TO FIND THE NATURAL GROUPING PRESENT IN A GIVEN DATA SET USING MST OF DATA POINTS

Objective of the work

- *To find the "natural grouping" of a given data set using MST of data points.*

- *Clustering techniques aim to extract such "natural groups"*

  present in a given data set and each such group is termed

  as a cluster.

CLUSTERING

Let the set of n patterns $S=\{x_1, x_2, \ldots, x_n\} \in \Re^m$ *and* K *clusters are represented by* $C_1, C_2, \ldots, C_K$ *then*

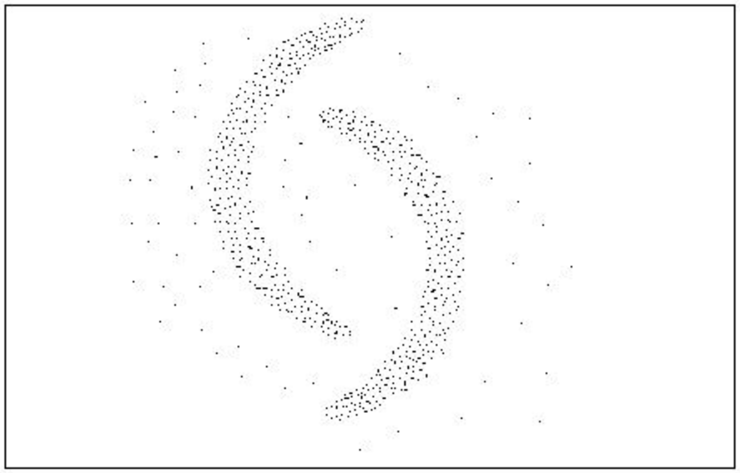
1. $C_i \neq \varphi$, *for* $i= 1, 2, \ldots, K$
2. $C_i \cap C_j = \varphi$ *for* $i \neq j$ *and*
3. $\cup_{i=1}^{K} C_i = S$ *where* φ *represents null set.*

What is Natural Grouping?

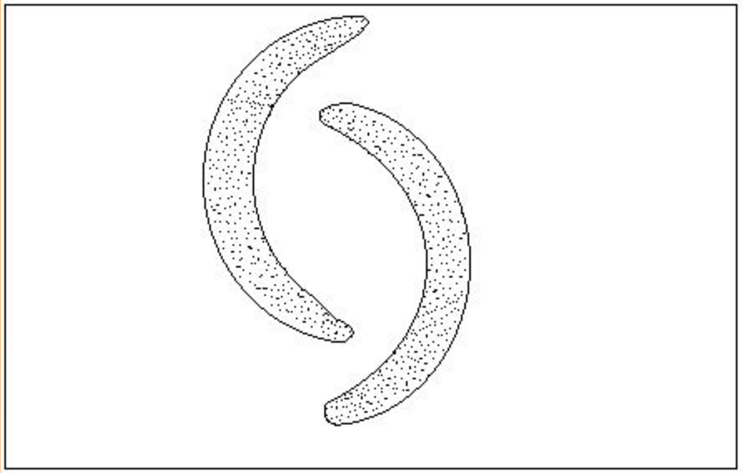
- *For a data set* $S = \{x_1, x_2, \ldots, x_n\} \in \Re^m$ *, what one perceives to be the groups present in* S *by viewing the scatter diagram*

  of S, is termed as natural groups of S.

Natural Grouping

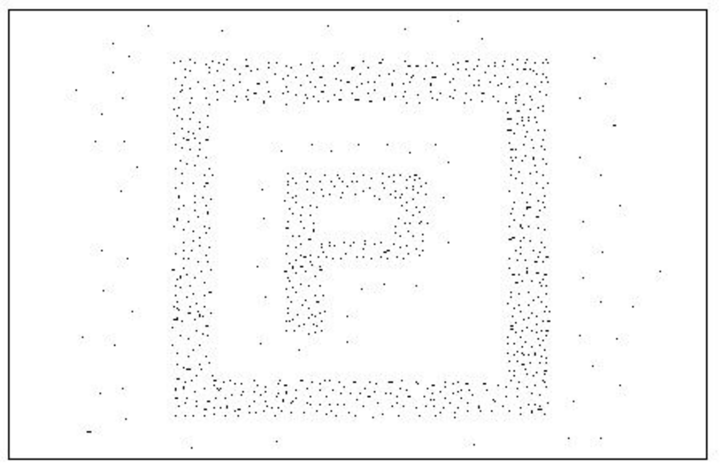


Scatter Diagram

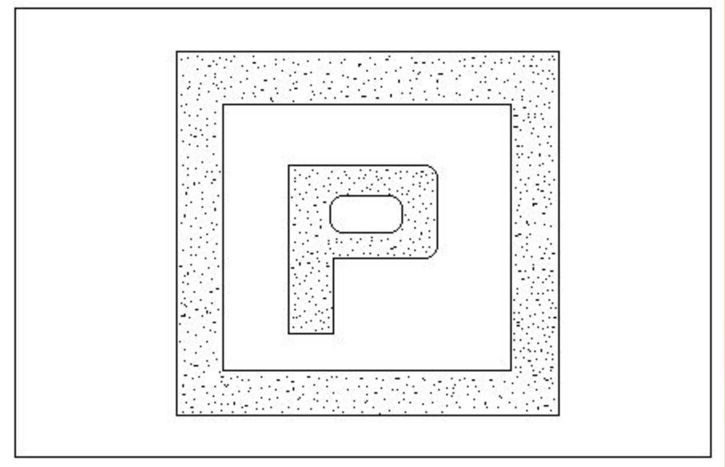


Natural Grouping

Natural Grouping



Scatter Diagram



Natural Grouping

Widely used Algorithms

- *K-Means Algorithm.*
- *ISODATA Algorithm.*

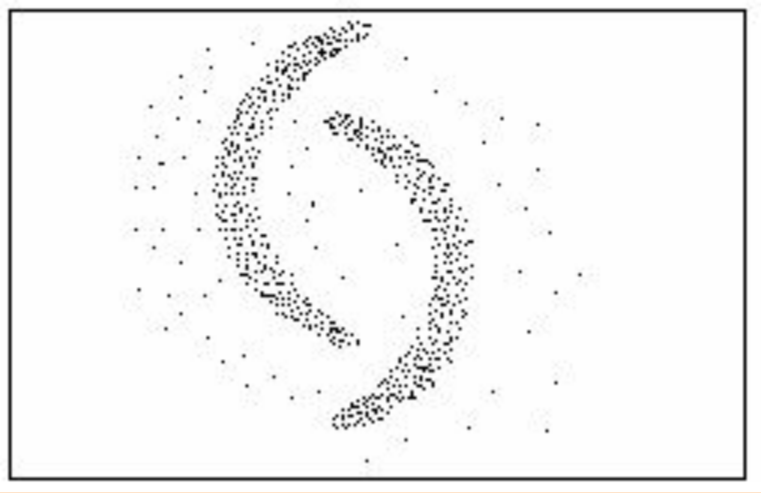
Disadvantage of K-Means Algorithm

- *It needs the number of clusters to be known a priori.*
- *It can find the grouping if clusters exhibit characteristics pocket and are not placed very close to each other.*
- *It may stack at a local minima.*
- *It can not provide proper grouping in case of some data having typical shape and size.*

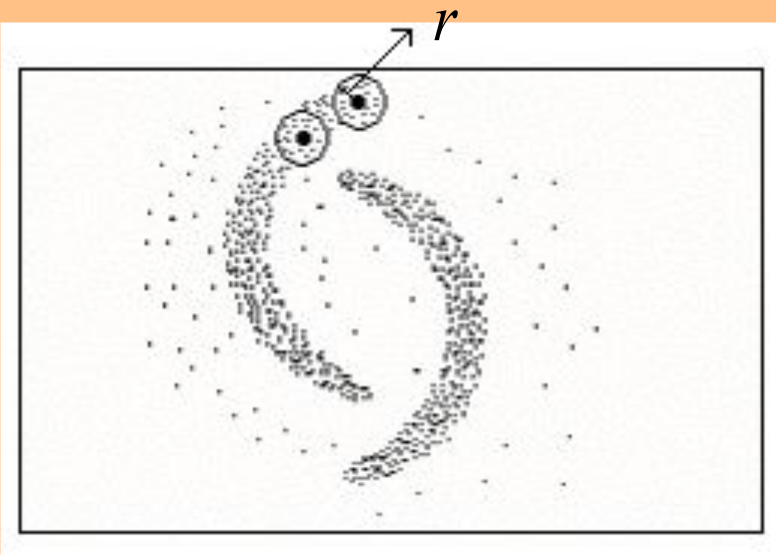
To find the natural grouping based on local densities of the data points

- *To find high density regions of a given data set.*
- *To merge those high density regions "suitably" along with the data*

  points of other regions to result in clustering.
- *To eliminates noise if any from the final clustering.*



Scatter Diagram

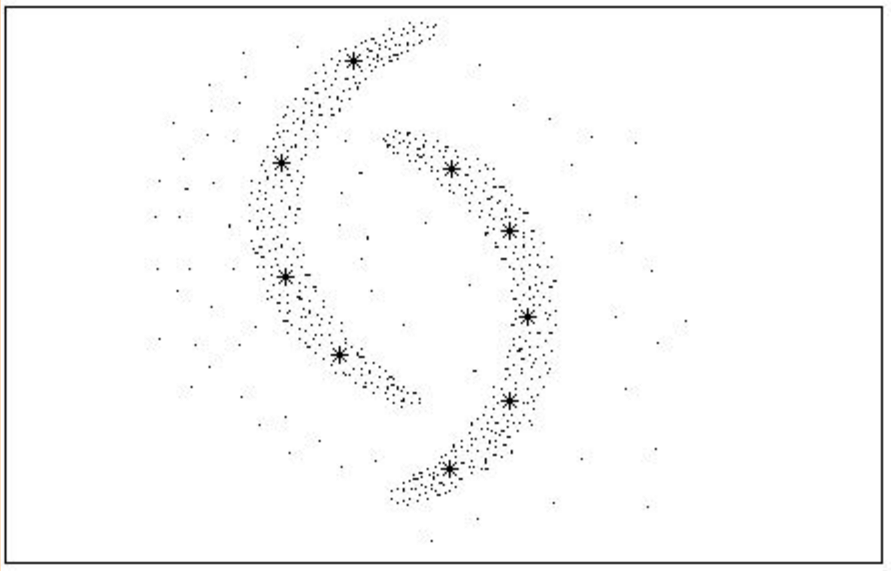


Finding density of each data points

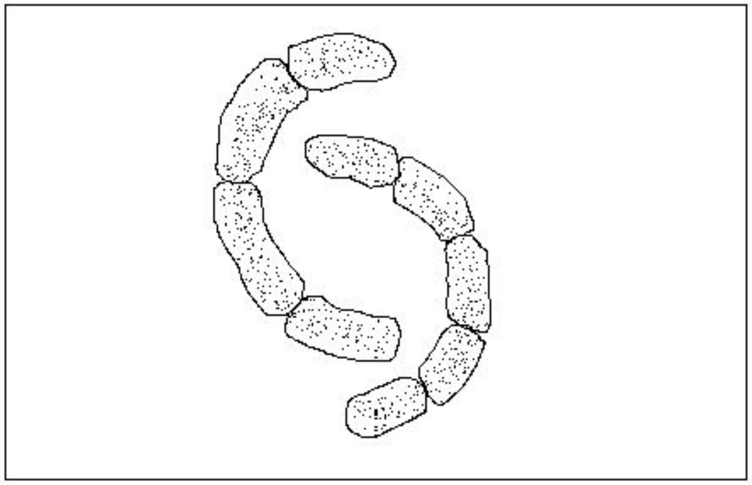
Radius for the open disk to compute the density of each data point is taken to be

$$r = h_n = \left(\frac{l_n}{n} \right)$$

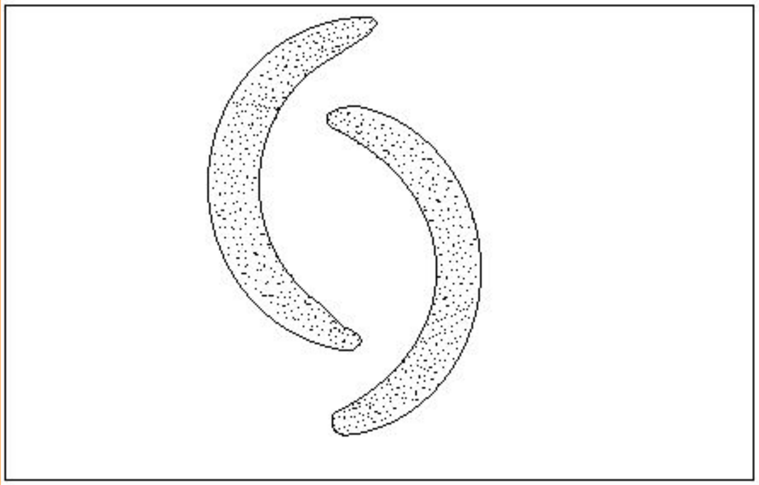
l_n → Sum of edge weights of minimal spanning tree of S.
n → Number of data points in S.
The edge weight is taken to be the Euclidean distance.



Scatter diagram with seed points



Scatter Diagram with 9 Groups



Natural Grouping By Proposed Method

Algorithm AL-1:

Step 1: Let $S=\{x_1,x_2,\ldots\ldots,x_n\}\subseteq\Re^m\ (m\ge 2)$. *Find the MST of* S *with the edge weight as the Euclidean distance. Let* $h_n=\left(\frac{l_n}{n}\right)$ *and the radius* $r=h_n$.

Step 2: Compute the density (the number of points) for each datum x as the number of other data units within an open disc of radius h_n *with x as center.* m_i *denote the density of the point* x_i, *i=1, …, n. In other words, let*

$$A_i=\{y:\|x_i-y\|\le h_n,\ y\in S\},\ i=1,2,\ldots,n$$

and

$$m_i=\#A_i,\ i=1,2,\ldots,n.$$

(#A means the number of points of the set A).

Step 3: Rearrange $m_1, m_2, \ldots\ldots, m_n$ in increasing order. Let the rearrangement be $m_1^*, m_2^*, \ldots\ldots, m_n^*$. Let $p_j, j = 1, 2, \ldots., n$ represent the corresponding cumulative sums of $m_1^*, m_2^*, \ldots\ldots, m_n^*$. i.e.

$$p_j = \sum_{i=1}^{j} m_i^*, j = 1, 2, \ldots.., n.$$

Step 4: Compute $M = \left[\frac{w}{100} \times n \right]$ where [a] means integral part of 'a'

i.e. the largest integer ≤ a. Find the value of i for which p_i is nearest to M.

Choose $k = m_i^*$ for that i.

If $p_i < M < p_{i+1}$ and $M - p_i = p_{i+1} - M$ then choose $k = m_{i+1}^*$.

We have taken the value of w to be 85.

Step 5: *Find the set $S_1 \subseteq S$ such that every point in S_1 has density at least equal to k i.e. Let* $S_1 = \{x_i : m_i \geq k,\ x_i \in S\} \subseteq S$. S_1 *represents the set of high density points of S.*

Step 6: *Arrange the points of S_1 according to their density. Choose the point having highest density as the first seed point.*

Step 7: *Choose subsequent seed points from the array of points from S_1 subject to the stipulation that each new seed point is at least at a distance $2h_n$ from all other previously chosen seed points. Continue until all remaining data units of S_1 are exhausted. Let V be the set of seed points of S. Let t = #V. Let V = { z_j , j=1, 2, …,t }. V is the*

Step 8: *Stop.*

ALGORITHM AL-2

Step 1: *Let* $V = \{z_j, j = 1, 2, ..., t\}$ *be the set of seed points of*

$$S = \{x_1, x_2, ..., x_n\} \subseteq \Re^m (m \geq 2).$$

Step 2: *Divide the n points of S into t groups in the following way:*

Put x_i *in the* j_{th} *group* C_j *utilizing the minimum squared Euclidean distance classifier concept: i. e.* $x_i \in z_j$ *if*

$$\left\| x_i - z_j \right\|^2 < \left\| x_i - z_q \right\|^2 \quad \forall q \in \{1, 2, ..., t\},\ q \neq j.$$

and $\quad \cup_{q=1}^{t} C_q = S.$

Step 3: *For two groups C_i and C_j find d_{ij} where* $d_{ij} = Min\{d(x_{m1}, x_{m2})\}$, $x_{m1} \in C_i, x_{m2} \in C_j$. *If* $d_{ij} \le h_n$ *then merge those two groups C_i and C_j into one group and name it as C_i.*

Step 4: *Repeat Step 3 for all possible pairs of i and j.*

Step 5: *Stop.*

Algorithm to Eliminate Noise

Step 1: Let $C_i, \ \{i = 1, 2, ..., K\}$ be the K clusters of

$$S = \{x_1, x_2, ..., x_n\} \subseteq \Re^m (m \geq 2).$$

Step 2: For each group C_i, compute the distance $d_j, \forall x_j \in C_i$

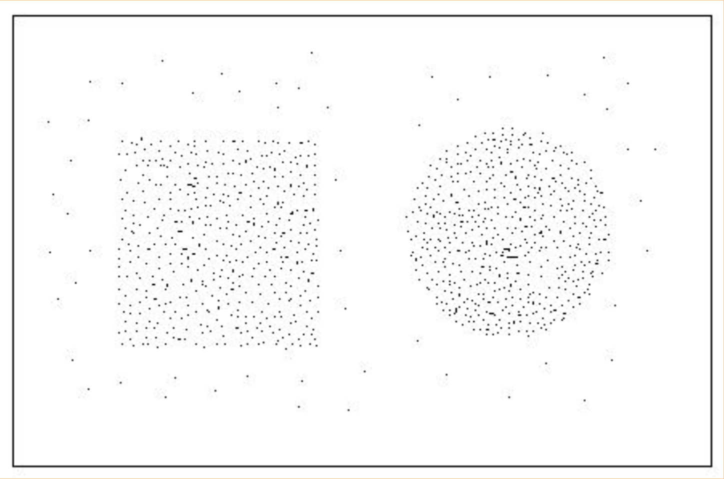
where $d_j = Min\{d(x_j, x_l)\}, \ j \neq l, \ x_l \in C_i$. If $d_j > 2h_n$

then remove the data point x_j from the clustering obtained by AL-2.

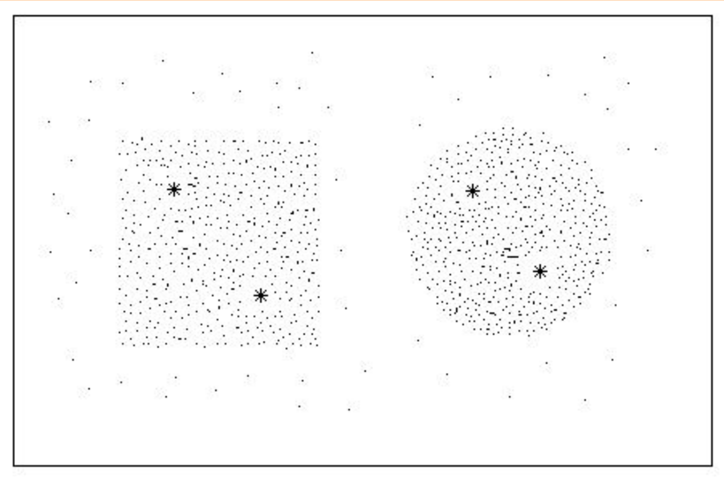
Step 3: Repeat Step 2 for all possible pairs of i and j.

Step 4: Stop.

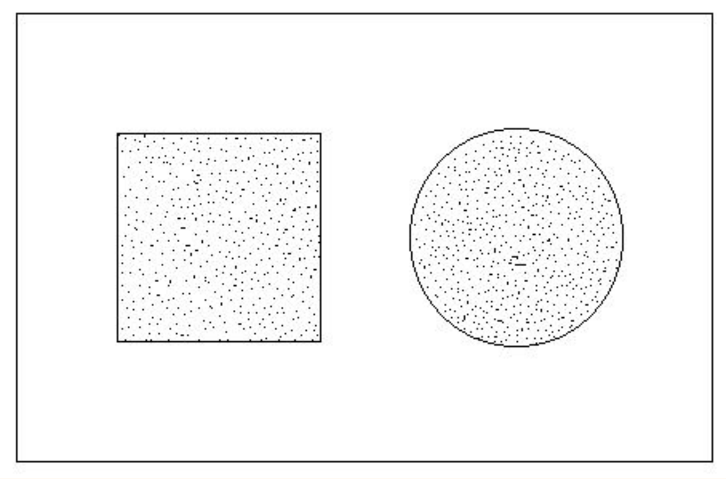
Experimental Results



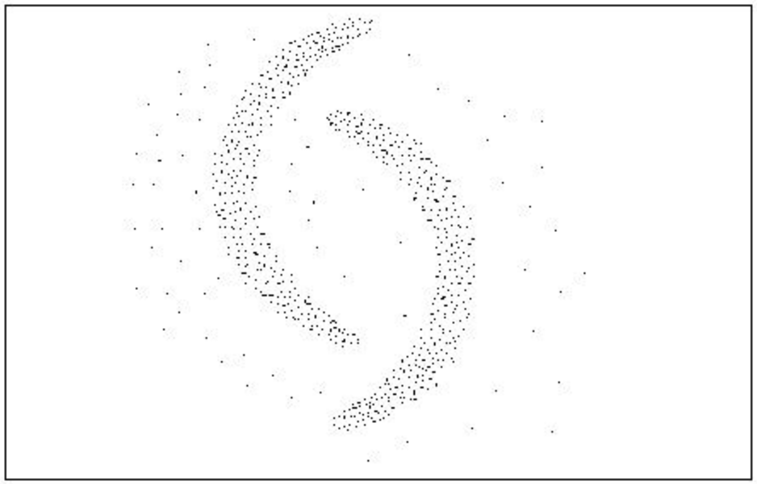
Scatter diagram of synthetic data with noise



Scatter diagram with seed points



Clustering by the proposed method



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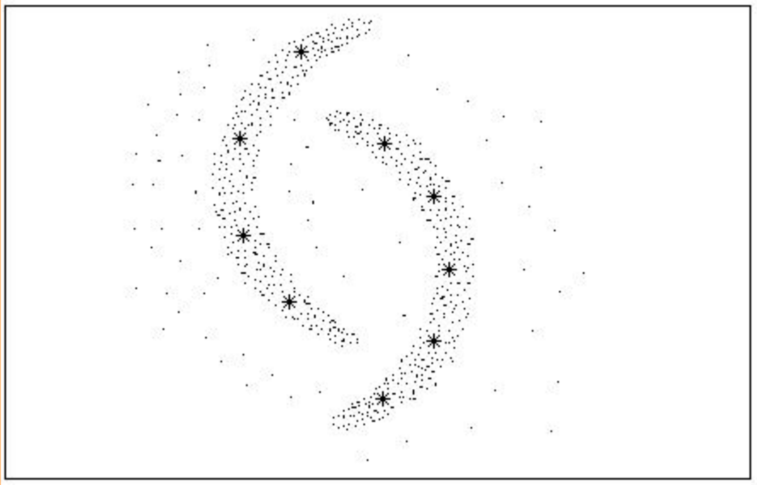

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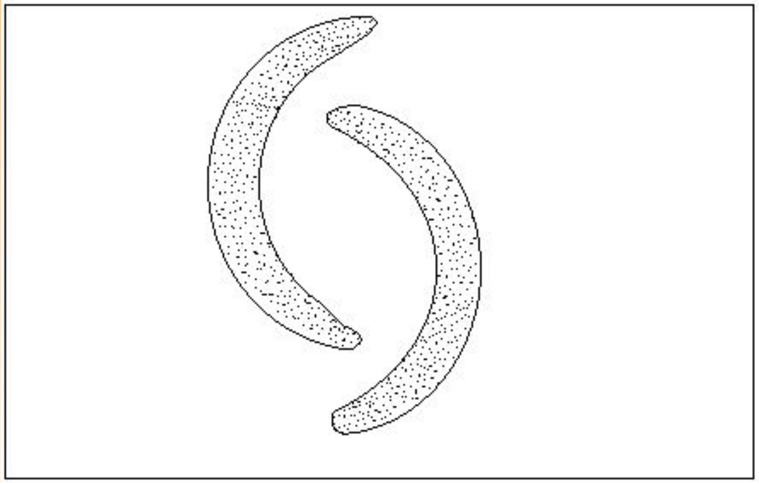

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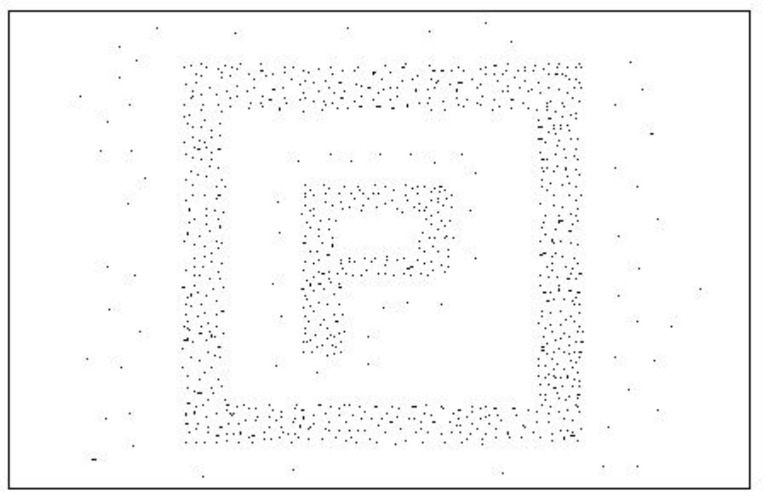

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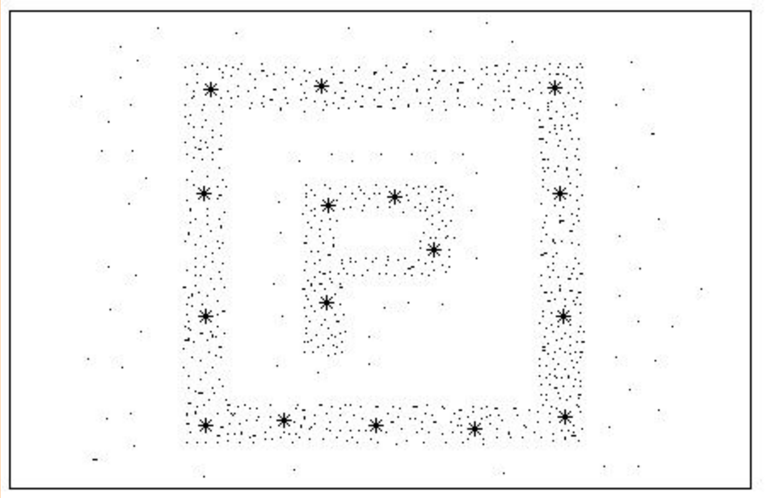

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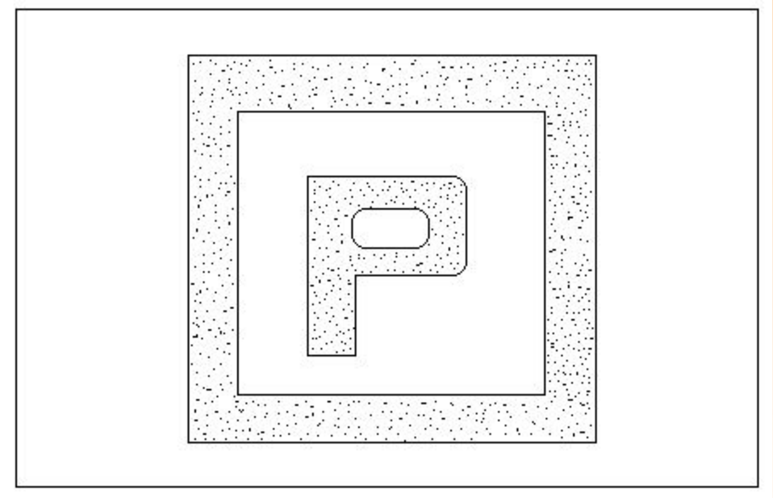

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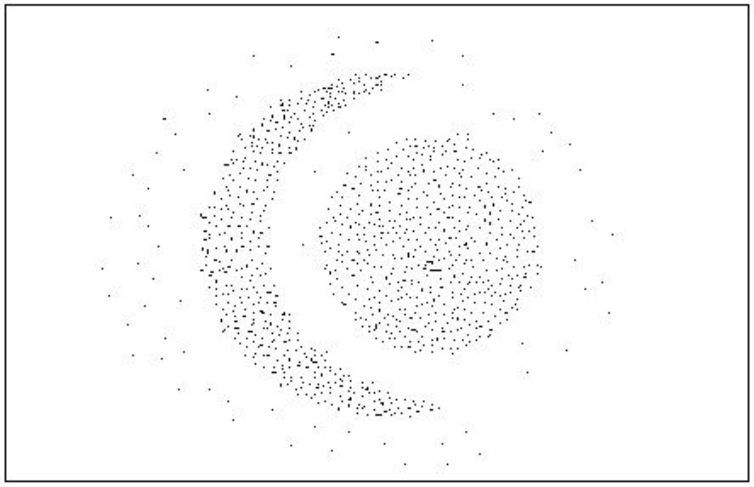

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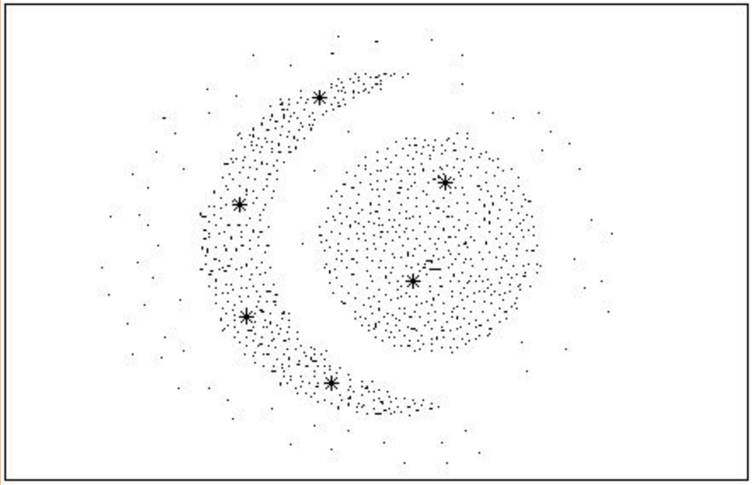

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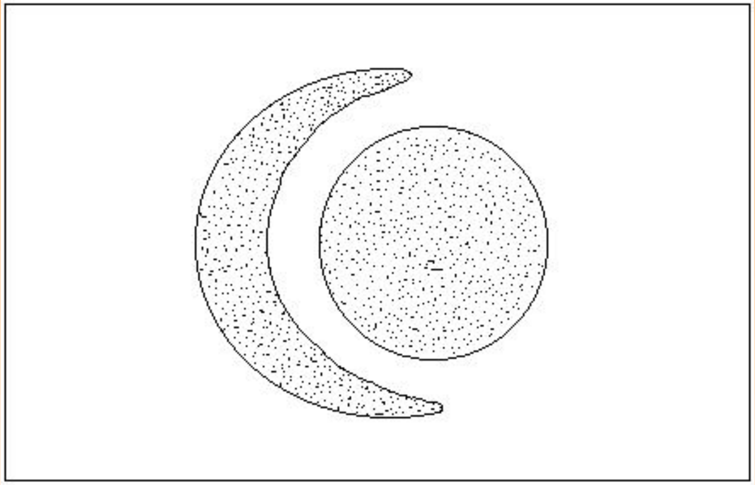

Clustering by the proposed method

Thank You