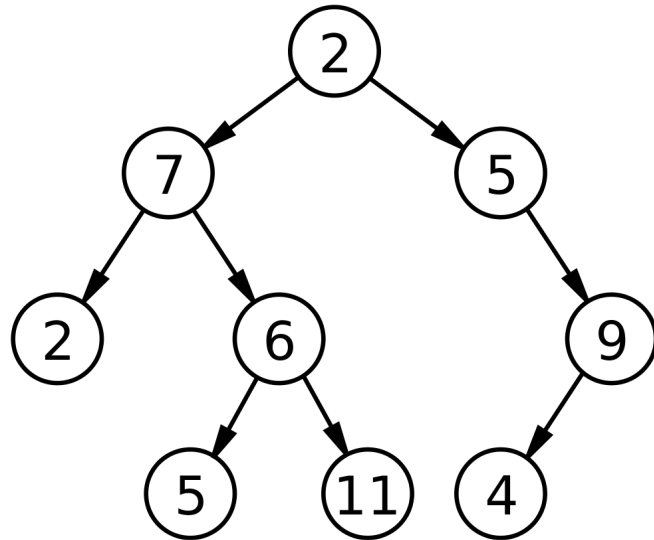

Binary Trees

Introduction

Binary Tree

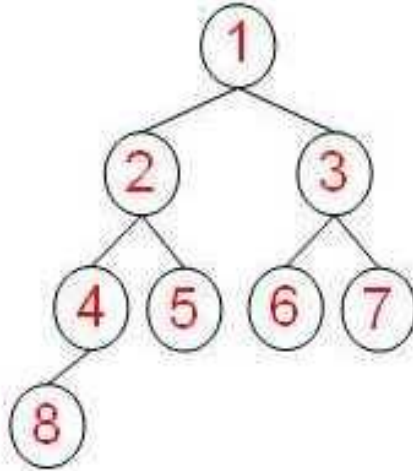
A **binary tree** is a tree data structure in which each node has at most two children, which are referred to as the *left child* and the *right child*.



Types of Binary Trees

Complete Binary Tree:

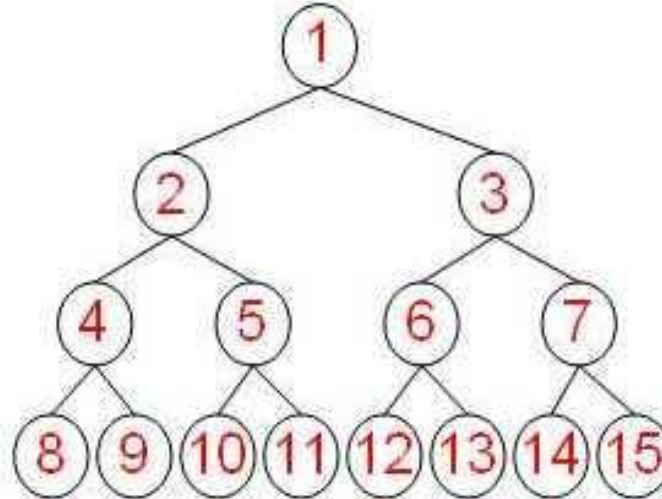
In a **complete** binary tree every level, *except possibly the last*, is completely filled, and all nodes in the last level are as far left as possible.



(a). Complete Tree

Full Binary Tree:

A **full** binary tree (sometimes referred to as a **proper** or **plane** binary tree) is a tree in which every node has either 0 or 2 children.



(b). Full Tree

Binary Tree Traversals

Tree Traversal: Process of visiting (checking and/or updating) each node in a tree data structure, exactly once.

Binary Tree Traversals

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Unlike linked lists, one-dimensional arrays etc., which are canonically traversed in linear order, trees may be traversed in multiple ways. They may be traversed in depth-first or breadth-first order.

Binary Tree Traversals

Tree Traversal: Process of visiting (checking and/or updating) each node in a tree data structure, exactly once.

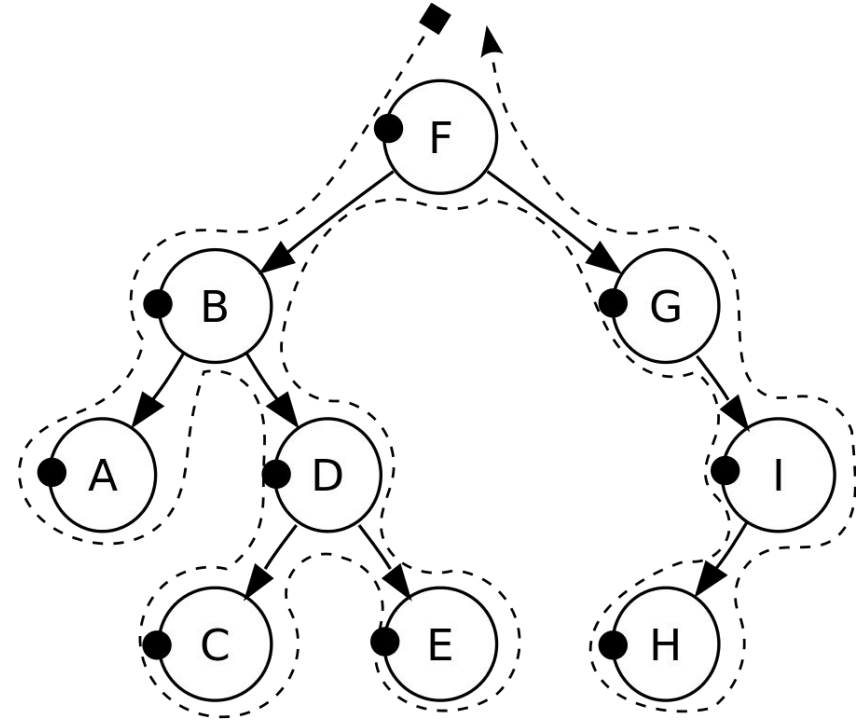
Unlike linked lists, one-dimensional arrays etc., which are canonically traversed in linear order, trees may be traversed in multiple ways. They may be traversed in depth-first or breadth-first order.

There are three common ways to traverse them in depth-first order: in-order, pre-order and post-order.

Pre-order Traversal

1. Check if the current node is empty / null.
2. Display the data part of the root (or current node).
3. Traverse the left subtree by recursively calling the pre-order function.
4. Traverse the right subtree by recursively calling the pre-order function.

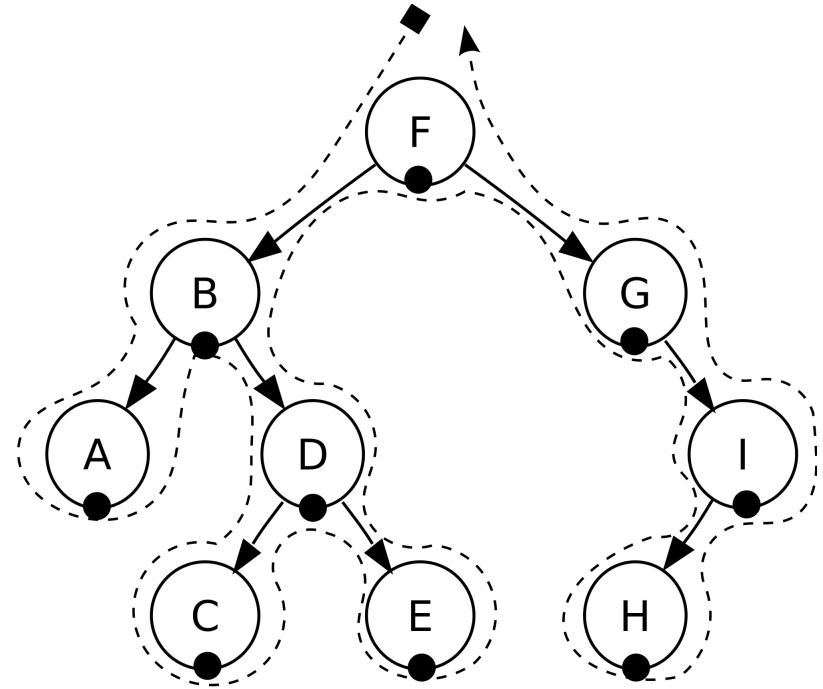
Pre-order: F, B, A, D, C, E, G, I, H.



In-order Traversal

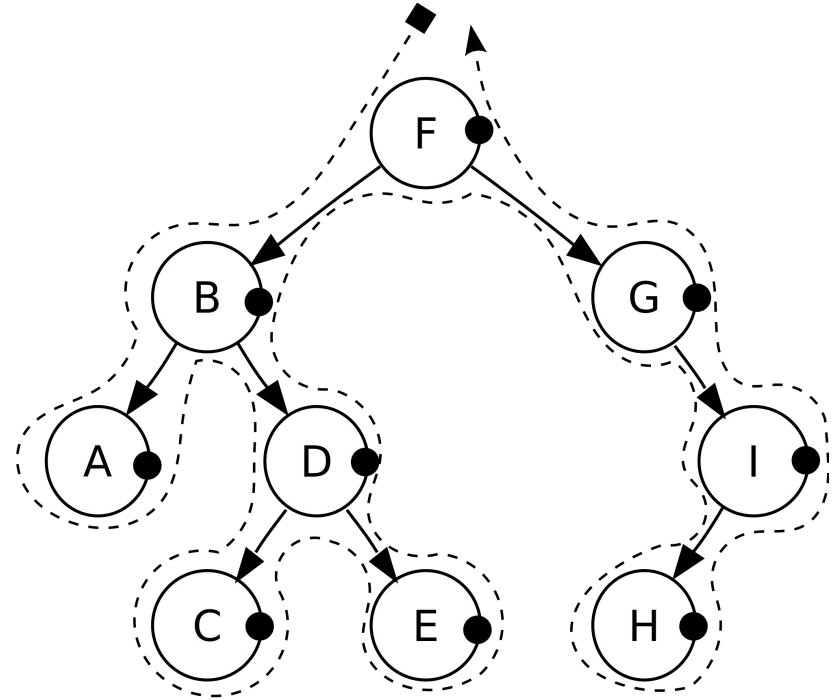
1. Check if the current node is empty / null.
2. Traverse the left subtree by recursively calling the in-order function.
3. Display the data part of the root (or current node).
4. Traverse the right subtree by recursively calling the in-order function.

In-order: A, B, C, D, E, F, G, H, I.



Post-order Traversal

1. Check if the current node is empty / null.
2. Traverse the left subtree by recursively calling the post-order function.
3. Traverse the right subtree by recursively calling the post-order function.
4. Display the data part of the root (or current node).

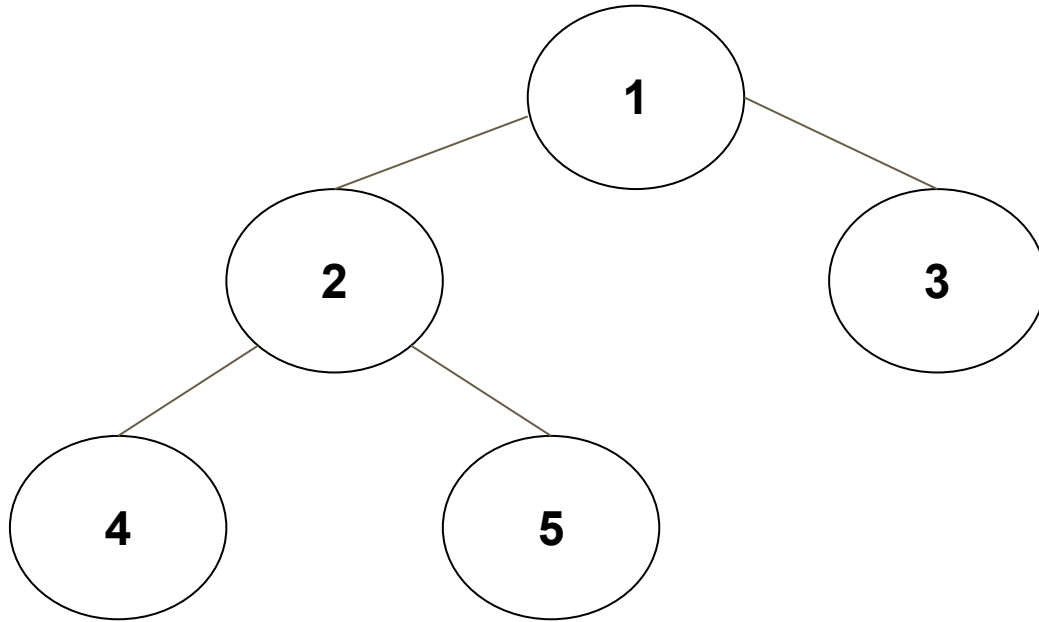


Post-order: A, C, E, D, B, H, I, G, F.

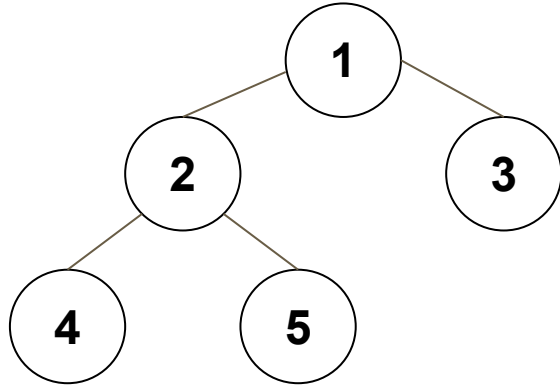
Level-order Traversal

Level-order traversal

Level-order traversal:
1,2,3,4,5

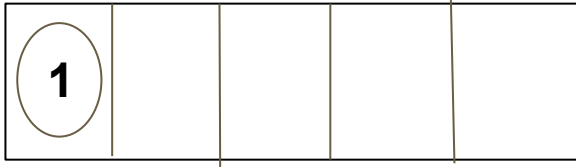


Level-order traversal

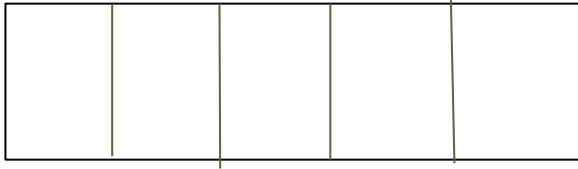
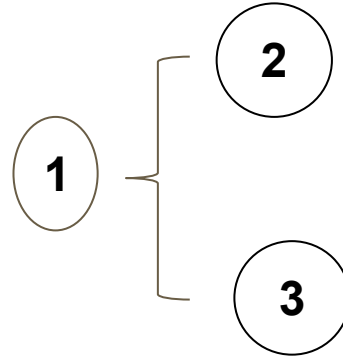
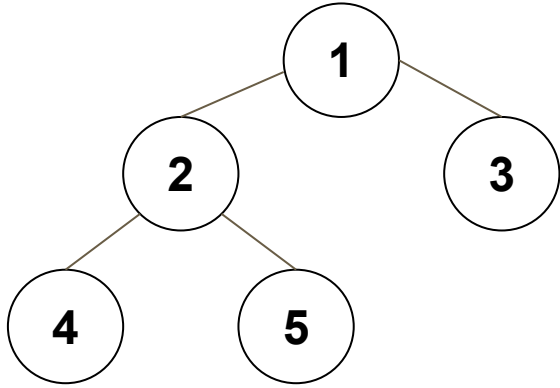


Level-order traversal:

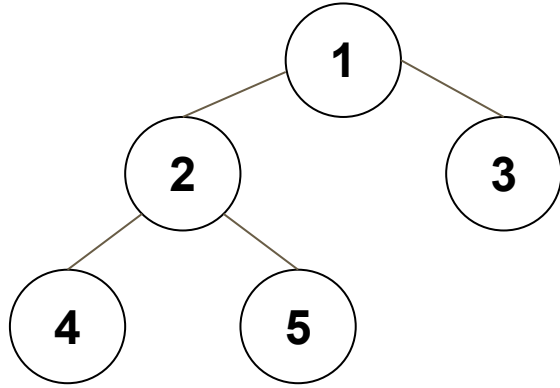
1,



Level-order traversal



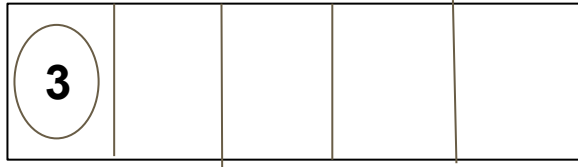
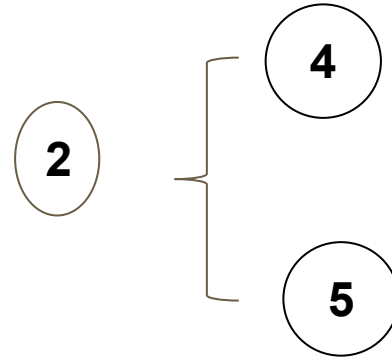
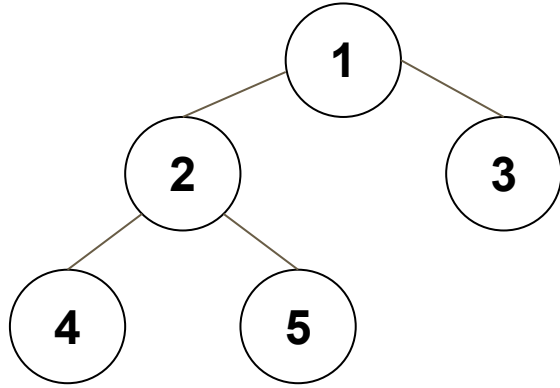
Level-order traversal



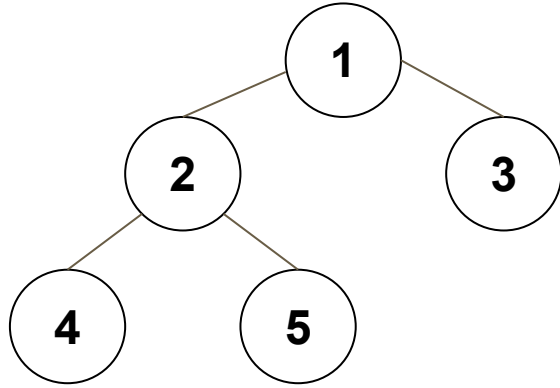
Level-order traversal:
1,2



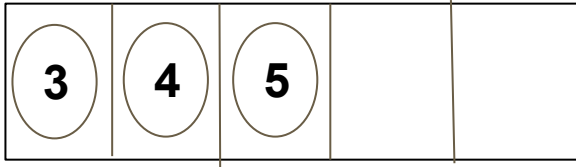
Level-order traversal



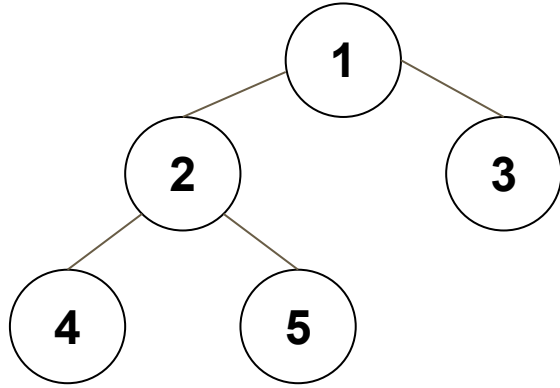
Level-order traversal



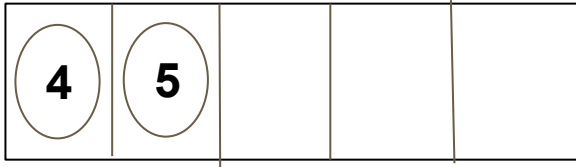
Level-order traversal:
1,2,3



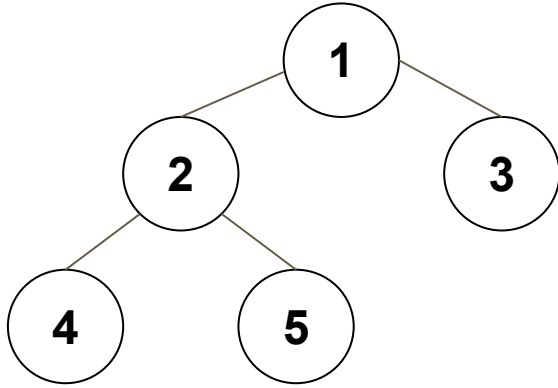
Level-order traversal



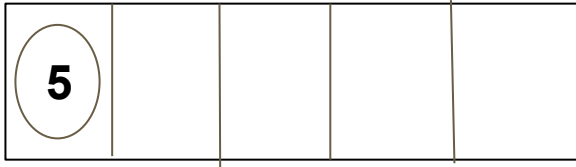
Level-order traversal:
1,2,3,4



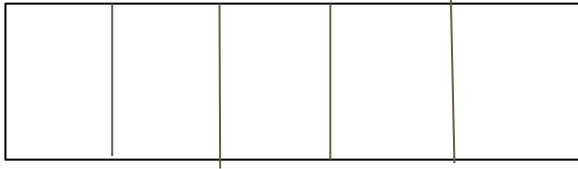
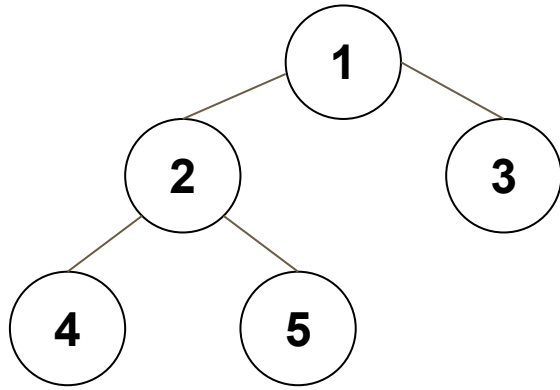
Level-order traversal



Level-order traversal:
1,2,3,4,5



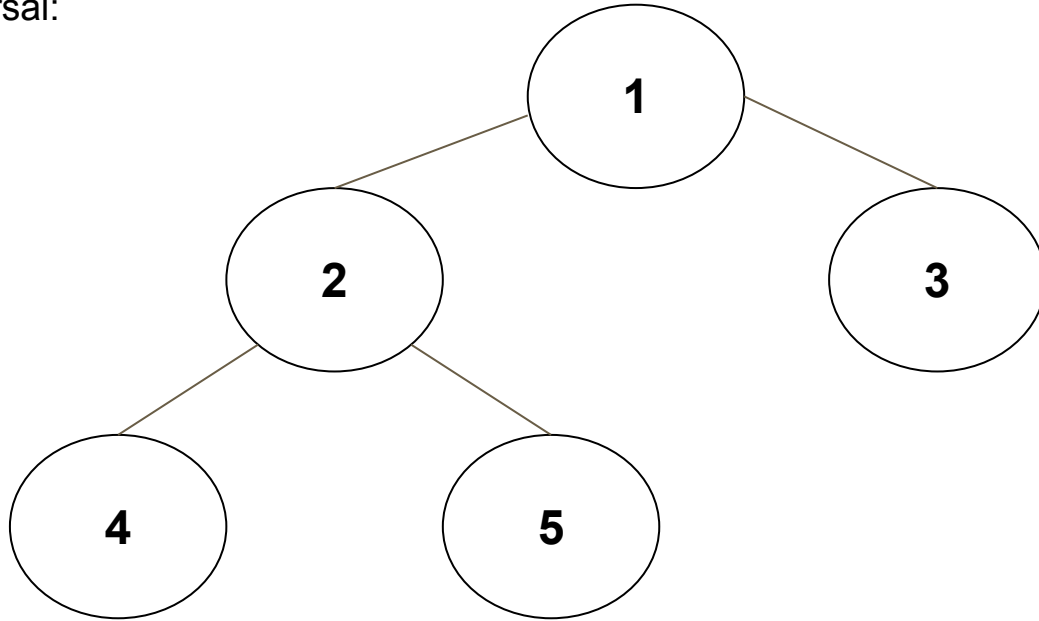
Level-order traversal



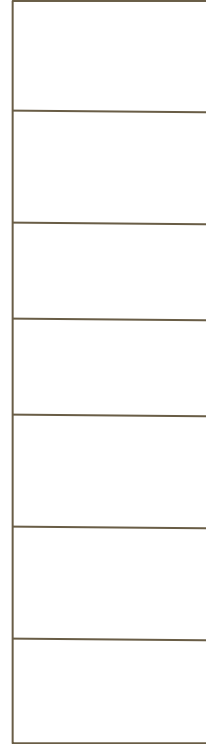
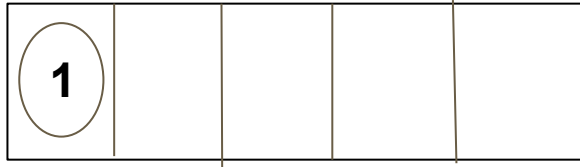
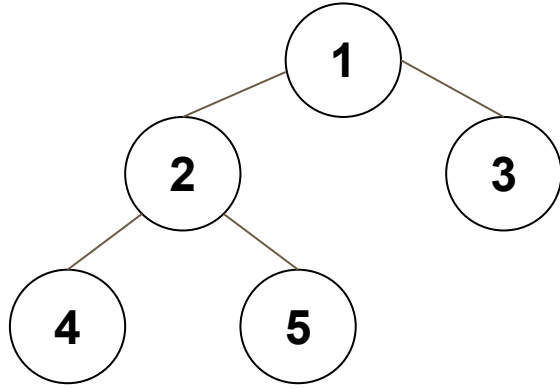
Reverse Level-order Traversal

Reverse Level-order traversal

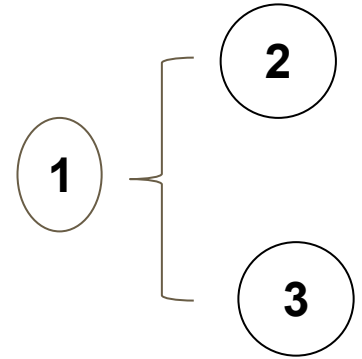
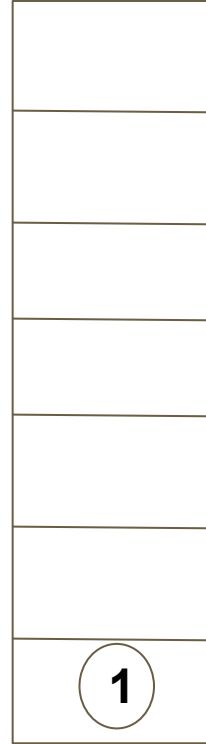
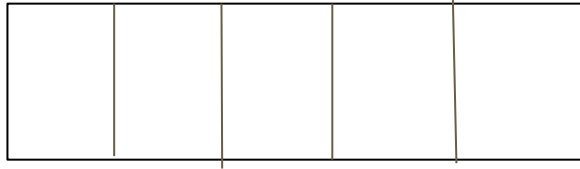
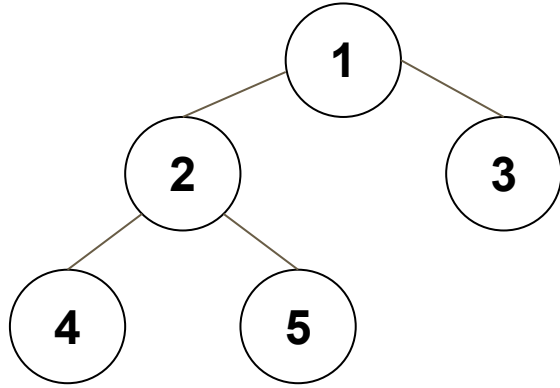
Reverse Level-order traversal:
4,5,2,3,1



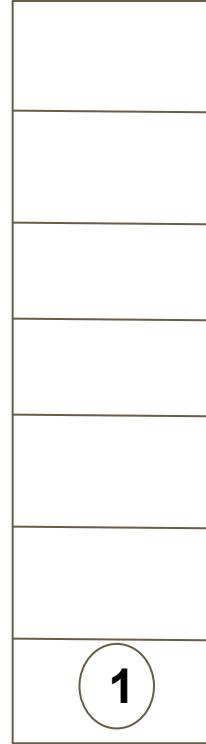
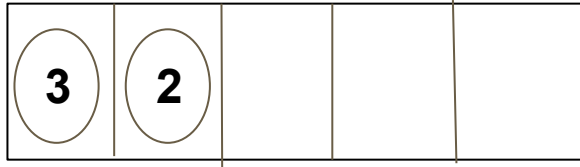
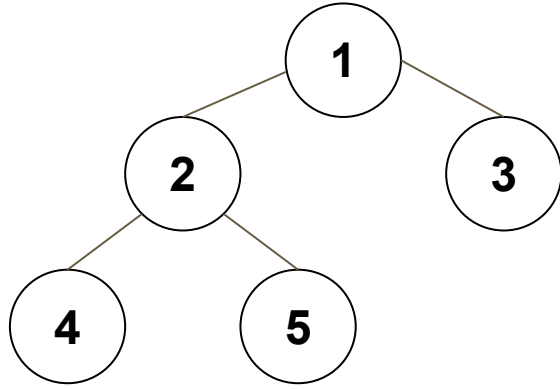
Reverse Level-order traversal



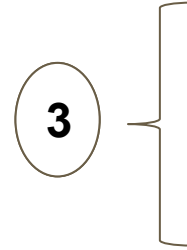
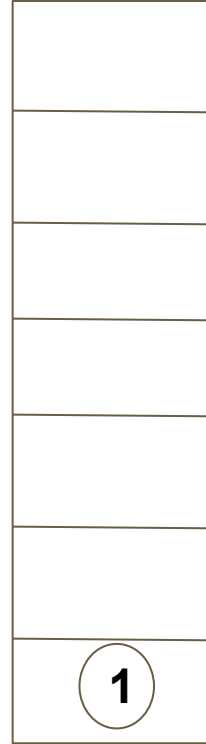
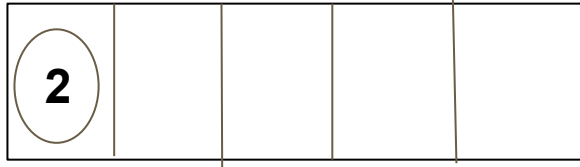
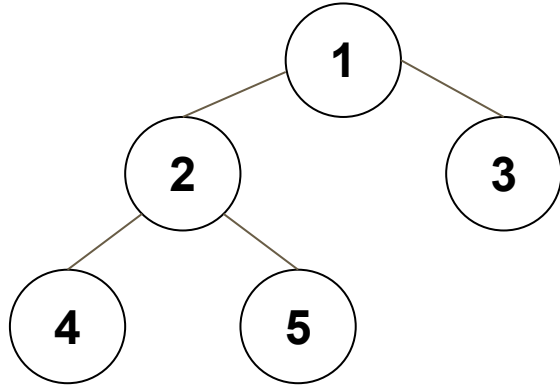
Reverse Level-order traversal



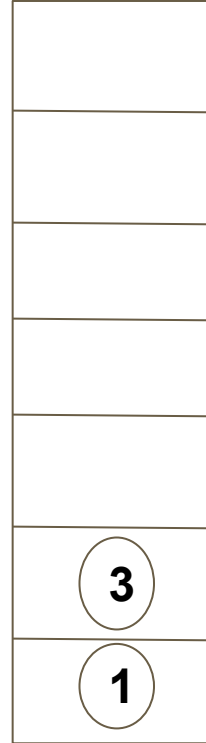
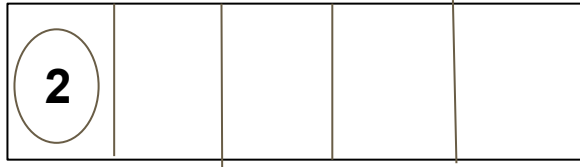
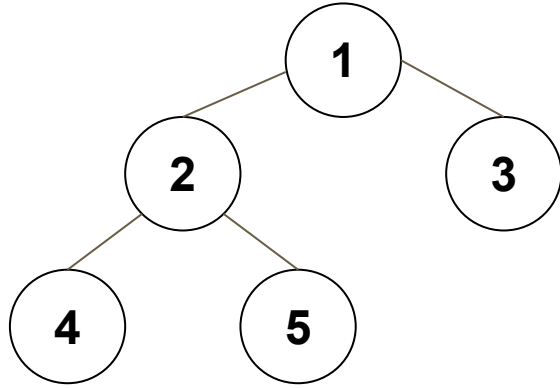
Reverse Level-order traversal



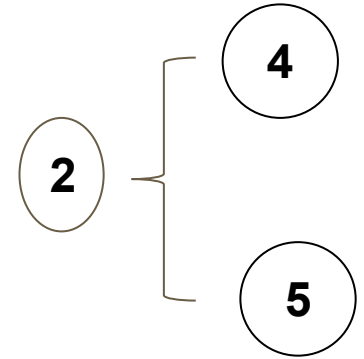
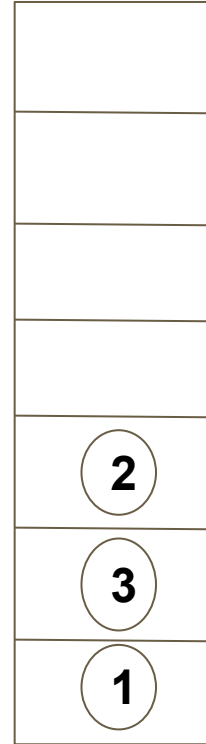
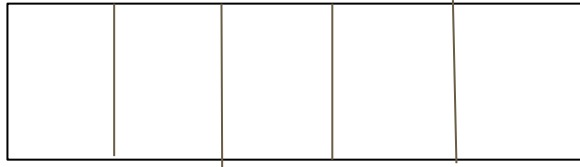
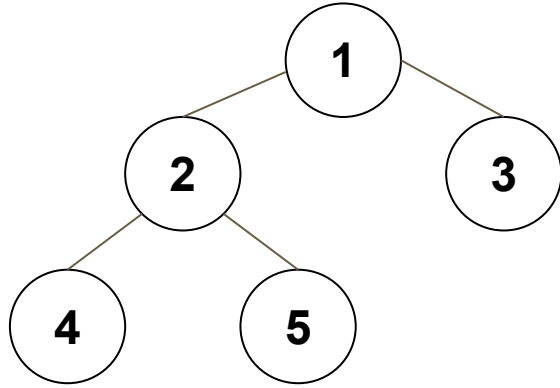
Reverse Level-order traversal



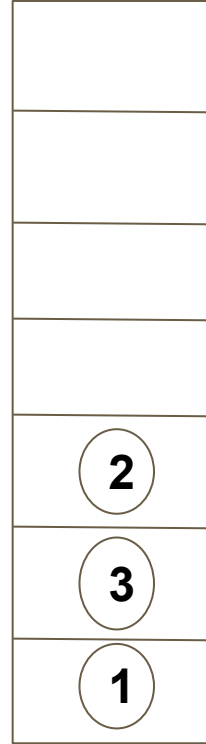
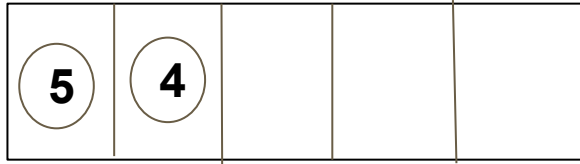
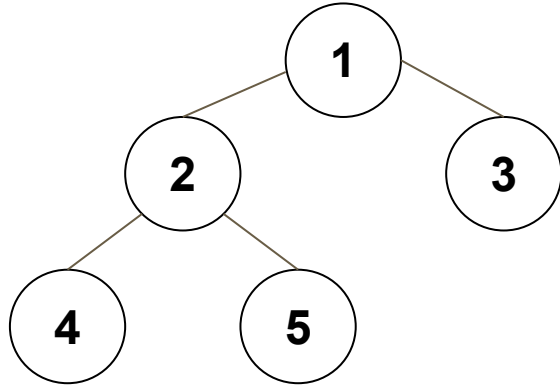
Reverse Level-order traversal



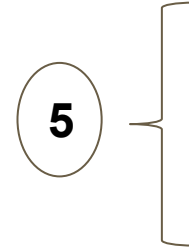
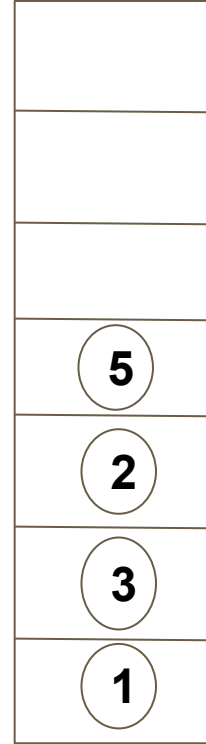
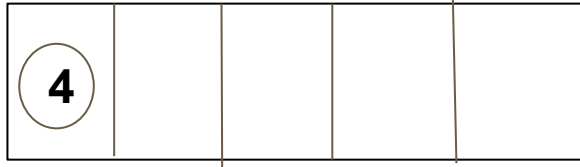
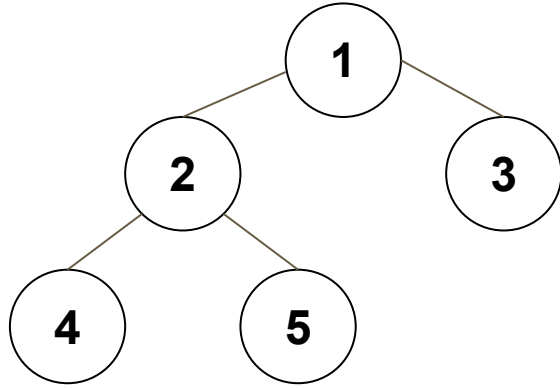
Reverse Level-order traversal



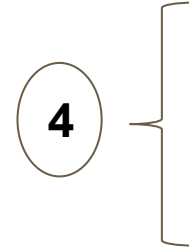
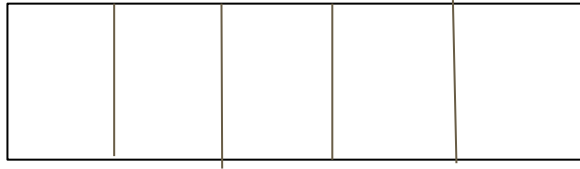
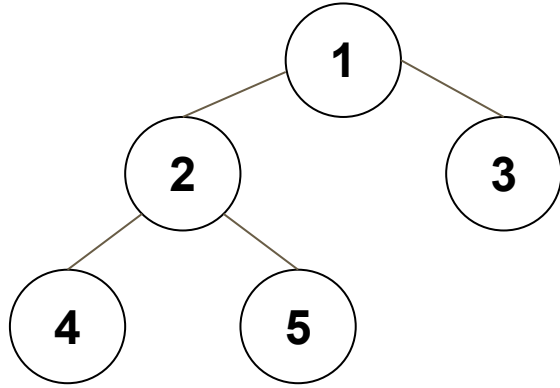
Reverse Level-order traversal



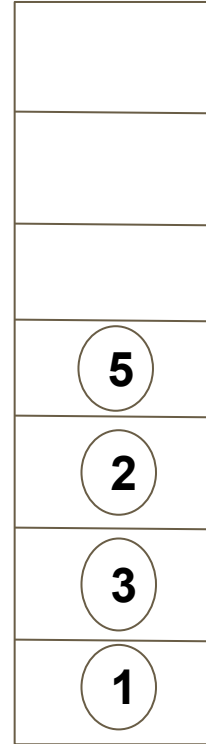
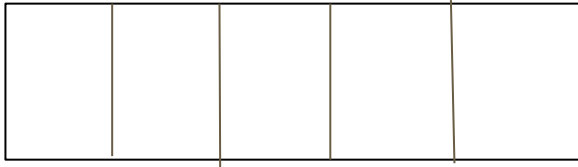
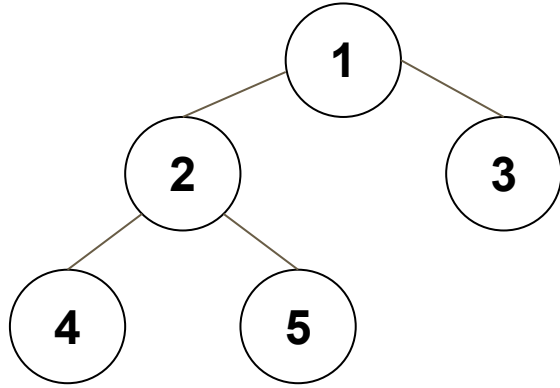
Reverse Level-order traversal



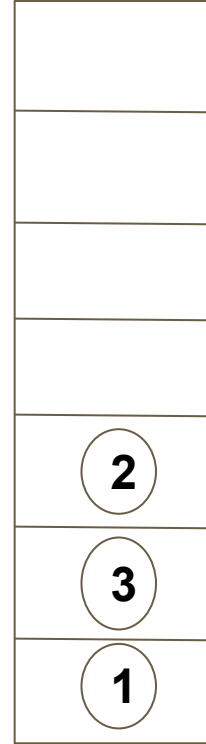
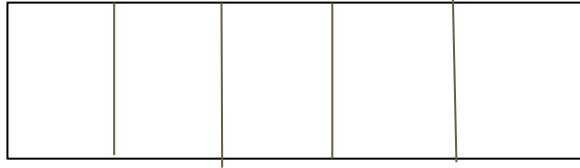
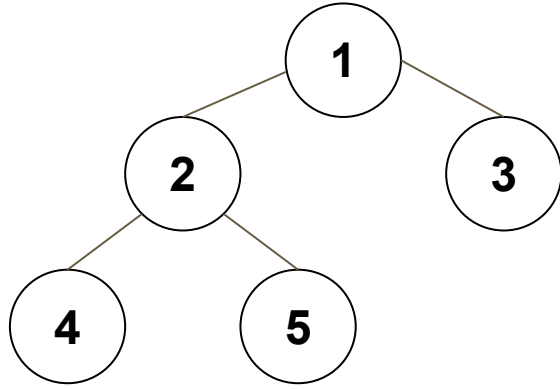
Reverse Level-order traversal



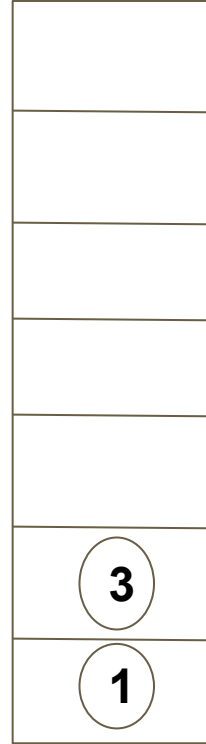
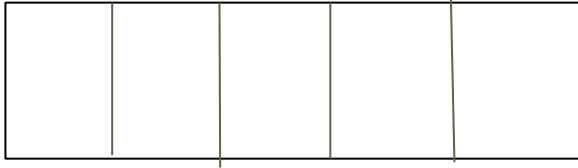
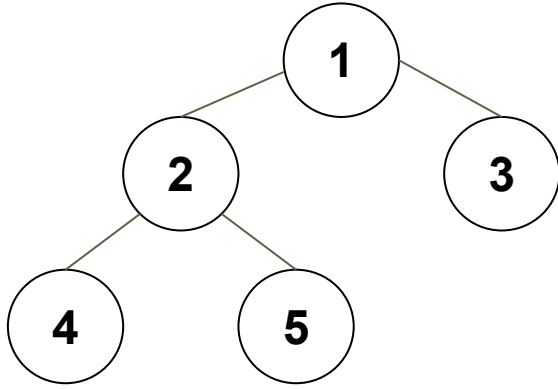
Reverse Level-order traversal



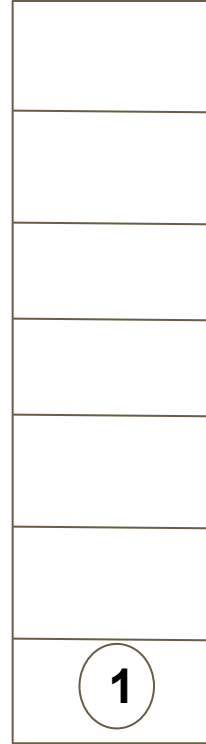
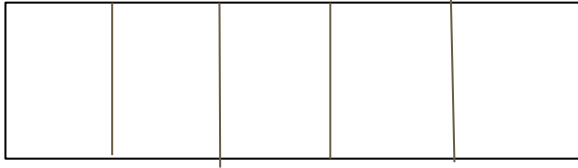
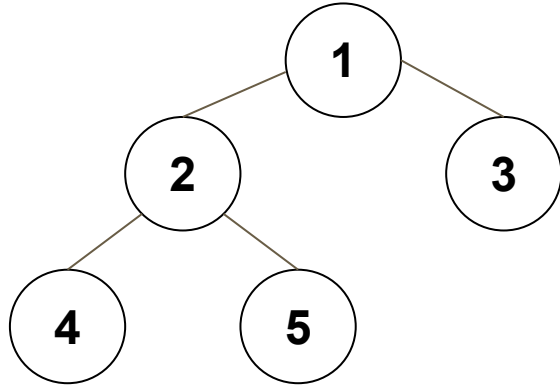
Reverse Level-order traversal



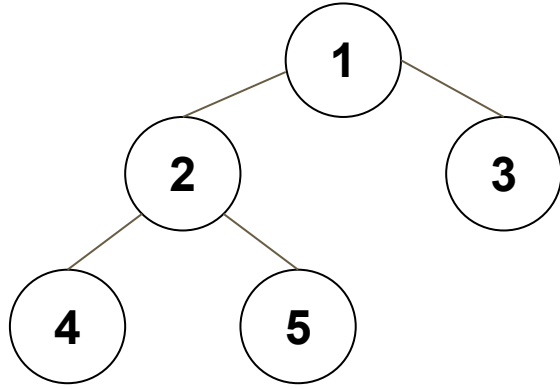
Reverse Level-order traversal



Reverse Level-order traversal



Reverse Level-order traversal



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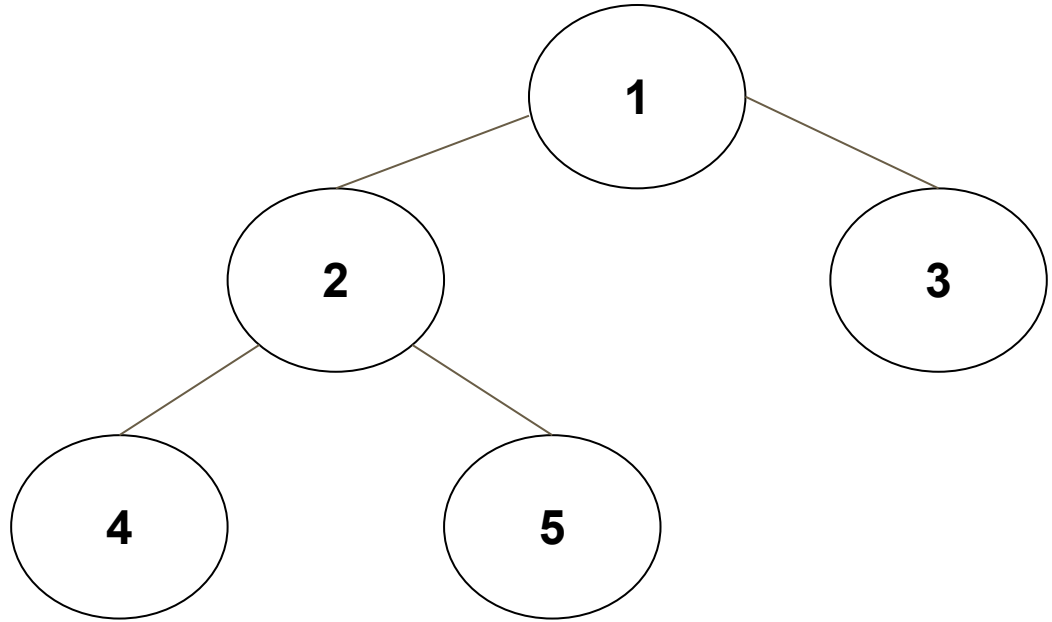


Size of Binary Tree

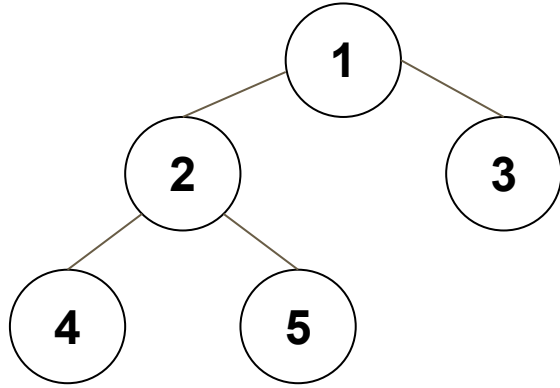
Calculate Size of Binary Tree

Size of Tree:

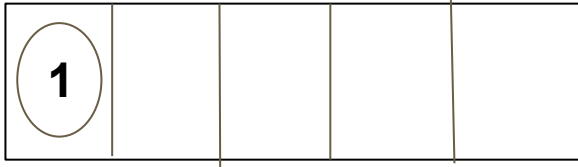
The total number of nodes in the tree.



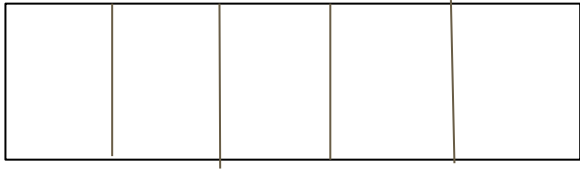
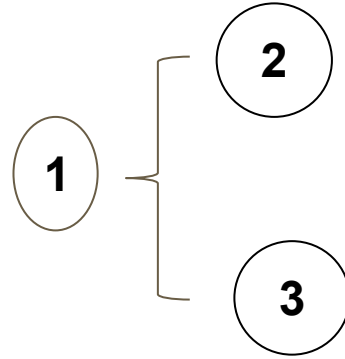
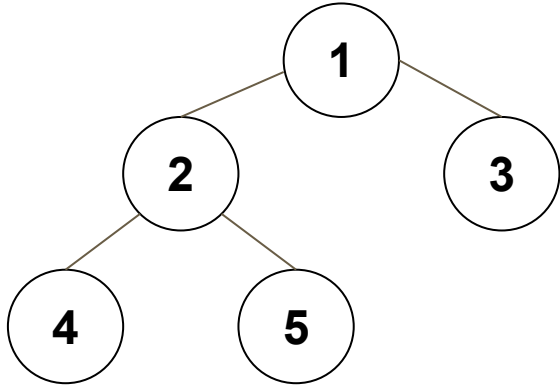
Size of Binary Tree



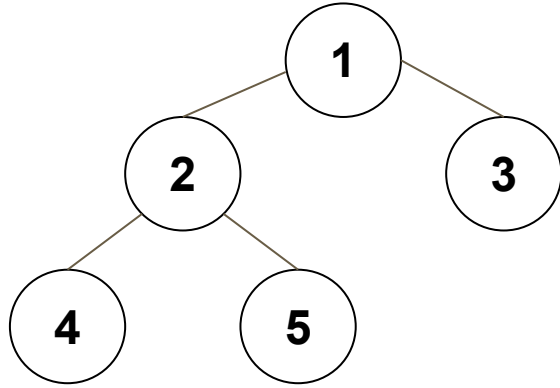
Size: 1



Size of Binary Tree



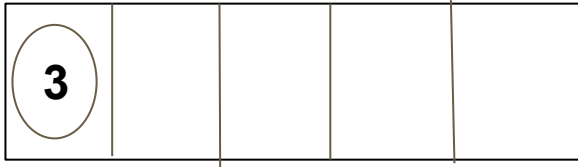
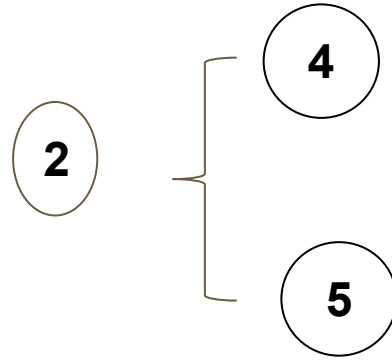
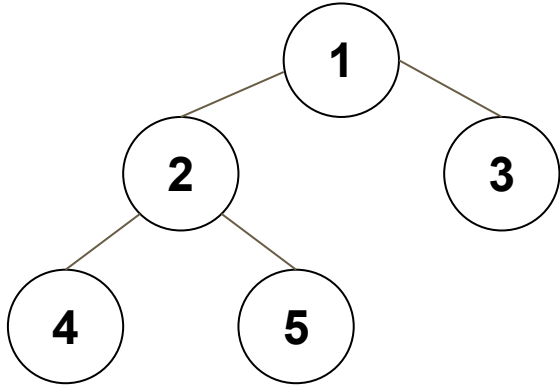
Size of Binary Tree



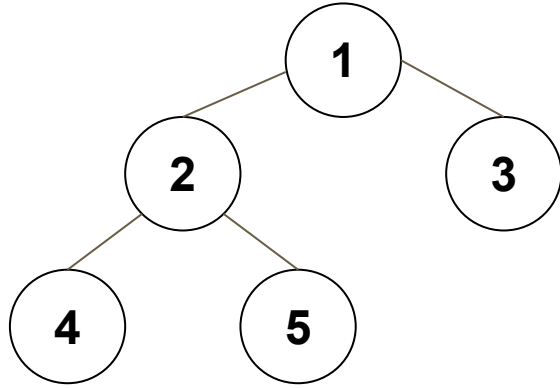
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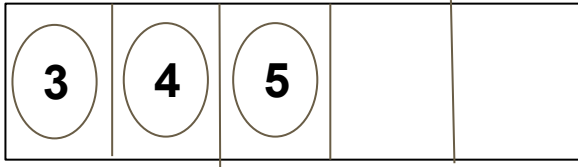
Size of Binary Tree



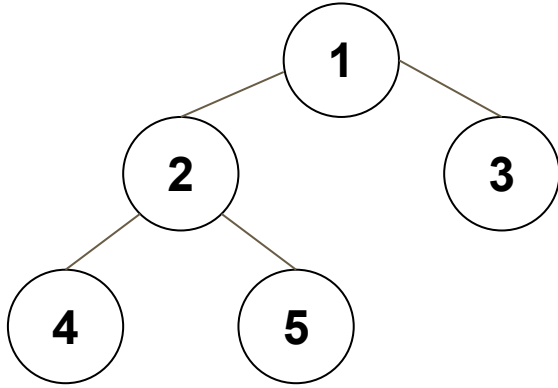
Size of Binary Tree



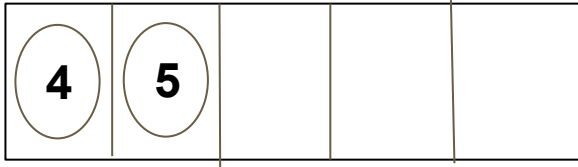
Size: 5



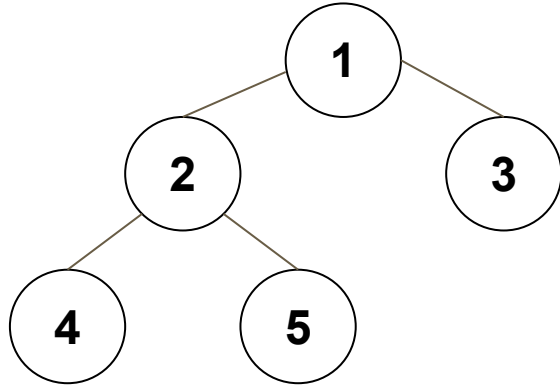
Size of Binary Tree



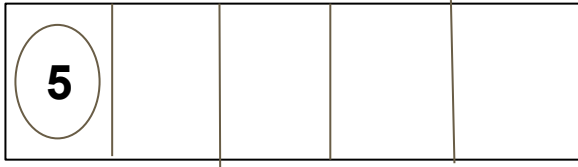
Size: 5



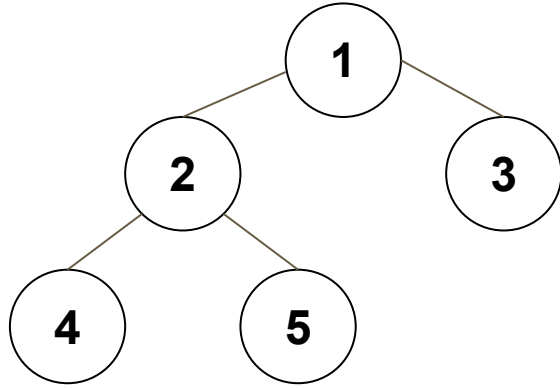
Size of Binary Tree



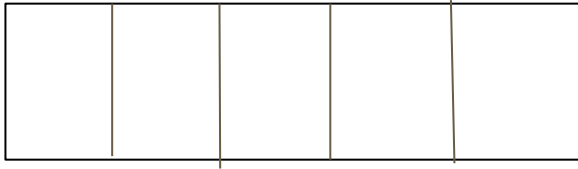
Size: 5



Size of Binary Tree



Size: 5



Height of Binary Tree

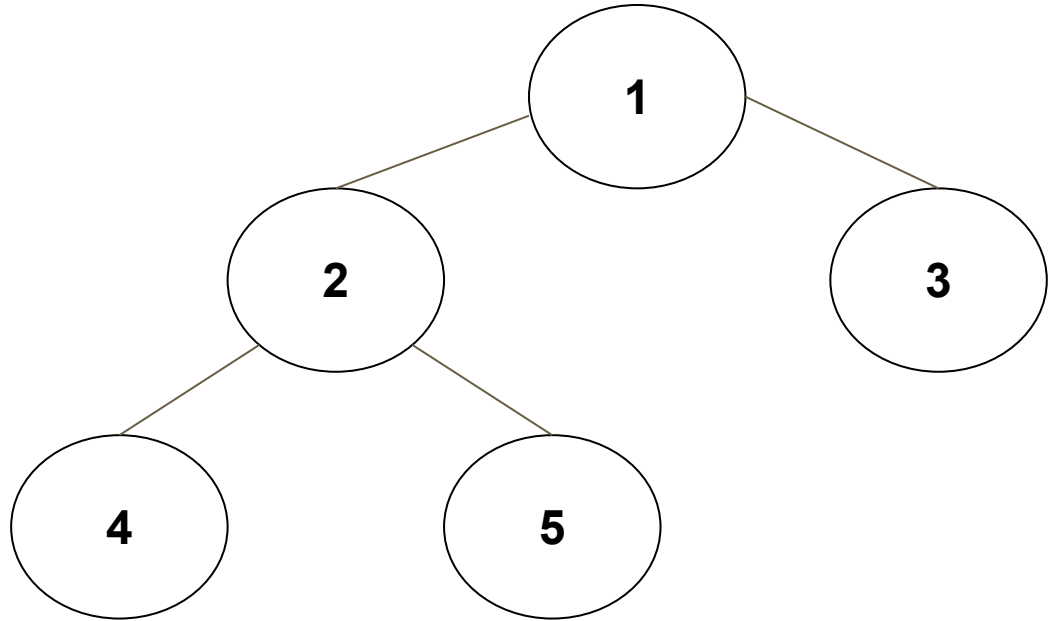
Calculate Height of Binary Tree

Height of Tree:

The height of a tree is the height of its root node.

Height of Node:

The height of a node is the number of edges on the longest path between that node and a leaf.



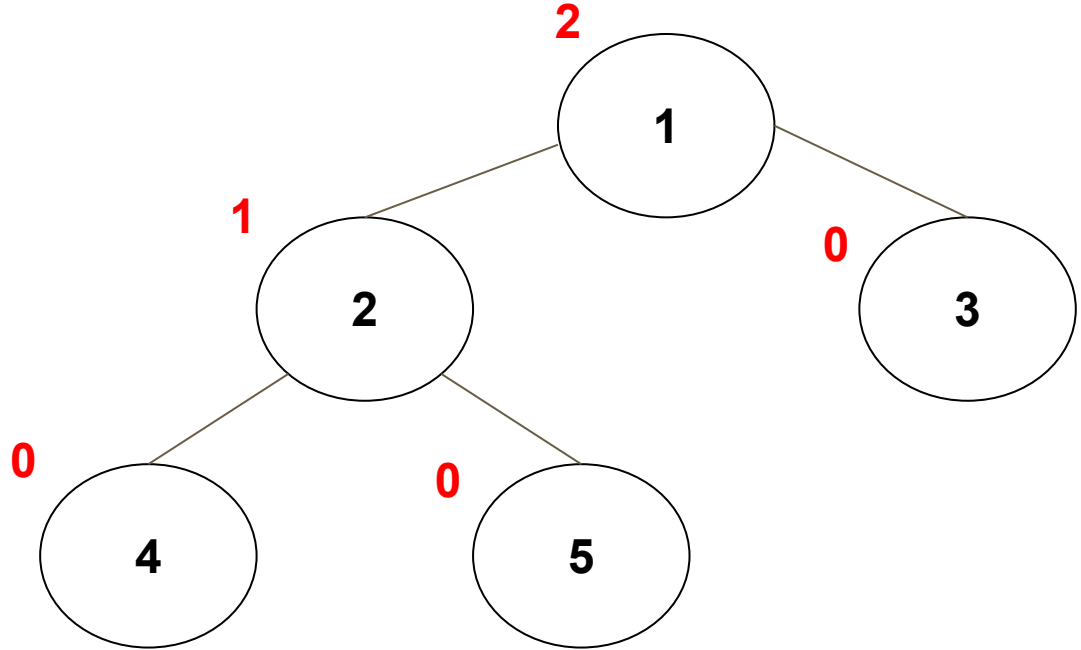
Calculate Height of Binary Tree

Height of Tree:

The height of a tree is the height of its root node.

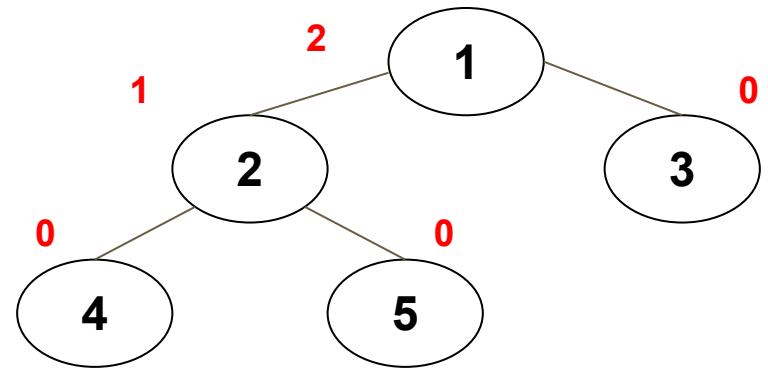
Height of Node:

The height of a node is the number of edges on the longest path between that node and a leaf.




```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

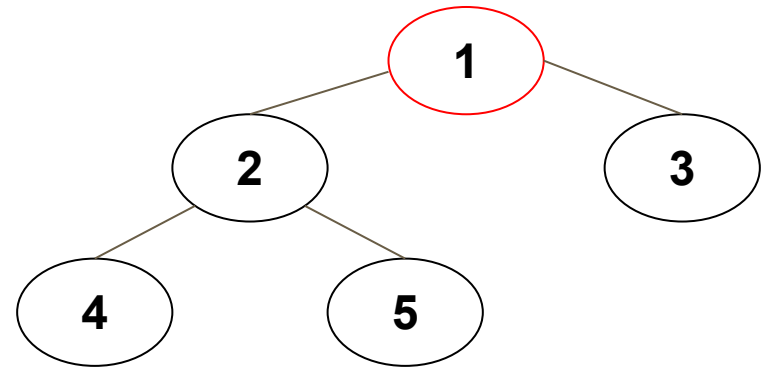
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

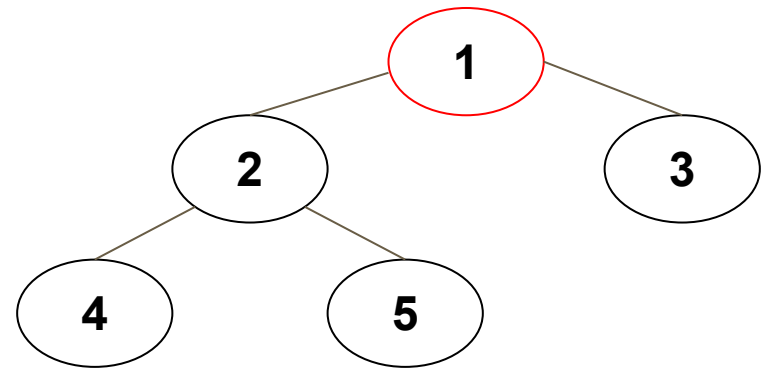
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

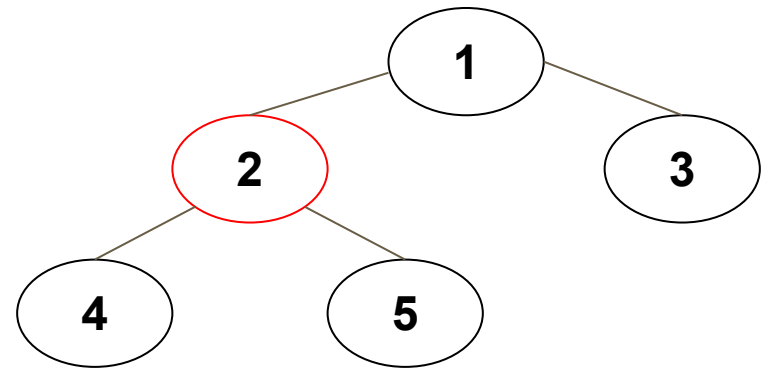
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

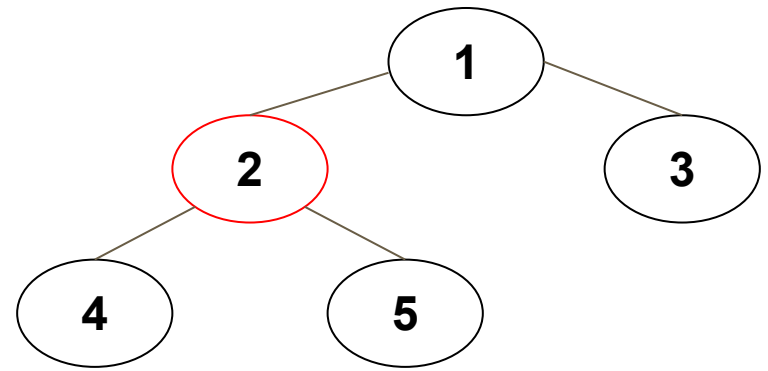
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

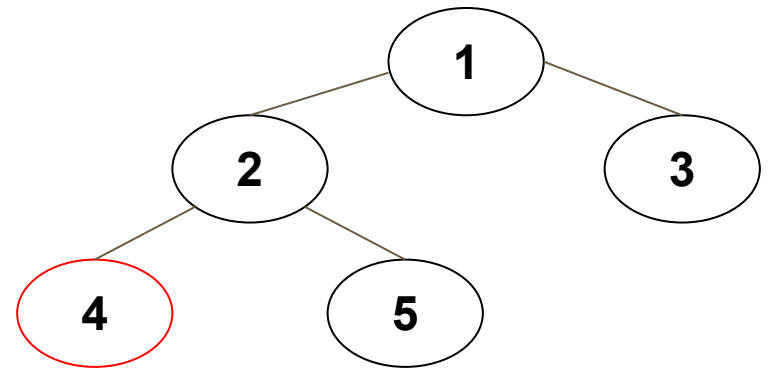
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

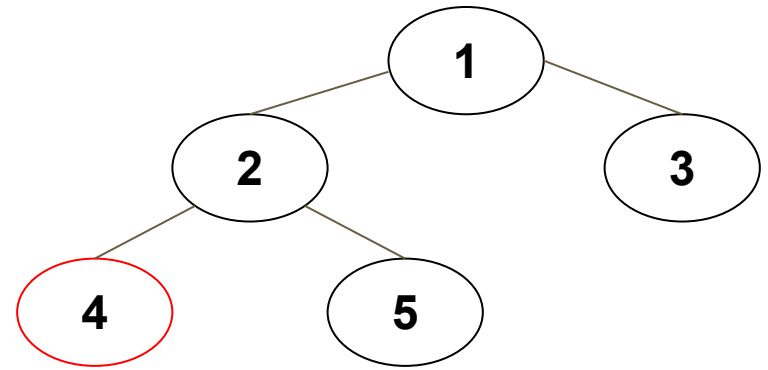
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
4		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

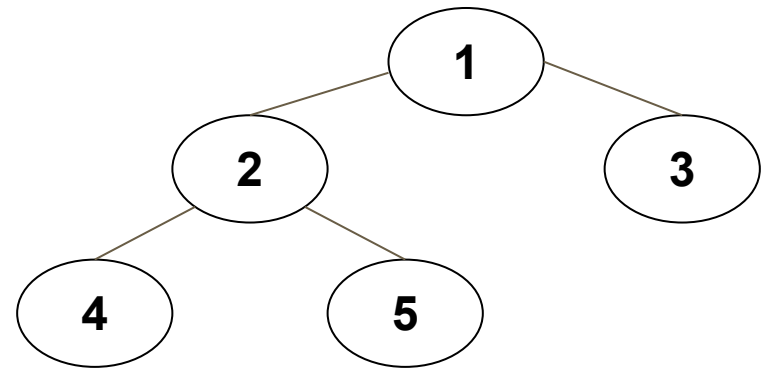
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
4		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

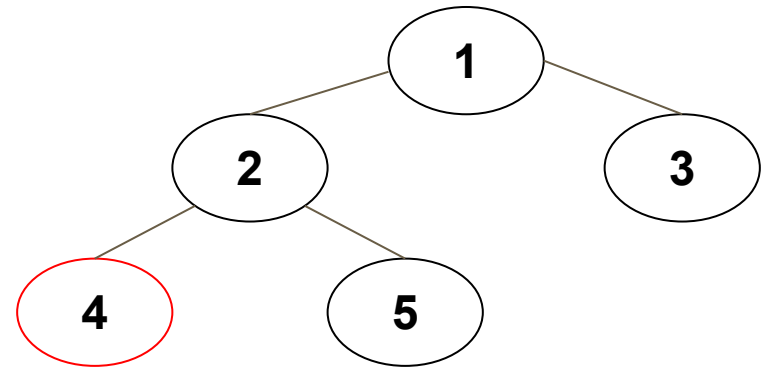
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
4		
None	-1	


```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

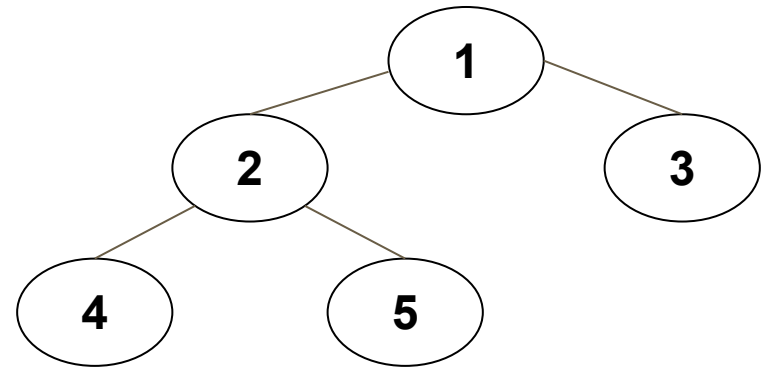
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
4		
None	-1	

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

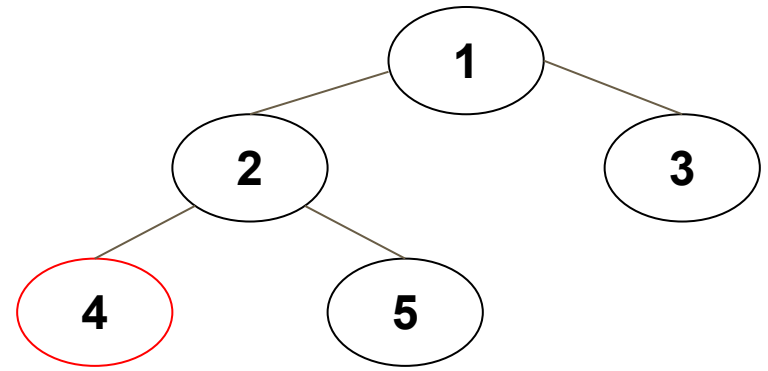
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
4		
None	-1	-1

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

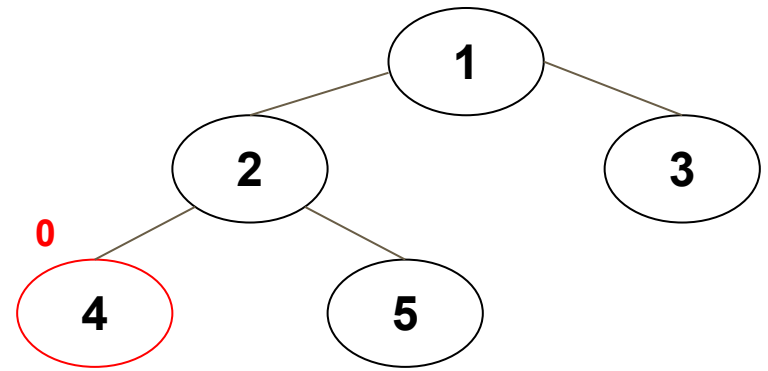
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
4		
None	-1	-1

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

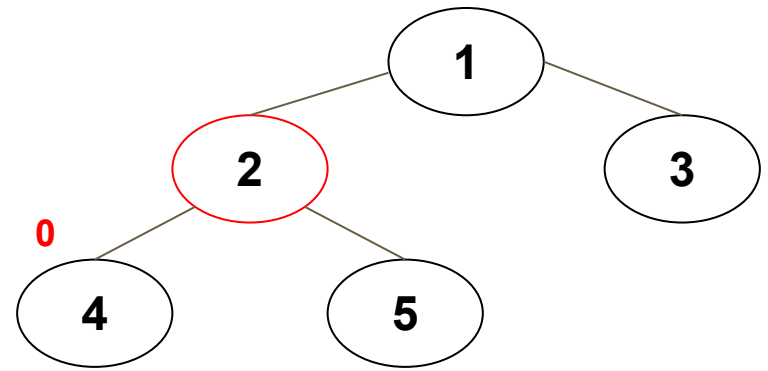
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
4		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

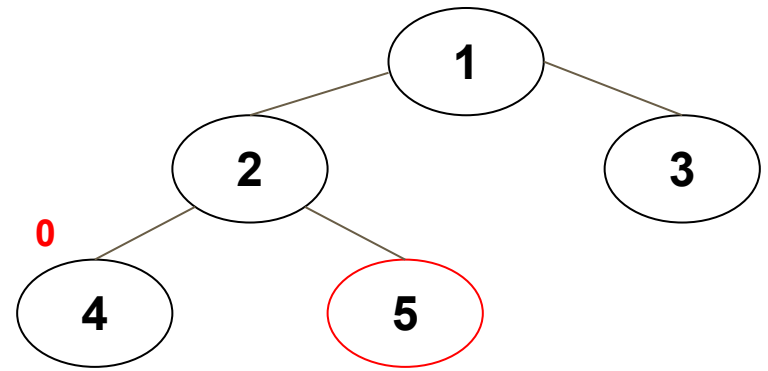
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

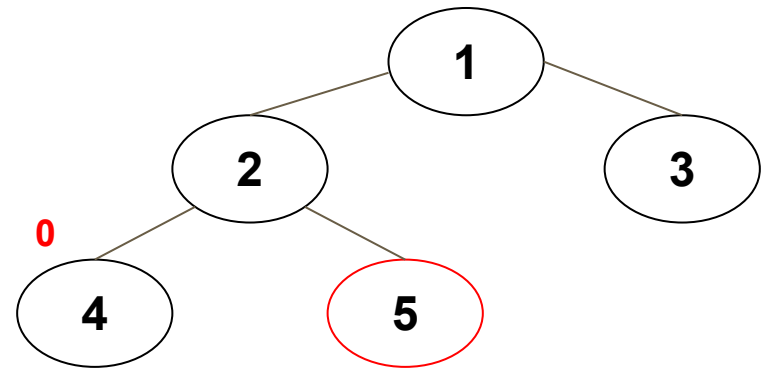
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
5		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

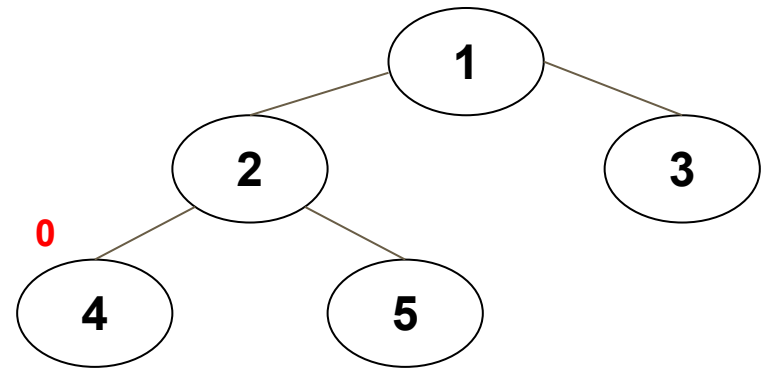
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
5		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

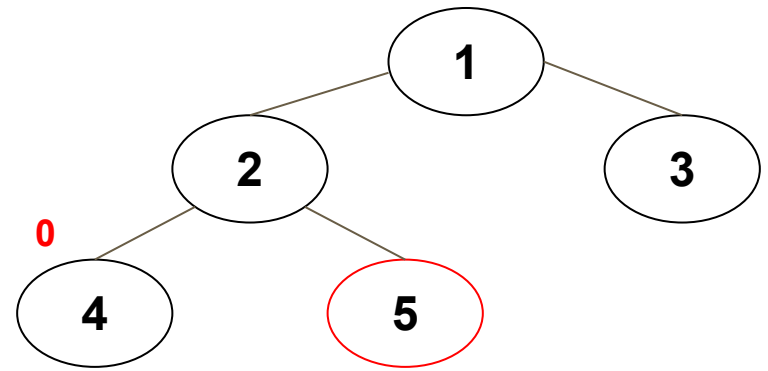
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
5		
None	-1	


```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

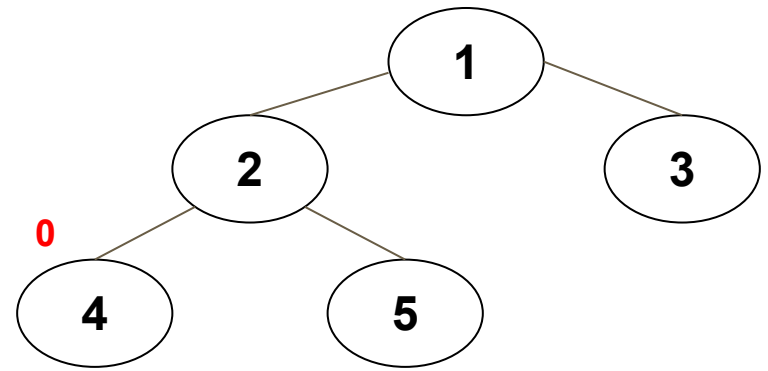
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
5		
None	-1	

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

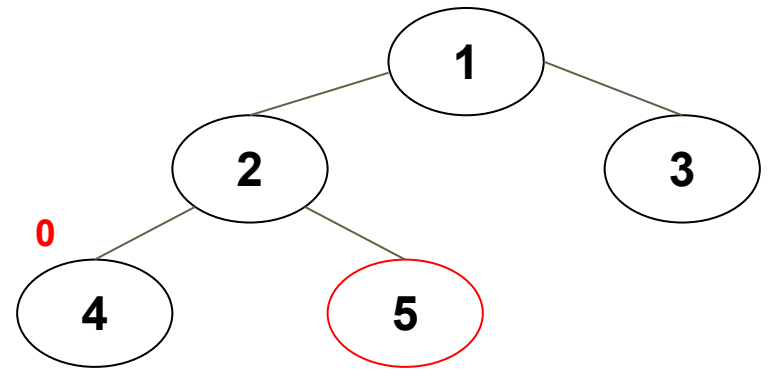
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
5		
None	-1	-1

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

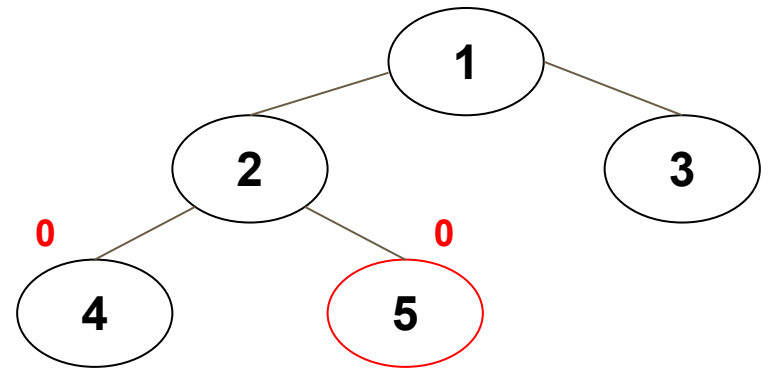
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
5		
None	-1	-1

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

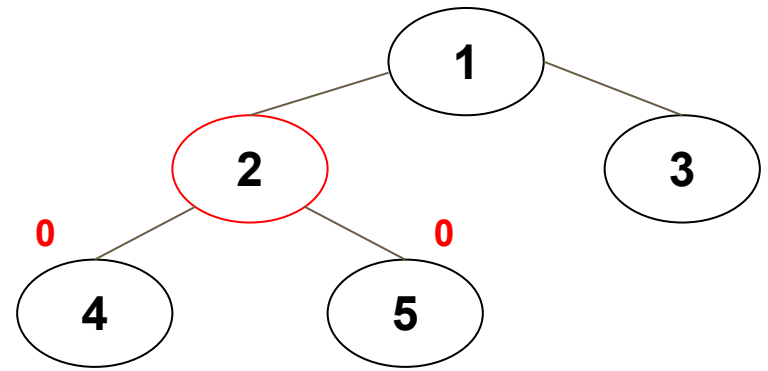
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		
5		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

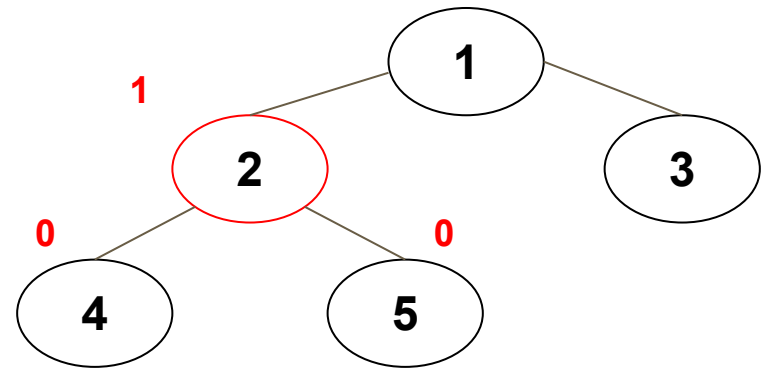
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

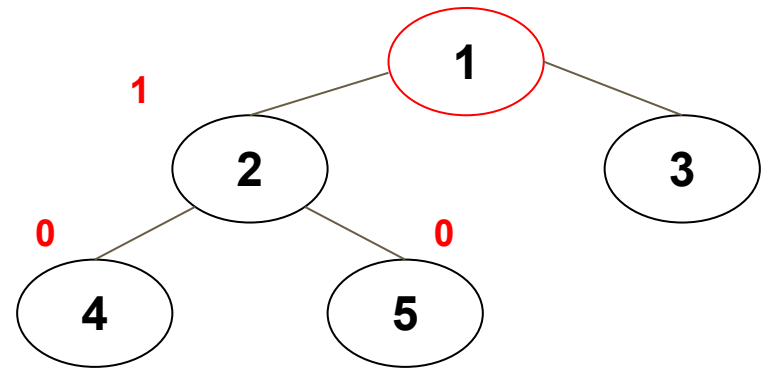
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1		
2		

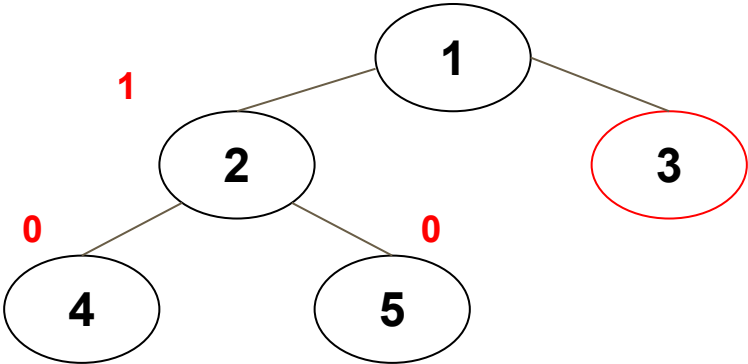
```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1	1	

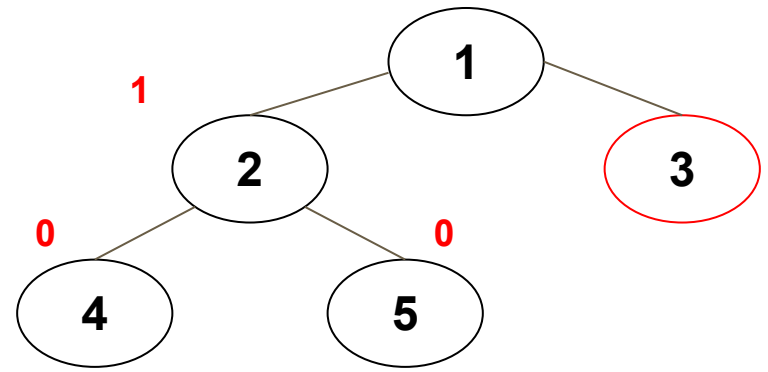
```
def height(node):  
    if node is None:  
        return -1  
    left_height = height(node.left)  
    right_height = height(node.right)  
  
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1	1	
3		


```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

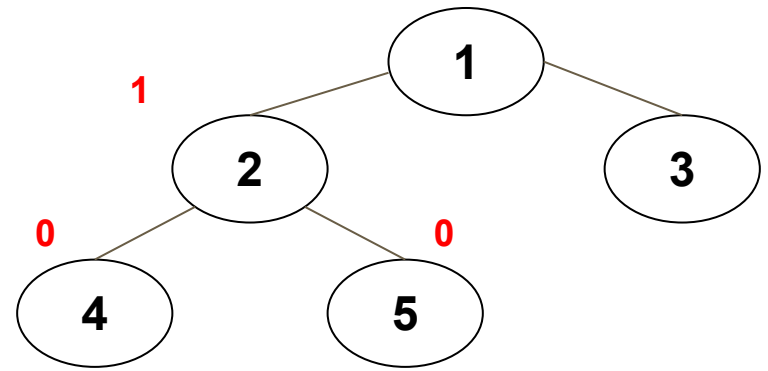
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1	1	
3		
None		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

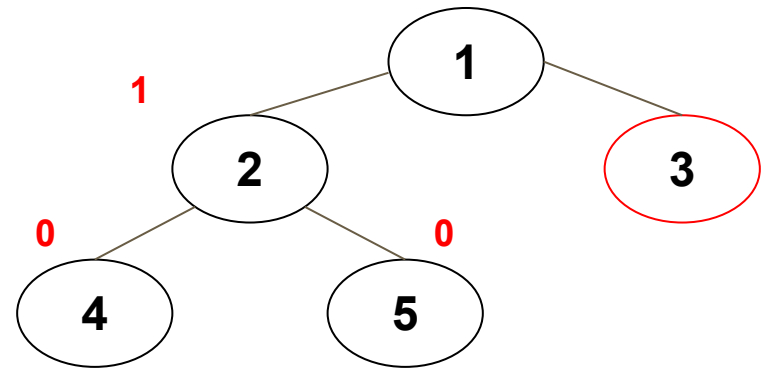
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1	1	
3		
None	-1	

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

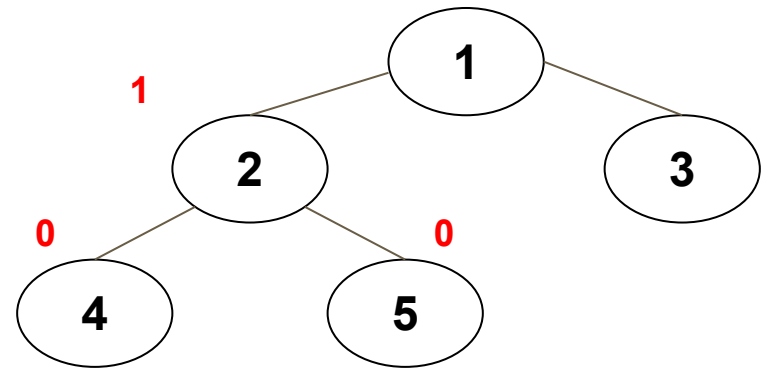
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1	1	
3		
None	-1	

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

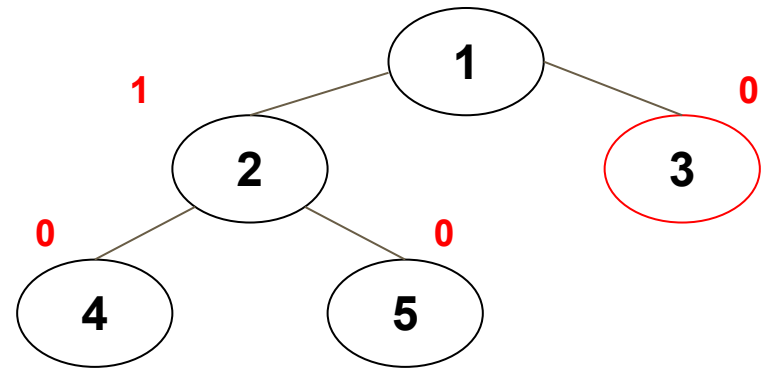
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1	1	
3		
None	-1	-1

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

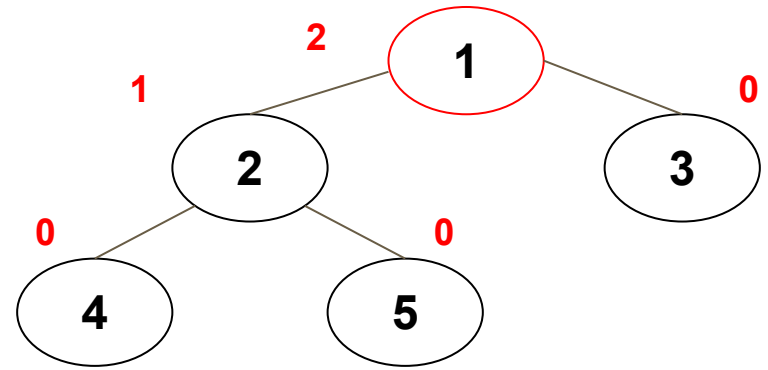
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1	1	
3		

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

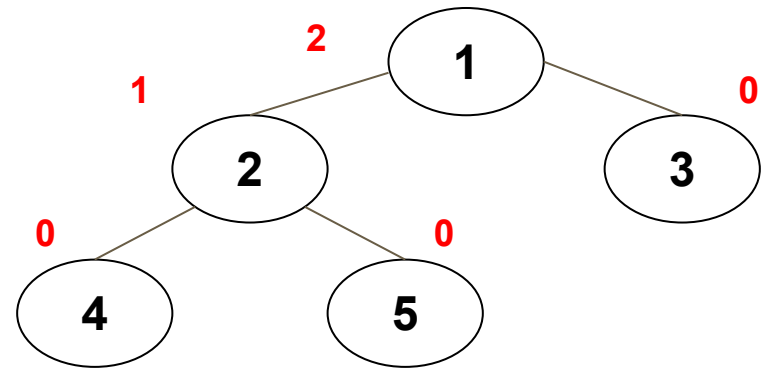
    return 1 + max(left_height, right_height)
```



node:	left_height	right_height
1	1	0

```
def height(node):
    if node is None:
        return -1
    left_height = height(node.left)
    right_height = height(node.right)

    return 1 + max(left_height, right_height)
```

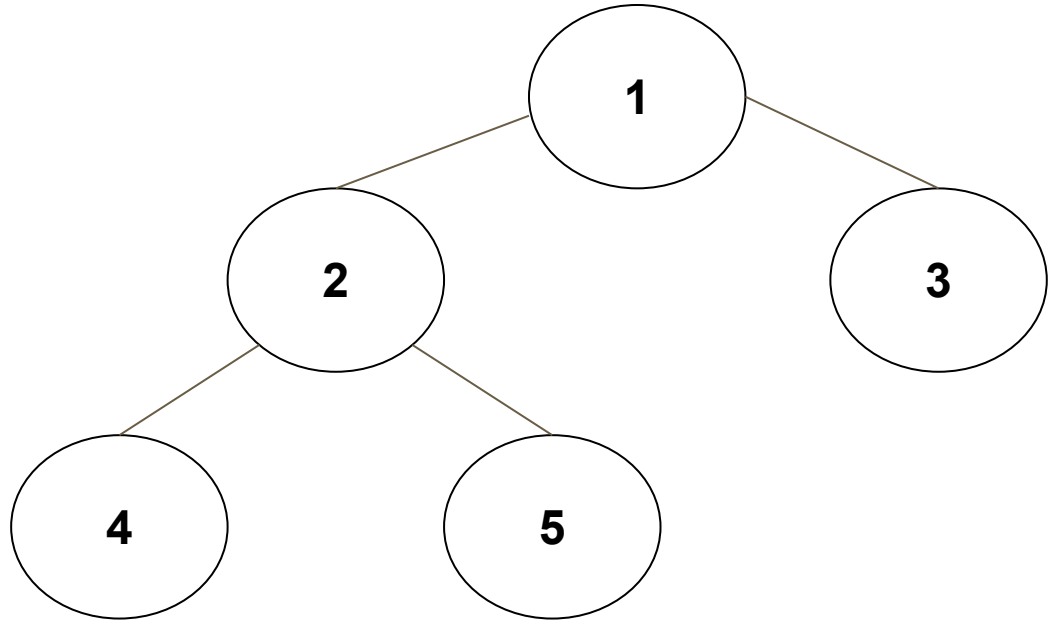


node:	left_height	right_height

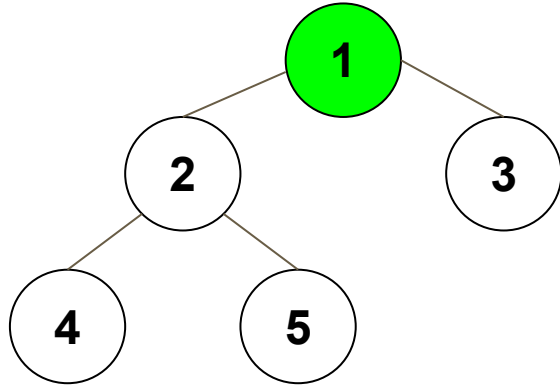
Pre-order Iterative

Pre-order Iterative

**Pre-order
output:
[1,2,4,5,3]**

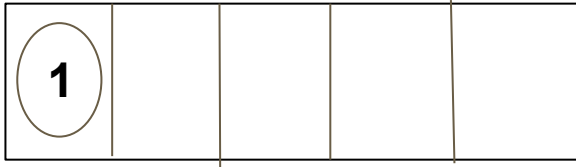


Pre-order traversal

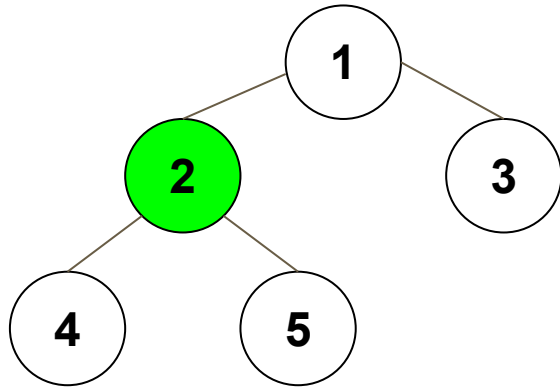


Pre-order traversal:

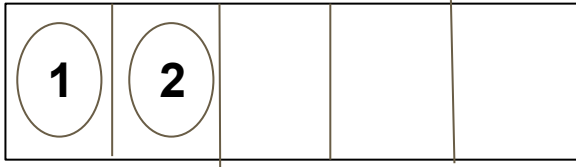
1,



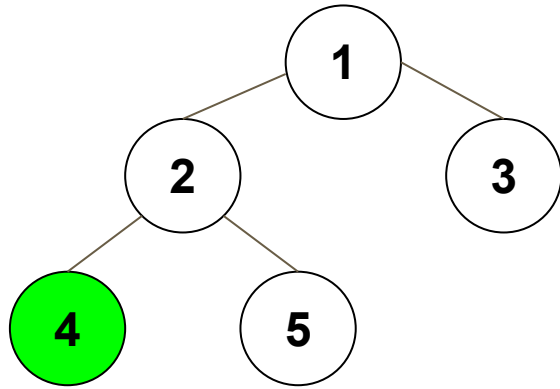
Pre-order traversal



Pre-order traversal:
1,2



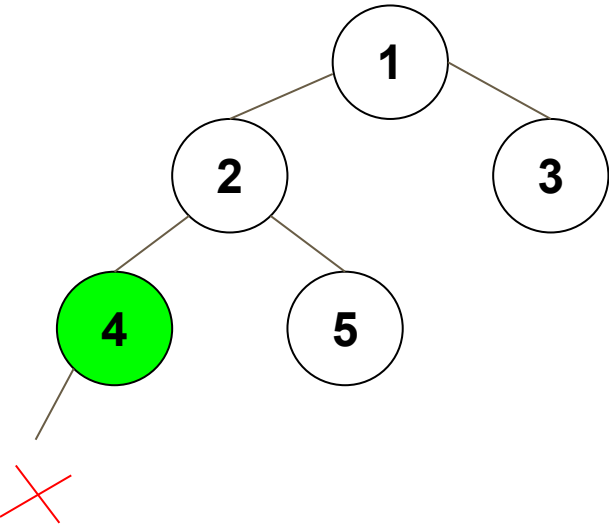
Pre-order traversal



Pre-order traversal:
1,2,4

1	2	4		
---	---	---	--	--

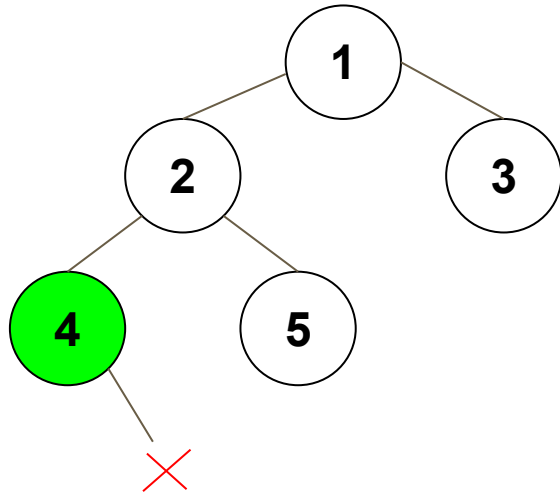
Pre-order traversal



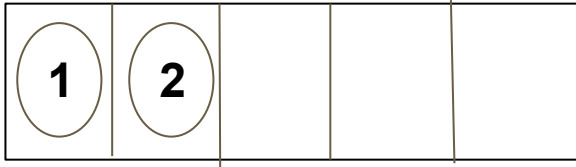
Pre-order traversal:
1,2,4

1	2	4		
---	---	---	--	--

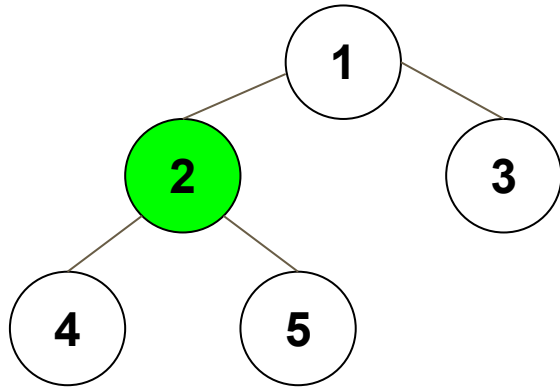
Pre-order traversal



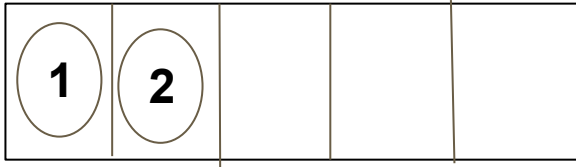
Pre-order traversal:
1,2,4



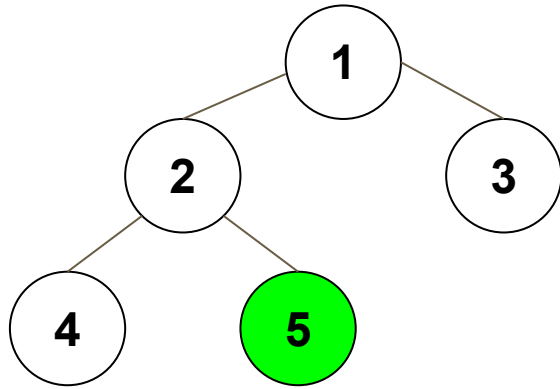
Pre-order traversal



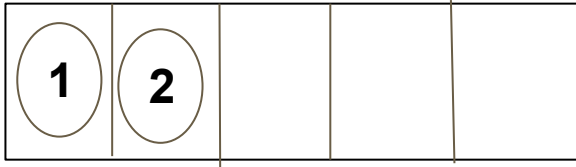
Pre-order traversal:
1,2,4



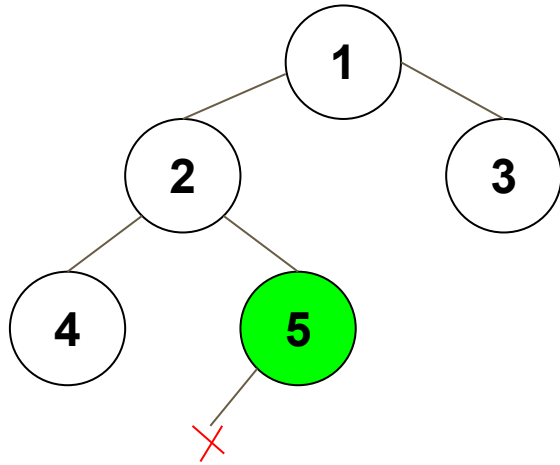
Pre-order traversal



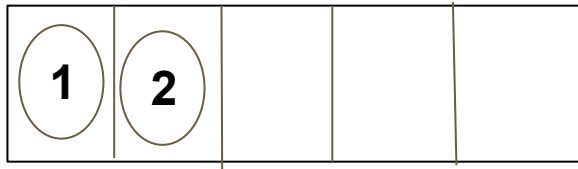
Pre-order traversal:
1,2,4,5



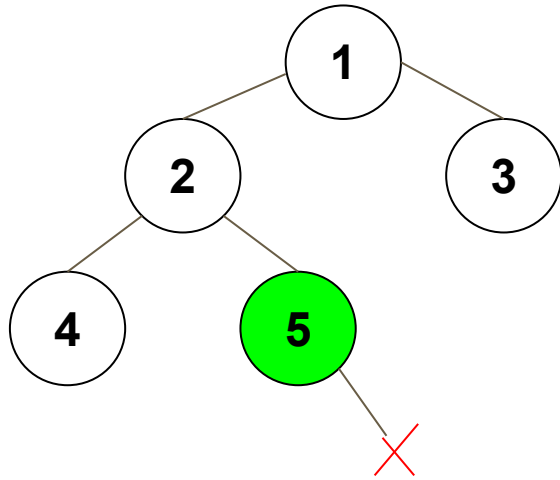
Pre-order traversal



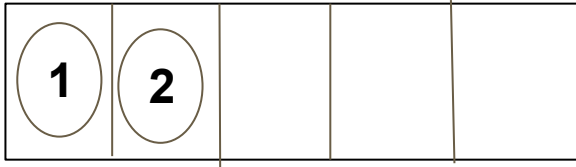
Pre-order traversal:
1,2,4,5



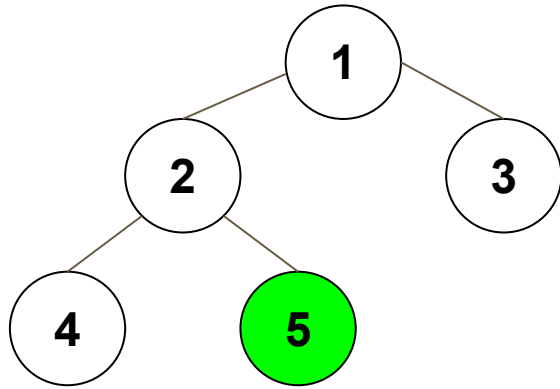
Pre-order traversal



Pre-order traversal:
1,2,4,5



Pre-order traversal



Pre-order traversal:
1,2,4,5

