

# MODERN PORTFOLIO THEORY

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## What is Modern Portfolio Theory (MPT)?

Modern Portfolio Theory also popularly known as MPT is a theory that proposes a solution to make a balanced portfolio that provides the highest return on a particularly given risk of the investor. This is a very basic portfolio theory that was developed by [Harry Markowitz](#). This was published in the Journal of Finance in 1952. He was awarded the Nobel prize for his work. The significance of this theory is its simplicity. The simple use of statistical methods for predicting the optimal balance for a portfolio is what it aims for. After the introduction of this theory, it went very popular all over the world. It initiated a wave of investment and financial research. This theory was so popular that every fund manager used this to make their portfolio at that time. Even today the initial approximate weights for the portfolio are calculated using this theory.

## Main points MPT focuses on are:

- Do not keep all your eggs in one basket. (Famous quote by Warren Buffet)  
The point of this quote is to invest your money in different asset classes to minimize the risk and try to maximize the return. This concept is known as **Diversification of the Portfolio** and this is the main focus of this portfolio theory.
- To find the percentage or weights of each asset class in a portfolio to maximize the return and minimize the risk. This is done by using simple statistical methods.

## Modern Portfolio Theory Methodology

The mathematical relation which was given to find the optimal weights is:  
He used **variance and covariance**.

$$\text{Variance: } \sigma^2 = \frac{\sum_{i=1}^n (x_i - \mu)^2}{n-1} \quad (\text{proposed risk associated with an asset})$$

$$\text{Covariance} = \frac{\text{var}(xy)}{\text{var}(x) \text{var}(y)} \quad (\text{relationship between two assets})$$

Covariance is generally between -1 to 1 (completely opposite to complete parallel)

The algebra followed in this theory is:

$$R = \begin{pmatrix} R_A \\ R_B \\ \vdots \\ R_n \end{pmatrix}, x = \begin{pmatrix} x_A \\ x_B \\ \vdots \\ x_n \end{pmatrix} \quad R_{p,x} = x^T \cdot R$$
$$E[R] = E \left[ \begin{pmatrix} R_A \\ R_B \\ \vdots \\ R_n \end{pmatrix} \right] = \begin{pmatrix} E[R_A] \\ E[R_B] \\ \vdots \\ E[R_n] \end{pmatrix} = \begin{pmatrix} \mu_A \\ \mu_B \\ \vdots \\ \mu_n \end{pmatrix} = \mu$$

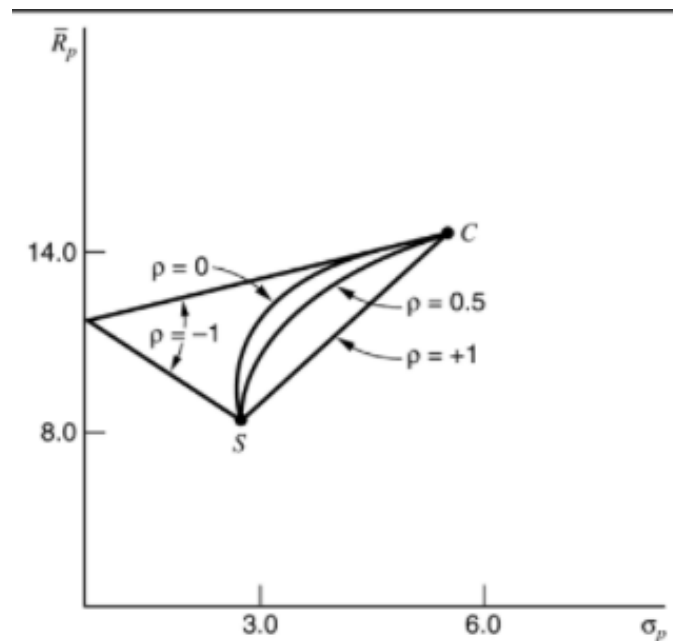
$$\begin{aligned}\mu_{p,x} &= E[x^T R] = x^T E[R] = x^T \mu \\ &= (x_A, x_B, \dots, x_n) \begin{pmatrix} \mu_A \\ \mu_B \\ \vdots \\ \mu_n \end{pmatrix}\end{aligned} \quad \begin{pmatrix} \sigma_A^2 & \sigma_{AB} & \sigma_{AC} \\ \sigma_{BA} & \sigma_B^2 & \sigma_{BC} \\ \sigma_{CA} & \sigma_{CB} & \sigma_C^2 \end{pmatrix} = \Sigma$$

$$\begin{aligned}\sigma_{p,x}^2 &= \text{var}(x^T R) = x^T \Sigma x \\ &= (x_A, x_B, x_C) \begin{pmatrix} \sigma_A^2 & \sigma_{AB} & \sigma_{AC} \\ \sigma_{BA} & \sigma_B^2 & \sigma_{BC} \\ \sigma_{CA} & \sigma_{CB} & \sigma_C^2 \end{pmatrix} \begin{pmatrix} x_A \\ x_B \\ x_C \end{pmatrix}\end{aligned}$$

The symbols have the usual meanings here.

The weight vector  $x$  is chosen randomly and for each weight Expected Return vs Risk is plotted. This plot is known as **Efficient Frontier**.

The Efficient Frontier looks like:



A parameter known as **Sharpe Ratio** was defined for this theory. It is the ratio of Returns / Risk of the portfolio. So, we need to find the weight which gives the highest Sharpe Ratio.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

**where:**

$R_p$  = return of portfolio

$R_f$  = risk-free rate

$\sigma_p$  = standard deviation of the portfolio's excess return

## Advantages of MPT

- MPT became very famous after its release in 1952. After introduction of MPT people started optimizing their portfolios in some rational way. Almost all the fund manager at that time used MPT to balance and re-balance the weights of their portfolio.
- People started using MPT for creating portfolio of ETFs (which is a collection of stocks which is traded in market as stocks. Like, you can sell and buy ETFs same as stock, you need not to hold it for years like Mutual Funds). This helped people to truly diversify their portfolio and increase their expected return.

## Disadvantages of MPT

- The biggest fault of this theory was that it used variance or volatility for risk calculation. Upside Volatility is preferable to the investors so it is not rational to account it in the risk calculation. To solve this, we should only account **downside-volatility or downside-risk which means the variance of that results only when there was a loss. This was proposed by Brian M. Rom and Kathleen Ferguson in 1991 as Post Modern Portfolio Theory (PMPT).**
- It was also assumed in MPT that the market is ideal. All the investors in the market are rational it is not affected by any factor or the effect is gradual. Which is also a major downside of this theory because we all know that markets are never ideal. It functions majorly on sentiments. For e.g. Covid brought up a negative sentiment and market crashed.
- Third major downside of this theory is it works on historical data only. So, it is very difficult to believe that the portfolio made on historical data will sustain the unpredictable future.

## Challenges on how to use it in today's scenario?

MPT can give you only weights of the stocks in the portfolio hence leaving us with a great challenge of selecting stocks.

There are certain guidelines to do this:

- The most logical hack is to select assets with negative co-relation. Like you can mix govt. issued bonds with small-cap and large-cap. Bonds are pretty stable and almost considered risk-free.
- The higher your selection co-relation is near to -1 the better your portfolio will be.
- Another popular hack is to apply MPT to already created 2 efficient portfolios. This is known as **TWO-FUND THEOREM**.

## **TWO-FUND THEOREM**

This theorem states that you can hold a weighted portfolio of 2 already available efficient mutual funds. (Mutual funds means already created pool of assets). This can create another efficient portfolio to invest on. Advantages of this is that it is easier to have such a portfolio because the mutual funds will be low in cost than holding each individual asset. And the portfolio will be easily maintainable by the investor because he needs to see only the two fund performances.

It can be shown that if you want your expected return between the 2 efficient funds than you can hold them both in certain weights (determined by MPT) to expect the return in comparatively less risk. It is similar to holding two of your favorite stocks in a portfolio.

### **Why MPT lost its significance**

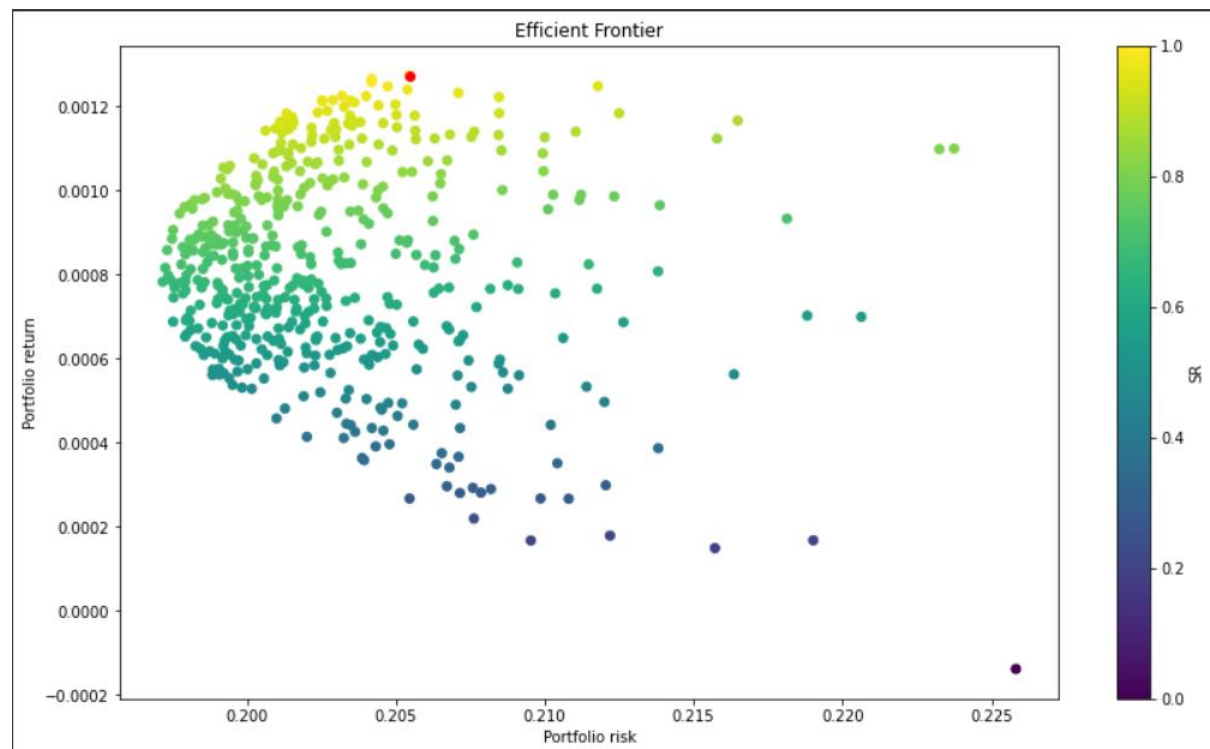
MPT lost its significance as more complicated research started in finance industry. Many researchers started proposing better methods to evaluate risk-return considering the psychology of the investors and financial markets. This was termed as **behavioral finance**.

But, MPT served as the starting point for portfolio optimization studies and encouraged the study of financial markets more critically. Some examples are **CAPM, SML, etc.**

## Code for MPT

<https://colab.research.google.com/drive/1Alxd53MZH94XAG8Si4Ce61E1YRSbTLYI?usp=sharing>

Assets chosen for a sample run was `assets = ['AAPL', 'MSFT', 'GOOG', 'AMZN']` between 2021-01-31 to present. The efficient frontier derived from this run was



The red point in the efficient frontier is the point of highest Sharpe Ratio. The weights for this point is

```
array([0.22252147, 0.01210444, 0.53133647, 0.23403762])
```

We invested \$10,000 on 2021-10-01 and ran the algorithm to find the returns and percentage returns.

```
marketreturn = portfoliovalue - startamt  
marketreturn
```

```
456.9062523647517
```

```
pctreturn = (marketreturn/startamt)*100  
pctreturn
```

```
4.569062523647517
```

**Returns: \$456.9**

**Percentage Return: 4.56% in almost 4 months.**

## **Interview of Harry Markowitz**

**<https://youtu.be/wdeolPCFtDU>**

This interview is conducted by MIT Financial Laboratory. Interested Readers can watch.