Real valued Functions ($\mathbb{R}_v$)

Study functions $f : D \longrightarrow \mathbb{R}$
where $D \subseteq \mathbb{R}$

(need not be the range)

Natural Domain of a function

Let $y = f(x)$ be a function

The Natural Domain of f is the largest subset of $\mathbb{R}$ on which f is defined. Denote by ND_f

Thus, if D is the natural domain of f, then

$x_0 \notin D \Rightarrow f$ is not defined at x_0

Examples.

1) Natural Domain for
$$f(x) = x^2 - 3x + 1$$
is $\mathbb{R}$

2) Natural domain of

$$f(x) = \frac{x^2+1}{x-1}$$

is $\mathbb{R} \setminus \{1\}$

3) Natural domain of

$$f(x) = \sin x$$

is $\mathbb{R}$

while that of

$$g(x) = \frac{1}{\sin x}$$ is

$$\mathbb{R} \setminus \{ m\pi : m \in \mathbb{Z} \}$$

Algebra of real valued function

Because we can add, multiply and divide real numbers, we can do the same for real valued functions using the notion

"point-wise" operation.

Let $f : D_1 \longrightarrow \mathbb{R}$ and

$g : D_2 \longrightarrow \mathbb{R}$

Define $h : D_1 \cap D_2 \longrightarrow \mathbb{R}$ as

$x_1 = x_2$, then $h(x_1) = h(x_2)$

$$h(x) = f(x) + g(x)$$

Then $h \in \mathbb{R}_v$, and denoted by

$h = f + g$ (the sum of f & g)

Similarly,

Define $p : D_1 \cap D_2 \rightarrow \mathbb{R}$ by

$$p(x) = f(x) \cdot g(x)$$

Then $p \in \mathbb{R}_v$, called the product of f & g

Next,

<u>Define</u> $q : D_1 \cap D_2 \to \mathbb{R}$ by

$$q(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

omit $x \in D_1 \cap D_2$ for which $g(x) = 0$.

<u>Examples</u>

$$f(x) = \sqrt{x}, \quad g(x) = \sin x$$

- $(f+g)(x) = \sqrt{x} + \sin x$

  Domain $= \mathbb{R}_{\geq 0}$

- $(f \cdot g)(x) = \sqrt{x} \sin x$
- $(f/g)(x) = \sqrt{x}/\sin x$

  Domain $= \mathbb{R}_{\geq 0} \setminus \{n\pi : n \in \mathbb{Z}\}$

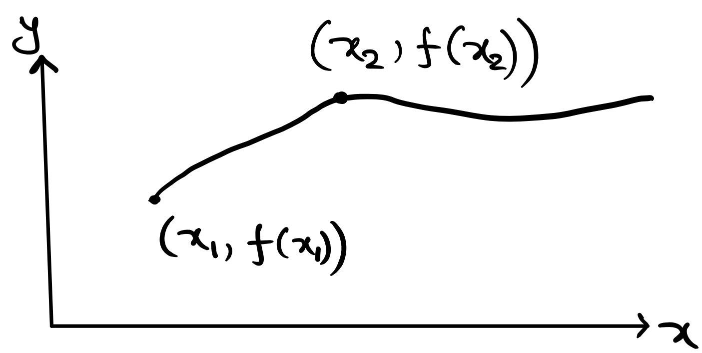
Graph of a function

Let $f : D \to \mathbb{R}$

The set $G_f = \{(x, f(x)) : x \in D\}$

$\subseteq \mathbb{R} \times \mathbb{R}$

is called the graph of $f(x)$

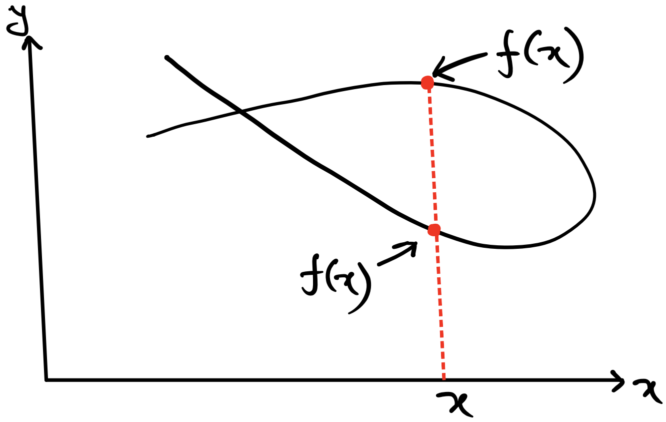


view of G_f in the Cartesian Plane

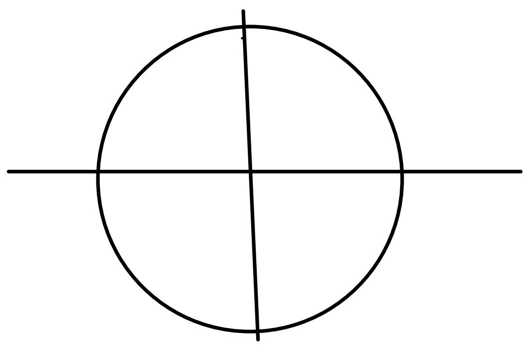
with $y = f(x)$

How to recognize a function from its graph?

vertical line test



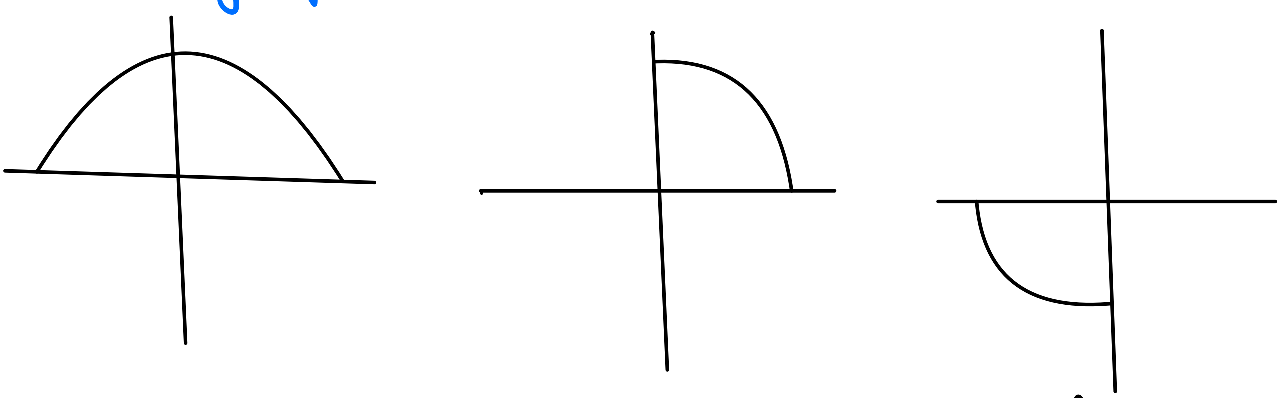
Not a graph of a fn.



$y = \pm\sqrt{1-x^2}$

A circle is not a fn.
nevertheless, it has an eqn.

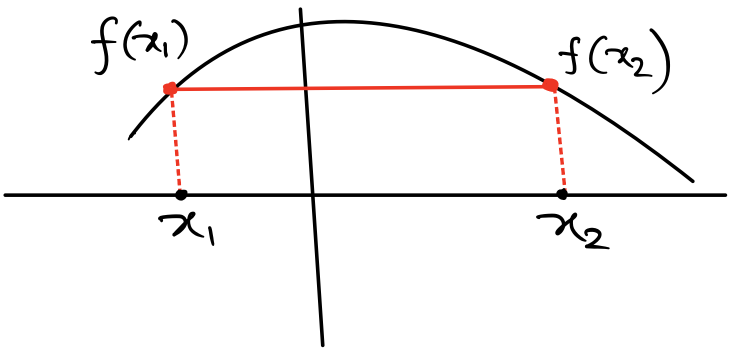
$y = \sqrt{1-x^2}$



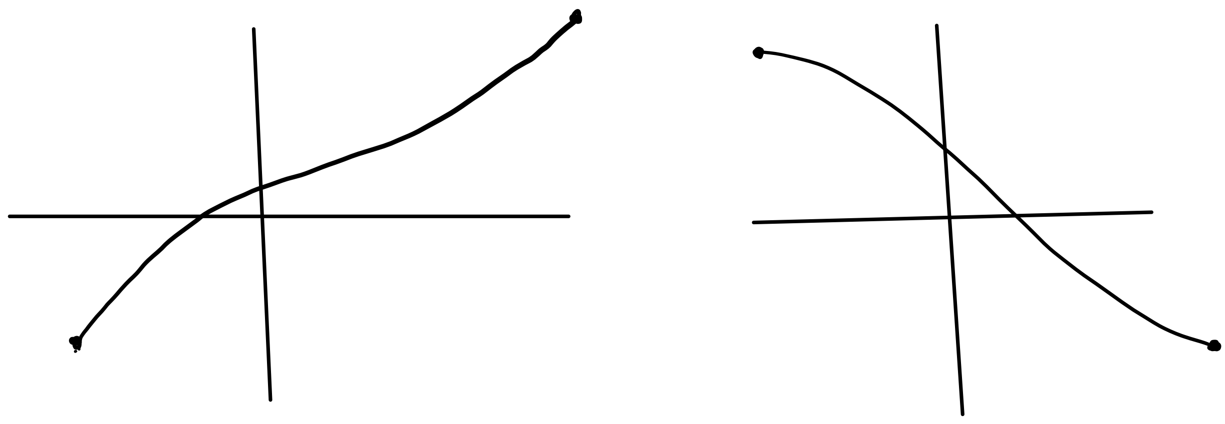
All of these are functions

Graphs of injective functions

horizontal line test



Not an injective fn.



Both functions are injective

Monotone Functions

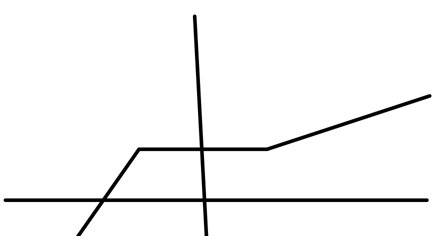
Definition. A function $f : D \rightarrow \mathbb{R}$ is increasing if for any x_1, x_2 in D with $x_1 < x_2$, one has $f(x_1) < f(x_2)$

and, f is decreasing if for x_1, x_2 in D with $x_1 < x_2$, one has $f(x_1) > f(x_2)$

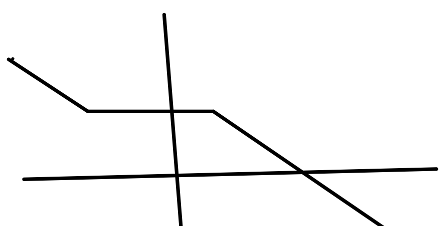
Similarly, a function non-decreasing if for x_1, x_2 in D with $x_1 < x_2$, one has $f(x_1) \leq f(x_2)$

and, f is non-increasing if

if for x_1, x_2 in D with $x_1 < x_2$, one has $f(x_1) \geq f(x_2)$



non-decreasing

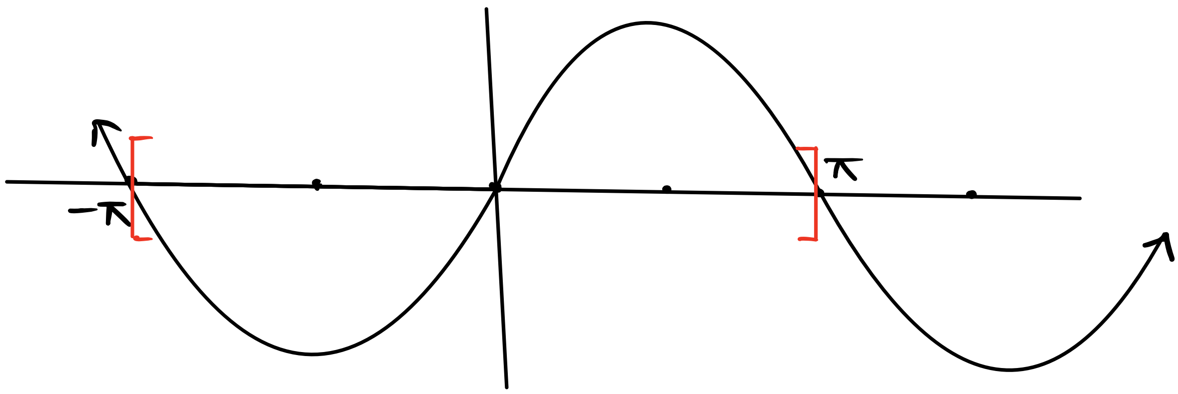


non-increasing

Restricted Domains

Let $f: D \longrightarrow \mathbb{R}$

if $D' \subseteq D$, then D' is a restricted domain of f.



$$f(x) = \sin x$$

Suffices to analyze in $[-\pi, \pi]$

Equality of functions

Suppose, $f: D_1 \longrightarrow \mathbb{R}$

and $g: D_2 \longrightarrow \mathbb{R}$

and $D \subseteq D_1 \cap D_2$ is such that

$$f(x) = g(x) \quad \forall \ x \in D.$$

Then, we say that $f = g$ on (the restricted domain) D.

Examples. • $\sin x = x$ on $\{0\}$

• $\sin \pi x = (x - \lfloor x \rfloor)$ on $\mathbb{Z}$

• $f(x) = 0 \quad \forall\, x \in \mathbb{R}$

$g(x) = \lfloor x \rfloor$

Then $f(x) = g(x)$ on $[0, 1)$