EP 1108 Practise Problem

1. In a collision between a proton at rest (rest mass equal to m_p) and a moving proton, another particle of rest mass M is produced, in addition to the two protons. Find the minimum total energy the moving proton must have in order for this process to take place. (Hint: The minimum total energy corresponds to the case where all the particles after the collision are at rest in the Center of Mass frame where total momentum is equal to 0)

Solution:
Since we are analyzing the dynamics in the lab frame (before collision) and COM frame (after collision) we cannot equate the individual momenta and energies before and after the collisions. However, we can still use the fact that the invariant rest mass is constant and independent of the frame in which the collision is analyzed. For simplification of algebra, we assume that $c = 1$ and our final answer will not contain c. But we can add it at the end to make it dimensionally consistent.

Let m_p be the rest mass of the proton and E_p and p denote the energy and momentum of the moving proton.

Before Collision (Lab frame):
Total energy $= E_p+m_p$
Total momentum $=p$
Invariant rest mass square $= (E_p + m_p)^2 - p^2$
Since $E_p^2 = m_p^2 + p^2$, the above term becomes

$$2E_p m_p + 2m_p^2$$

After Collision (COM frame):
Total energy $= 2m_p + M$
Total momentum $= 0$
Invariant rest mass squared $= (2m_p + M)^2$

Equating this with the invariant rest mass before the collision, we get
$E = m_p + 2M + \frac{M^2}{2m_p}$
To make it dimensionally consistent we then add the missing factor of c^2 and get

$$\boxed{E = (m_p + 2M + \frac{M^2}{2m_p})c^2}$$

Note that you will also get this answer at the outset if you don't put $c = 1$