Quiz-3

1. Question: Identify all the convergent series. 3 points

Check all that apply.

$n\sqrt{n} \geq 1 = n\sqrt{n}+1 \leq 2n\sqrt{n}$

Thus $\frac{\sqrt{n}}{n\sqrt{n}+1} \geq \frac{\sqrt{n}}{2n\sqrt{n}} = \frac{1}{2n}$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n\sqrt{n}+1}$$

Since $\sum_n \frac{1}{2n}$ diverges, hence the series diverges

☒ Option 1

$\frac{1+n}{n^2} > \frac{n}{n^2} = \frac{1}{n}$

hence diverges

$$\sum_{n=1}^{\infty} \frac{1+n}{n^2}$$

☒ Option 2

$\frac{\sqrt{n+1}-\sqrt{n}}{n} = \frac{1}{n(\sqrt{n+1}+\sqrt{n})} < \frac{1}{2n\sqrt{n}}$

since $\sum_n \frac{1}{2n\sqrt{n}}$ converges,

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n}$$

hence the series converges.

☑ Option 3

$\frac{n^{1/4}-1}{n\sqrt{n}} < \frac{n^{1/4}}{n\sqrt{n}} = \frac{1}{n^{1+1/4}}$

hence convergent

$$\sum_{n=1}^{\infty} \frac{n^{1/4}-1}{n\sqrt{n}}$$

☑ Option 4

2. Question: Identify all the convergent series. 2 points

Check all that apply.

It was proved that
$\ln n < 2\sqrt{n} \ \forall n$
Thus, $\sqrt{n}\ln n \le 2n$

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln n}$$

$= \frac{1}{\sqrt{n}\ln n} > \frac{1}{2n}$, hence divergent

☒ Option 1

Convergent by Integral Test
Do it!

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1.001}}$$

☑ Option 2

It was proved that
$\sin\frac{1}{n} > \frac{1}{2n} \ \forall n$
Thus $n\sin\frac{1}{n} > 2 \ \forall n$

$$\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$$

hence the series diverges.

☒ Option 3

similarly as in option 3,
$\sin\frac{1}{n^2} \ge \frac{1}{2n^2} \ \forall n$

$$\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n^2}\right)$$

Thus $n\sin\frac{1}{n^2} \ge \frac{1}{2n}$,
hence diverges.

☒ Option 4

3. Question: Identify all the convergent series. 3 points

Check all that apply.

Convergent by Root Test

$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^n = \frac{1}{e} < 1$

$$\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$$

- [x] Option 1

$\frac{1}{(1+n)^n} < \frac{1}{n^n} \leq \frac{1}{n^2} \; \forall \; n \geq 2$

hence convergent by

$$\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^{n}$$

comparison Test.

- [x] Option 2

Divergent by Root Test

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e > 1$

$$\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2}$$

- [ ] Option 3

$\frac{1}{n^{1+\ln n}} \leq \frac{1}{n^2} \; \forall \; n \geq 3.$

So convergent by

$$\sum_{n=1}^{\infty} \frac{1}{n^{1+\ln n}}$$

comparison Test.

- [x] Option 4

4. Question: Identify all the convergent series. 3 points

Check all that apply.

Apply Ratio Test

$$\frac{a_{n+1}}{a_n} = \frac{n+1}{e} \longrightarrow \infty$$

$$\sum_{n=1}^{\infty} \frac{n!}{e^n}$$

hence divergent.

☒ Option 1

Apply Ratio Test

$$\frac{a_{n+1}}{a_n} = \frac{5^{n+1}\left((n+1)!\right)^2}{(2n+2)!} \cdot \frac{2n!}{5^n (n!)^2}$$

$$\sum_{n=1}^{\infty} \frac{5^n (n!)^2}{2n!}$$

$$= \frac{5(n+1)}{2(2n+1)} \longrightarrow \frac{5}{4} > 1$$

hence divergent.

☒ Option 2

Apply Ratio Test

$$\frac{a_{n+1}}{a_n} = \frac{2(n+1)}{2(2n+1)} \longrightarrow \frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{2n!}$$

hence convergent

☑ Option 3

Apply Root Test

$$a_n^{1/n} = \frac{2^n}{n^{2/n}} \longrightarrow \infty$$

$$\sum_{n=1}^{\infty} \frac{2^{n^2}}{n^2}$$

hence divergent.

☒ Option 4

5. Which among the following series is/are absolutely convergent? 2 points

Check all that apply.

$\sin\left(\frac{(2n+1)\pi}{2}\right) = (-1)^{n+1}$

Thus, the series is $\sum_n \frac{(-1)^{n+1}}{n^2}$

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n+1)\pi}{2}\right)}{n^2}$$

Obviously absolutely conv. since $\sum_n \frac{1}{n^2}$ is conv.

- [x] Option 1

In this case

$S^+ = \sum_n \frac{1}{n}$, not conv.

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n+1)\pi}{2}\right)}{n}$$

hence the series is not absolutely covg.

- [ ] Option 2

Same as option 2

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n}$$

- [ ] Option 3

$S^+ = \sum_n \frac{1}{n^{1.2}}$, convergent

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^{1.2}}$$

- [x] Option 4

6. Which among the following series is/are conditionally convergent? 3 points

Check all that apply.

since absolutely converg. hence not conditionally convg.

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n+1)\pi}{2}\right)}{n^2}$$

☒ Option 1

convergent by Alternating Series Test.

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n+1)\pi}{2}\right)}{n}$$

since not absolutely convg., hence conditionally convg.

☑ Option 2

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n}$$

Same as option 2

☑ Option 3

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^{1.2}}$$

argument same as option 1.

☒ Option 4

7. Question

3 points

Let $\{a_n\}_n$ be a sequence of positive terms such that $\sum_{n=1}^{\infty} a_n$ is convergent.

Which of the following series is always convergent?

Check all that apply.

$\frac{a_n}{n} \leq a_n$

hence convg. by Comparison Test

$$\sum_{n=1}^{\infty} \frac{a_n}{n}$$

☑ Option 1

$\frac{a_n}{\ln(1+n)} \leq a_n \quad \forall\ n \geq 3$

hence convergent

$$\sum_{n=1}^{\infty} \frac{a_n}{\ln(1+n)}$$

☑ Option 2

$\sum_n \frac{1}{n^2}$ is convergent

by $\sum_n n \cdot \frac{1}{n^2} = \sum_n \frac{1}{n}$ is not convg.

$$\sum_{n=1}^{\infty} n a_n$$

☒ Option 3

$\frac{a_n}{1+a_n} < a_n$ since $a_n > 0$

hence convergent

$$\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$$

☑ Option 4

8. Question 2 points

Suppose $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive terms. Pick the correct alternative(s).

Check all that apply.

Proved in class

$$\lim_{n\to\infty} a_n = 0$$

- [x] Option 1

$\sqrt{a_n} < \varepsilon$ whenever $a_n < \varepsilon^2$ and happens after a stage n_0

$$\lim_{n\to\infty} \sqrt{a_n} = 0$$

Thus $\sqrt{a_n} < \varepsilon \ \forall\ n \geq n_0$
hence $\sqrt{a_n} \to 0$

- [x] Option 2

$\sum_n \frac{1}{n^2}$ converges
but $\lim_{n\to\infty} \frac{1}{n^{2/n}} = 1$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = 0$$

- [ ] Option 3

$a_n < 1$ after a stage n_0.
Then
$0 < a_n^n \leq a_n \ \forall\ n \geq n_0$

$$\lim_{n\to\infty} a_n^n = 0$$

Thus, $a_n^n \to 0$ by Sandwich Theorem.

- [x] Option 4

9. Question 2 points

If $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive terms. Then the series $\sum_{n=1}^{\infty} \sin a_n$ is absolutely convergent.

Mark only one oval.

- (x) True
- ( ) False

$|\sin a_n| \leq a_n$

hence $\sum_n |\sin a_n|$ is convergent

$\Rightarrow \sum_n \sin a_n$ is absolutely convergent.

10. Question 2 points

If $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive terms. Then the series $\sum_{n=1}^{\infty} \sin a_n$ is conditionally convergent.

Mark only one oval.

- ( ) True
- (X) False

An absolutely convergent series cannot be conditionally convergent!

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