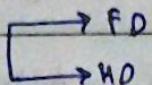


AES Algorithm (Advanced Encryption Standard)

invented by NIST in 2001

- bit level transposition and substitution.
- 128 and 192 bit keys.

* Applications :-

[Secure Access Program] uses AES 128 bit alg.

* 3 versions of AES,

- 1) AES 128 → 128 bit key, 10 Rounds. ✓
- 2) AES 192 → 192 bit key No. of Round 12,
- 3) AES 256 → 256 bit 14 Rounds.

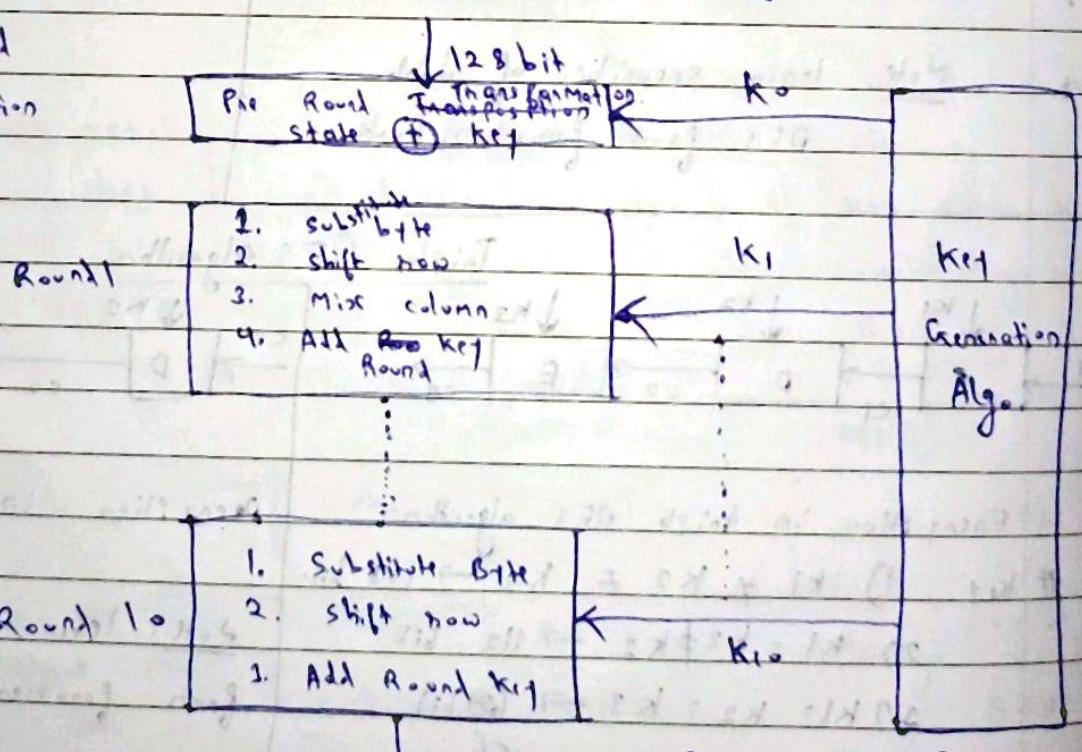
In syllabus only AES 128

- if forgot password, no recovery mechanism.
- In iOS and iPad OS uses AES 128 bit.
- AES is lightweight, in mobile phones or servers.

Block diagram of AES Algorithm 128 bit

State:- 4x4 matrix

of plaintext and

4x4 key representation
as matrix

Pre-round Transformationstate \oplus KeyState: 4×4 Matrix

$$\begin{matrix} a_0 & a_4 & a_8 & a_{12} \\ a_1 & a_5 & a_9 & a_{13} \\ a_2 & a_6 & a_{10} & a_{14} \\ a_3 & a_7 & a_{11} & a_{15} \end{matrix}$$

Input: ABCD, ... total 16

$$\begin{matrix} A & E & I & M & Q \\ B & F & J & P & R \\ C & G & K & O & S \\ D & H & L & T & U \end{matrix}$$

State = 128 bit

* In exam ^{state} key and input will be 2×2 matrix

Q. Key: - DJSA NKGHVNICoEcomp

$$\begin{matrix} D & N & I & C \\ J & G & C & O \\ S & H & O & M \\ A & V & E & P \end{matrix}$$

4×4 Matrix Key

Note: it will be in uppercase
characters only, but if
lowercase is given convert to
uppercase.

state \oplus Key

$$\begin{matrix} A & & D \\ \downarrow & & \downarrow \\ 00 & , & 03 \end{matrix}$$

$$0000 \quad 0000 \quad \oplus \quad 0000 \quad 0011$$

$$0000 \quad 000011$$

= 0

13/03/24

* Key expansion in AES Algorithm *

K₀

b ₀	b ₄	b ₈	b ₁₂
b ₁	b ₅	b ₉	b ₁₃
b ₂	b ₆	b ₁₀	b ₁₄
b ₃	b ₇	b ₁₁	b ₁₅

 \downarrow w₁ \downarrow w₂ \downarrow w₃ \downarrow w₄

K ₀ (16x16)				128 bit Algo. 4x4, K ₁			
c ₀	c ₄	c ₈	c ₁₂	c ₀	c ₄	c ₈	c ₁₂
c ₁	c ₅	c ₉	c ₁₃	c ₁	c ₅	c ₉	c ₁₃
c ₂	c ₆	c ₁₀	c ₁₄	c ₂	c ₆	c ₁₀	c ₁₄
c ₃	c ₇	c ₁₁	c ₁₅	c ₃	c ₇	c ₁₁	c ₁₅

$$Rij = x^{i-1} \bmod \text{prime}$$

$$\omega_4 = \omega_0 \oplus g(\omega_3) \dots$$

$$\omega_5 = \omega_1 \oplus g(\omega_4) \dots$$

$$\omega_6 = \omega_2 \oplus g(\omega_5) \dots$$

$$\omega_7 = \omega_3 \oplus g(\omega_6) \dots$$

$$\text{Round 1} = 01 \quad 9$$

$$\text{Round 2} = 02$$

$$3 = 04$$

$$4 = 08$$

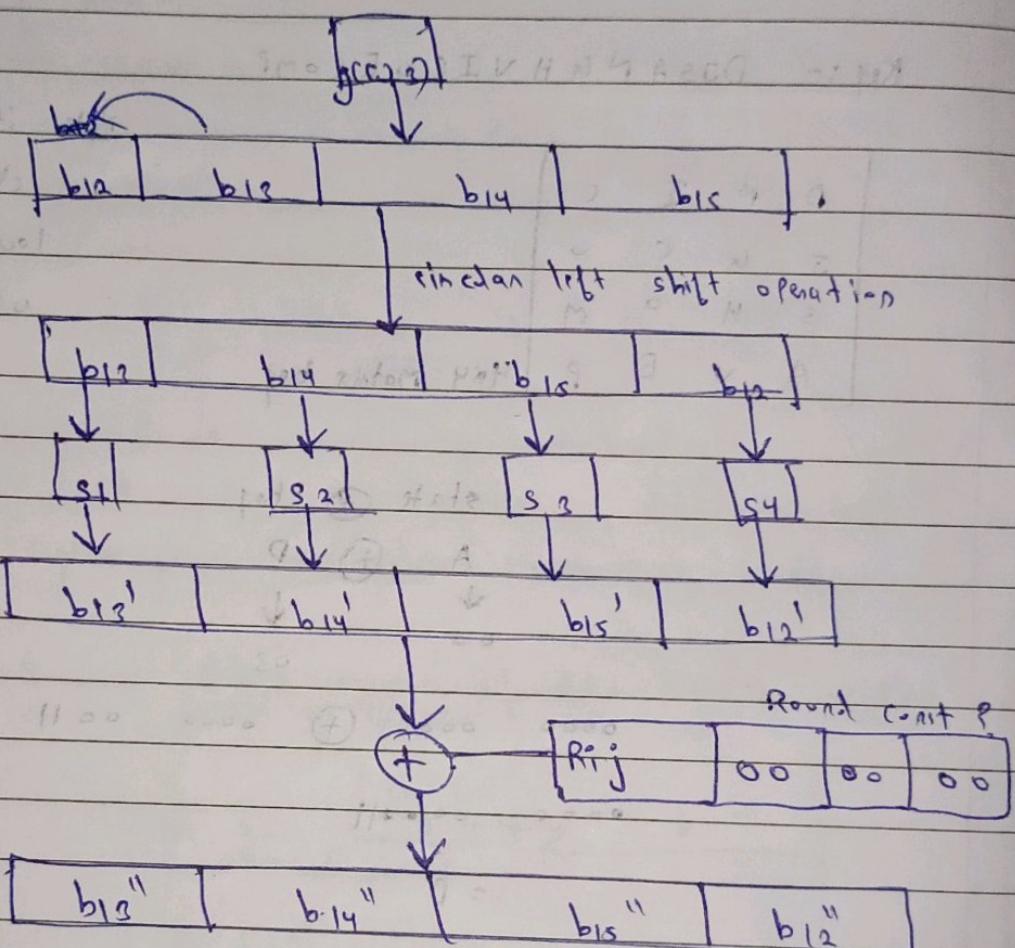
$$5 = 10$$

$$\text{Round } 10^6 = 26$$

$$\text{Hexa}$$

$Rij = \text{Round}$

$$\omega_3 = \{ b_{12}, b_{13}, b_{14}, b_{15} \}$$



* using Key Expansion AES 128 bit technique generate ω_4, ω_5

$$\text{given } \omega_0 = \{ 29, 78, A2, B3 \}$$

$$\omega_3 = \{ 13, AA, 54, 87 \}$$

$$\omega_1 = \{ 34, 75, 56, 88 \}$$

S-Box

13	AA	54	87
Ac	20	17	7D

→ Map this to
this \leftarrow charged value

24 →	0010	0100	{ } w0
7S →	0111	10101	
A2 -	1010	0010	{ } w1
B3 -	1011	0011	
13 -	0001	0011	{ } w2
AA -	1010	1010	
54 -	0101	0100	{ } w3
87 -	1000	0111	
34 -	0011	0100	{ } w4
75 -	0111	0101	
56 -	0101	0110	{ } w5
88 -	1000	1000	

AC - 1010 1101 0010 0011 0110

20 - 0010 0000

17 - 0001 0111

7D - 0111 1101

13	AA	54	87
----	----	----	----

→ Take w3

1 byte left shift circular

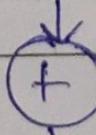
AA	54	87	12
----	----	----	----

→ 12 zero on left

S-box

20	17	7D	AC
----	----	----	----

→ given



Round 1

x_1^-	x_2^-	x_3^-	x_4^-
---------	---------	---------	---------

$$x_1 = 20 \oplus 01 = 21$$

$$x_2 = 17 \oplus 00 = 17$$

$$x_3 = 70 \oplus 00 = 70$$

$$x_4 = AC \oplus 00 = AC$$

This we got $g(x_3) = 21, 17, 70, AC$.

Now by formula,

$$w_4 = w_0 \oplus g(x_3)$$

Given.

$$w_4 = \{24, 75, A2, B3\} + \{21, 17, 70, AC\}$$

$$w_4 = \{05, 62, DF, 1F\} \rightarrow 16 \text{ bit hexa decimal}$$

$$w_5 = w_1 \oplus w_4 \text{ by formula}$$

$$w_5 = \{39, 75, 56, 83\} \oplus \{05, 62, DF, 1F\}$$

$$w_5 = \{31, 17, 89, 97\}$$

① 18/03/24

AES

- Sub byte
- Shift Row
- Mix column
- Add Round Key

H sub byte

{ 2A, C1, 3B, 12 }

4x4 Hexadecimal

01 2	03	01	01	number only now 1. → then only CRS.
01	012	03	01	1 byte circular RS.
01	01	02	02	1 byte circular RS.
03	01	01	02	

if state = 4×4 then assume constant matrix 4×4
if given or not given.

If if then state = 2×2 the constant matrix 2×2
if given or not given.

* Mix Column operation numerical *

Given state = $\begin{bmatrix} b_3 & b_7 \\ b_2 & b_6 \end{bmatrix}$ constant matrix = $\begin{bmatrix} 02 & 03 \\ 01 & 02 \end{bmatrix}$

Resultant Matrix $b = T_{\text{final}}^{-1} \begin{bmatrix} b_0 & b_2 \\ b_1 & b_3 \end{bmatrix}$ {Output of columnar transp.}

Solution : Step 1 Constant matrix \times column 1

$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 02 & 03 \\ 01 & 02 \end{bmatrix} \times \begin{bmatrix} b_3 \\ b_2 \end{bmatrix}$$

$$b_0 = (02 \times b_3) \oplus (03 \times b_2)$$

$$b_1 = (01 \times b_3) \oplus (02 \times b_2)$$

row 1 \times C1row 2 \times C2

No normal multiplication,

it is Finite Field Arithmetic operation. $\text{GF}(2^8) \rightarrow$ Galois Field# Maximum shuffling of data / info.
will be computed by $\text{GF}(2^8)$

- To Add AES algorithm multiplication of two hexa decimal values is performed by using finite field. Arithmetic operation i.e. Galois Field multiplication operation ($\text{GF}(2^8)$).
- This finite field Arithmetic operation is supporting maximum scanning/shuffling of data.

$$\begin{aligned} b_0 &= (02 \times 63) \oplus (02 \times f_2) \\ 02 &= 0000 \quad 0010 \\ 63 &= 0110 \quad 0011 \end{aligned}$$

Next step finding polynomialic values for the 8 binary data

$\text{GF}(2^8)$ = default standard = {0000 0000 ... 1111 1111}

represented $\text{GF}(2^8)$ $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

$$\begin{aligned} \{02\} &= 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \\ &= x \end{aligned}$$

$$\begin{aligned} \{63\} &= 0 \quad 01 \quad 01 \quad 0 \quad 0 \quad 0 \quad 11 \\ &= x^6 + x^5 + x + 1 \end{aligned}$$

$$\begin{aligned} (02 \times 63) &= x \times (x^6 + x^5 + x + 1) \\ &= x^7 + x^6 + x^2 + x \quad \text{removing its binary} \\ &\underline{11000110} \quad \text{if present -1} \\ &= CC \quad \text{not present = 0.} \\ &\qquad\qquad\qquad x^7 = 1 \\ &\qquad\qquad\qquad x^6 = 1 \\ &\qquad\qquad\qquad x^5 = 0 \quad \text{not... 0} \\ &\qquad\qquad\qquad x^4 = 0 \\ &\qquad\qquad\qquad x^3 = 0 \\ &\qquad\qquad\qquad x^2 = 1 \\ &\qquad\qquad\qquad x^1 = 1 \end{aligned}$$

$$\begin{aligned} (03 \times f_2) &= \\ 03 &= 0000 \quad 0011 \\ f_2 &= 1111 \quad 0010 \end{aligned}$$

$$\begin{array}{ccccccccc}
 x^8 & + & x^6 & + & x^5 & + & x^4 & + & x^3 & + x^2 & + x^1 & + 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0
 \end{array}$$

100011.

$x+1$

$$x^7 + x^6 + x^5 + x^4 + x^3$$

(00011010)

$$\begin{aligned}
 &= x^{11} \times (x^7 + x^6 + x^5 + x^4 + x^3) \\
 &= \text{first multiplies all by } x \text{ then all with 1} \\
 &= (x^8 + x^7 + x^6 + x^5 + x^4) + (x^7 + x^6 + x^5 + x^4 + x^3) \text{ with 1.}
 \end{aligned}$$

Now it is $x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x^1 + x^0$

Now cut the same terms
 ~~$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x^1 + x^0$~~

$$(x^r) = x^8 + x^4 + x^2 + x^0$$

The degree of above polynomial is 8 (x^8) which is not part of LCF(28) and maximum degree supported by LCF(28) is x^7 i.e., we need to convert above polynomial into reduce polynomial and for that purpose we have to divide above polynomial by irreducible polynomial.
 irreducible polynomial is a fixed polynomial:

$$x^8 + x^4 + x^3 + x^2 + x^1$$

LCF(28) = 100011011

$$\text{Term} = \text{Ans} = 100011011$$

$$\begin{array}{r}
 100011011 \\
 - 100010110 \\
 \hline
 01000011011 \\
 \hline
 000000000
 \end{array}$$

$(03 \times Bf_2)$, 00

$$\begin{aligned}
 b_0 &= (02 \times b_3) \oplus (03 \times f_2) \\
 &= CB \oplus 00 \\
 b_0 &= CB
 \end{aligned}$$

$$b_1 = (01 \times b_3) \oplus (02 \times f_2)$$

$$01 = 0000 \quad 0001$$

$$b_3 = 0110 \quad 0011$$

$$\begin{aligned}
 GF(2^8) &= x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x^1 + 1 \\
 \{01\} &= 0 \quad 1 \\
 &\quad - \quad | \\
 &\quad 1
 \end{aligned}$$

$$\begin{aligned}
 \{b_3\} &= 0 \quad 11 \quad 0 \quad 00 \quad 11 \\
 &\quad x^6 + x^5 + x + 1
 \end{aligned}$$

$$\begin{aligned}
 \{01 \times b_3\} &= x^6 + x^5 + x + 1 \\
 &= 01100011 \\
 b_0 &= b_3
 \end{aligned}$$

$$\begin{aligned}
 (02 \times f_2) & \quad 02 = 0000 \quad 0010 \\
 & \approx \quad b_2 = 111 \quad 0010 \\
 & \quad x^4 \cdot GF(2^8)
 \end{aligned}$$

$$\{02\} = x$$

$$\{f_2\} = x^7 + x^6 + x^5 + x^4 + x$$

$$(c_2 * f_2) = x(x^3 + x^6 + x^5 + x^4 + x)$$

$$= x^8 + x^7 + x^6 + x^5 + x^2$$

$$w(x) = x^8 + x^7 + x^6 + x^5 + x^2$$

$$\underline{\hspace{2cm} \hspace{2cm} \hspace{2cm} \hspace{2cm} \hspace{2cm}}$$

$$p(x) = x^8 + x^4 + x^3 + x + 1$$

$$= 100011011$$

$$\begin{array}{r} w(x) \\ p(x) \end{array} \quad \begin{array}{r} \underline{\hspace{2cm}} \hspace{2cm} \hspace{2cm} \hspace{2cm} \hspace{2cm} \\ 100011011 \end{array}$$

$$\begin{array}{r} \\ \\ 100011011 \\ \hline \end{array} \quad \left| \begin{array}{r} \underline{\hspace{2cm} \hspace{2cm} \hspace{2cm} \hspace{2cm} \hspace{2cm}} \\ - 100011011 \\ \hline 011111111 \end{array} \right.$$

$$b_1 = (ff)$$

$$(b_3 \oplus ff)$$

$$= \begin{array}{r} 0110 \quad 0011 \\ \hline \end{array} \quad \begin{array}{r} x \oplus R \\ \oplus R \end{array} \quad \begin{array}{r} \underline{\hspace{2cm} \hspace{2cm} \hspace{2cm} \hspace{2cm} \hspace{2cm}} \\ 1111 \quad 1111 \end{array}$$

$$= \underline{\hspace{2cm} \hspace{2cm} \hspace{2cm} \hspace{2cm} \hspace{2cm}}$$

$$\begin{array}{r} 0110 \quad 0011 \\ + \underline{1111} \quad 1111 \\ \hline 1001 \quad 1100 \end{array}$$

$$b_2 = (c_2 \times b_3) + (c_3 \times f_2)$$

with column 2

constant matrix

$$b_2 = \begin{bmatrix} c_2 & c_3 \\ c_1 & c_2 \end{bmatrix} \times \begin{bmatrix} 47 \\ 9C \end{bmatrix}$$

$$b_2 = (c_2 \times 47) + (c_3 \times 9C)$$

$$b_3 = (c_1 \times 47) + (c_2 \times 9C)$$

$$\textcircled{1} \quad b_2 = (c_2 \times 47)$$

$$\begin{array}{r} 02 = 0000 \quad 0010 \\ 47 = 0100 \quad 00111 \end{array}$$

$$GF(2^8) = x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

0 0 0 0 0 1 0

$$\{02\} = x$$

$$\{47\} = x^6 + x^2 + x + 1$$

$$(02 \times 47) = x(x^6 + x^2 + x + 1)$$

$$= x^7 + x^3 + x^2 + x$$

$$= 10001110$$

$$\underline{b_2} = 8E$$

$$\textcircled{1} \quad \underline{b_3} = (03 \times 9C)$$

$$= 03 = 0000 \quad 0011$$

$$9C = 1001 \quad 1100$$

$$GF(2^8) = x^7 \quad x$$

$$\{03\} = x + 1$$

$$\{9C\} = x^7 + x^4 + x^3 + x^2$$

$$(03 \times 9C) = (x+1) \times (x^7 + x^4 + x^3 + x^2)$$

$$= x^8 + x^5 + x^4 + x^3 + x^7 + x^4 + x^3 + x^2$$

$$G(x) = x^8 + x^5 + x^7 + x^2$$

110100100

$$P(x) = x^8 + x^4 + x^3 + x + 1$$

100011011

$$= G(x) = \underline{\underline{110100100}}$$

P(x) 100011011

$$\begin{array}{r}
 1000 \ 11011 \\
 + 1000 \ 1101 \\
 \hline
 0101 \ 11111
 \end{array}
 \quad
 \begin{array}{c}
 \text{B} \\
 \text{F}
 \end{array}$$

$$b_2 = 8E \times BF$$

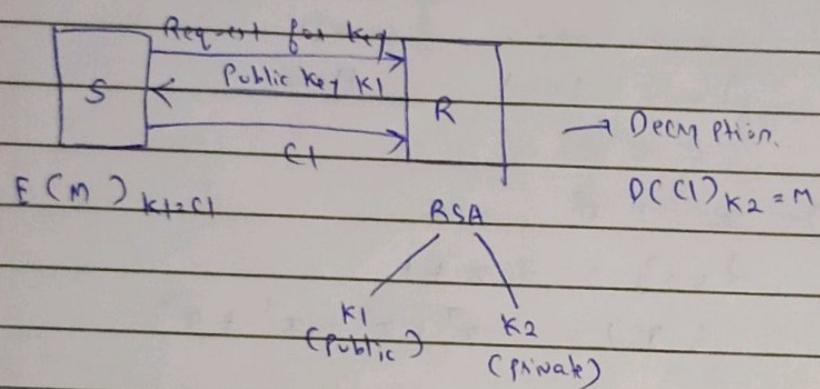
$$b_2 = 31$$

To find: b_3

02/09/24.

* Knapsack Cryptosystem, *

- Public Key Cryptography \rightarrow Public Key.
 \rightarrow Private Key



* RSA algorithm *

Step 1: Selecting 2 prime numbers

p, q

public key
(e, n)

Step 2: $n = p \times q$

Step 3: Removing $z = \phi(n)$ private key
 $= (p-1) * (q-1)$ small

(d, n)

\hookrightarrow Euler Totient Function.

Euler Totient Function for non-prime number $\phi(10)$

$\phi(10)$

①

②

writing till that ϕ value

removing the factors that coming into that