**SVKM's**
**D. J. Sanghvi College of Engineering**

**Program: B.Tech in Electronics & Telecommunication Engineering** **Academic Year: 2022** **Duration: 3 hours**
**Date: 19.01.2023**
**Time: 09:00 am to 12:00 pm**
**Subject: Engineering Mathematics III (Semester III)** **Marks: 75**

**Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover page of the Answer Book, which is provided for their use.**

1. This question paper contains two pages.
2. **All Questions are Compulsory.**
3. All questions carry equal marks.
4. **Answer to each new question is to be started on a fresh page.**
5. **Figures in the brackets on the right indicate full marks.**
6. **Assume suitable data wherever required, but justify it.**
7. Draw the neat labelled diagrams, wherever necessary.

| Question No. | | Max. Marks |
|---|---|---|
| Q1 (a) | Find the Laplace transform of $\int_0^\infty e^{-t}\left(\int_0^t u^2 \sin hu \cos hu \ du\right) dt$ | [07] |
| | **OR** | |
| (a) | Evaluate $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}\ dt$ by using Laplace transform. | [07] |
| Q1 (b) | i. Evaluate $L\{\frac{e^{-t}\sin 2t \ \sin h\, t}{t}\}$ | [04] |
| | ii. Evaluate $L\{\sin 2t \ \cos t \ \cos h\, 2t\}$ | [04] |
| Q2 (a) | Solve using Laplace Transform<br>$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}$ given $y(0) = 4\ and\ y'(0) = 2$ | [07] |
| | **OR** | |
| (a) | Find the inverse Laplace transform of<br>$\frac{s^2 + 16s - 24}{s^4 + 20s^2 + 64}$ | [07] |
| Q2 (b) | i. Find $L^{-1}\left\{\frac{s^2}{(s^2-a^2)^2}\right\}$ by using Convolution theorem. | [04] |
| | ii. Find inverse Laplace transform of | [04] |

| | | |
|---|---|---|
| | $L^{-1}\left\{\tan^{-1}\frac{(s+a)}{b}\right\}$ | |
| Q3 (a) | i. Find the Fourier series for $f(x)=\frac{3x^2-6x\pi+2\pi^2}{12}$ $in$ $(0,2\pi)$. Hence deduce that $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$. | [07] |
| | **OR** | |
| (a) | Find a half range cosine series of $f(x)=\sin x$ in $0\le x\le\pi$. | [07] |
| Q3 (b) | i. Obtain the complex form of Fourier series for $f(x)=e^{-ax}$ $in$ $(-\pi,\pi)$. | [04] |
| | ii. Verify the set of functions $\{\sin x, \sin 3x, \sin 5x, \ldots\}$ i.e. $\sin(2n+1)x$ $n+0,1,2,\ldots$ is orthogonal or not over $\left[0,\frac{\pi}{2}\right]$. If orthogonal then construct the orthonormal set of functions. | [04] |
| Q4 (a) | Verify Green's theorem in the plane for $\int_C (xy+y^2)dx+x^2\,dy$ where C is the closed curve of the region bounded by $y=x$ $and$ $y=x^2$. | [07] |
| | **OR** | |
| (a) | Use Stokes' theorem to evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F}=(y^2+z^2-x^2)\mathrm{i}+(z^2+x^2-y^2)\mathrm{j}+(x^2+y^2-z^2)\mathrm{\kappa}$ over the boundary of the surface $x^2+y^2-2ax+az=0$ above the plane $z=0$. | [07] |
| Q4 (b) | Find $div\,\bar{F}$ $and$ $curl\,\bar{F}$ where $\bar{F}=\frac{xi-yj}{x^2+y^2}$. | [08] |
| Q5 (a) | By Milne-Thompson method find the imaginary part of the analytic function whose real part is $e^{2x}(x\cos 2y-y\sin 2y)$ and also verify that $v$ is harmonic. | [07] |
| | **OR** | |
| (a) | Find the analytic function $f(z)=\frac{\cos x+\sin x-e^{-y}}{2\cos x-e^{y}-e^{-y}}$ $when$ $f\left(\frac{\pi}{2}\right)=0$. | [07] |
| Q5 (b) | i. Find the constants a, b, c, d, e if $f(z)=(ax^3+bxy^2+3x^2+cy^2+x)+i(dx^2y-2y^3+exy+y)$ is analytic. | [04] |
| | ii. Find the Bilinear Transformation which maps the points 2, *i*, -2 of z-plane onto the points 1, *i*, -1 of w-plane by using the cross-ratio property. | [04] |