ME 30040/ME 50210

Mathematical Elements for Geometrical Modelling/CAD

Assignment 3

ME21BTECH11001

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Question 1

Write a code to generate a B-spline curve for n and k where n+1 is the number of points and k -1 is the degree of the polynomial curve.

a. Assume n = 6 and k = 4 and use open uniform knot vector as discussed in the class. Let the 7 points be (0,1), (2,6), (4,8), (5,2), (7,8), (9,7) and (10,4). Plot the B spline curve and the points in the same graph.

b. Assume n = 6 and k = 3. Using the same set of points, plot the B spline curve in the same graph.

c. Assume n = 6 and k = 2. Using the same set of points, plot the B spline curve in the same graph.

Code:

```
%Abhishek Ghosh
%ME21BTECH11001
%Question 1



function B_Spline_Plot()
    % Define control points
    control_points = [0 1; 2 6; 4 8; 5 2; 7 8; 9 7; 10 4];
    n = 6; % n + 1 is the number of control points

    % Plot separately
    figure;
    hold on;
    % a) part
    % Degree k = 4
    k = 4;
    plotBSpline(control_points, n, k, 'r', 'B-Spline: n=6, k=4');
    plot(control_points(:,1), control_points(:,2), 'o--', 'DisplayName', 'Control
Points');
    legend;
    title('B-Spline with Degree 4');
    xlabel('X-axis');
    ylabel('Y-axis');
    hold off;
    % b) part
    %Degree k = 4 & 3
    figure;
    hold on;
```

```
    % Degree k = 4
    k = 4;
    plotBSpline(control_points, n, k, 'r', 'B-Spline: n=6, k=4');

    % Degree k = 3
    k = 3;
    plotBSpline(control_points, n, k, 'b', 'B-Spline: n=6, k=3');

    plot(control_points(:,1), control_points(:,2), 'o--', 'DisplayName', 'Control
Points');

    % Finalize plot
    legend;
    title('B-Splines with Different Degrees');
    xlabel('X-axis');
    ylabel('Y-axis');
    hold off;
    % c) part
    % Plot all together
    figure;
    hold on;

    % Degree k = 4
    k = 4;
    plotBSpline(control_points, n, k, 'r', 'B-Spline: n=6, k=4');

    % Degree k = 3
    k = 3;
    plotBSpline(control_points, n, k, 'b', 'B-Spline: n=6, k=3');

    % Degree k = 2
    k = 2;
    plotBSpline(control_points, n, k, 'g', 'B-Spline: n=6, k=2');

    % Plot control points
    plot(control_points(:,1), control_points(:,2), 'o--', 'DisplayName', 'Control
Points');

    % Finalize plot
    legend;
    title('B-Splines with Different Degrees');
    xlabel('X-axis');
    ylabel('Y-axis');
    hold off;
end

function curve = b_spline_curve(control_points, n, k)
    % Generate knot vector
    knots = [zeros(1, k), linspace(1, n - k + 1, n - k + 1), repmat(n - k + 2, 1,
k)];

    % Define number of points to plot
    num_points = 100;
    t_values = linspace(0, n - k + 2, num_points);
    curve = zeros(num_points, 2);

    for j = 1:num_points
        t = t_values(j);
        point = [0, 0];
        for i = 1:n + 1
            b = basis_function(i - 1, k, t, knots); % MATLAB is 1-indexed
            point = point + b * control_points(i, :);
        end
        curve(j, :) = point;
    end
end

function b = basis_function(i, k, t, knots)
```

```
    if k == 1
        if knots(i + 1) <= t && t < knots(i + 2)
            b = 1.0;
        else
            b = 0.0;
        end
    else
        denom1 = knots(i + k) - knots(i + 1);
        denom2 = knots(i + k + 1) - knots(i + 2);

        if denom1 ~= 0
            term1 = ((t - knots(i + 1)) / denom1) * basis_function(i, k - 1, t,
knots);
        else
            term1 = 0.0;
        end

        if denom2 ~= 0
            term2 = ((knots(i + k + 1) - t) / denom2) * basis_function(i + 1, k -
1, t, knots);
        else
            term2 = 0.0;
        end

        b = term1 + term2;
    end
end

function plotBSpline(control_points, n, k, color, label)
    curve = b_spline_curve(control_points, n, k);
    plot(curve(1:end-1,1), curve(1:end-1,2), color, 'DisplayName', label);
end
```

Plots

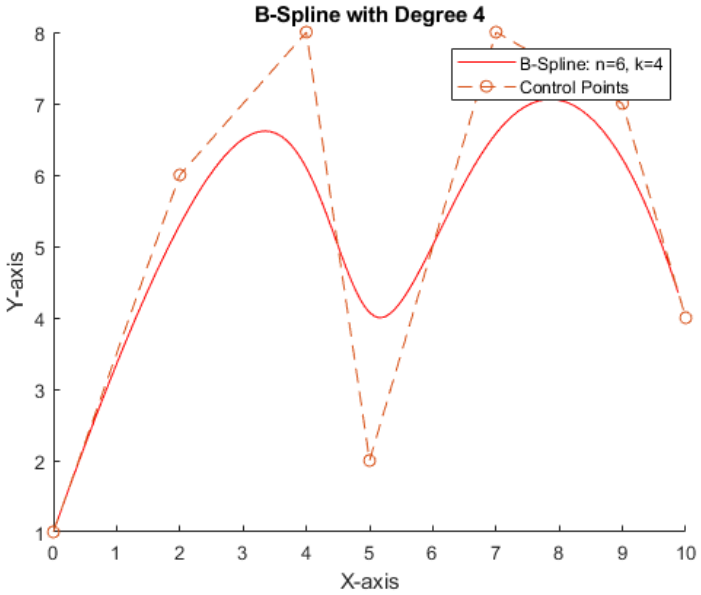

Fig: B Spline Curve with Degree 4

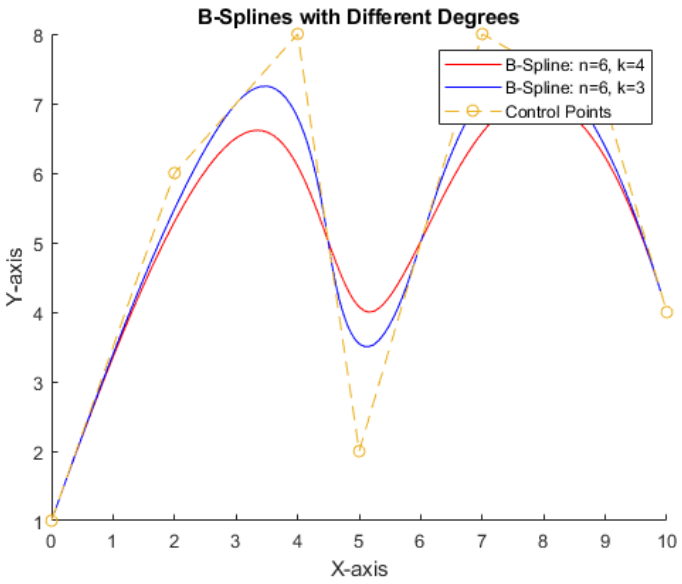

Fig: B Spline Curve with Degree 4 & 3

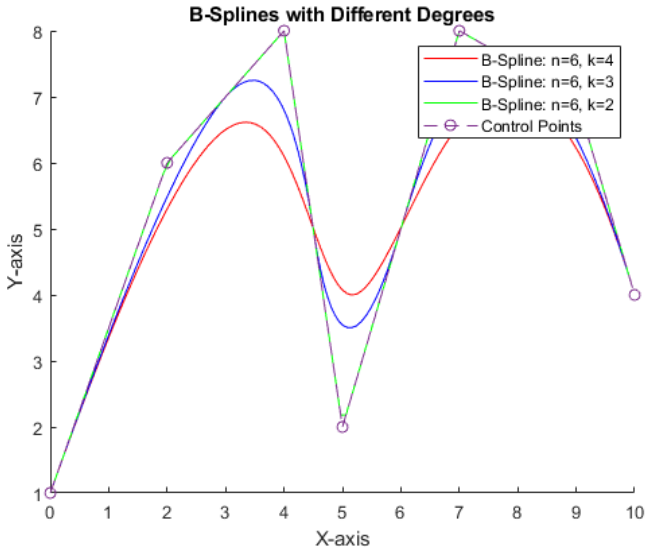

Fig: B Spline Curve with Degree 4,3 and 2

Question 2

Effect of multiple vertices on a B spline curve: Use the same code as above for the following questions.

a. Assume k=4. Plot the B spline curve for the following points: (0,1), (2,6), (4,8), (5,2)

b. Assume k=4. Plot the B spline curve in the same graph for the following points: (0,1), (2,6), (2,6), (4,8), (5,2) (The 2nd point is repeated once).

c. Assume k=4. Plot the B spline curve in the same graph for the following points: (0,1), (2,6), (2,6), (2,6), (4,8), (5,2) (The 2nd point is repeated twice).

Comment on the effect of multiple vertices on a B spline curve.

Code:

```
%Abhishek Ghosh
%ME21BTECH11001
%Question 2



function B_Spline_Plot()
    % Define control points
    control_points = [0 1; 2 6; 4 8; 5 2];
    n = 3; % n + 1 is the number of control points

    % Plot separately
    figure;
    hold on;
    % a) part
    n = 3;
    k = 4;
    plotBSpline(control_points, n, k, 'r', 'B-Spline: n=3, k=4');
    plot(control_points(:,1), control_points(:,2), 'o--', 'DisplayName', 'Control
Points');
    legend;
    title('B-Spline with n = 3');
    xlabel('X-axis');
    ylabel('Y-axis');
    hold off;


    % b) part
```

```
    %n = 3 & 4
    figure;
    hold on;
    % n = 3
    control_points = [0 1; 2 6; 4 8; 5 2];
    k = 4;
    n = 3;
    plotBSpline(control_points, n, k, 'r', 'B-Spline: n=3, k=4');

    % n = 4
    control_points = [0 1; 2 6;2 6; 4 8; 5 2];
    k = 4;
    n = 4;
    plotBSpline(control_points, n, k, 'b', 'B-Spline: n=4, k=4');

    plot(control_points(:,1), control_points(:,2), 'o--', 'DisplayName', 'Control
Points');

    % Finalize plot
    legend;
    title('B-Splines with Different n');
    xlabel('X-axis');
    ylabel('Y-axis');
    hold off;
    % c) part
    % Plot all together
    figure;
    hold on;
    % n = 3
    control_points = [0 1; 2 6; 4 8; 5 2];
    k = 4;
    n = 3;
    plotBSpline(control_points, n, k, 'r', 'B-Spline: n=3, k=4');

    % n = 4
    control_points = [0 1; 2 6;2 6; 4 8; 5 2];
    k = 4;
    n = 4;
    plotBSpline(control_points, n, k, 'b', 'B-Spline: n=4, k=4');

    % n = 5
    control_points = [0 1; 2 6;2 6;2 6; 4 8; 5 2];
    k = 4;
    n = 5;
    plotBSpline(control_points, n, k, 'g', 'B-Spline: n=5, k=4');

    plot(control_points(:,1), control_points(:,2), 'o--', 'DisplayName', 'Control
Points');

    % Finalize plot
    legend;
    title('B-Splines with Different n');
    xlabel('X-axis');
    ylabel('Y-axis');
    hold off;
end

function curve = b_spline_curve(control_points, n, k)
    % Generate knot vector
    knots = [zeros(1, k), linspace(1, n - k + 1, n - k + 1), repmat(n - k + 2, 1,
k)];

    % Define number of points to plot
    num_points = 100;
    t_values = linspace(0, n - k + 2, num_points);
    curve = zeros(num_points, 2);

    for j = 1:num_points
```

```
        t = t_values(j);
        point = [0, 0];
        for i = 1:n + 1
            b = basis_function(i - 1, k, t, knots); % MATLAB is 1-indexed
            point = point + b * control_points(i, :);
        end
        curve(j, :) = point;
    end
end

function b = basis_function(i, k, t, knots)
    if k == 1
        if knots(i + 1) <= t && t < knots(i + 2)
            b = 1.0;
        else
            b = 0.0;
        end
    else
        denom1 = knots(i + k) - knots(i + 1);
        denom2 = knots(i + k + 1) - knots(i + 2);

        if denom1 ~= 0
            term1 = ((t - knots(i + 1)) / denom1) * basis_function(i, k - 1, t,
knots);
        else
            term1 = 0.0;
        end

        if denom2 ~= 0
            term2 = ((knots(i + k + 1) - t) / denom2) * basis_function(i + 1, k -
1, t, knots);
        else
            term2 = 0.0;
        end

        b = term1 + term2;
    end
end

function plotBSpline(control_points, n, k, color, label)
    curve = b_spline_curve(control_points, n, k);
    plot(curve(1:end-1,1), curve(1:end-1,2), color, 'DisplayName', label);
end
```

Plots:

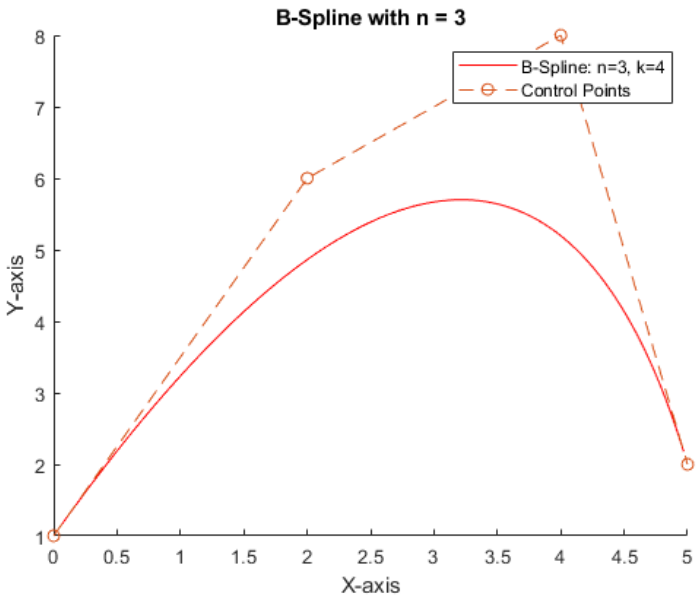

Fig: B Spline Curve with n=3 & k=4

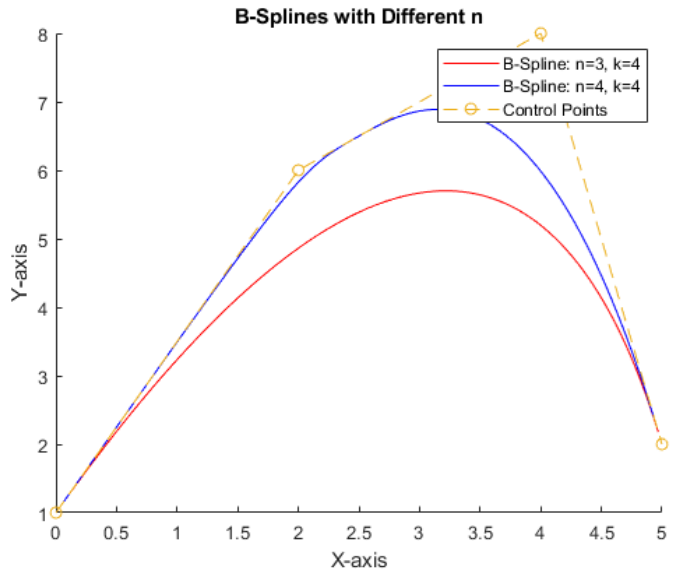

Fig: B Spline Curve with n=3 & 4 with k=4

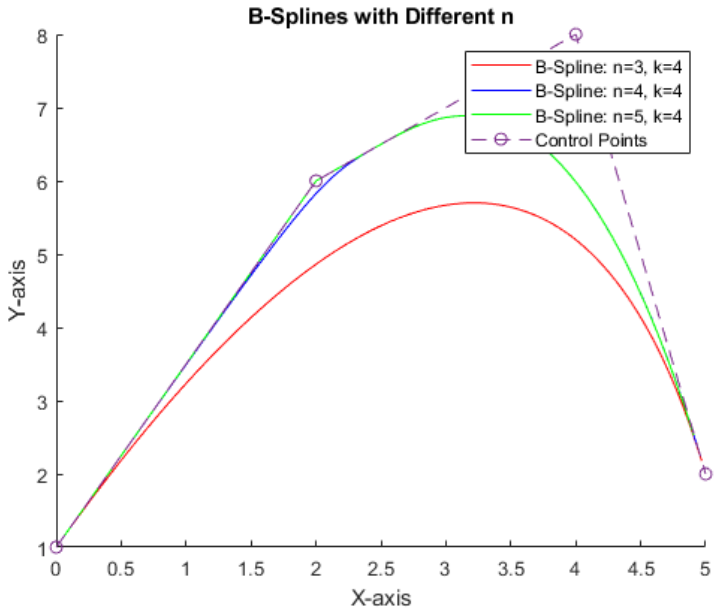

Fig: B Spline Curve with n = 3,4 & 5 with k=4

Effect of multiple vertices on B Spline Curve: If one control point is repeated then it has more influence that is it has more pulling effect towards that point creating sharp transitions at that point.