Shri Vile Parle Kelavani Mandal's

DWARKADAS J. SANGHVI COLLEGE OF ENGINEERING

(Autonomous College Affiliated to the University of Mumbai)

NAAC Accredited with "A" Grade (CGPA : 3.18)

Academic Year (2022-23)

Year: 3 Semester: V

Program: B. Tech. (EXTC) **Max. Marks: 75**

Subject: Mathematics and Statistics for Artificial Intelligence & Machine Learning

Time: 10: 30 am to 1:30 pm

Date: **Duration: 3 Hours**

REGULAR EXAMINATION

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover page of the Answer Book, which is provided for their use.

(1) This question paper contains two pages.
(2) All Questions are Compulsory.
(3) All questions carry equal marks.
(4) Answer to each new question is to be started on a fresh page.
(5) Figures in the brackets on the right indicate full marks.
(6) Assume suitable data wherever required, but justify it.
(7) Draw the neat labelled diagrams, wherever necessary.

Question No.		Max. Marks
Q1 (a)	Solve the following linear systems $\mathbf{Ax} = \mathbf{b}$ by using **LU** factorization for: $$A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ 10 & 12 & 34 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 5 \\ -11 \\ 84 \end{pmatrix}$$	[07]
Q1 (b)	Transform the following basis vectors in $\mathbb{R}^3$ to an orthonormal basis for $\mathbb{R}^3$ with respect to the dot product using Gram-Schmidt method. $$v_1 = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}^T, v_2 = \begin{pmatrix} -1 & 0 & -1 \end{pmatrix}^T \text{ and } v_3 = \begin{pmatrix} -1 & 2 & 3 \end{pmatrix}^T$$	[08]
	OR	
	Determine the matrices U, D and V using SVD, such that $A = UDV^T$ for the following: $$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$	[08]
Q2 (a)	i. For the following vectors u and v in $\mathbb{R}^4$ determine the angle between them in degrees. $$\mathbf{u} = (2 \quad 3 \quad -8 \quad 1)^T, \mathbf{v} = (-1 \quad 2 \quad -5 \quad -3)^T$$	[03]
	ii. For the following matrix find an orthogonal matrix Q which diagonalizes the given matrix. Also check that $Q^TAQ = D$ where D is a diagonal matrix. $$A = \begin{pmatrix} -5 & \sqrt{3} \\ \sqrt{3} & -3 \end{pmatrix}$$	[06]
	OR	
	Find the dimension and basis for the solution space of the system.	[06]

	$3x_1 + x_2 + x_3 + x_4 = 0$ $5x_1 - x_2 + x_3 - x_4 = 0$	
Q2 (b)	A manufacturing firm produces steel pipes in three plants with daily production volumes of 500, 1000, and 2,000 units respectively. According to past experience, it is known that the fractions of defective output produced by the three plants are respectively 0.005, 0.008, and0.010. If a pipe is selected from a day's total production and found to be defective, find out what is the probability that it came from the first plant?	[06]
Q3 (a)	A random sample of size 16 has the sample mean 53. The sum of the squares of deviation taken from the mean value is 150. Can this sample be regarded as taken from the population having 56 as its mean? Obtain 95 per cent and 99 per cent confidence limits of the sample mean.	[07]
	OR	
	Find the sample covariance matrix for the following table	[07]
Q3 (b)	Theory predicts that the proportion of beans in the four groups A, B, C, D should be 9 : 3 : 3: 1 in an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental results support the theory at 5% level of significance?	[08]

Math (X)	Science (Y)
92	80
60	30
100	70

Q4 (a)	Let the random variable X be five possible symbols {α, β, γ, δ, ε}. Consider two probability distributions p(x) and q(x) over these symbols.	[08]

Symbol	$p(x)$	$q(x)$
α	1/2	1/2
β	1/4	1/8
γ	1/8	1/8
δ	1/16	1/8
ϵ	1/16	1/8

Calculate (i) H(p), (i) H(q), (iii) Relative entropy (Kullback-Leibler distance) D(p||q) , (iv) Relative entropy (Kullback-Leibler distance) D(q||p).

Q4 (b)	Explain Gradient Descent algorithm.	[07]
	OR	
	Explain the steps involved in Expectation Maximization (EM) algorithm	[07]
Q5 (a)	Consider the two dimensional patterns (2, 1), (3, 5), (4, 3), (5, 6), (6, 7), (7, 8). Compute the principal component using PCA Algorithm.	[10]
Q5 (b)	Find steady state matrix, for the given transition probability matrix of a Markov chain $$P = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$$	[05]