Parametrized curves: $x = x(t)$, $y = y(t)$ $\quad \boxed{\alpha \leq t \leq \beta}$

$x, y : [\alpha, \beta] \to \mathbb{R}$ are continuous

Arc length:

$$\int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Special Cases: $\boxed{y = f(x)}$, f continuous on $[\alpha, \beta]$

Arc length:

$$\int_{\alpha}^{\beta} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

$\boxed{x = g(y)}$, g is continuous on $[\alpha, \beta]$

Arc length:

$$\int_{\alpha}^{\beta} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

① $y = 1 + 6x^{3/2}$. $\quad 0 \le x \le 1$.

Arc length: $\displaystyle\int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$.

$$= \int_0^1 \sqrt{1+\left(\frac{3}{2}\cdot 6\,x^{1/2}\right)^2}\,dx$$

$$= \int_0^1 \sqrt{1+\left(9x^{1/2}\right)^2}\,dx$$

$$= \int_0^1 \sqrt{1+81x}\,dx$$

Substitution/FTC(2).

$$= \left.\frac{(1+81x)^{3/2}}{3/2}\cdot\frac{1}{81}\right]_0^1$$

$$= \frac{2}{243}\left(82^{3/2}-1\right).$$

2. $x = \frac{1}{3}\sqrt{y}(y-3)$ $\quad 1 \le y \le 9$

Arc length $= \int_1^9 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy.$

$$= \int_1^9 \sqrt{1+\left(\frac{9}{2}y^{1/2} - \frac{1}{2}y^{-1/2}\right)^2}\, dy$$

$= \frac{32}{3}$ (check!!)

Find the length of the arct of the curve $y=\frac{1}{2}x^2$ from the point $P=(-1,\frac{1}{2})$ to $Q=(1,\frac{1}{2})$

$y=\frac{1}{2}x^2.$

Arc length. $\int_{-1}^{1}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,.\,dx$

$$=\int_{-1}^{1}\sqrt{1+x^2}\,dx$$

$$=2\int_{0}^{1}\sqrt{1+x^2}\,dx.$$

FTC(2)

$$=2\left(\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\ln\left(x+\sqrt{1+x^2}\right)\right)\Big]_0^1$$

$$=\sqrt{2}+\ln\left(1+\sqrt{2}\right).$$