BCSE IVth Year - 2016
1st Semester
FORMAL LANGUAGE & AUTOMATA THEORY

Time: Three hours | Full Marks: 100

Answer any ***five*** questions

1. (a) For each language L, define the language

$$2L = \{a_1a_1a_2a_2 \cdots a_na_n : a_1a_2 \cdots a_n \in L\}$$

i.e each string of $2L$ is obtained by repeating each symbol of a string of L twice. Prove that if L is accepted by a DFA, then $2L$ is also accepted by a DFA.

(b) For each language L, define the language

$$\text{MIN}(L) = \{x \in L : \text{every proper prefix of } x \notin L\}.$$

Prove that if L is accepted by a DFA, then MIN(L) is also accepted by a DFA.

10+10

2. (a) Let M be a DFA without any unreachable state. Assuming that the equivalence classes of its states are already known, explain how the minimum state DFA equivalent to M can be constructed.

(b) Prove the consistency and correctness of the equivalent minimum state DFA construction.

5+15

3. (a) Let L_n = {all strings of a, b whose nth symbol from the right hand end is a}. Prove that any DFA accepting L_n must have at least 2^n states.
Explain if this bound remains true when all such strings over the alphabet $\{a, b, c\}$ are considered.

(b) Define the equivalence relation among the states of a DFA. Prove that this relation is indeed an equivalence relation.
Let M be a DFA without any unreachable state. If no two states of M are equivalent, prove that M is a minimum state DFA for the language $L(M)$.

8+12

4. (a) Describe an algorithm for determination of equivalent states of a DFA, explaining the data structure to be used.
Explain how the equivalence classes can be found from the table used at the end of the algorithm.

(b) Prove the correctness of the algorithm.

6+14

5. (a) Develop grammar for the language $\{a^nb^nc^n : n \geq 1\}$with necessary proof.

(b) Develop grammar for the complement of the language $L = \{ww : w \in \{a, b\}^*\}$ with necessary proof.

10+10

6. Construct Push Down Automatas to accept the following languages with necessary justifications :

(a) $\{a^ib^jc^k : i = j \text{ or } j = k\}$.

(b) $\{ww^R : w \in \{a, b\}^*\}$

In each case explain the mode of acceptance and if the PDA is deterministic. 10+10

7. (a) Prove that a language is accepted by a PDA with empty stack if and only if it is accepted by a PDA with final states.

   (b) Prove that the intersection of a regular language with a context-free language is context-free.

15+5