10} + 0.2 = 0.47$$

$$k_5 = \frac{30 \times 0.47 \times 5 \times 10^{-3}}{0.2} = 0.3525 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \times 10^6 \, N/m$$

(c) code

(d) Reactn force ⇒ By force balance = P

(e) in code.

$$-R + P = 0$$

$$R = P$$

```
% Abhishek Ghosh
```

```
% ME21BTECH11001
% ME3180 Assignment 4 q2

E = 30*1e6;
L = 1;
n = 5
;
P = 10000;
cm =zeros(n,3);

% connectivity matrix
for i=1:n
    cm(i,1) = i;
    cm(i,2) = i;
    cm(i,3) = i+1;
end

k = [0.1725 0.2175 0.2625 0.3075 0.3525];
k = k.*1e6;

% Global Stiffness matrix
A = zeros(6,6);
% Force matrix
B = zeros(6,1);

for i=1:n
    x = cm(i,2);
    y = cm(i,3);

    A(x,x) = A(x,x) + k(i);
    A(x,y) = A(x,y) + -1*k(i);
    A(y,x) = A(y,x) + -1*k(i);
    A(y,y) = A(y,y) + k(i);
    B(i) = P;
end


sigma = zeros(6,1);

A(1,1)=1;
A(1,2)=0;
A
B(6) = P;

% Boundary condition for u1=0
B(1)=0;

% displacement vector
x=A\B;
x

l = L/n;

sigma = x.*(E/l);
sigma
```

```
sigma -> stresses developed  (Boudary condition i.e x=0 displacement = 0 )

A -> Global Stiffness matrix

X -> Displacement vector
```

```
A =

           1           0           0           0           0           0
     -172500      390000     -217500           0           0           0
           0     -217500      480000     -262500           0           0
           0           0     -262500      570000     -307500           0
           0           0           0     -307500      660000     -352500
           0           0           0           0     -352500      352500


x =

   -0.0000
    0.2899
    0.4738
    0.5880
    0.6531
    0.6815


sigma =

   1.0e+08 *

   -0.0000
    0.4348
    0.7106
```

```
0.8821

0.9796

1.0222
```

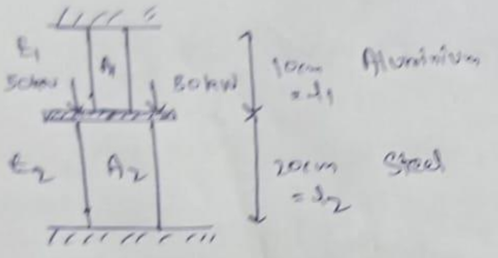
Question 3 ->

Question 3

Roll no. = 01

$1\%3 = 1 = q$

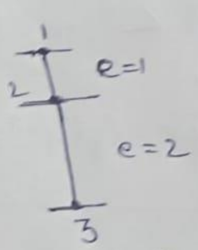
$q+1 = 2 \Rightarrow$ case 2



$A_1 = 60\ mm^2$

$A_2 = 600\ mm^2$

Let $k_1 = \dfrac{E_1 A_1}{l_1}$ & $k_2 = \dfrac{E_2 A_2}{l_2}$

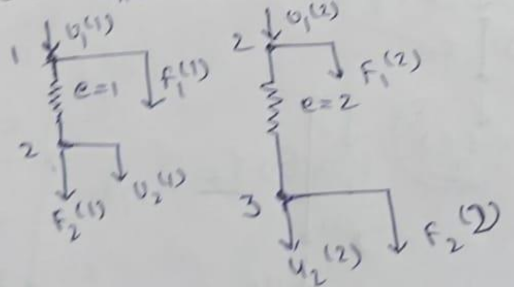


$E_1 = E_{Al} = 70\ GPa$

$E_2 = E_s = 200\ GPa$

Using two linear elements (2 noded elem)

e1 = Aluminium & e=2 Steel



$$CM = \begin{bmatrix} e & n\#1 & n\#2 \\ 1 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

Using inter element continuity

$F_1^{(2)} + F_2^{(1)} = P = 100\ kN$

& $u_2^{(1)} = u_1^{(2)} = u_2$

$u_1^{(1)} = u_1$ & $u_2^{(2)} = u_3$

Equations $\rightarrow$ $F_1^{(1)} = k_1 u_1 - k_1 u_2$ & $F_2^{(1)} = k_1 u_2 - k_1 u_1$

$F_1^{(2)} = k_2 u_2 - k_2 u_3$ & $F_2^{(2)} = k_2 u_3 - k_2 u_2$

writing in matrix form

for $e=1$

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} f_1^{(1)} \\ f_2^{(1)} \\ 0 \end{bmatrix}$$

for $e=2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 0 \\ f_1^{(2)} \\ f_2^{(2)} \end{bmatrix}$$

Adding up

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} f_1^{(1)} \\ f_2^{(1)} + f_1^{(2)} \\ f_2^{(2)} \end{bmatrix}$$

Displacement at ends, $u_1 = 0$, $u_3 = 0$ } Boundary conditions

Also $f_2^{(1)} + f_1^{(2)} = P = 100\ kN$

we get, $\delta = u_2 = \dfrac{P}{k_1+k_2} = \dfrac{100 \times 10^3}{\frac{E_1 A_1}{l_1} + \frac{E_2 A_2}{l_2}}$

$\frac{E_1 A_1}{l_1} \rightarrow 426 \times 10^{+3}$, $\frac{E_2 A_2}{l_2} \rightarrow 6000 \times 10^{+3}$

$$= \frac{100}{420 + 6000} = \frac{100}{642}$$

$= \cancel{0.015\ mm}$ $0.155 mm$

Stresses, $f_2^{(1)} = k_1 u_2 = \dfrac{E_1 A_1}{l_1} u_2 \Rightarrow \sigma_{Al} = \dfrac{E_1}{l_1} u_2$

$= 108.5 MPa$ (Tensile)

$\sigma_S = \dfrac{E_2}{l_2} u_2 = 155\ MPa$ (Comp)