Ordinary Differential Equations (MA-1150)

Assignment 2 (Second and Higher Order Linear ODEs)

Note: Submit the assignment by solving any two questions from each Q. (1) and Q. (2). Rest of the problems are for your practice only. To get more problems, you can try to solve some examples from the book of S. L. Ross.

1. Solve the following differential equations: **(Homogeneous linear ODEs with constant Coefficients)**

(a) $2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$

(b) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

(c) $\frac{d^4y}{dx^4} = m^4 y$

(d) $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$

(e) $\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

(f) $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$, given that when $t = 0, x = 0$ and $\frac{dx}{dt} = 0$

(g) $\frac{d^2y}{dx^2} + y = 0$, given that $y = 2$ for $x = 0$ and $y = -2$ for $x = \frac{\pi}{2}$

(h) Solve $(D^4 - n^4)y = 0$. Now if $Dy = y = 0$ when $x = 0$ and $x = l$, prove that

$y = c_1(\cos nx - \cosh nx) + c_2(\sin nx - \sinh nx)$ and $\cos nl \cosh nl = 1$.

(i) $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$, given that $Q = Q_0$ and $\frac{dQ}{dt} = 0$ when $t = 0$ and that $CR^2 < 4L$.

(j) $m\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + c\theta = 0$, given that $k^2 < 4mc$. Also find the solution when k is so small that $\frac{k^2}{mc}$ is negligible.

2. Solve the following differential equations: **(non-homogeneous linear ODEs with constant Coefficients)**

(a) $\frac{d^2y}{dx^2} + a^2 y = \operatorname{cosec} ax$

(b) $\frac{d^2y}{dx^2} + a^2 y = \tan ax$

(c) $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4y = x\, e^{-2x}$

(d) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 20\, e^{-2x}$

(e) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 24\, e^{-3x}$

(f) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$

(g) $(D^2 + 5D + 6)\, y = e^{-2x} \sin 2x$

(h) $(D^3 - D^2 + 3D + 5)y = e^x \cos 3x$

(i) $(D^2 - 1)\, y = x \sinh x$

(j) $(D^2 + 2)y = x^2 e^{3x} + e^x \cos 2x$

(k) $(D^2 + 1)y = 3\cos^2 x + 2\sin^3 x$

(l) $(D^4 - 2\, D^3 + D^2)y = x^3$

(m) $(D^3 + 1)y = e^{2x} \sin x + e^{\frac{x}{2}} \sin \frac{x\sqrt{3}}{2}$

(n) $\frac{d^2r}{dt^2} - \omega^2 r = -g \sin \omega t$, ($g, \omega$ positive constants)

3. Solve the following differential equations: (Use Method of Variation of Parameters) (non-homogeneous linear ODEs with constant & variable coefficients)

(a) $(D^2 - 2D)y = e^x \cos x$

(b) $(D^2 + 2D + 1)y = \frac{e^{-x}}{x^2}$

(c) $(D^2 - 3D + 2)y = \frac{e^x}{1+e^x}$

(d) $(D^3 + D)y = \operatorname{cosec} x$

(e) $(2x+1)(1+x)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = (1+2x)^2$, where $y = x$ and $y = \frac{1}{x+1}$ are two linearly independent solutions of corresponding homogeneous equation.

(f) $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = x^3$, where $y = x$ and $y = x\,e^x$ are two linearly independent solutions of corresponding homogeneous equation

.

(g) $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = \log x$, $(x > 0)$, where $y = x$ and $y = \frac{1}{x}$

are two linearly independent solutions of corresponding homogeneous equation.

.

(h) $(x^3D^3 + 3x^2D^2)\,y = 1,\ x > 0,$ where $y = \frac{1}{x},\ y = 1$ and $y = x$

are three linearly independent solutions of corresponding homogeneous equation.

.

(i) $(x^2 + x)\frac{d^2y}{dx^2} + (2 - x^2)\frac{dy}{dx} - (2 + x)\,y = x\,(1 + x)^2$, where $y = e^x$ and $y = \frac{1}{x}$ are two linearly independent solutions of corresponding homogeneous equation.

(j) If you want to solve more problems, then go for solving Q. 2 in this assignment by the Method of Variation of Parameters.

4. **Solve the following differential equations: (Euler-Cauchy Equations)**

(non-homogeneous linear ODEs with variable coefficients)

(a) $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

(b) $(x + a)^2\frac{d^2y}{dx^2} - 4(x + a)\frac{dy}{dx} + 6y = x$

(c) $(2 + x)^2\frac{d^2y}{dx^2} + (x + 2)\frac{dy}{dx} + 4y = 2\sin\{2\log(2 + x)\}$

(d) $x^4\frac{d^4y}{dx^4}+6x^3\frac{d^3y}{dx^3}+4x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\cos(\log x)$

(e) $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$

(f) $z^2\frac{d^2y}{dz^2}-3z\frac{dy}{dz}+y=\frac{(\log z\sin(\log z)+1)}{z}$.

(g) $((x+1)^2D^2+(1+x)\,D)\,y=(2x+3)(2x+4)$

(h) $[x^2D^2-(2m-1)xD+(m^2+n^2)]\,y=n^2x^m\log x$

(i) $\frac{d^2V}{dr^2}+\frac{1}{r}\frac{dV}{dr}=4\pi\rho,\ \ \rho$ is constant.