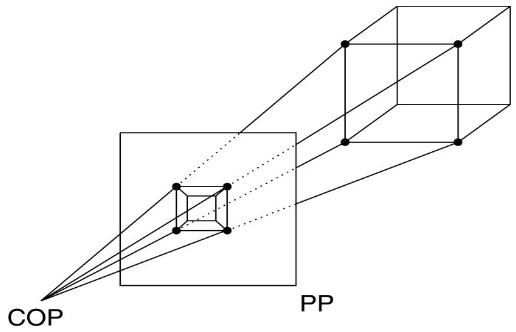
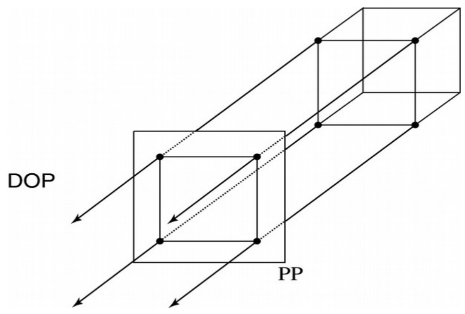
Computer Graphics 8: Projections

Projections

Projections transform points in *n*-space to *m*-space, where $m<n$.

In 3-D, we map points from 3-space to the **projection plane** (PP) (image plane) along **projectors** (viewing rays) emanating from the center of projection (COP):





There are two basic types of projections:

- Perspective – distance from COP to PP finite
- Parallel – distance from COP to PP infinite

Parallel projections

For parallel projections, we specify a **direction of projection** (DOP) instead of a COP.

We can write orthographic projection onto the *z=0* plane with a simple matrix, such that x'=x, y'=y.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \qquad\qquad \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Normally, we do not drop the z value right away. Why not?

Properties of parallel projection

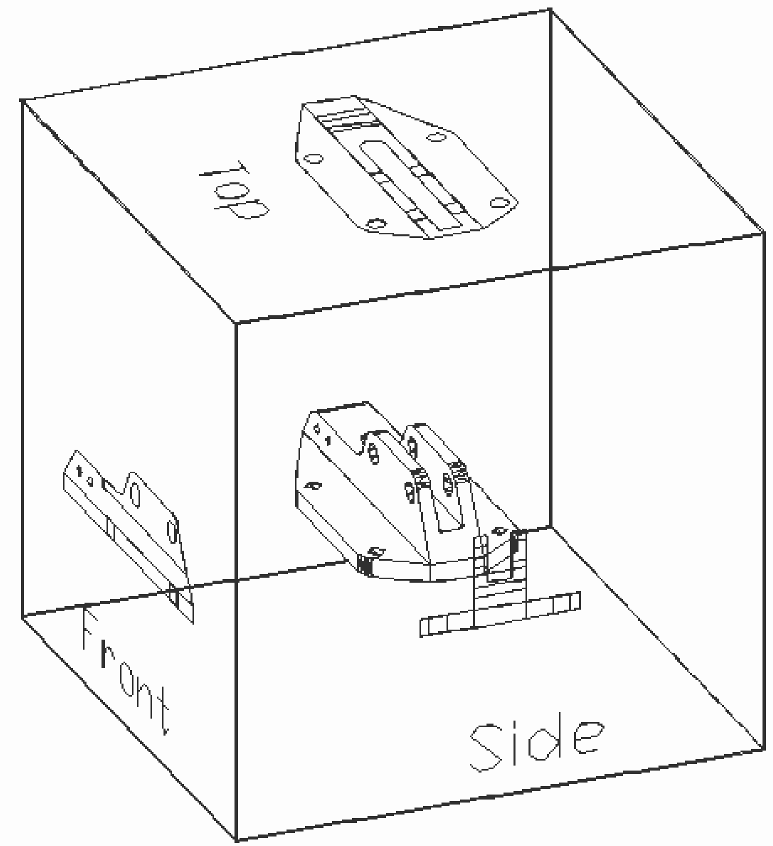
Properties of parallel projection:

- Are actually a kind of affine transformation
  - Parallel lines remain parallel
  - Ratios are preserved
  - Angles not (in general) preser
- Not realistic looking
- Good for exact measurements,

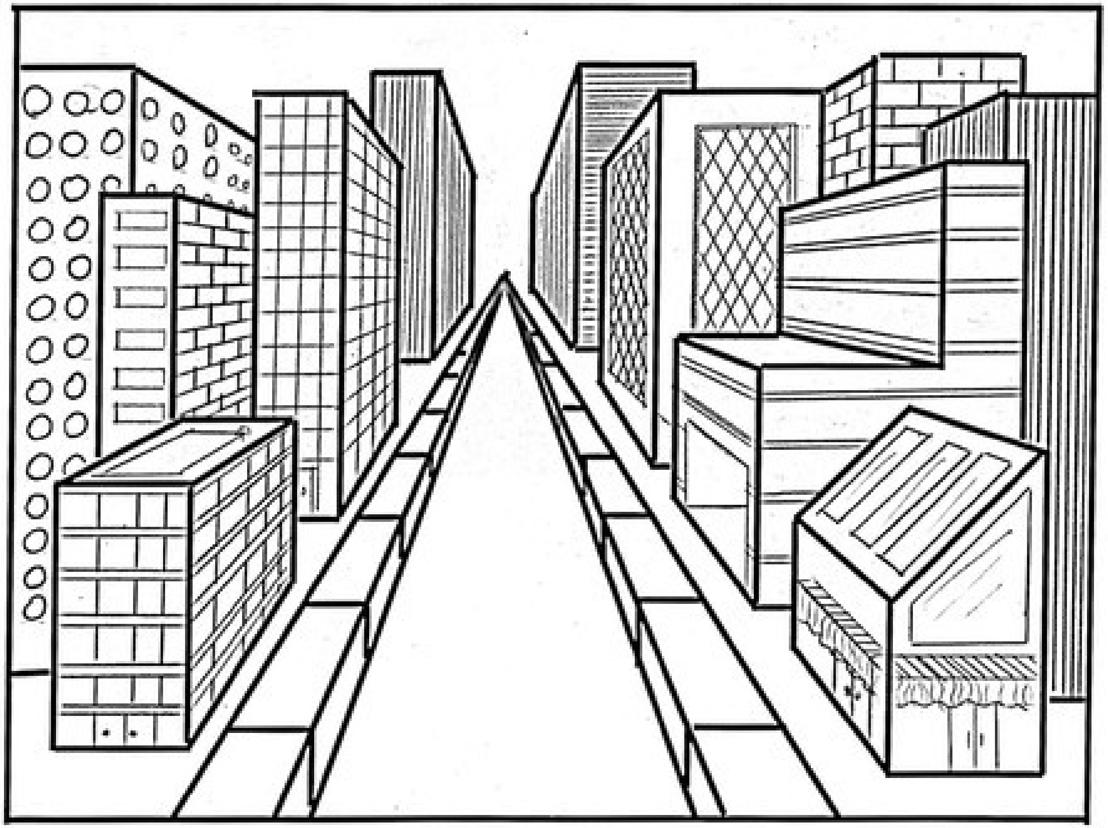
  Most often used in
  - CAD,
  - architectural drawings,
  - etc.,

  where taking exact measurement

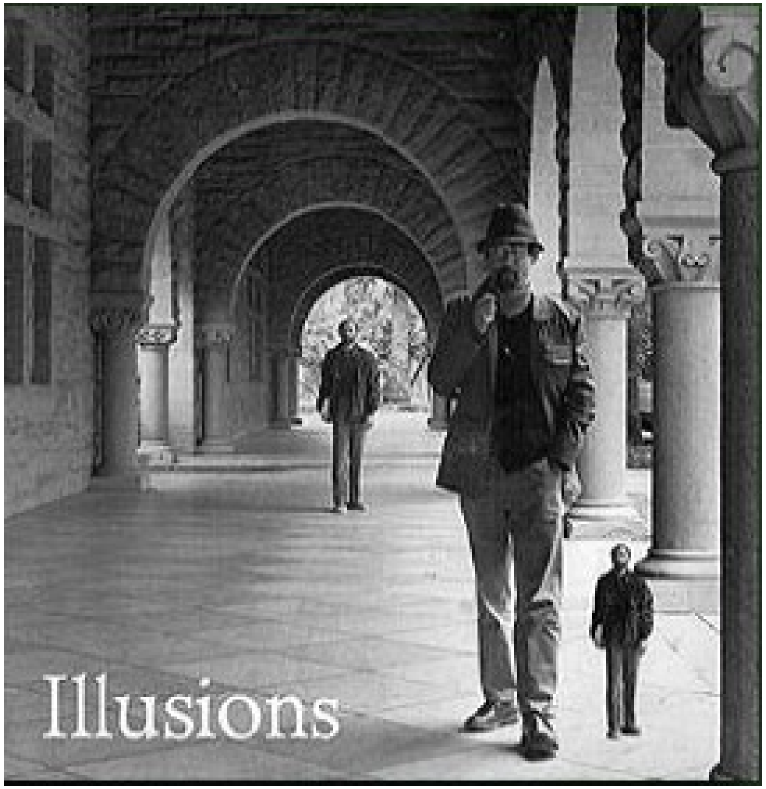
  is important



Perspective effect



Perspective Illusion



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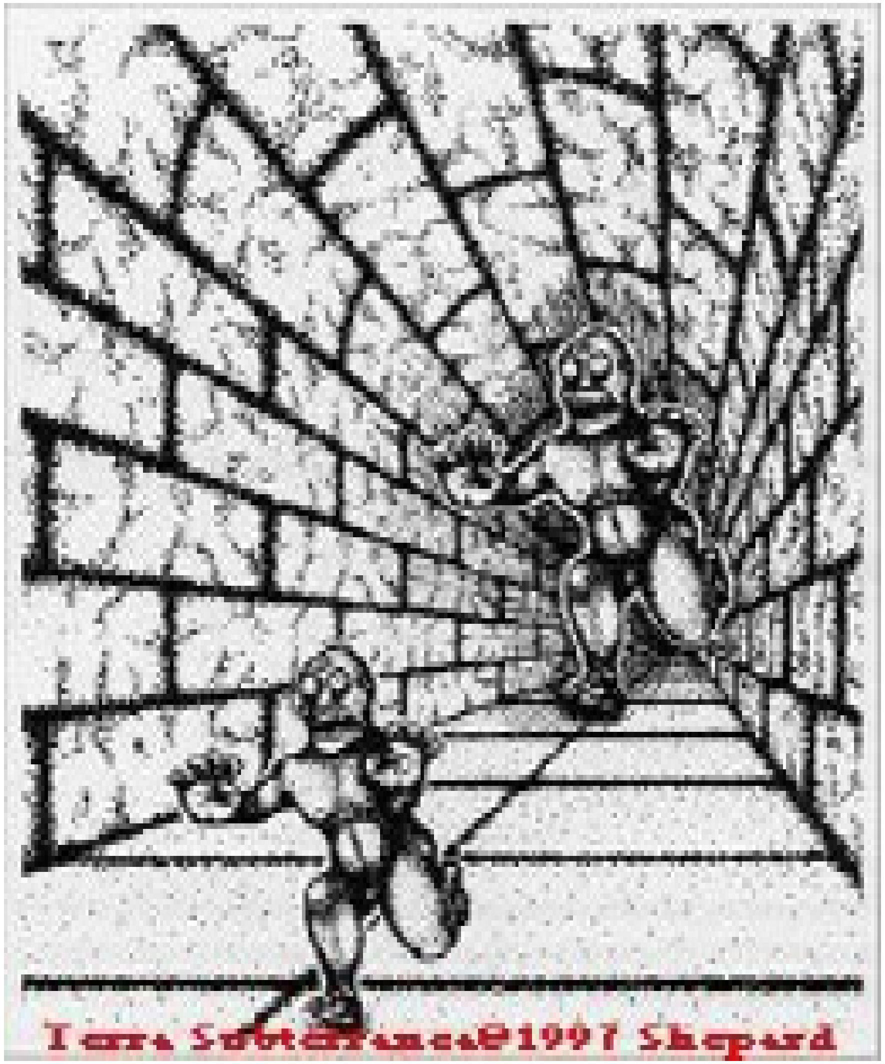
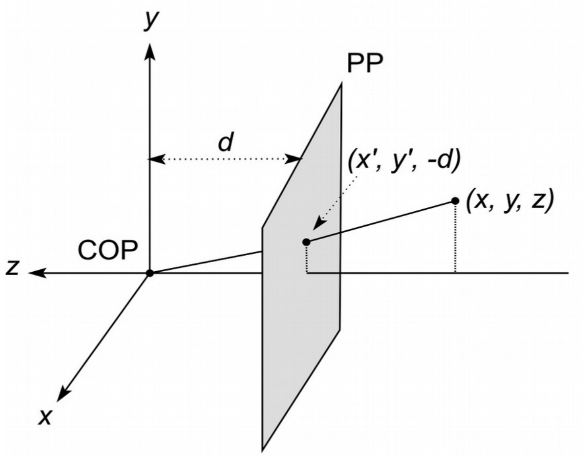


Derivation of perspective projection

Consider the projection of a point onto the projection plane:



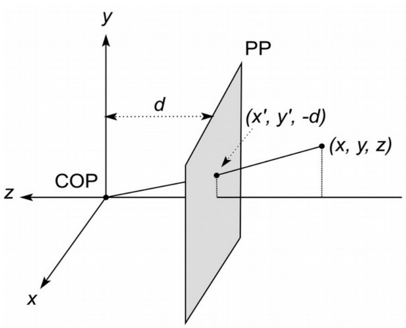
By similar triangles, we can compute how much the *x* and *y* coordinates are scaled:

$$\frac{y'}{d} = \frac{y}{z} \quad \Longrightarrow \quad y' = \frac{d}{z} y$$

$$x' = \frac{d}{z} x$$

Homogeneous coordinates and perspective projection

Now we can re-write the perspective projection as a matrix equation:



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} (d/z)x \\ (d/z)y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

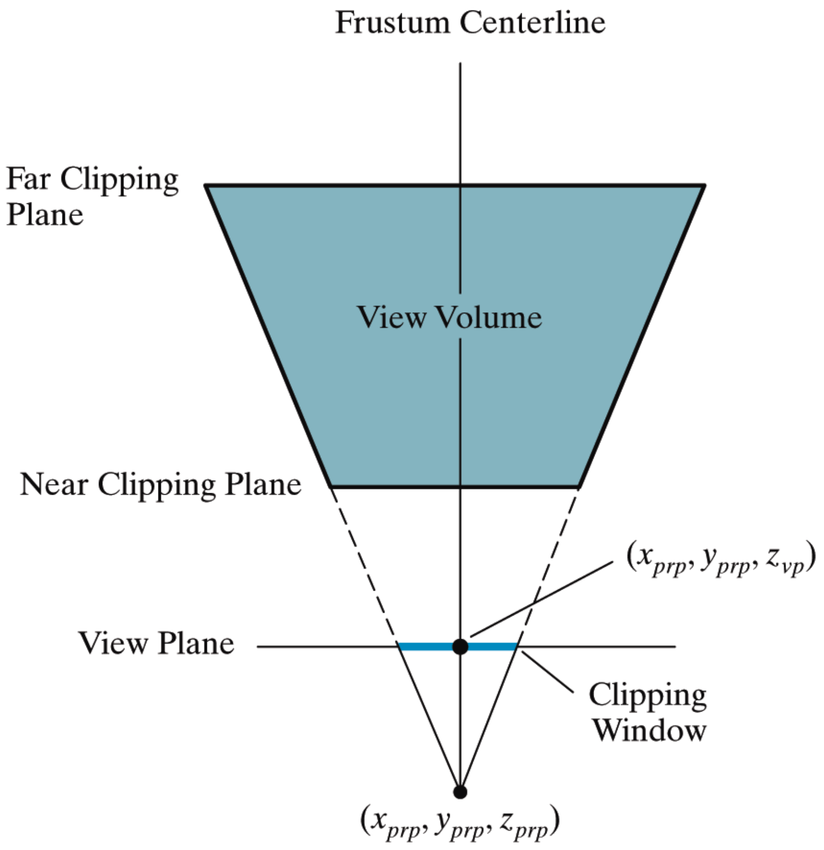
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Orthographic projection

$$= \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} (d/z)x \\ (d/z)y \\ d \\ 1 \end{bmatrix}$$

Setting Up A Perspective Projection

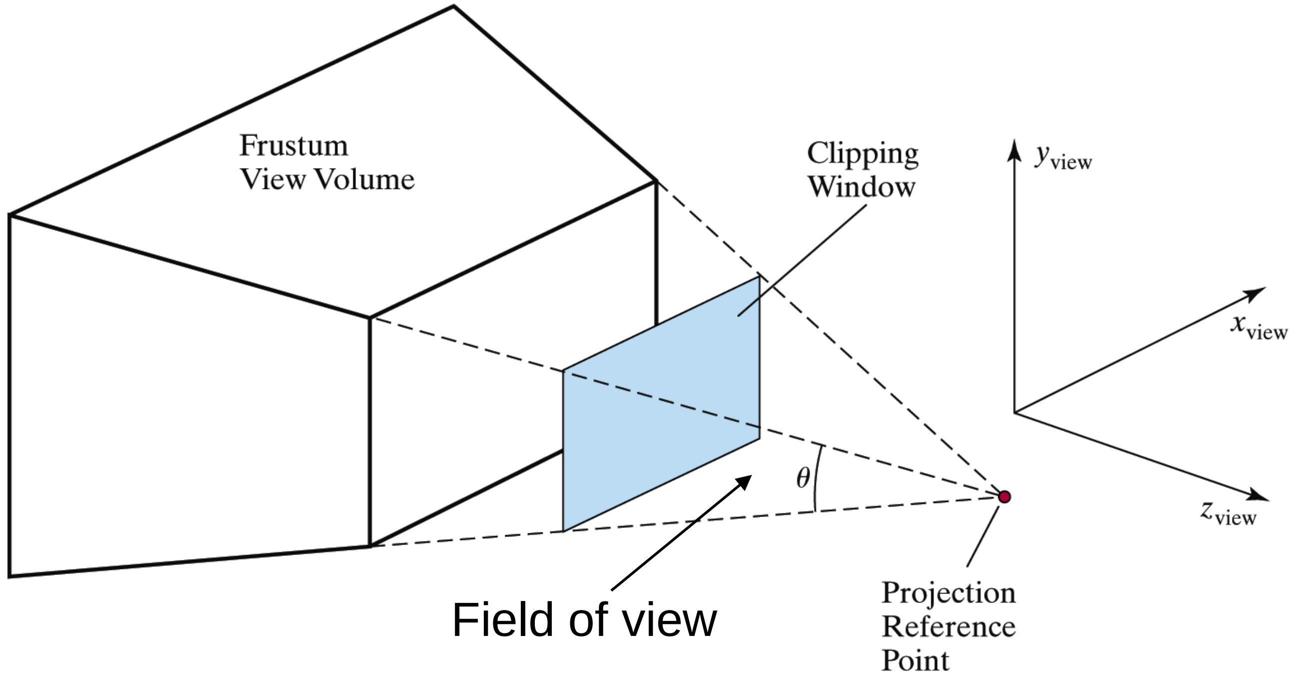
A perspective projection can be set up by specifying the position and size of the view plane and the position of the projection reference point



Setting Up A Perspective Projection (cont…)

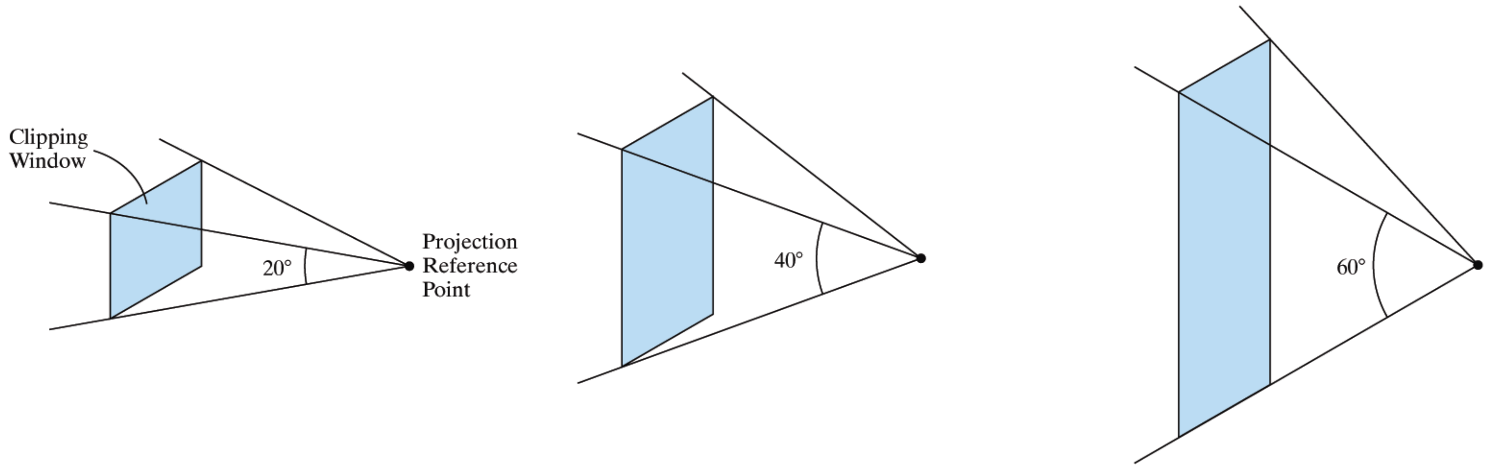
The *field of view* angle can be a more intuitive way to specify perspective projections

This is analogous to choosing a lense for a camera



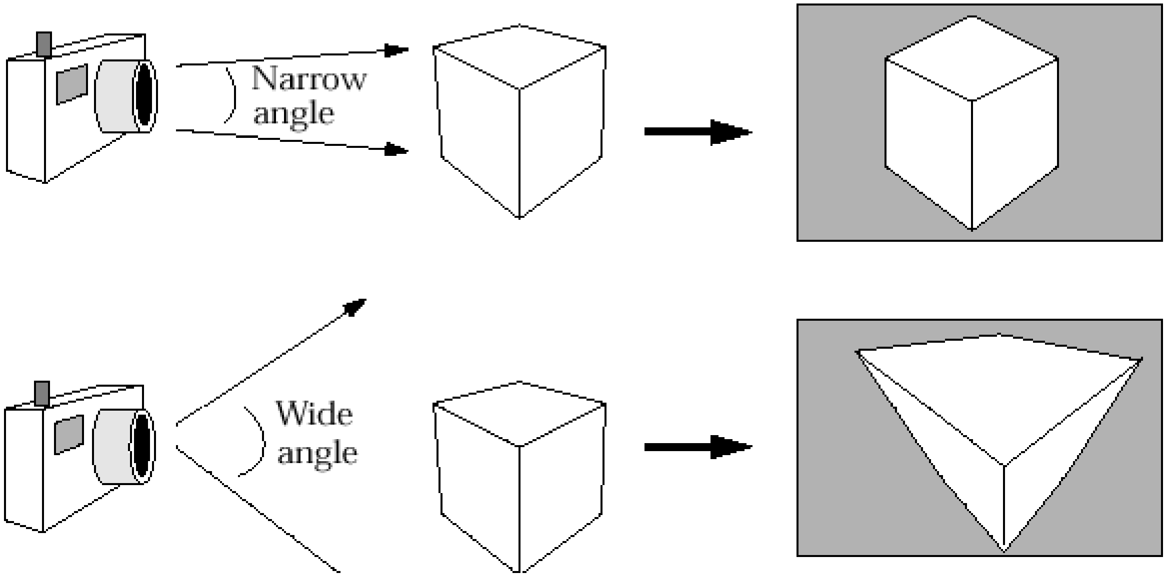
Setting Up A Perspective Projection (cont…)

Increasing the field of view angle increases the height of the view plane and so increases *foreshortening*



Setting Up A Perspective Projection (cont…)

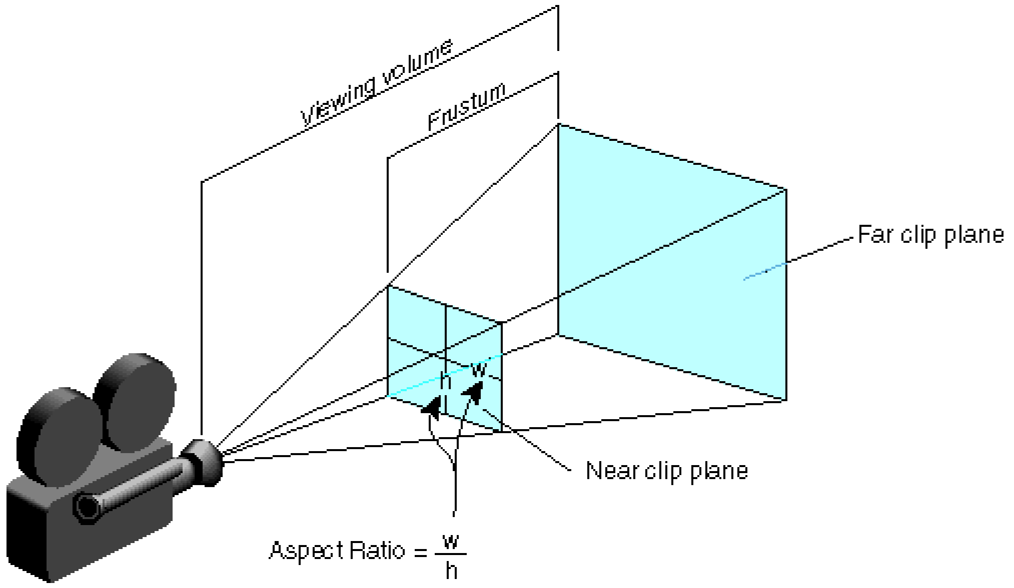
The amount of foreshortening that is present can greatly affect the appearance of our scenes



Setting Up A Perspective Projection (cont…)

We need one more thing to specify a perspective projections using the filed of view angle

The aspect ratio gives the ratio between the width sand height of the view plane



Demo

Summary

In today’s class we looked at the detail of generating a perspective projection of a three dimensional scene