SVKM's
D. J. Sanghvi College of Engineering

Program: B.Tech in Mechanical Engineering **Academic Year: 2022** **Duration: 3 hours**
Date: 19.01.2023
Time: 09:00 am to 12:00 pm
Subject: Engineering Mathematics III (Semester III) **Marks: 75**

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover page of the Answer Book, which is provided for their use.
(1) This question paper contains 02 pages.
(2) All Questions are Compulsory.
(3) All questions carry equal marks.
(4) Answer to each new question is to be started on a fresh page.
(5) Figures in the brackets on the right indicate full marks.
(6) Assume suitable data wherever required but justify it.
(7) Draw the neat-labelled diagrams, wherever necessary.

Question No.		Max. Marks
Q1(A)	Find the Fourier expansion for $f(x) = xsin(x)$ in (0 ,2π)	07
	OR	
	Obtain half rang cosine series for $f(x) = \begin{cases} x & 0 < x < (\frac{\pi}{2}) \\ (\pi - x) & (\frac{\pi}{2}) < x < \pi \end{cases}$	07
Q1(B)	Evaluate using Laplace transform $\int_0^\infty e^{-t}(\int_0^t u^2 \sinh(u)cosh(u)du)dt$	08
Q2(A)	Find the Bilinear transformation which maps the points $2, i, -2$ onto the points $1, i, -1$ by using cross – ratio property.	07
	OR	
	Find the analytic function $f(z) = u + iv$ such that $u + v = \frac{2\sin(2x)}{e^{2y}+e^{-2y}-2cos(2x)}$	07
Q2(B)	Solve Using Laplace transform $\frac{d^2y}{dt^2} + 9y = 18t$ given that y (0) = 0 and y (π/2) = 0.	08
Q3(A)	Evaluate $\int_C \frac{z^2}{z^4-1} dz$ where C is the circle (i) $\lvert z \rvert = \frac{1}{2}$ (ii) $\lvert z - 1 \rvert = 1$	03 04

	OR	
	Evaluate using residues $\int_0^{2\pi} \frac{cos(3\theta)}{5-4cos(\theta)} d\theta$	07
Q3(B)	Obtain Taylor's and Laurent's expansion of $f(z) = \frac{(z-1)}{(z^2-2z-3)}$ indicating regions of convergence.	08
Q4(A)	Obtain complex form of Fourier series of f(x) = cosh (3x) + sinh (3x) in (-3,3)	07
	OR	
	Show that the set of functions $\{\sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), - - - - -\}$ is orthogonal over $(0, L)$ Hence Construct corresponding orthonormal set.	07
Q4(B)	Using Crank-Nicholson method Solve $\frac{\partial^2 u}{\partial t^2} - 16\frac{\partial u}{\partial t} = 0 \quad 0 < x < 1$, $t > 0$ given $u(x,0) = 0, u(0,t) = 0, u(1,t) = 200t$ Compute u for one step in division taking $h = \frac{1}{4}$.	08
Q5(A)	Fit a second-degree curve (parabola) to the following data x: 0, 1, 2, 3, 4 y: 1.0, 1.5, 1.5, 2.5, 3.5	07
	OR	
	Find Karl Pearson's correlation coefficient from the following data X: 23, 27, 28, 29, 30, 31, 33, 35, 36, 39 Y: 18, 22, 23, 24, 25, 26, 28, 29, 30, 32	07
Q5(B)	i) Find Spearman's rank correlation coefficient from the following data X: 18, 20, 34, 52, 12 Y: 39, 23, 35, 18, 46	04
	ii) Solve $\frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method ,given $u(0,t) = 0, \quad u(4,t) = 0, \quad u(x,0) = x(4-x)$, Assume $h = 1$ and find the values of u upto $t = 3$.	04

All the Best!