EP1108 Nuclear Physics Part 2

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Statistics of Radioactive decay

The *activity* of a sample of any radioactive material is the rate at which the constituent nuclei decay. if N is the number of nuclei present at a certain time in the sample, the activity R is given by:

$$R = -\frac{dN}{dt}$$

R is usally positive. R is expressed in terms of *curie*, where
1 curie $= 3.7 \times 10^{10}$ disintegrations/sec
1 millicurie $= 3.7 \times 10^{7}$ disintegrations/sec
1 μcurie $= 3.7 \times 10^{4}$ disintegrations/sec

Activity as a function of time

Empirically it was observed that activity decreases exponentially as a function of time. So this can be written as

$$R = R_0 e^{-\lambda t}, \tag{1}$$

where λ is known as decay constant. One can also derive the relation between half-life $T_{1/2}$ and λ

$$T_{1/2} = \frac{0.693}{\lambda}$$

One can also define a mean lifetime $\bar{T}$ which is defined as

$$\bar{T} = \frac{1}{\lambda}$$

Radioactive decays are statistical in nature. No apriori way of knowing *which* nuclei will decay in a particular time span.

Derivation of empirical activity law

The empirical activity law follows from the assumption of constant probability per unit time (λ) for the decay of each nucleus. Consider a sample containing N non-decayed nuclei. The number dN that will decay in a time dt is given by the product of number of nuclei N and the probability λdt that each will decay in dt.

$$dN = -N\lambda dt$$

The minus sign is because N decreases with increasing t. The above equation can be rewritten as

$$\frac{dN}{N} = -\lambda dt$$

$$\implies N = N_0 e^{-\lambda t}$$

Activity of a radioactivity sample is given by $R = -\frac{dN}{dt}$

Derivation of empirical activity law (contd)

$$R = \lambda N_0 e^{-\lambda t}$$

This agrees with the empirical activity law if

$$R_0 = \lambda N_0$$

and in general one can write $R = \lambda N$

Example

Calculate the activity of a 1-gm sample of ${}_{38}Sr^{90}$ whose lifetime for beta decay is 23 years.
$\lambda = 0.693/T_{1/2} = 7.8 \times 10^{-10} s^{-1}$ 1 gm of Sr contains (1/90) moles.
Total no of nuclei in 1 g of Sr = $(1/90) \times N_A$ where N_A= Avogadro's number. Therefore total no of nuclei (N) is equal to 6×10^{21}.
Activity of the sample = $\lambda N = 6 \times 10^{21} \times 7.8 \times 10^{-10} = 141$ curies.

Radioactive series

In nature, one observes several decays of radioactive elements, some of which were created in the early universe with others formed by the bombardment of cosmic rays.

Since all nuclei with $A > 209$ are unstable and undergo either α, β or γ decay. Since the changes in the number of nucleons are only due to α decay, the mass numbers A of the series of nuclei produced in a series are related by $A = a + 4n$. Therefore, four such series exist corresponding to $a = 0, 1, 2, 3$ and n is an integer. They are summarized below.

Series	A	Longest lived nuclei	Final stable nucleus
Thorium	$4n$	$^{232}Th(1.39 \times 10^{10})$	^{208}Pb
Neptunium	$4n + 1$	$^{237}Np(2.25 \times 10^{6})$	^{209}Bi
Uranium	$4n + 2$	$^{238}U(4.51 \times 10^{9})$	^{206}Pb
Actinium	$4n + 3$	$^{235}U(7.07 \times 18^{6})$	^{207}Pb

Table 1: Four Radioactive series

Radioactive Series(Contd)

- Of all the above, thorium, uranium and actinium series are found in nature.
-Neptunium series is not observed in nature, since its longest lived nuclei has a lifetime of $\sim 10^6$ years and whatever created in the early universe would have decayed by now.
- Neptinium can however be artificially produced from ^{236}U by the capture of a neutron followed by β decay.
-All these elements undergo a series of α or β decay till they are reduced to the final stable nucleus (Details of each chain can be found in Beiser)

Age determination

The occurrence of radioactive elements in nature provides a means of determining the ages of elements in our planetary system. Assuming that ^{238}U and ^{235}U were initially created in about the same quantities, the ratio of these elements found in nature is given by:

$$\frac{N_{235}(t)}{N_{238}(t)} \approx \exp[-t(1/\tau_{235} - 1/\tau_{238})] \tag{2}$$

Using the observed ratio of $1/140$ for the relative abundance and $\tau_{235} \approx 1.02 \times 10^9$ yrs, $\tau_{238} \approx 6.51 \times 10^9$ yrs, the age of the elements is given by $t = 5.98 \times 10^9$ yrs.

References for these sets of notes

All these slides have been obtained from the Modern Physics book by Beiser and Patil (mentioned in the first nuclear Physics lecture)