ME1020 Homework 6

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Question 1:-

a)

$Given\ position\ of\ mill\ at\ time\ t\ is\ x(t) = 2t - 6t^3\ cm$

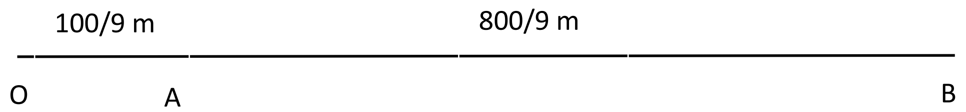
$Velocity, v(t) = \frac{dx}{dt} = \frac{d(2t - 6t^3)}{dt} = \frac{d(2t)}{dt} - \frac{d(6t^3)}{dt} = 2 - 18t^2$

$Velocity\ at\ t = 5s\ is\ v(5) = 2 - 18(5)^2 = \mathbf{-448\ cm\ s^{-1}}$

$Acceleration, a(t) = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{d(2 - 18t^2)}{dt} = \frac{d(2)}{dt} - \frac{d(18t^2)}{dt} = -36t$

$Acceleration\ at\ t - 5s\ is\ a(5) = -36(5) = \mathbf{-180\ cm\ s^{-2}}$

b)



$Given\ initial\ velocity\ u = 20km\ hr^{-1} = \frac{50}{9} m\ s^{-1}$

$The\ car\ moves\ with\ constant\ velocity\ for\ 2\ s\ , OA = \frac{50}{9} X\ 2 = \frac{100}{9} m$

$For\ next\ 100 - \frac{100}{9} = \frac{800}{9} m\ it\ moves\ with\ constant\ a = 3\ m\ s^{-2}$

$Given\ traffic\ light\ takes\ 5s\ to\ turn\ red\ from\ point\ O\ to\ B$

$Time\ taken\ by\ car\ to\ travel\ AB\ is\ given\ by\ eqtn: -$

$$s = ut + \frac{1}{2}at^2$$

$$\frac{800}{9} = \frac{50}{9}t + \frac{1}{2}(3)t^2$$

$Solving\ the\ quadratic\ to\ get\ t$

$$t = \frac{-100 + \sqrt{100^2 + 4(27)(1600)}}{2(27)} = 6.06\ s$$

$As\ the\ time\ t\ is\ more\ than\ the\ time\ taken\ to\ turn\ thr\ traffic\ signal\ red.$

$The\ car\ won't be\ able\ ot\ reach\ on\ time.$

$Velocity\ at\ B\ is\ given\ by\ equation: -$

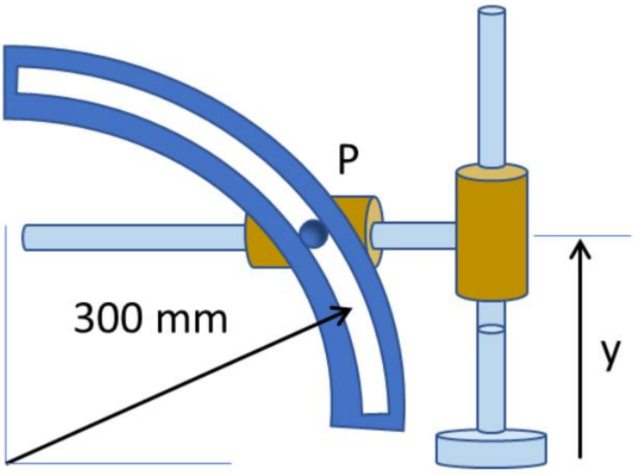
$$v^2 = u^2 + 2as$$

$$v = \sqrt{\left(\frac{50}{9}\right)^2 + 2(3)\left(\frac{800}{9}\right)}$$

$$\boldsymbol{v = 23.75\ m\ s^{-1}}$$

<u>Question 2:-</u>

<u>a)</u>



$$Given\ y = 200mm; \frac{dy}{dt} = 200\frac{mm}{s} = v_y\ ; \frac{d^2y}{dt^2} = 0 = a_y$$

$Equation\ of\ the\ curve\ is\ x^2 + y^2 = 300^2$ ...... (1)

$Differentiating\ the\ curve\ wrt\ t\ we\ get$

$$\frac{d(x^2 + y^2)}{dt} = \frac{d(300^2)}{dt}$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

.........(2)

$Again\ differentiating\ wrt\ t\ to\ get\ acceleration$

$$\left(\frac{dx}{dt}\right)^2 + x\frac{d^2x}{dt^2} + \left(\frac{dy}{dt}\right)^2 + y\frac{d^2y}{dt^2} = 0$$

$$v_x^2 + xa_x + v_y^2 + ya_y = 0$$

……(3)

$From\ (1) we\ get\ x = \sqrt{300^2 - 200^2} = 100\sqrt{5}\,mm$

$From\ (2) we\ get\ v_x = -\frac{y}{x}\frac{dy}{dt} = \frac{200}{100\sqrt{5}}(200) = 80\sqrt{5}\,mm\ s^{-1}$

$Putting\ these\ values\ in\ (3) we\ get$

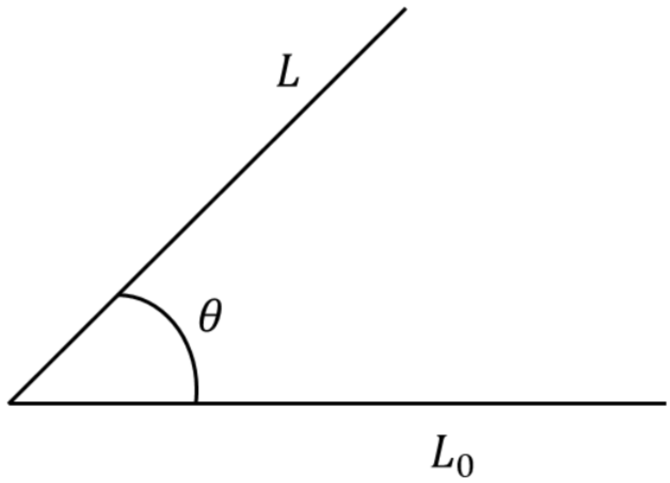
$$a_x = -\frac{v_x^2 + v_y^2}{x} = -321.99\ mm\ s^{-2}$$

$Total\ velocity\ |v| = \sqrt{v_X^2 + v_y^2} = \sqrt{\left(80\sqrt{5}\right)^2 + 200^2}$

$= \mathbf{268.32\ mm\ s^{-1}}$

$Total\ acceleration\ |a| = \sqrt{a_x^2 + a_y^2} = |a_x| = \mathbf{321.99\ mm\ s^{-2}}$

<u>b)</u>



$Given\ angular\ acceleratin\ \alpha(t) = 2 - 1.5t\ rad\ s^{-2}$

$At\ t = 0\ ;\ \omega(0) = 5\ rad\ s^{-1}\ ;\ \theta(0) = 0\ rad;$

$Also\ angular\ velocity\ \omega\ and\ \alpha\ has\ relation$

$$\alpha = \frac{d\omega}{dt}$$

$$d\omega = \alpha dt$$

$$\int_{\omega(0)}^{\omega(t)} d\omega = \int_0^t (2 - 1.5t)dt$$

$$\omega(t) - \omega(0) = \left[2t - 1.5\frac{t^2}{2}\right]_0^t = 2t - \frac{3}{4}t^2$$

$$\omega(t) = 2t - \frac{3}{4}t^2 + 5$$

$$\omega(3) = 6 - \frac{27}{4} + 5 = \boldsymbol{4.25\ rad\ s^{-1}}$$

$Realtion\ between\ angle\ \theta\ and\ \omega\ is$

$$\omega = \frac{d\theta}{dt}$$

$$d\theta = \omega dt$$

$$\int_{\theta(0)}^{\theta(t)} d\theta = \int_0^t (\ 2t - \frac{3}{4}t^2 + 5)dt$$

$$\theta(t) - \theta(0) = \left[t^2 - \frac{t^3}{4} + 5t\right]_0^t$$

$$\theta(t) = t^2 - \frac{t^3}{4} + 5t$$

$$\theta(3) = 9 - \frac{27}{4} + 15 = \boldsymbol{17.25\ rad}$$