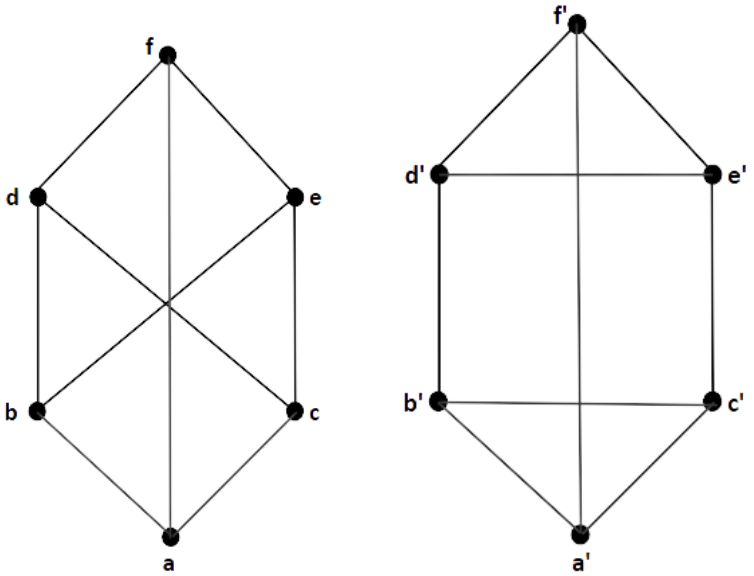
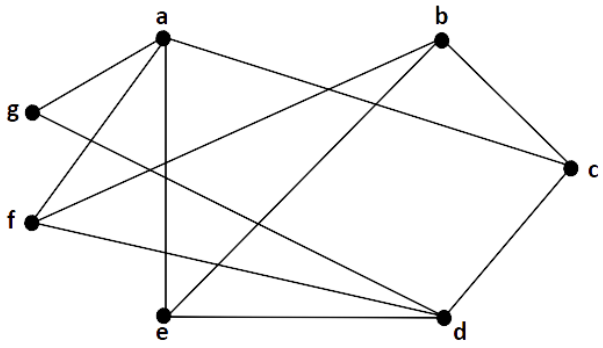
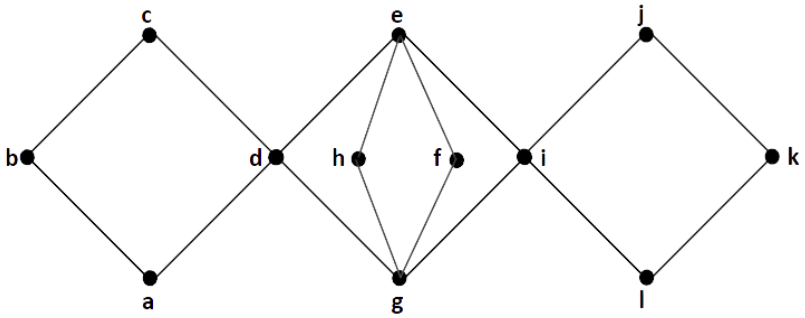
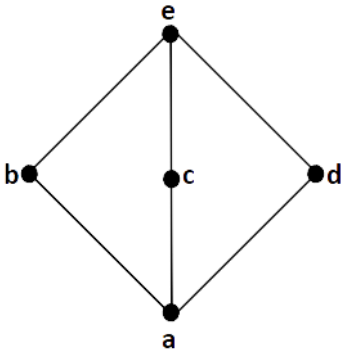
SVKM's
D. J. Sanghvi College of Engineering

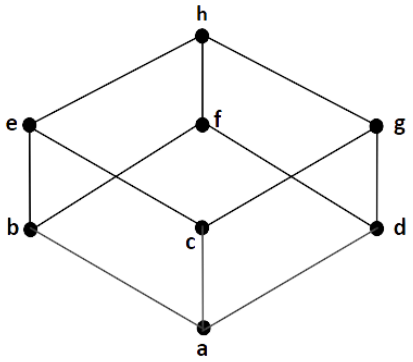
Program: B.Tech in Information Technology **Academic Year: 2022** **Duration: 3 hours**
Date: 19.01.2023
Time: 09:00 am to 12:00 pm
Subject: Discrete Structures (Semester III) **Marks: 75**

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover page of the Answer Book, which is provided for their use.
(1) This question paper contains 4 pages.
(2) All Questions are Compulsory.
(3) All questions carry equal marks.
(4) Answer to each new question is to be started on a fresh page.
(5) Figures in the brackets on the right indicate full marks.
(6) Assume suitable data wherever required but justify it.
(7) Draw the neat, labelled diagrams, wherever necessary.
(8) Use of scientific calculator is allowed.

Question No.	Questions	Marks
Q.1. a	Consider the elements $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ from $SL_2(\mathbb{R})$. Find $ord(A), ord\ (B), ord(AB)$.	07
Q.1. b	For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other. i) the set of all local trains from Borivali to Dadar, the set of all slow local trains from Borivali to Dadar. ii) the set of people who speak Marathi, the set of people who speak Gujrati. iii) the set of flying squirrels, the set of living creatures that can fly.	08
	OR	
	Find the number of elements in A1 ∪ A2 ∪ A3 and $A1 \cap A2 \cap A3$ if there are 205 elements in A1, 2050 in A2, and 20,500 in A3 if i) A1 ⊆ A2 and A2 ⊆ A3. ii) the sets are pairwise disjoint. iii) there are two elements common to each pair of sets and one element in all three sets.	08
Q.2. a	Find the subgroups generated by the elements of the group $\mathbb{Z}_4 = \{0,1,2,3\}$ under addition modulo 4 and the group $U(8) = \{1,3,5,7\}$ under multiplication modulo 8. Is $\mathbb{Z}_4$ isomorphic to $U(8)$?	07
Q.2. b	Find the reflexive closure, symmetric closure and transitive closure of a relation whose matrix representation is given by $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$.	08
	OR	

	i. Let $f: \mathbb{R}\backslash\{2022\} \longrightarrow \mathbb{R}\backslash\{0\}$, be a function defined as $f(x) = \frac{1}{x-2022}$. Is f invertible? If it is, find f^{-1}. ii. Let A denote the set of all students of DJ Sanghvi college of Engineering. Let R be a relation on A defined as aRb if and only if a and b are in the same branch. Determine whether R is reflexive, symmetric, transitive relation. Hence conclude whether R is an equivalence relation.	04 04
Q.3. a	Find the Hamming distance between x and y i. $x = 110110, y = 000101$ ii. $x = 001100, y = 010110$ iii. $x = 1100010, y = 1010001$ iv. $x = 0100101, y = 0110010$ v. $x = 00111001, y = 10101001$ vi. $x = 101001101, y = 101001101$	07
Q.3. b	Use Huffman algorithm to construct a $2-$ tree of minimum weighted path length for the following data. Symbols | A | E | L | I | M | O | S | T Frequencies | 20 | 32 | 14 | 11 | 16 | 12 | 7 | 3 Find the minimum path length of the tree constructed, encode the symbols using Huffman coding and give an encoding for the words: TEST and LEAST. OR Determine whether the given pair of graphs are isomorphic. Justify your answer. 	08 08
Q.4. a	i. Check whether the given graph is bipartite. If it is bipartite, then give a bipartition of the vertex set. If not, then justify. 	04 03

	ii. Check whether the following graph has Euler path and Euler circuit. If it has, then trace it. If not, then justify.  	
Q.4. b	Let $G = S_3 = \left\{e = \begin{pmatrix}1 & 2 & 3\\ 1 & 2 & 3\end{pmatrix},\ \sigma_1 = \begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix}, \sigma_2 = \begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}, \sigma_3 = \begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix}, \phi_1 = \begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}, \phi_2 = \begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}\right\}$ be a group under composition. Let $H = \{e, \sigma_1\}$. Find all left and right cosets of H. Is H a normal subgroup of G ?	08
	OR	
	i. Let d be the $(6, 2)$ decoding function of $(m, 3m)$ encoding function. Determine $d(y)$ for the following words y of B^6. 1. $y = 111011$ 2. $y = 010100$ 3. $y = 100001$ 4. $y = 100101$	04
	ii. Find the minimum distance of the $(2,4)$ encoding function $e: B^2 \rightarrow B^4$ defined as $e(00) = 0000, e(01) = 1011, e(10) = 0110, e(11) = 1100$.	04
Q.5. a	i. Let $G = \mathbb{Q}\backslash\{1\}$ and let $*$ be a binary operation defined on G as $a * b = a + b - ab$. Does $*$ follows associativity? Justify.	03
	ii. Consider the following assumptions: S_1: All dictionaries are useful. S_2: Mary owns only horror novels. S_3: No horror novel is useful. Use a Venn diagram to determine the validity of the following conclusions: a. Horror novels are not dictionaries. b. Mary does not own a dictionary. c. All useful books are dictionaries.	04
Q.5. b	Draw the join $(\vee)$ and the meet $(\wedge)$ table of the POSET corresponding to the following Hasse diagram. Is it a Lattice?  	08

	OR	04
	i. Draw the directed graph and Hasse diagram of a relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $a\ R\ b$ if and only if $a \mid b$ $(a\ divides\ b)$.	04
	ii. Let the Hasse diagram of a POSET $(\{a, b, c, d, e, f, g, h\}, \leq)$ be as follows:  Find the upper bound/s, lower bound/s, least upper bound and greatest lower bound of the pairs $\{e, g\}, \{f, d\}, \{a, b\}, \{c, d\}, \{h, a\}, \{f, c\}$.	