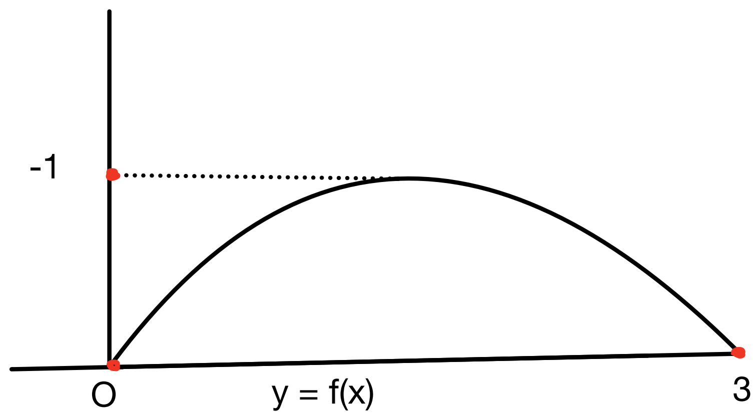
WORKSHEET - 1

1. Let $f(x) = \sqrt{x+1}$ and $g(x) = 1/x$. Determine the functions $g\,o\,f$ and $f\,o\,g$, their natural domains and the respective ranges.

2. Given the graph of the function $y = f(x)$ below, sketch the graph of the indicated functions (a) – (h).



(a) $f(x) + 2$

(b) $f(x) - 1$

(c) $3f(x)$

(d) $-f(x)$

(e) $f(x+2)$

(f) $f(x-1)$

(g) $f(-x)$

(h) $-f(x+1) + 1$

2. Is it possible that the graphs of f and g are not straight lines but the graph of $g\,o\,f$ is a straight line?

3. Let $f(x)$ be a function with a domain D that is symmetric about the origin (if $x \in D$, then $-x \in D$). Show that there is an even function $E_f(x)$ and and odd function $O_f(x)$ such that $f = E_f + O_f$ on D.

The following problems require abstract mathematical reasoning.

4. Let $f : X \to Y$ and $g : Y \to X$ be such that. $g\, o\, f$ is identity on X and $f\, o\, g$ is identity on Y. Prove that f is bijective with $g = f^{-1}$. Does the same conclusion hold if one of $g\, o\, f$ and $f\, o\, g$ is not an identity function?

5. Let x and y are arbitrary positive real numbers. Use AP to show that there is a natural number n such that $nx > y$.

6. Let $x > 0$ be a positive real number. Using AP show that there is a natural number n such that $1/2^n < x$.

7. Show that every interval contains infinitely many rational numbers.

8. Prove that between any two real numbers there is an irrational number.

9. Suppose x is a real number satisfying $0 < x < 1$. Show that there is a natural number n such that $1/n < x \leq 1/(n-1)$.

10. Let $X \subset \mathbb{R}$ be a nonempty finite set. Suppose $f : X \to X$ is a function satisfying $f(x) \leq x$ for every $x \in X$. Argue as to why there is a $x \in X$ such that $f(x) = x$.