ME3030
Modelling and Simulation
Assignment 5

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ME21BTECH11001

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Given, $m_1 = 1$ $m_2 = 2$ $J_1 = 1$ $J_2 = 2$ $g = 10$ $a = 0.2$ $b = 0.2$

$\theta_1(0) = \pi/2$ $\theta_2(0) = \pi/4$

$$r_{O_1}^0 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad r_{O_2}^0 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \quad r_P^0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad r_Q^1 = \begin{bmatrix} -a \\ -b \end{bmatrix}$$

$$r_P^1 = \begin{bmatrix} a \\ b \end{bmatrix} \quad r_Q^2 = \begin{bmatrix} a \\ b \end{bmatrix}$$

Governing eqn:—

$$r_{O_1}^0 + R(\theta_1) r_Q^1 = r_{O_2}^0 + R(\theta_2) r_Q^2 \quad \text{—— (I)}$$

$$r_{O_1}^0 + R(\theta_1) r_P^1 = r_P^0 \quad \text{—— (II)}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} -a \\ -b \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \quad \text{—— (I)}$$

$$\begin{bmatrix} x_1 - a\cos\theta_1 + b\sin\theta_1 \\ y_1 - a\sin\theta_1 - b\cos\theta_1 \end{bmatrix} = \begin{bmatrix} x_2 + a\cos\theta_2 - b\sin\theta_2 \\ y_2 + a\sin\theta_2 + b\cos\theta_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 + a\dot{\theta}_1\sin\theta_1 + b\dot{\theta}_1\cos\theta_1 \\ \dot{y}_1 - a\dot{\theta}_1\cos\theta_1 + b\dot{\theta}_1\sin\theta_1 \end{bmatrix} = \begin{bmatrix} \dot{x}_2 - a\dot{\theta}_2\sin\theta_2 - b\dot{\theta}_2\cos\theta_2 \\ \dot{y}_2 + a\dot{\theta}_2\cos\theta_2 - b\dot{\theta}_2\sin\theta_2 \end{bmatrix}$$

Double Diff eq (I)

$$\ddot{x}_1 + a\dot{\theta}_1^2\cos\theta_1 + a\ddot{\theta}_1\sin\theta_1 + b\dot{\theta}_1^2(-\sin\theta_1) + b\ddot{\theta}_1\cos\theta_1$$
$$= \ddot{x}_2 - a\ddot{\theta}_2\sin\theta_2 - \dot{\theta}_2^2\cos\theta_2 - b\dot{\theta}_2^2(-\sin\theta_2)$$
$$- b\ddot{\theta}_2\cos\theta_2$$

$$\ddot{y}_1 + a\dot{\theta}_1^2\sin\theta_1 - a\ddot{\theta}_1\cos\theta_1 + b\dot{\theta}_1^2\cos\theta_1 + b\ddot{\theta}_1\sin\theta_1$$
$$= \ddot{y}_2 + a\ddot{\theta}_2\cos\theta_2 - a\dot{\theta}_2^2\sin\theta_2 - b\ddot{\theta}_2\sin\theta_2$$
$$- b\dot{\theta}_2^2\cos\theta_2$$

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$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{— (VI)}$$

$$\begin{bmatrix} \dot{x}_1 - a\dot{\theta}_1 \sin\theta_1 - b\dot{\theta}_1 \cos\theta_1 \\ \dot{y}_1 + a\dot{\theta}_1 \cos\theta_1 - b\dot{\theta}_1 \sin\theta_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Double Diff eq (VI)

$$\ddot{x}_1 - a\ddot{\theta}_1 \sin\theta_1 - a\dot{\theta}_1^2 \cos\theta_1 + b\dot{\theta}_2^2 \sin\theta_1 - b\ddot{\theta}_1 \cos\theta_1 = 0$$

$$\& \quad \ddot{y}_1 + a\ddot{\theta}_1 \cos\theta_1 - a\dot{\theta}_1^2 \sin\theta_1 - b\dot{\theta}_2^2 \cos\theta_1 - b\ddot{\theta}_2 \sin\theta_1 = 0$$

$$\ddot{x}_1 + (-a\sin\theta_1 - b\cos\theta_1)\ddot{\theta}_1 = a\dot{\theta}_1^2 \cos\theta_1 - b\dot{\theta}_1^2 \sin\theta_1$$

$$\& \quad \ddot{y}_1 + (a\cos\theta_1 - b\sin\theta_1)\ddot{\theta}_1 = a\dot{\theta}_1^2 \sin\theta_1 + b\dot{\theta}_1^2 \cos\theta_1$$

$$q = \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ \ddot{\theta}_2 \end{bmatrix} \quad m = \begin{bmatrix} m_1 & & & & & \\ & m_1 & & & & \\ & & J_1 & & & \\ & & & m_2 & & \\ & & & & m_2 & \\ & & & & & J_2 \end{bmatrix} \quad f = \begin{bmatrix} 0 \\ -m_1 g \\ 0 \\ 0 \\ -m_2 g \\ 0 \end{bmatrix}$$

$$u_{4\times6} \cdot q_{6\times1} = v_{4\times1}$$

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Final eqⁿ :-

$$\ddot{x}_1 - \ddot{x}_2 + \ddot{\theta}_1 (a\sin\theta_1 + b\cos\theta_1) + \ddot{\theta}_2 (a\sin\theta_2 + b\cos\theta_2)$$
$$= \dot{\theta}_1^2 [b\sin\theta_1 - a\cos\theta_1] + \dot{\theta}_2^2 [b\sin\theta_2 - a\cos\theta_2]$$

$$\ddot{y}_1 - \ddot{y}_2 + \ddot{\theta}_1 (b\sin\theta_1 - a\cos\theta_1) + \ddot{\theta}_2 (b\sin\theta_2 - a\cos\theta_2)$$
$$= \dot{\theta}_1^2 [-a\sin\theta_1 - b\cos\theta_1] + \dot{\theta}_2^2 [-a\sin\theta_2 - b\cos\theta_2]$$

$$\ddot{x}_1 + \ddot{\theta}_1 (-a\sin\theta_1 - b\cos\theta_1) = \dot{\theta}_1^2 [a\cos\theta_1 - b\sin\theta_1]$$

$$\ddot{y}_1 + \ddot{\theta}_1 (a\cos\theta_1 - b\sin\theta_1) = \dot{\theta}_1^2 [b\cos\theta_1 + a\sin\theta_1]$$

$$u = \begin{bmatrix} 1 & 0 & a\sin\theta_1 + b\cos\theta_1 & -1 & 0 & a\sin\theta_2 + b\cos\theta_2 \\ 0 & 1 & b\sin\theta_1 - a\cos\theta_1 & 0 & -1 & b\sin\theta_2 - a\cos\theta_2 \\ 1 & 0 & -a\sin\theta_1 - b\cos\theta_1 & 0 & 0 & 0 \\ 0 & 1 & a\cos\theta_1 - b\sin\theta_1 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} \dot{\theta}_1^2 (b\sin\theta_1 - a\cos\theta_1) + \dot{\theta}_2^2 [b\sin\theta_2 - a\cos\theta_2) \\ \dot{\theta}_1^2 (-a\sin\theta_1 - b\cos\theta_1) + \dot{\theta}_2^2 [-a\sin\theta_2 - b\cos\theta_2) \\ \dot{\theta}_1^2 (a\cos\theta_1 - b\sin\theta_1) \\ \dot{\theta}_1^2 (b\cos\theta_1 + a\sin\theta_1) \end{bmatrix}$$

$$Z = [x_1 \; y_1 \; \theta_1 \; x_2 \; y_2 \; \theta_2 \; \dot{x}_1 \; \dot{y}_1 \; \dot{\theta}_1 \; \dot{x}_2 \; \dot{y}_2 \; \dot{\theta}_2]$$

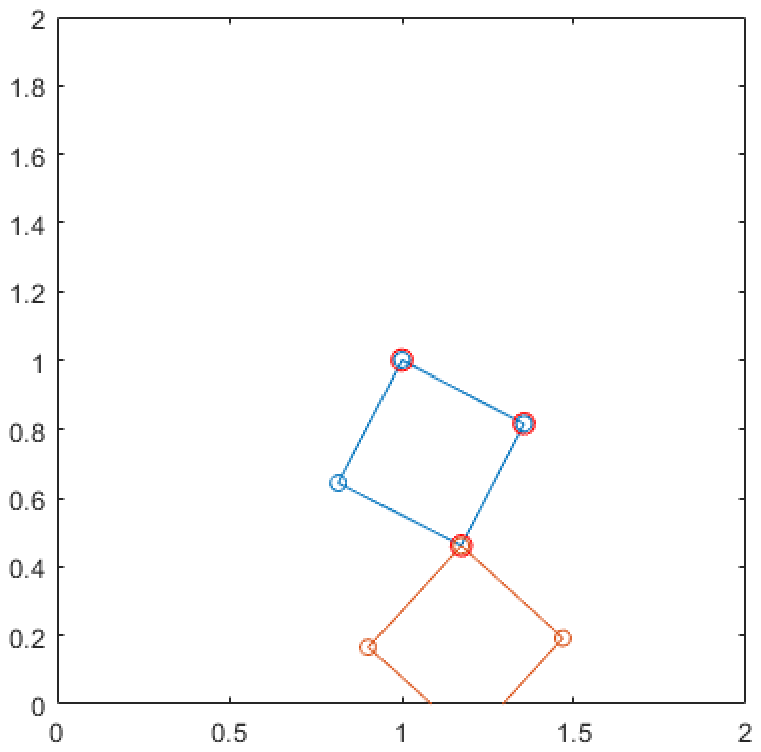
Code:-

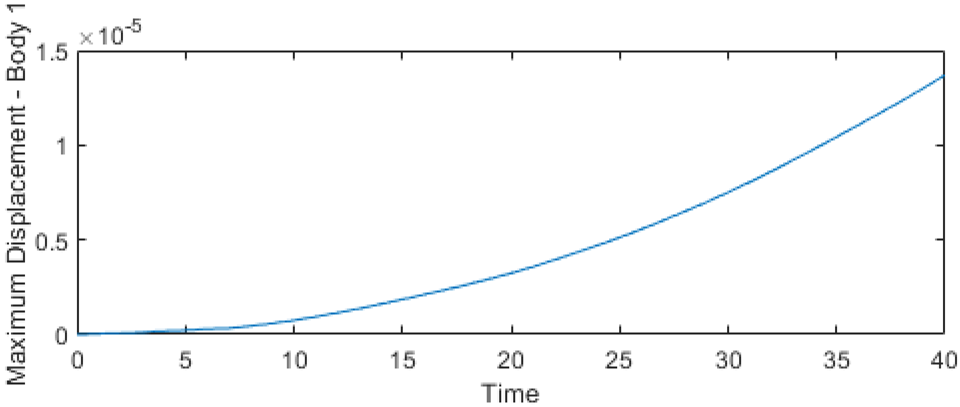
```
% ME21BTECH11001 Abhishek Ghosh
% Modelling and Simulation Assignment 5

clc
clear all

% Declare global variables
global m1 m2 J1 J2 g a b

% Assign values to global parameters
m1 = 1;         % Mass for body 1
m2 = 2;         % Mass for body 2
J1 = 1;         % Moment of inertia for body 1
J2 = 2;         % Moment of inertia for body 2
g = 10;         % Gravity
a = 0.2;        % Distance from the center of mass to the front
b = 0.2;        % Distance from the center of mass to the rear

% Set initial conditions
theta1_init = pi/2;    % Initial angle for body 1
theta2_init = pi/4;    % Initial angle for body 2
rpin = [1 1]';         % Initial position of the pin
Rinit1 = [cos(theta1_init) -sin(theta1_init); sin(theta1_init) cos(theta1_init)]; %
Initial rotation matrix for body 1
Rinit2 = [cos(theta2_init) -sin(theta2_init); sin(theta2_init) cos(theta2_init)]; %
Initial rotation matrix for body 2
rcg1 = rpin - Rinit1 * [a b]'; % Initial position of the center of mass for body 1
rcg2 = rpin - Rinit2 * [a b]'; % Initial position of the center of mass for body 2
init = [rcg1(1) rcg1(2) theta1_init rcg2(1) rcg2(2) theta2_init 0 0 0 0 0 0]; %
Initial state vector
tspan = 0:0.1:40; % Time span
options = odeset('Reltol', 1e-8, 'AbsTol', 1e-8); % ODE solver options

% Solve the system of ODEs using ode15s
[t, z] = ode15s(@BES, tspan, init, options);

% Extract states from the solution
xcg1 = z(:, 1);
ycg1 = z(:, 2);
theta1 = z(:, 3);
xdcg1 = z(:, 7);
ydcg1 = z(:, 8);
thetad1 = z(:, 9);

xcg2 = z(:, 4);
ycg2 = z(:, 5);
theta2 = z(:, 6);
xdcg2 = z(:, 10);
ydcg2 = z(:, 11);
thetad2 = z(:, 12);

% Animation loop
figure;

for i = 1:length(t)
    % Compute the positions of the four corners of body 1
    rcg1 = [xcg1(i) ycg1(i)]';

    % Rotation Matrix
    R1 = [cos(theta1(i)) -sin(theta1(i)); sin(theta1(i)) cos(theta1(i))];
    R2 = [cos(theta2(i)) -sin(theta2(i)); sin(theta2(i)) cos(theta2(i))];
    r11 = rcg1 + R1 * [a b]';
    r21 = rcg1 + R1 * [-a b]';
    r31 = rcg1 + R1 * [-a -b]';
```

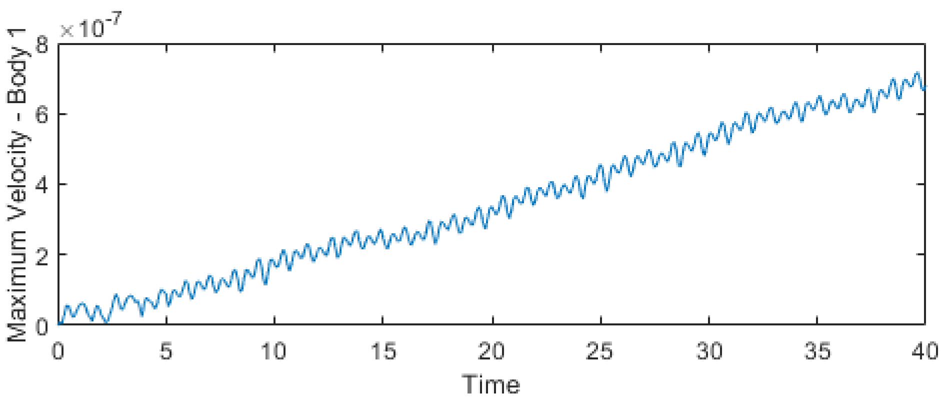
```
    r41 = rcg1 + R1 * [a -b]';

    % Location of P and Q for body 1
    r0P1 = rcg1;
    r1P1 = rcg1 + R1 * [a b]';
    r1Q1 = rcg1 + R1 * [-a -b]';
    r2Q1 = rcg1 + R1 * [a -b]';

    % Location of P and Q for body 2 (fixed at Q for body 1)
    rcg2 = rcg1 + R1*[-a -b]' - R2*[a b]'; % Fixed at Q for body 1
    R2 = [cos(theta2(i)) -sin(theta2(i)); sin(theta2(i)) cos(theta2(i))];
    r12 = rcg2 + R2 * [a b]';
    r22 = rcg2 + R2 * [-a b]';
    r32 = rcg2 + R2 * [-a -b]';
    r42 = rcg2 + R2 * [a -b]';

    % Plot both bodies and their pinned locations
    plot([r11(1) r21(1) r31(1) r41(1) r11(1)], [r11(2) r21(2) r31(2) r41(2)
r11(2)], 'o-');
    hold on;
    plot([r12(1) r22(1) r32(1) r42(1) r12(1)], [r12(2) r22(2) r32(2) r42(2)
r12(2)], 'o-');
    plot(r1P1(1), r1P1(2), 'ro', 'MarkerSize', 8);  % Pin location Q for body 1
    plot(r1Q1(1), r1Q1(2), 'ro', 'MarkerSize', 8);
    plot(r2Q1(1), r2Q1(2), 'ro', 'MarkerSize', 8);
    plot(r1Q1(1), r1Q1(2), 'ro', 'MarkerSize', 8);  % Fixed location Q for body 2
    plot(r1P1(1), r1P1(2), 'ro', 'MarkerSize', 8);  % Pin location Q for body 2
    hold off;

    axis equal
    xlim([0 2])
    ylim([0 2])
    pause(0.1)
end


% Calculate and plot the maximum displacements for body 1
C1 = zeros(1, length(t));
Cd1 = zeros(1, length(t));

for i = 1:1:length(t)
    % Calculate current position and velocity of the pin for body 1
    xc1 = 1;
    yc1 = 1; % Assuming A and omega are not specified for body 1

    xcd1 = 0;
    ycd1 = 0;

    % Calculate the current position and velocity of the center of mass for body 1
    rcg1 = [xcg1(i) ycg1(i)]';
    vcg1 = [xdcg1(i) ydcg1(i)]';

    % Calculate maximum displacements for body 1
    rc1 = [xc1 yc1]';
    rcd1 = [xcd1 ycd1]';
    R1 = [cos(theta1(i)) -sin(theta1(i)); sin(theta1(i)) cos(theta1(i))];
    Rd1 = thetad1(i) * [-sin(theta1(i)) -cos(theta1(i)); cos(theta1(i)) -
sin(theta1(i))];

    C1(i) = max(abs(rcg1 + R1 * [a b]' - rc1));
    Cd1(i) = max(abs(vcg1 + Rd1 * [a b]' - rcd1));
end

% Plot the results for body 1
figure;
subplot(2,1,1);
plot(t, C1)
xlabel('Time')
```

```
ylabel('Maximum Displacement - Body 1')

subplot(2,1,2);
plot(t, Cd1)
xlabel('Time')
ylabel('Maximum Velocity - Body 1')
```

Plots :-







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Code for BES function:-

```
% Abhishek Ghosh ME21BTECH11001
% ME3030 Assignment 5

% function for zdot
function zdot=BES(t,z)
global m1 m2 J1 J2 a b g A omega

%initial conditions & derivatives
xc1=1;
yc1=1;
xc2=1;
yc2=1;
xcd1=0;
ycd1=0;
xcdd1=0;
ycdd1=0;
xcd2=0;
ycd2=0;
xcdd2=0;
ycdd2=0;

% Mass matrix
M=diag([m1 m1 J1 m2 m2 J2]);
% Force matrix
F=[0 -m1*g 0 0 -m2*g 0]';
x1=z(1);
y1=z(2);
theta1=z(3);

x2=z(4);
y2=z(5);
theta2=z(6);

x1d=z(7);
y1d=z(8);
theta1d=z(9);

x2d=z(10);
y2d=z(11);
theta2d=z(12);
% Uqdd=v

% u-> coeeficient of qdd matrix

U=[1 0 a*sin(theta1)+b*cos(theta1) -1 0 a*sin(theta2)+b*cos(theta2);
   0 1 b*sin(theta1)-a*cos(theta1) 0 -1 b*sin(theta2)-a*cos(theta2);
   1 0 -a*sin(theta1)-b*cos(theta1) 0 0 0;
   0 1 a*cos(theta1)-b*sin(theta1) 0 0 0];

% v-> independent of qdd terms

v=[theta1d^2*(b*sin(theta1)-a*cos(theta1)) + theta2d^2*(b*sin(theta2)-
a*cos(theta2));
   theta1d^2*(-a*sin(theta1)-b*cos(theta1)) + theta2d^2*(-a*sin(theta2)-
b*cos(theta2));
   theta1d^2*(a*cos(theta1)-b*sin(theta1));
   theta1d^2*(b*cos(theta1)+a*(sin(theta1)))];
```

```
% acc-> derivatives of x1 y1 theta1 x2 y2 theta2
acc=M\F+(M^(-0.5))*pinv(U*(M^(-0.5)))*(v-U*(M\F));
zdot=[z(7) z(8) z(9) z(10) z(11) z(12) acc']';
```