

FORMAL LANGUAGE & AUTOMATA THEORY

Time: Three hours

Answer any **five** questions

Full Marks: 100

1. Construct Deterministic Finite Automatas to accept the following languages with necessary justifications :
- (a) All binary strings of length ≤ 3 and all binary strings where every substring of length 4 contains at least three 1's.
 - (b) All binary strings where the absolute difference between the number of 0's and 1's in every prefix is at most 1.
2. (a) State and prove the Pumping Lemma for regular languages. 12+8
Why it is useful?
- (b) Prove that for each positive integer n , there exists a language L_n over $\{a, b\}$ such that L_n can be accepted by a NDFA with $n + 1$ states but any DFA accepting L_n must have at least 2^n states.
3. (a) Explain the concept of ϵ -closure of a set of states of a NDFA. 10+10
(b) Prove that for each non-deterministic finite automata, there exists an equivalent deterministic automata.
4. (a) Develop regular expressions for all strings of $\{a, b\}$ that contain an even number of a 's and odd number of b 's. 3+17
(b) Prove that the language accepted by a DFA can always be described by a regular expression.
5. (a) Construct a Push Down Automata to accept the language $\{a^i b^j c^k : i = j \text{ or } j = k, i, j, k \geq 0\}$ with necessary justifications. 8+12
What is the mode of acceptance of your PDA?
Explain all sources of non-determinism present in the transitions of this PDA.
(b) Prove that if a language L is accepted by a NDFA, then the language L^* can also be accepted by a NDFA.
6. (a) Explain when is a Push Down Automata called *deterministic*. 10+10
Prove that languages accepted by DPDA's with empty stack are *prefix-free* languages.
(b) Prove that a language is accepted by a PDA with empty stack *if and only if* it is accepted by a PDA with final states.
7. (a) Find out the language generated by the grammar whose production rules are : 6+14
(i) $S \rightarrow aSbS$, (ii) $S \rightarrow bSaS$, (iii) $S \rightarrow \epsilon$
Give necessary justifications.
(b) Develop a grammar for the language $\{a^i b^j c^k d^l : i + j = k + l, i, j, k, l \geq 0\}$ with necessary proof. 12+8