ME 3030

Modelling and Simulation

Assignment 1

ME21BTECH11001

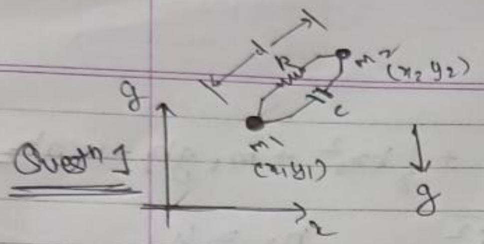
Abhishek Ghosh

Question 1 :-

a)

ME21BTECH11001 Abhishek Ghosh

Questn 1



$$r_1 = \begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} \qquad r_2 = \begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix}$$

~~For m_1 :-~~

~~Spring force, F_{spr}~~

(a) Spring force, $f_s = k(|r_2 - r_1| - L)\left(\frac{r_2 - r_1}{|r_2 - r_1|}\right)$

Damping force, $f_c = -c(\dot{r}_2 - \dot{r}_1)$

for $m_1 \rightarrow$

$$m_1 \begin{Bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \end{Bmatrix} = \frac{k(|r_2 - r_1| - L)}{|r_2 - r_1|} \begin{Bmatrix} x_2 - x_1 \\ y_2 - y_1 \end{Bmatrix} + c \begin{Bmatrix} \dot{x}_2 - \dot{x}_1 \\ \dot{y}_2 - \dot{y}_1 \end{Bmatrix} - m_1 g \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

for $m_2 \rightarrow$

$$m_2 \begin{Bmatrix} \ddot{x}_2 \\ \ddot{y}_2 \end{Bmatrix} = -\frac{k(|r_2 - r_1| - L)}{|r_2 - r_1|} \begin{Bmatrix} x_2 - x_1 \\ y_2 - y_1 \end{Bmatrix} - c \begin{Bmatrix} \dot{x}_2 - \dot{x}_1 \\ \dot{y}_2 - \dot{y}_1 \end{Bmatrix} - m_2 g \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

where $|r_2 - r_1| = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

(c) Energy of the system

$$E = KE + PE$$

where PE = spring + gravitational energy

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}k x_0^2 + m_1 g h_1 + m_2 g h_2$$

where $v_1 = \sqrt{\dot{x}_1^2 + \dot{y}_1^2}$ $\quad h_1 = y_1$

$v_2 = \sqrt{\dot{x}_2^2 + \dot{y}_2^2}$ $\quad h_2 = y_2$

$x_0 = |r_2 - r_1| - L$

b) The MATLAB code :-

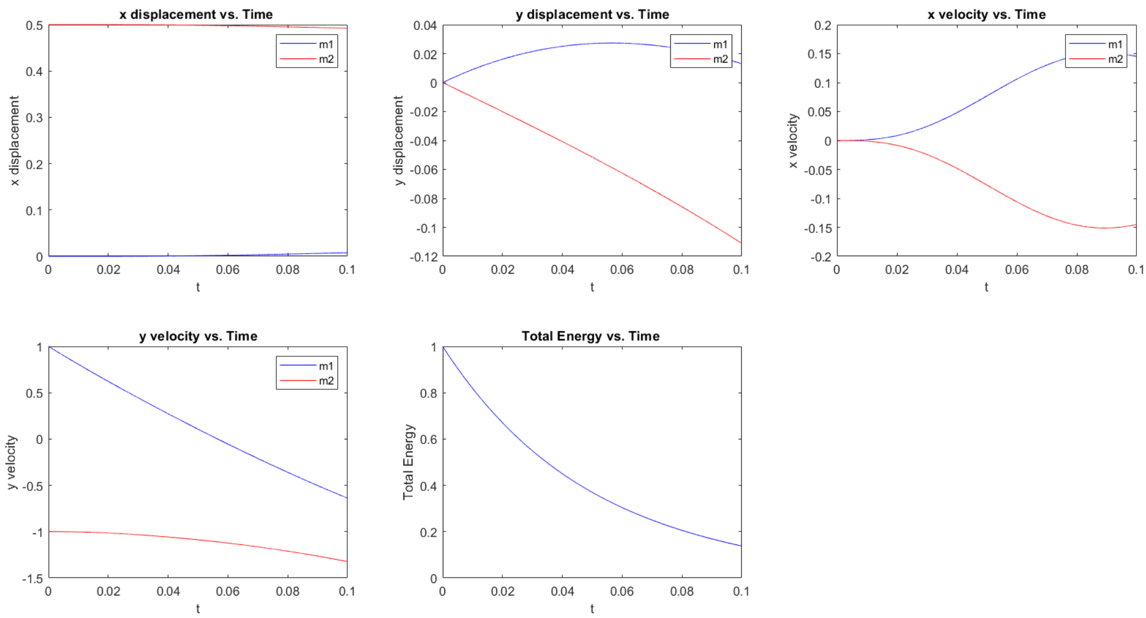
```
% Define system parameters
m1 = 1.0;       % Mass of m1 in kg
m2 = 1.0;       % Mass of m2 in kg
k = 1000.0;     % Spring stiffness in N/m
c = 5.0;        % Damping coefficient in Ns/m
l = 0.5;      % Free length of the spring in m
g = 9.81;     % Acceleration due to gravity in m/s^2


% Initial conditions
x1_i = 0.0;
y1_i = 0.0;
vx1_i = 0.0;
vy1_i = 1.0;


x2_i = 0.5;
y2_i = 0.0;
vx2_i = 0.0;
vy2_i = -1.0;


% Time step
dt = 1.0e-5;


% Number of time steps
num_steps = 10000;
```

```matlab
% Time array
time = (0:num_steps-1) * dt;


% Initialize arrays to store positions and velocities
x1 = zeros(1, num_steps);
y1 = zeros(1, num_steps);
vx1 = zeros(1, num_steps);
vy1 = zeros(1, num_steps);


x2 = zeros(1, num_steps);
y2 = zeros(1, num_steps);
vx2 = zeros(1, num_steps);
vy2 = zeros(1, num_steps);


% Initialize distance between masses array
distance_between_masses = zeros(1, num_steps);


% Initialize kinetic energy, spring potential energy, gravitational potential
energy and total energy array
kinetic_energy = zeros(1, num_steps);
spring_pot_energy = zeros(1, num_steps);
grav_pot_energy = zeros(1, num_steps);
total_energy = zeros(1, num_steps);


% Set initial conditions
x1(1) = x1_i;
y1(1) = y1_i;
vx1(1) = vx1_i;
vy1(1) = vy1_i;


x2(1) = x2_i;
y2(1) = y2_i;
vx2(1) = vx2_i;
vy2(1) = vy2_i;


distance_between_masses(1) = sqrt((x2(1) - x1(1))^2 + (y2(1) - y1(1))^2);
kinetic_energy(1) = 0.5*m1*((vx1(1))^2 + (vy1(1))^2) + 0.5*m2*((vx2(1))^2 +
(vy2(1))^2);
spring_pot_energy(1) = 0.5*k*((distance_between_masses(1) - l)^2);
grav_pot_energy(1) = m1*g*y1(1) + m2*g*y2(1);
total_energy(1) =  kinetic_energy(1) + spring_pot_energy(1) +
grav_pot_energy(1);


% disp(total_energy(1));
```

```
% Using Euler's explicit scheme
for i = 1:num_steps-1
    % Forces
    spring_force = k * (distance_between_masses(i) - l);
    damper_force_x = c * (vx2(i) - vx1(i));
    damper_force_y = c * (vy2(i) - vy1(i));

    % Accelerations
    ax1 = (spring_force * (x2(i) - x1(i))) / (m1 * distance_between_masses(i))
+ damper_force_x / m1;
    ay1 = (spring_force * (y2(i) - y1(i))) / (m1 * distance_between_masses(i))
+ damper_force_y / m1 - g;

    ax2 = (spring_force * (x1(i) - x2(i))) / (m2 * distance_between_masses(i))
- damper_force_x / m2;
    ay2 = (spring_force * (y1(i) - y2(i))) / (m2 * distance_between_masses(i))
- damper_force_y / m2 - g;

    % Update velocities and positions of m1
    vx1(i+1) = vx1(i) + ax1 * dt;
    vy1(i+1) = vy1(i) + ay1 * dt;

    x1(i+1) = x1(i) + vx1(i+1) * dt;
    y1(i+1) = y1(i) + vy1(i+1) * dt;

    % Update velocities and positions of m2
    vx2(i+1) = vx2(i) + ax2 * dt;
    vy2(i+1) = vy2(i) + ay2 * dt;

    x2(i+1) = x2(i) + vx2(i+1) * dt;
    y2(i+1) = y2(i) + vy2(i+1) * dt;


    % Update distance
    distance_between_masses(i+1) = sqrt((x2(i+1) - x1(i+1))^2 + (y2(i+1) -
y1(i+1))^2);


    % Update energy
    kinetic_energy(i+1) = 0.5*m1*((vx1(i+1))^2 + (vy1(i+1))^2) +
0.5*m2*((vx2(i+1))^2 + (vy2(i+1))^2);
    spring_pot_energy(i+1) = 0.5*k*((distance_between_masses(i+1) - l)^2);
    grav_pot_energy(i+1) = m1*g*y1(i+1) + m2*g*y2(i+1);
    total_energy(i+1) =  kinetic_energy(i+1) + spring_pot_energy(i+1) +
grav_pot_energy(i+1);
end


for i = 1:num_steps
    kinetic_energy(i) = round(kinetic_energy(i), 3);
```

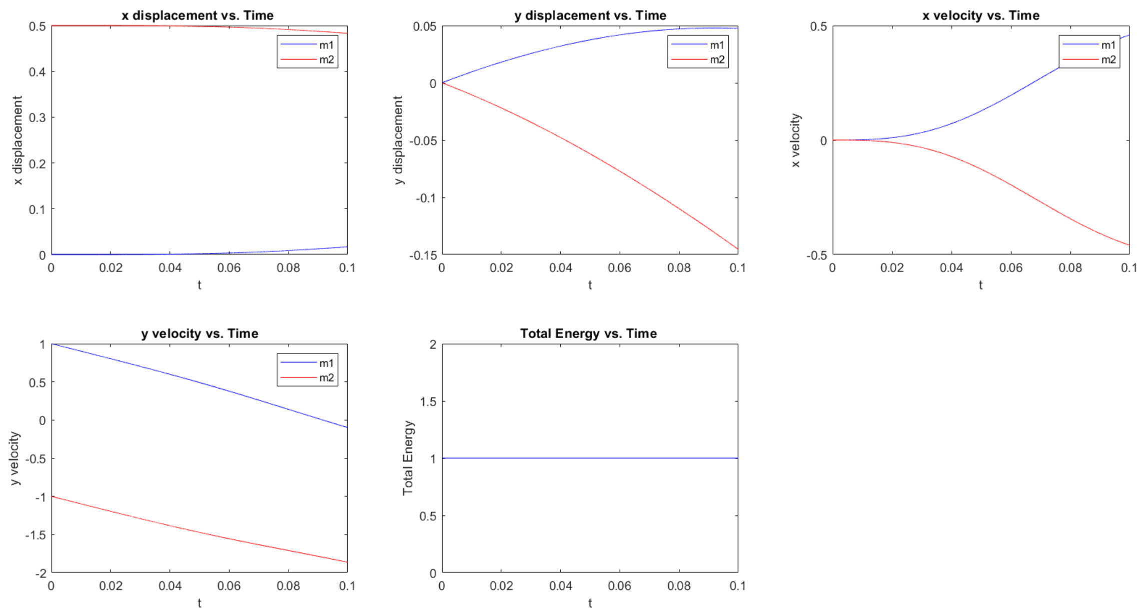
```
    spring_pot_energy(i) = round(spring_pot_energy(i), 3);
    grav_pot_energy(i) = round(grav_pot_energy(i), 3);
    total_energy(i) = round(total_energy(i), 3);
end


% Plot results in 1 plot
figure;


% x vs time plot
subplot(2,3,1);
plot(time, x1, 'b',time,x2,'r');
xlabel('t');
ylabel('x displacement');
legend('m1', 'm2');
title('x displacement vs. Time ');


% y vs time plot
subplot(2,3,2);
plot(time, y1, 'b',time,y2,'r');
xlabel('t');
ylabel('y displacement');
legend('m1', 'm2');
title('y displacement vs. Time');


% vx vs time plot
subplot(2,3,3);
plot(time, vx1, 'b',time,vx2,'r');
xlabel('t');
ylabel('x velocity');
legend('m1', 'm2');
title('x velocity vs. Time');


% vy vs time plot
subplot(2,3,4);
plot(time, vy1, 'b',time,vy2,'r');
xlabel('t');
ylabel('y velocity');
legend('m1', 'm2');
title('y velocity vs. Time');


% Energy vs time plot
subplot(2,3,5);
plot(time, total_energy, 'b');
xlabel('t');
ylabel('Total Energy');
title('Total Energy vs. Time ');
```

The graphs are as follows: -

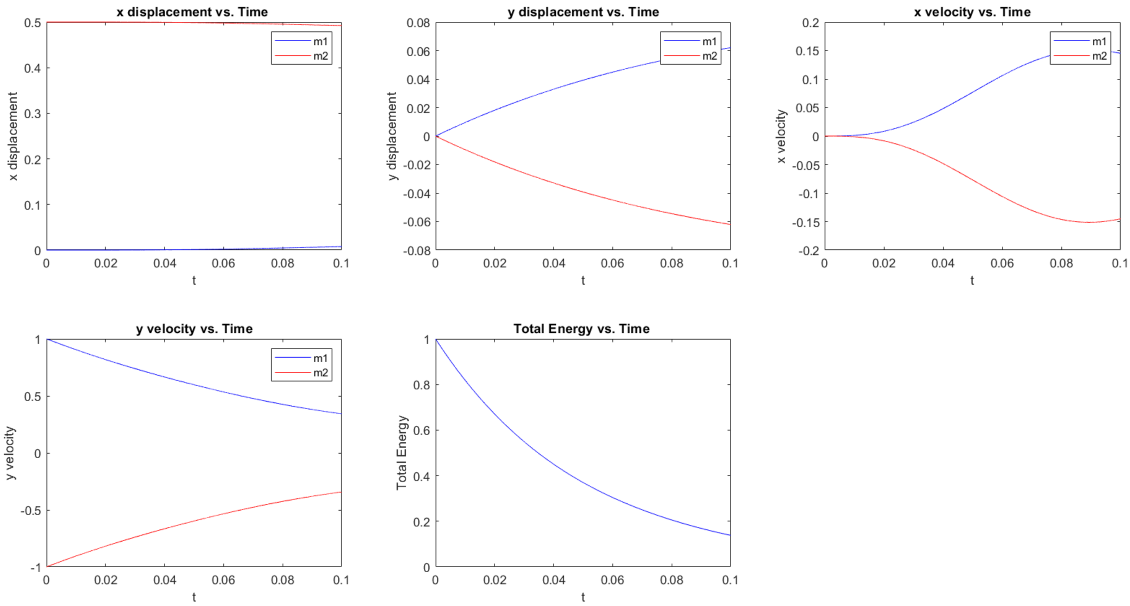


c) at c=0, the graphs are as follows

The total energy becomes constant without damping.



d) at g=0, the graphs are as follows: -


x displacement vs. Time
x displacement
t
m1
m2
y displacement vs. Time
y displacement
t
m1
m2
x velocity vs. Time
x velocity
t
m1
m2
y velocity vs. Time
y velocity
t
m1
m2
Total Energy vs. Time
Total Energy
t