# Batch : ELITE (CAT)

## Subject : Quantitative Aptitude

### Topic : Inequalities L2 & Function L1

**DPP - 07**

**1.** $(x-1)(x-2)^2(x-3)^3(x-4)^4 < 0$, then how many integral values 4x can assume?

**2.** Arrange the numbers in ascending order.

$6^{\frac{1}{6}}, 2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 8^{\frac{1}{8}}$

(a) $6^{\frac{1}{6}} > 2^{\frac{1}{2}} > 3^{\frac{1}{3}} > 8^{\frac{1}{8}}$ (b) $2^{\frac{1}{2}} > 3^{\frac{1}{3}} > 6^{\frac{1}{6}} > 8^{\frac{1}{8}}$

(c) $3^{\frac{1}{3}} > 2^{\frac{1}{2}} > 6^{\frac{1}{6}} > 8^{\frac{1}{8}}$ (d) $2^{\frac{1}{2}} > 2^{\frac{1}{2}} > 6^{\frac{1}{6}} > 8^{\frac{1}{8}}$

**3.** If $(x^4 - 16x^3 + 86x^2 - 176x + 105) < 0$, then the number of integral values 3x can assume is:

**4.** If $(x-1)^{2017}(x-3)^{2019}(x-5)^{2021}(x-7)^{2023} < 0$, then how many integral values 3x can assume?

**5.** If $x^4 - 22x^3 + 159x^2 - 418x + 300 < 20$, then 3x can assume how many integral values?

**6.** If $x(x^2 - 16x + 65) > 50$ and $x^2 - 10x + 36 \leq 20$, then find the values x can assume.

(a) $\pi$ (b) $\frac{\pi}{\sqrt{3}}$

(c) $\pi\sqrt{3}$ (d) $2\pi$

**7.** If $x(x-7)^2 < 64 - 7x$ and $x(x-8)^2 < 50 - x$, then what can be the value of x if $x > 1$?

(a) $\pi$ (b) $\pi\sqrt{3}$

(c) $\frac{\pi}{\sqrt{3}}$ (d) $\pi^2$

**8.** If $x(x-7)^2 < 64 - 7x$ and $x(x-8)^2 > 50 - x$, then what can be the value of x?

(a) $\pi$ (b) $\pi\sqrt{2}$

(c) $\pi\sqrt{3}$ (d) $\frac{\pi}{\sqrt{2}}$

**9.** If $a + 3b + 5c + 7d < 31$ where a, b, c, d are distinct natural numbers, then $\frac{(x-a)(x-d)^d}{(x-b)(x-c)+\frac{(b+c)^d}{a}}$ can assume which of the below values:

1. $-\frac{\pi}{3}$ 2. $\frac{\pi}{3}$

3. $-\pi$ 4. $\frac{\pi}{4}$

(a) Only 1 and 2

(b) Only 2, 3, and 4

(c) Only 1 and 3

(d) Only 1 and 4

**10.** If $a + 3b + 5c + 7d < 31$, where a, b, c, d are distinct natural numbers and if $\frac{(x-a)^a(x-d)^d}{(x-b)^b(x-c)^c} < 0$, then x can assume which of the below values:

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{3}$

(c) 2 (d) $\frac{\pi}{4}$

**11.** If $x(x-6)^2 < 28 - 3x$, $x(x-9)^2 < 120 - 11x$ and $(x-4)(x-8) < 5$, then x can assume which of the following values?

(a) $\pi\sqrt{1}$ (b) $\pi\sqrt{3}$

(c) $\pi^2$ (d) $\pi\sqrt{4}$

**12.** If $x(x-6)^2 > 28 - 3x$, $x(x-9)^2 < 120 - 11x$ and $(x-4)(x-8) < 5$, then x can assume which of the below values?

(a) $\pi\sqrt{1}$ (b) $\pi\sqrt{3}$

(c) $\pi\sqrt{4}$ (d) $\pi\sqrt{6}$

**13.** If $(x^3 - 3^4) < 9(x-1)(2x-9)$ and $x^2(x-10)^2 < 9(10-x)(3x-2)$, then x can assume which of the below values?

1. $\pi$
2. $2\pi$
3. $\frac{\pi}{2}$
4. $\sqrt{\pi}$

(a) Only 1 and 2 (b) Only 2 and 3
(c) Only 1 and 3 (d) Only 2, 3 and 4

**14.** If $(x^3 - 3^4) > 9(x-1)(2x-9)$ and $x^2(x-10)^2 < 9(10-x)(3x-2)$, then x can assume which of the below values?

1. $\pi$
2. $2\pi$
3. $3\pi$
4. $\pi^2$

(a) Only 1 and 2 (b) Only 2 and 3
(c) Only 3 and 4 (d) Only 2 and 4

**15.** If $a + 3b + 5c + 7d < 31$, where a, b, c, d are distinct natural numbers and if $\frac{(x-a)^a (x-d)^d}{(x-b)^b (x-c)^c} > 0$, then x can assume which of the below values:

1. $-\frac{\pi}{3}$
2. $\pi$
3. 4
4. $\frac{\pi}{4}$

(a) Only 1 and 2 (b) Only 1, 2, and 4
(c) Only 1 and 3 (d) Only 2 and 4

**16.** If $\frac{x^3 - 12x^2 + 39x - 28}{x^3 - 18x^2 + 99x - 162} < 0$, then x can assume

1. $\pi$
2. $\frac{\pi}{2}$
3. $\pi\sqrt{3}$
4. $3\pi$

(a) Only 1 and 2 (b) Only 2 and 3
(c) Only 2, 3, and 4 (d) Only 1 and 3

**17.** If $(x^3 - 3^4) < 9(x-1)(2x-9)$ and $x^2(x-10)^2 > 9(10-x)(3x-2)$, then x can assume which of the below values?

1. $\pi$
2. $\frac{\pi}{2}$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{4}$

(a) Only 1, 2 and 5 (b) Only 1 and 4
(c) Only 3 and 4 (d) Only 4

**18.** If $\sqrt{3-x} > \sqrt{x+1}$, then how many integral values 4x can assume?

**19.** If $\sqrt{20-x^2} > \sqrt{x+8}$, then how many integral values x can assume?

**20.** If $\sqrt{28-7x} > \sqrt{x(x-4)(x-8)}$, then how many values x can assume?

1. 0
2. 1
3. $\frac{\pi}{3}$
4. $\frac{\pi}{4}$

(a) Only 1 and 3 (b) Only 1 and 4
(c) Only 2 and 3 (d) Only 2 and 4

**21.** If $\sqrt{28-7x} < \sqrt{x(x-4)(x-8)}$, then how many values x can assume

1. 0
2. $\pi$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{4}$

(a) Only 1 and 3 (b) Only 1 and 4
(c) Only 2 and 3 (d) Only 2 and 4

**22.** If $\sqrt{x+9} > \sqrt{16-x}$, then how many integral values x can assume?

**23.** If $\sqrt{36-x^2} > \sqrt{x+16}$, then find which of the below values x can assume?

1. $\pi$
2. $-\pi\sqrt{2}$
3. $\pi\sqrt{2}$
4. $\pi^2$

(a) Only 2 (b) Only 1 and 2
(c) Only 2, 3 and 4 (d) Only 3

**24.** If $x^4 - 8x^3 + 14x^2 + 8x < 15$, then $x\sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}}$ can assume how many integral values?

**25.** Let

$$\left(\sqrt{x}+\sqrt{4}\right)^2 \left(\sqrt{x}+\sqrt{2}\right)^3 \left(\sqrt{x}-\sqrt{2}\right)^4 \left(\sqrt{x}-\sqrt{4}\right)^5 < 0$$

If $P = x^{\frac{1}{\sqrt{2\sqrt{2\sqrt{2...\infty}}}}}$ is a real number, then

(a) $-\sqrt{4} < P < \sqrt{4}$

(b) $0 < P\sqrt{4}$

(c) $0 \le P < \sqrt{2}, \sqrt{2} < P < \sqrt{4}$

(d) $\sqrt{2} < P < \sqrt{4}$

**26.** If $\left(x^2 - 17\right)^{\left(x^3 - 6x^2 + 11x - 6\right)} = 1$ and if $(x+4)^{2021}(x-5)^{2023} < 0$, then find the number of values x can assume.

**27.** If $x^3 - 9x^2 + 23x - 15 > 0$ and if $x^2 - 6x + 5 < 0$, then find the possible numbers of integer values 3x can assume.

**28.** If $|x^2 - 9x + 18| > x^2 - 9x + 18$, then which is true?

(a) $x \le 3$ or $x \ge 6$

(b) $3 \le x \le 6$

(c) $3 < x < 6$

(d) None of the above

**29.** $\left(\sqrt{x\sqrt{x\sqrt{x....\infty}}}\right)^{\sqrt{2+\sqrt{2+\sqrt{2+....\infty}}}}$ $(x-3)\left(x^2 - x + 4\right)^{\frac{3}{2}}$ $(x-9)^3 < 0$. So, find the number of integral values that x can assume.

**30.** If $\dfrac{x^3 - 12x^2 + 39x - 28}{x^3 - 18x^2 + 99x - 162} > 0$, then x can assume

1. $\pi$

2. $\dfrac{\pi}{2}$

3. $\pi\sqrt{3}$

4. $3\pi$

(a) Only 1 and 2

(b) Only 2 and 3

(c) Only 1, 3, and 4

(d) Only 1 and 4

# Answer Key

1. (6)
2. (c)
3. (10)
4. (10)
5. (16)
6. (a)
7. (b)
8. (b)
9. (d)
10. (b)
11. (d)
12. (d)
13. (d)
14. (c)
15. (b)
16. (b)
17. (d)
18. (8)
19. (6)
20. (b)
21. (c)
22. (13)
23. (b)
24. (10)
25. (c)
26. (5)
27. (5)
28. (c)
29. (5)
30. (d)

# Hints & Solutions

**1.** **(6)**

$(x-1)(x-2)^2(x-3)^3(x-4)^4 < 0$

As, $(x-2)^2 \geq 0$ and $(x-4)^4 \geq 0$

So, $(x-1)(x-3)^3 < 0$

$\Rightarrow (x-1)(x-3) < 0$ [Since, $(x-3)^2 \geq 0$]

$\Rightarrow 1 < x < 3$

But, $x \neq 2$ as $x = 2$ will give $(x-1)(x-2)^2(x-3)^3(x-4)^4 = 0$

So, $1 < x < 2$ and $2 < x < 3$

$\Rightarrow 4 < 4x < 8$ and $8 < 4x < 12$

So, 4x can assume values from 5, 6, 7, 9, 10, 11

Therefore, total 6 values can be assumed by 4x.

**2.** **(c)**

Let $a = 6^{\frac{1}{6}}$

$\Rightarrow a^6 = 6$ …. (i)

Again, $b = 2^{\frac{1}{2}}$

$b^2 = 2$

$\Rightarrow b^6 = 2^3 = 8$

$\Rightarrow b^6 > a^6$ …. (ii)

Also, $c = 3^{\frac{1}{3}}$

$\Rightarrow c^3 = 3$

$\Rightarrow c^6 = 3^2 = 9$

$\Rightarrow c^6 > b^6 > a^6$

$\Rightarrow c > b > a$ …. (iii)

Also, $d = 8^{\frac{1}{8}}$

$\Rightarrow d^8 = 8 = 2^3$

$\Rightarrow d^8 = b^6$

$\Rightarrow d^6 < b^6$

$\Rightarrow d < b$ …. (iv)

Again,

$d^{24} = 2^9 = 512$

$a^{24} = 6^4 > d^{24}$

$\Rightarrow a > d$ …. (v)

Hence, from (iii), (iv) and (v), we can conclude that, $c > b > a > d$

So, $3^{\frac{1}{3}} > 2^{\frac{1}{2}} > 6^{\frac{1}{6}} > 8^{\frac{1}{8}}$

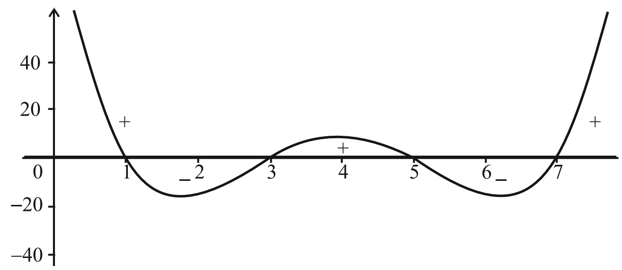
**3.** **(10)**

$x^4 - 16x^3 + 86x^2 - 176x + 105 < 0$

$\Rightarrow (x^4 - 4x^3 + 3x^2 - 12x^3 + 48x^2 - 36x + 35x^2 - 140x + 105) < 0$

$\Rightarrow (x^2 - 4x + 3)(x^2 - 12x + 35) < 0$

$\Rightarrow (x-1)(x-3)(x-5)(x-7) < 0$




So, $1 < x < 3$ and $5 < x < 7$.

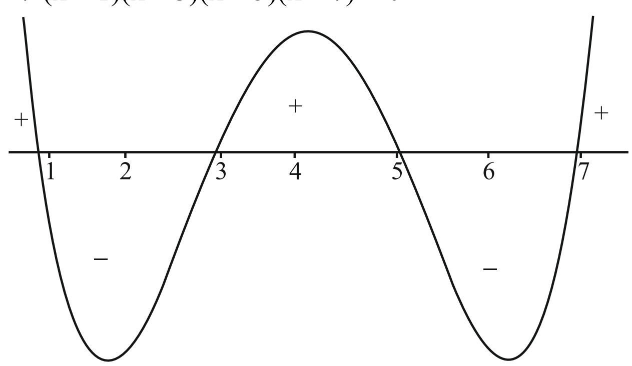
Thus, 3x can assume a total of 10 integer values.

**4.** **(10)**

As $(x-1)^{2016} \geq 0$, $(x-3)^{2018} \geq 0$, $(x-5)^{2020} \geq 0$ and $(x-7)^{2022} \geq 0$, so

$(x-1)^{2017}(x-3)^{2019}(x-5)^{2021}(x-7)^{2023} < 0$

$\Rightarrow (x-1)(x-3)(x-5)(x-7) < 0$




So, $1 < x < 3$ ; $5 < x < 7$

$\Rightarrow 3 < 3x < 9$ ; $15 < 3x < 21$

So, 3x can assume 10 integral values.

**5.** **(16)**

Given that,

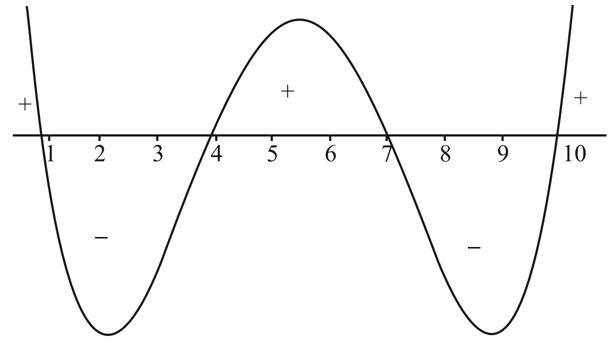
$x^4 - 22x^3 + 159x^2 - 418x + 300 < 20$

$\Rightarrow x^4 - 22x^3 + 159x^2 - 418x + 280 < 0$

$\Rightarrow (x^4 - 17x^3 + 70x^2 - 5x^3 + 85x^2 - 350x + 4x^2 - 68x + 280) < 0$

$\Rightarrow (x^2 - 5x + 4)(x^2 - 17x + 70) < 0$

$\Rightarrow (x-1)(x-4)(x-7)(x-10) < 0$




So, $1 < x < 4$ and $7 < x < 10$

$\Rightarrow 3 < 3x < 12$ and $21 < 3x < 30$

3x can assume 16 integral values.

**6.** **(a)**

Given that,

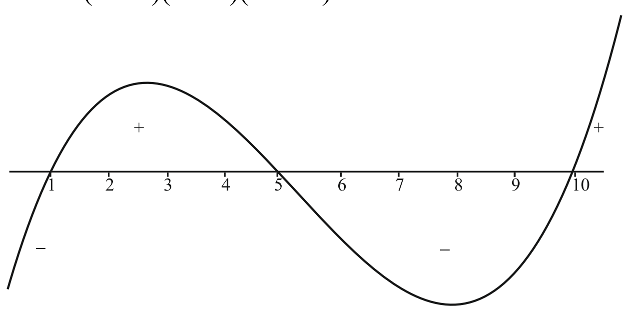
$x(x^2 - 16x + 65) > 50$

$\Rightarrow x^3 - 16x^2 + 65x - 50 > 0$

$\Rightarrow x^3 - 6x^2 + 5x - 10x^2 + 60x - 50 > 0$

$\Rightarrow (x^2 - 6x + 5)(x - 10) > 0$

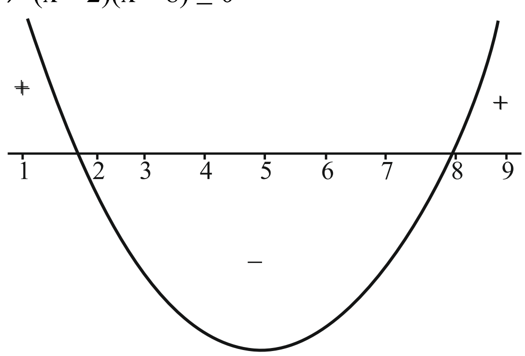
$\Rightarrow (x - 1)(x - 5)(x - 10) > 0$




Again, $x^2 - 10x + 36 \le 20$

$\Rightarrow x^2 - 10x + 16 \le 0$

$\Rightarrow (x - 2)(x - 8) \le 0$




Combining the two conditions, we can say that

$2 \le x < 5$

Hence, the only value of x satisfying this range is $\pi$.

**7.** **(b)**

Given that, $x(x - 7)^2 < 64 - 7x$

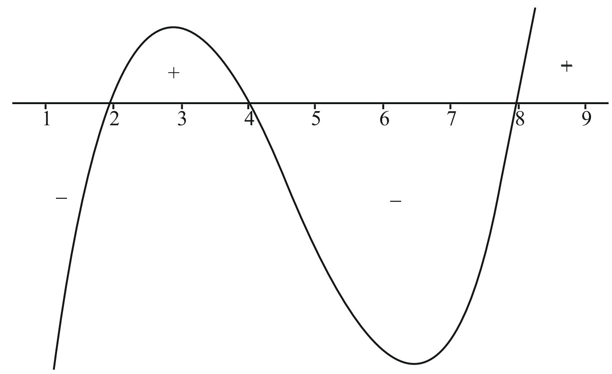
$\Rightarrow x^3 - 14x^2 + 49x < 64 - 7x$

$\Rightarrow x^3 - 14x^2 + 56x < 64$

$\Rightarrow x^3 - 6x^2 + 8x - 8x^2 + 48x - 64 < 0$

$\Rightarrow (x^2 - 6x + 8)(x - 8) < 0$

$\Rightarrow (x - 2)(x - 4)(x - 8) < 0$



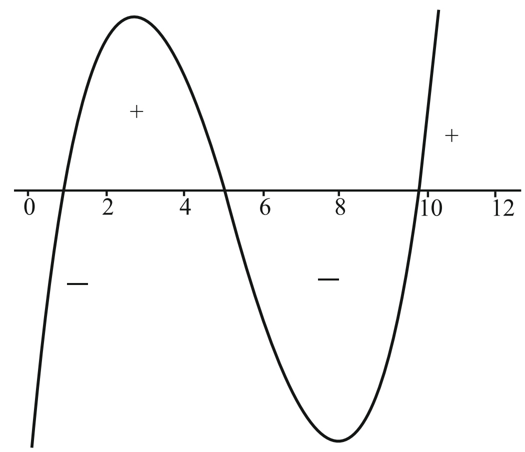

Again, $x(x - 8)^2 < 50 - x$

$\Rightarrow x^3 - 16x^2 + 65x < 50$

$\Rightarrow x^3 - 6x^2 + 5x - 10x^2 + 60x < 50$

$\Rightarrow (x^2 - 6x + 5)(x - 10) < 0$

$\Rightarrow (x - 1)(x - 5)(x - 10) < 0$




Hence, combining these two conditions, we get

$5 < x < 8$

In this range, the only x that can be satisfied is $\pi\sqrt{3}$.

**8.** **(b)**

The given inequation is

$x(x - 7)^2 < 64 - 7x$

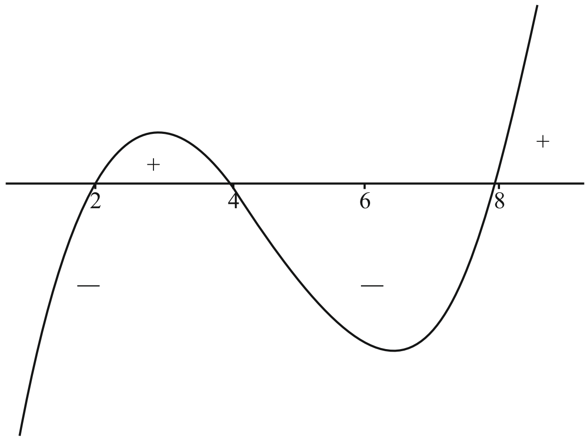
$\Rightarrow x^3 - 14x^2 + 49x < 64 - 7x$

$\Rightarrow x^3 - 14x^2 + 56x < 64$

$\Rightarrow x^3 - 6x^2 + 8x - 8x^2 + 48x - 64 < 0$

$\Rightarrow (x^2 - 6x + 8)(x - 8) < 0$

$\Rightarrow (x - 2)(x - 4)(x - 8) < 0$



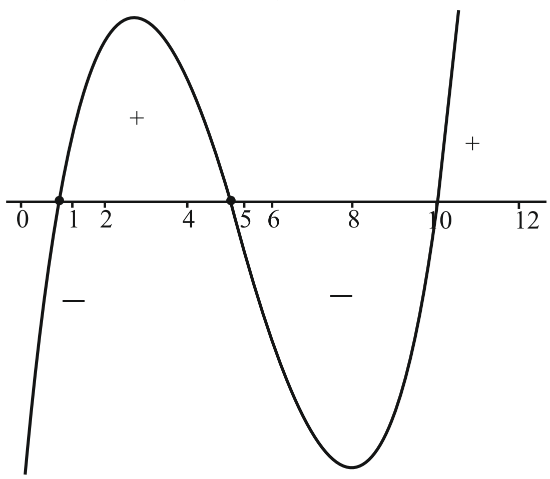

Again, $x(x-8)^2 > 50 - x$

$\Rightarrow x^3 - 16x^2 + 65x > 50$

$\Rightarrow x^3 - 6x^2 + 5x - 10x^2 + 60x > 50$

$\Rightarrow (x^2 - 6x + 5)(x - 10) > 0$

$\Rightarrow (x - 1)(x - 5)(x - 10) > 0$




Combining these two conditions, we get
$4 < x < 5$.

In this range, the only value of x will be $\pi\sqrt{2}$.

**9.** **(d)**

$a + 3b + 5c + 7d \le 30$ as a, b, c, d are distinct natural numbers.

Minimum value of the expression will be obtained if the lower number is multiplied with the higher coefficients.

So, Min $(a + 3b + 5c + 7d) = 4 + 9 + 10 + 7 = 30$ which satisfy the equation.

Thus, $a = 4$, $b = 3$, $c = 2$, and $d = 1$

Now, we have

$$\frac{(x-4)(x-1)}{(x-3)(x-2)+\frac{(3+2)^1}{4}}$$

$$=\frac{(x-4)(x-1)}{(x-3)(x-2)+\frac{5}{4}}$$

$$=\frac{x^2-5x+4}{x^2-5x+6+\frac{5}{4}}$$

$$=\frac{x^2-5x+4}{\left(x-\frac{5}{2}\right)^2+1}$$

$$=1-\frac{\frac{13}{4}}{\left(x-\frac{5}{2}\right)^2+1}$$

Minimum value of the expression can be obtained at $x=\frac{5}{2}$, where $\left(x-\frac{5}{2}\right)^2+1$ is minimum.

$$\text{So, Min}\left\{\frac{(x-4)(x-1)}{(x-3)(x-2)+\frac{5}{4}}\right\}=-\frac{9}{4}$$

Also, the maximum value of the expression will be obtained if $\left(x-\frac{5}{2}\right)^2+1$ is maximized.

Thus, maximum value of the expression will be very close to 1 (but less than 1) as x assumes a very high value.

$$\text{So, } -\frac{9}{4}\le\frac{(x-1)(x-4)}{(x-3)(x-2)+\frac{5}{4}}<1$$

**10.** **(b)**

$a + 3b + 5c + 7d \le 30$ as a, b, c, d are distinct natural numbers. Then, the minimum value of the expression will be obtained if the lower number is multiplied with the higher coefficients.

So, Min $(a + 3b + 5c + 7d) = 4 + 9 + 10 + 7 = 30$ which satisfies the equation.

Thus, $a = 4$, $b = 3$, $c = 2$, and $d = 1$

$$\text{Therefore, } \frac{(x-4)^4(x-1)}{(x-3)^3(x-2)^2}<0$$

$\Rightarrow \dfrac{x-1}{x-3} < 0$ [Since, the even powered expressions are non–negative]

$\Rightarrow \dfrac{(x-1)(x-3)}{(x-3)^2} < 0\,[x \neq 2, 3]$

$\Rightarrow (x - 1)\,(x - 3) < 0$

$\Rightarrow 1 < x < 3$ except $x = 2$ and $3$.

**11.** **(d)**

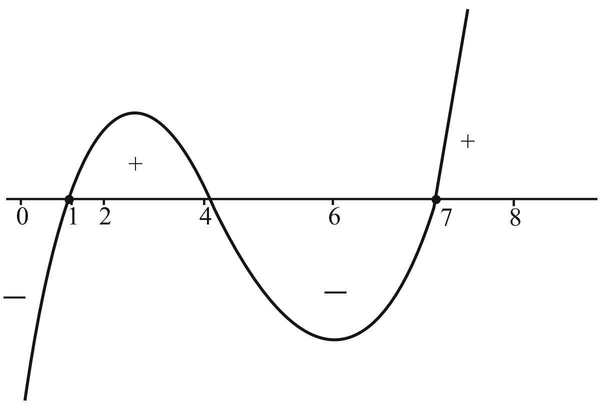
$x(x - 6)^2 < 28 - 3x$

$\Rightarrow x^3 - 12x^2 + 39x - 28 < 0$

$\Rightarrow x^3 - 5x^2 + 4x - 7x^2 + 35x - 28 < 0$

$\Rightarrow (x^2 - 5x + 4)(x - 7) < 0$

$\Rightarrow (x - 1)(x - 4)(x - 7) < 0$




$x(x - 9)^2 < 120 - 11x$

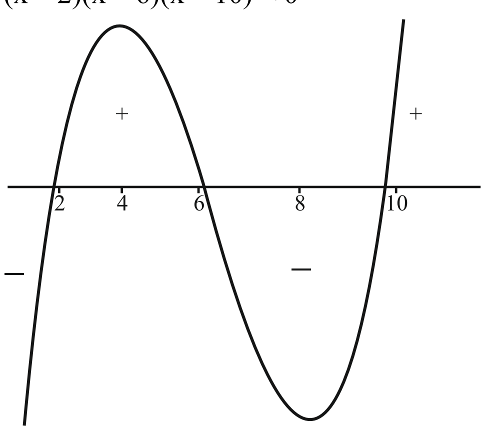
$\Rightarrow x^3 - 18x^2 + 81x < 120 - 11x$

$\Rightarrow x^3 - 18x^2 + 92x < 120$

$\Rightarrow x^3 - 8x^2 + 12x - 10x^2 + 80x < 120$

$\Rightarrow (x^2 - 8x + 12)(x - 10) < 0$

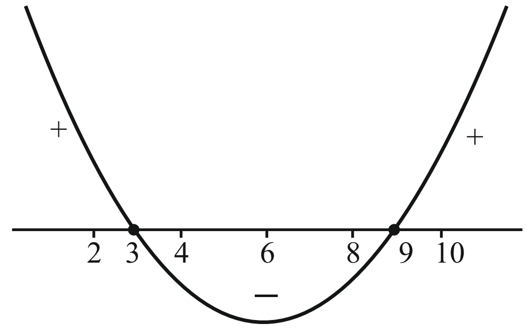
$\Rightarrow (x - 2)(x - 6)(x - 10) < 0$




$(x - 4)(x - 8) < 5$

$\Rightarrow x^2 - 12x + 27 < 0$

$\Rightarrow (x - 3)(x - 9) < 0$




Combining three conditions, we get

$6 < x < 7$.

Hence, the only value that satisfy this range is $\pi\sqrt{4}$.

**12.** **(d)**

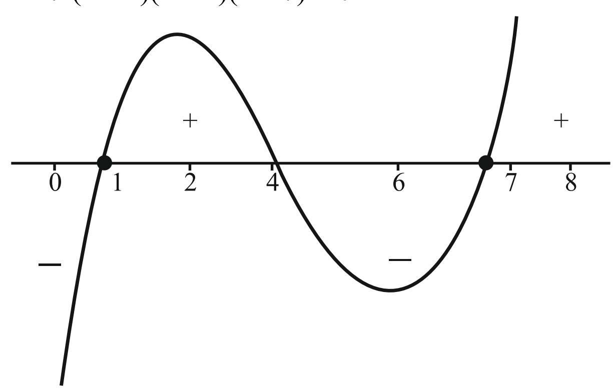
$x(x - 6)^2 > 28 - 3x$

$\Rightarrow x^3 - 12x^2 + 39x - 28 > 0$

$\Rightarrow x^3 - 5x^2 + 4x - 7x^2 + 35x > 28$

$\Rightarrow (x^2 - 5x + 4)(x - 7) > 0$

$\Rightarrow (x - 1)(x - 4)(x - 7) > 0$




$x(x - 9)^2 < 120 - 11x$

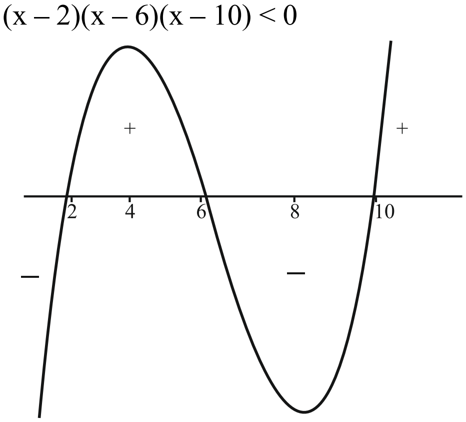
$\Rightarrow x^3 - 18x^2 + 81x < 120 - 11x$

$\Rightarrow x^3 - 18x^2 + 9^2x < 120$

$\Rightarrow x^3 - 8x^2 + 12x - 10x^2 + 80x < 120$

$\Rightarrow (x^2 - 8x + 12)(x - 10) < 0$

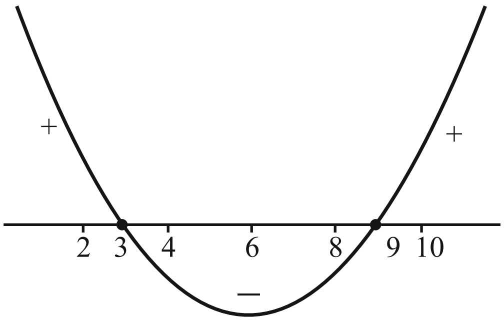
$\Rightarrow (x - 2)(x - 6)(x - 10) < 0$



$(x - 4)(x - 8) < 5$

$\Rightarrow x^2 - 12x + 27 < 0$

$\Rightarrow (x - 3)(x - 9) < 0$




Combining three conditions, we get

$7 < x < 9$.

Hence, the only value that satisfy this range is $\pi\sqrt{6}$.

**13.** **(d)**

Given that $(x^3 - 3^4) < 9(x - 1)(2x - 9)$

$$\Rightarrow \left(\frac{x^3}{9} - 9\right) < 2x^2 - 11x + 9$$

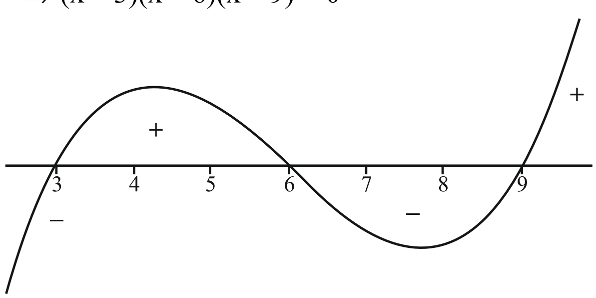
$$\Rightarrow \frac{x^3}{9} < 2x^2 - 11x + 18$$

$\Rightarrow x^3 - 18x^2 + 99x - 162 < 0$

$\Rightarrow x^3 - 9x^2 + 18x - 9x^2 + 81x - 162 < 0$

$\Rightarrow (x^2 - 9x + 18)(x - 9) < 0$

$\Rightarrow (x - 3)(x - 6)(x - 9) < 0$




Again, $x^2(x - 10)^2 < 9(10 - x)(3x - 2)$

$\Rightarrow x^2(x - 10)^2 + 9(x - 10)(3x - 2) < 0$

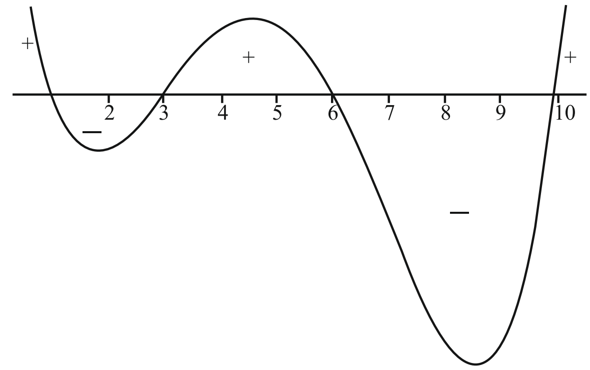
$\Rightarrow x^2(x - 10)^2 + 9(3x^2 - 30x - 2x + 20) < 0$

$\Rightarrow x^4 - 20x^3 + 127x^2 - 288x + 180 < 0$

$\Rightarrow x^4 - 4x^3 + 3x^2 - 16x^3 + 64x^2 - 48x + 60x^2 - 240x + 180 < 0$

$\Rightarrow (x^2 - 4x + 3)(x^2 - 16x + 60) < 0$

$\Rightarrow (x - 1)(x - 3)(x - 6)(x - 10) < 0$




Hence, combining these two conditions, we get

$6 < x < 9$ and $1 < x < 3$.

**14.** **(c)**

Given that $(x^3 - 3^4) > 9(x - 1)(2x - 9)$

$$\Rightarrow \left(\frac{x^3}{9} - 9\right) > 2x^2 - 11x + 9$$

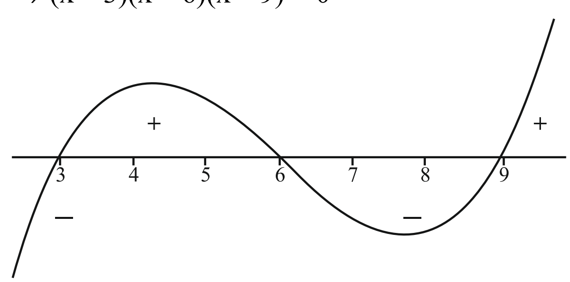
$$\Rightarrow \frac{x^3}{9} > 2x^2 - 11x + 18$$

$\Rightarrow x^3 - 18x^2 + 99x - 162 > 0$

$\Rightarrow x^3 - 9x^2 + 18x - 9x^2 + 81x > 162$

$\Rightarrow (x^2 - 9x + 18)(x - 9) > 0$

$\Rightarrow (x - 3)(x - 6)(x - 9) > 0$




Again, $x^2(x - 10)^2 < 9(10 - x)(3x - 2)$

$\Rightarrow x^2(x - 10)^2 + 9(x - 10)(3x - 2) < 0$

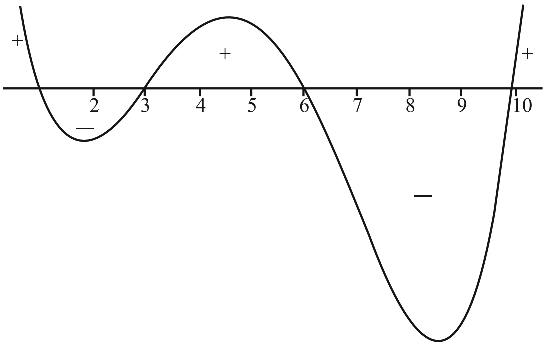
$\Rightarrow x^2(x - 10)^2 + 9(3x^2 - 30x - 2x + 20) < 0$

$\Rightarrow x^4 - 20x^3 + 127x^2 - 288x + 180 < 0$

$\Rightarrow x^4 - 4x^3 + 3x^2 - 16x^3 + 64x^2 - 48x + 60x^2 - 240x < -180$

$\Rightarrow (x^2 - 4x + 3)(x^2 - 16x + 60) < 0$

$\Rightarrow (x - 1)(x - 3)(x - 6)(x - 10) < 0$




Hence, combining these two conditions, we get $9 < x < 10$.

**15.** **(b)**

$a + 3b + 5c + 7d \leq 30$ as a, b, c, d are distinct natural numbers. Then, the minimum value of the expression will be obtained if the lower number is multiplied with the higher coefficients.

So, Min $(a + 3b + 5c + 7d) = 4 + 9 + 10 + 7 = 30$ which satisfies the equation.

Thus, $a = 4$, $b = 3$, $c = 2$, and $d = 1$

Therefore, $\dfrac{(x-4)^4(x-1)}{(x-3)^3(x-2)^2} > 0$

$\Rightarrow \dfrac{x-1}{x-3} > 0$ [Since, the even powered expressions are non–negative]

$\Rightarrow \dfrac{(x-1)(x-3)}{(x-3)^2} > 0$ $[x \neq 2, 3]$

$\Rightarrow (x-1)(x-3) > 0$

$\Rightarrow x < 1$ and $x > 3$ [$x \neq 4$ and 1 as it'll make the value 0]

**16.** **(b)**

The expression $x^3 - 18x^2 + 99x - 162$ can be written as

$x^3 - 18x^2 + 99x - 162$

$= x^3 - 9x^2 + 18x - 9x^2 + 81x - 162$

$= (x^2 - 9x + 18)(x - 9)$

$= (x - 3)(x - 6)(x - 9)$

Also, $x^3 - 12x^2 + 39x - 28$

$= x^3 - 5x^2 + 4x - 7x^2 + 35x - 28$

$= (x^2 - 5x + 4)(x - 7)$

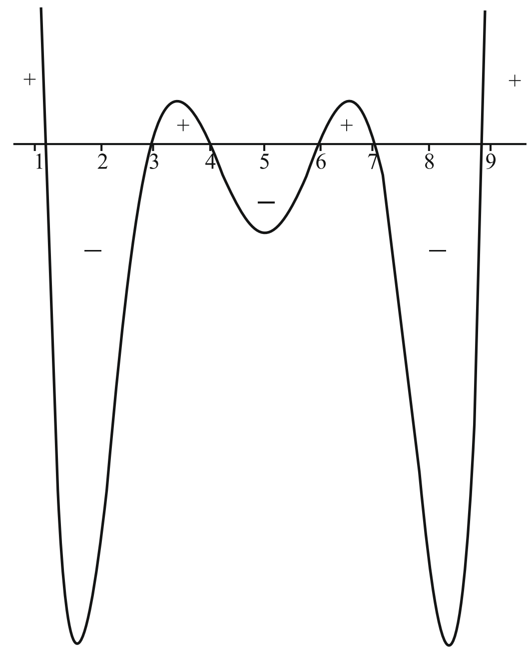
$= (x - 1)(x - 4)(x - 7)$

Now,

$$\frac{x^3 - 12x^2 + 39x - 28}{x^3 - 18x^2 + 99x - 162} < 0$$

$$\Rightarrow \frac{(x-1)(x-4)(x-7)}{(x-3)(x-6)(x-9)} < 0$$

$$\Rightarrow \frac{(x-1)(x-3)(x-4)(x-6)(x-7)(x-9)}{(x-3)^2(x-6)^2(x-9)^2} < 0$$




**17.** **(d)**

Given that $(x^3 - 3^4) < 9(x - 1)(2x - 9)$

$\Rightarrow \left(\dfrac{x^3}{9} - 9\right) < 2x^2 - 11x + 9$

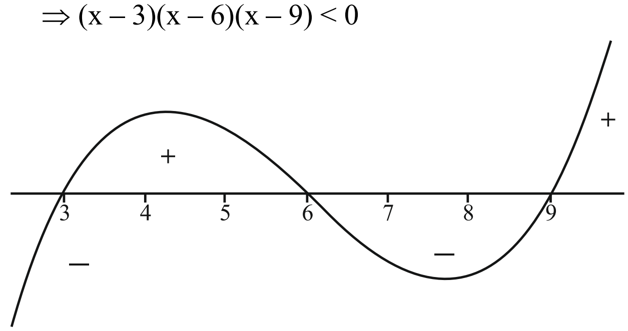
$\Rightarrow \dfrac{x^3}{9} < 2x^2 - 11x + 18$

$\Rightarrow x^3 - 18x^2 + 99x - 162 < 0$

$\Rightarrow x^3 - 9x^2 + 18x - 9x^2 + 81x < 162$

$\Rightarrow (x^2 - 9x + 18)(x - 9) < 0$

$\Rightarrow (x - 3)(x - 6)(x - 9) < 0$




Again, $x^2(x - 10)^2 > 9(10 - x)(3x - 2)$

$\Rightarrow x^2(x-10)^2 + 9(x-10)(3x-2) > 0$

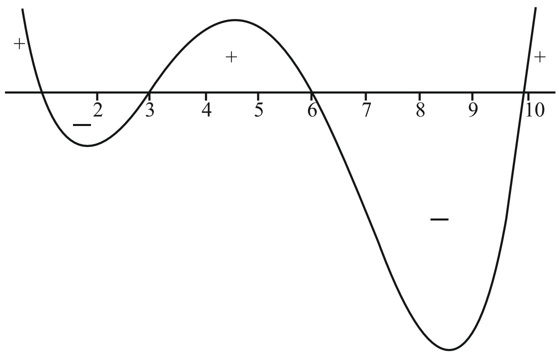
$\Rightarrow x^2(x-10)^2 + 9(3x^2 - 30x - 2x + 20) > 0$

$\Rightarrow x^4 - 20x^3 + 127x^2 - 288x + 180 > 0$

$\Rightarrow x^4 - 4x^3 + 3x^2 - 16x^3 + 64x^2 - 48x + 60x^2 - 240x > -180$

$\Rightarrow (x^2 - 4x + 3)(x^2 - 16x + 60) > 0$

$\Rightarrow (x-1)(x-3)(x-6)(x-10) > 0$




Hence, combining these two conditions, we get $x < 1$.

**18.** **(8)**

$\sqrt{3-x} > \sqrt{x+1}$

As square of any non–negative number $\geq 0$

$3 - x > x + 1$

$\Rightarrow 2 > 2x$

$\Rightarrow x < 1$

Also, $\sqrt{x+1}$ is a real number, so $x + 1 \geq 0$

$\Rightarrow x \geq -1$

So, $-1 \leq x < 1$

$\Rightarrow -4 \leq 4x < 4$

Thus, 4x can assume 8 integral values.

**19.** **(6)**

$\sqrt{20-x^2} > \sqrt{x+8}$

$\Rightarrow 20 - x^2 > x + 8$

$\Rightarrow 0 > x^2 + x - 12$

$\Rightarrow (x+4)(x-3) < 0$

$\Rightarrow -4 < x < 3$

So, x can assume 6 integral values.

**20.** **(b)**

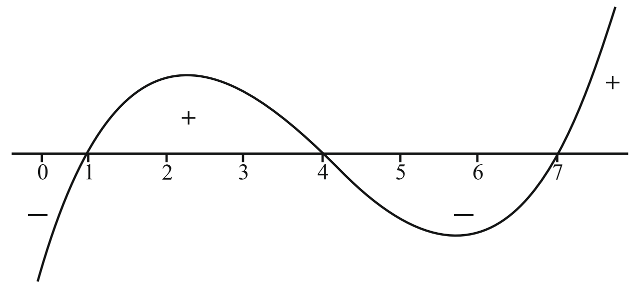
Given that, $\sqrt{28-7x} > \sqrt{x(x-4)(x-8)}$

$\Rightarrow x^3 - 12x^2 + 39x - 28 < 0$

$\Rightarrow x^3 - 5x^2 + 4x - 7x^2 + 35x - 28 < 0$

$\Rightarrow (x^2 - 5x + 4)(x - 7) < 0$

$\Rightarrow (x-1)(x-4)(x-7) < 0$



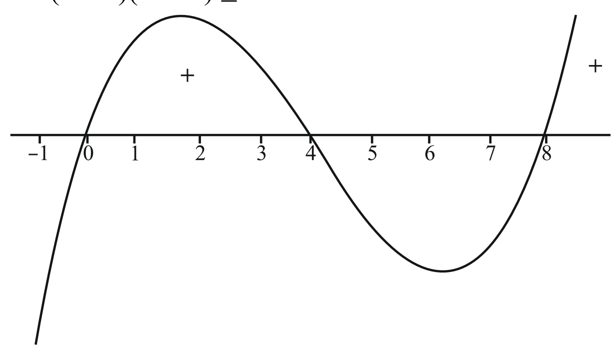

As, $\sqrt{28-7x}$ is a real number, so

$28 - 7x \geq 0$

$\Rightarrow x \leq 4$

As, $\sqrt{x(x-4)(x-8)}$ is a real number, so

$x(x-8)(x-4) \geq 0$




$\Rightarrow 0 \leq x \leq 4$ and $x \geq 8$.

Combining these two conditions, we get

$0 \leq x < 1$

**21.** **(c)**

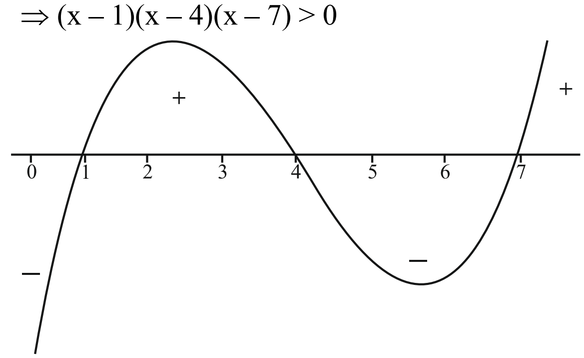
Given that, $\sqrt{28-7x} < \sqrt{x(x-4)(x-8)}$

$\Rightarrow x^3 - 12x^2 + 39x - 28 > 0$

$\Rightarrow x^3 - 5x^2 + 4x - 7x^2 + 35x - 28 > 0$

$\Rightarrow (x^2 - 5x + 4)(x - 7) > 0$

$\Rightarrow (x-1)(x-4)(x-7) > 0$



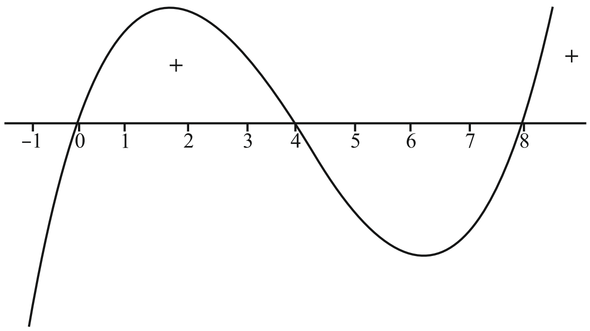

As, $\sqrt{28-7x}$ is a real number, so

$\Rightarrow 28 - 7x \geq 0$

$\Rightarrow x \le 4$

As, $\sqrt{x(x-4)(x-8)}$ is a real number, so

$x(x-8)(x-4) \ge 0$




$\Rightarrow 0 \le x \le 4$ and $x \ge 8$.

Combining these two conditions, we get

$1 < x < 4$.

**22.** **(13)**

Given that,

$\sqrt{x+9} > \sqrt{16-x}$

$x + 9 > 16 - x$

$\Rightarrow x > 3.5$

Also, $16 - x \ge 0$

$\Rightarrow x \le 16$

So, $3.5 < x \le 16$

Therefore, x can assume 13 integral values.

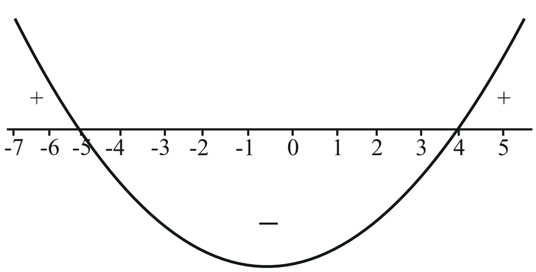
**23.** **(b)**

$\sqrt{36-x^2} > \sqrt{x+16}$

$\Rightarrow 36 - x^2 > x + 16$

$\Rightarrow x^2 + x - 20 < 0$

$\Rightarrow (x-4)(x+5) < 0$




Also, $36 - x^2 \ge 0$

$\Rightarrow -6 \le x \le 6$

And,

$x + 16 \ge 0$

$\Rightarrow x \ge -16$

Combining all the conditions, we get

$-5 < x < 4$

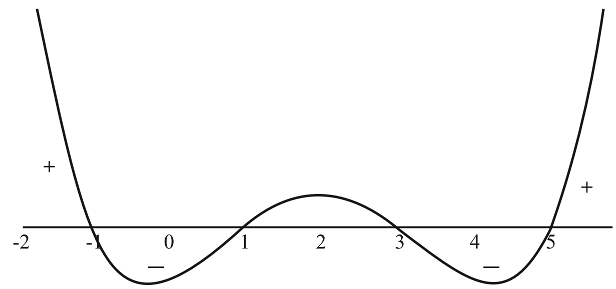
**24.** **(10)**

Given that, $x^4 - 8x^3 + 14x^2 + 8x < 15$

$\Rightarrow x^4 - 8x^3 + 15x^2 - x^2 + 8x - 15 < 0$

$\Rightarrow (x^2 - 1)(x^2 - 8x + 15) < 0$

$\Rightarrow (x + 1)(x - 1)(x - 3)(x - 5) < 0$




Again, let $\sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}} = y$

Then, $y^2 = 6 + y$

$y^2 - y - 6 = 0$

$\Rightarrow (y-3)(y+2) = 0$

$\Rightarrow y = 3$ [Since, $y > 0$]

So, 3x can assume

$-3 < 3x < 3$; $9 < 3x < 15$

So, 3x can assume 10 values.

**25.** **(c)**

Let $k = \sqrt{2\sqrt{2\sqrt{2\ldots\infty}}}$

$\Rightarrow k^2 = 2k$

$\Rightarrow k = 2$ (Since, $k \ne 0$)

So, $P = x^{\frac{1}{2}}$ is a real number.

Also, $\sqrt{x} \ge 0 \Rightarrow P \ge 0$

So, $\left(P+\sqrt{4}\right)^2\left(P+\sqrt{2}\right)^3\left(P-\sqrt{2}\right)^4\left(P-\sqrt{4}\right)^5 < 0$

$\Rightarrow \left(P+\sqrt{2}\right)\left(P-\sqrt{4}\right) < 0$

[Since, $\left(P+\sqrt{4}\right)^2 \ge 0, \left(P+\sqrt{2}\right)^2 \ge 0; \left(P-\sqrt{4}\right)^4$

$\ge 0, \left(P-\sqrt{2}\right)^4 \ge 0$]

$\Rightarrow -\sqrt{2} < P < 2$

As, $P \ge 0$, so $0 \le P < 2$, but $P \ne \sqrt{2}$ as

$\left(P-\sqrt{2}\right)^4 = 0$ and the expression will become 0.

So, $0 \le P < \sqrt{2}; \sqrt{2} < P < 2$

**26.** **(5)**

$x^3 - 6x^2 + 11x - 6$

$= (x^3 - 3x^2 + 2x - 3x^2 + 9x - 6)$

$= (x^2 - 3x + 2)(x - 3)$

$= (x - 1)(x - 2)(x - 3)$

Now, $(x^2 - 17)^{(x^3 - 6x^2 + 11x - 6)} = 1$ implies

either, $x^3 - 6x^2 + 11x - 6 = 0$

$\Rightarrow (x - 1(x - 2)(x - 3) = 0$

$\Rightarrow x = 1, 2, 3$

or, $x^2 - 17 = 1$

$\Rightarrow x = \pm 3\sqrt{2}$

or, $x^2 - 17 = -1$

$\Rightarrow x = \pm 4$

Therefore,

$(x + 4)^{2021} (x - 5)^{2023} < 0$

As, $(x + 4)^{2020} \ge 0$ and $(x - 5)^{2022} \ge 0$

So, $(x + 4)(x - 5) < 0$

$\Rightarrow -4 < x < 5$

So, the values of x in this range are 1, 2, 3, 4, $3\sqrt{2}$

Hence, x can assume 5 values.

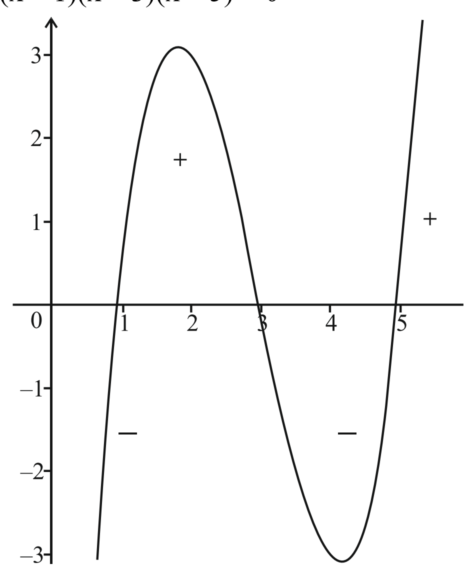
**27.** **(5)**

$x^3 - 9x^2 + 23x - 15 > 0$

$\Rightarrow x^3 - 4x^2 + 3x - 5x^2 + 20x - 15 > 0$

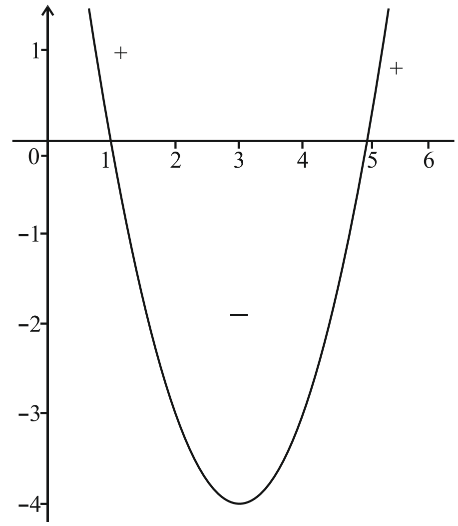
$\Rightarrow (x^2 - 4x + 3)(x - 5) > 0$

$\Rightarrow (x - 1)(x - 3)(x - 5) > 0$




Also, $x^2 - 6x + 5 < 0$

$\Rightarrow (x - 1)(x - 5) < 0$




$\Rightarrow 1 < x < 3$

So, $3 < 3x < 9$.

Hence, 3x can assume total of 5 integer values.

**28.** **(c)**

We have the inequality

$|x^2 - 9x + 18| > x^2 - 9x + 18$

First let, $x^2 - 9x + 18 \ge 0$

$\Rightarrow (x - 3)(x - 6) \ge 0$

$\Rightarrow x \le 3$ or $x \ge 6$.

Again, if $x^2 - 9x + 18 < 0$

$\Rightarrow (x - 3)(x - 6) < 0$

$\Rightarrow 3 < x < 6$

Therefore, if $x^2 - 9x + 18 > 0$, then $x^2 - 9x + 18 > x^2 - 9x + 18$ and there is no solution.

If $x^2 - 9x + 18 < 0$, then $-(x^2 - 9x + 18) > x^2 - 9x + 18$

$\Rightarrow -x^2 + 9x - 18 > 0$

$\Rightarrow -(x - 3)(x - 6) > 0$

$\Rightarrow 3 < x < 6$

Hence, option (c) is correct.

**29.** **(5)**

Let $\sqrt{x\sqrt{x\sqrt{x \ldots \infty}}} = p$

Then, $p^2 = xp$

$\Rightarrow p = x$ [Since, $x \ne 0$]

Also, $\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots \infty}}} = q$

$\Rightarrow q^2 = 2 + q$

$\Rightarrow q^2 - q - 2 = 0$

$\Rightarrow (q-2)(q+1)=0$

$\Rightarrow q = 2$, since $q > 0$

So, $\left(\sqrt{x\sqrt{x\sqrt{x\ldots\infty}}}\right)^{\sqrt{2+\sqrt{2+\sqrt{2+\ldots\infty}}}}$

$$(x-3)\left(x^2-x+4\right)^{\frac{3}{2}}(x-9)^3<0$$

$$\Rightarrow x^2\left(x^2-x+4\right)^{\frac{3}{2}}(x-3)(x-9)^3<0$$

Now $x^2-x+4=\left(x-\frac{1}{2}\right)^2+\frac{15}{4}>0$

or $\left(x^2-x+4\right)^{\frac{3}{2}}>0$

Also $x^2 \geq 0$

So, $(x-3)(x-9)^3 < 0$

Because $(x-9)^2 \geq 0$

So, $(x-3)(x-9) < 0$

So, $3 < x < 9$

So, x can assume integral values from 4 to 8.

So, total of 5 values can be assumed by x.

**30.** **(d)**

The expression $x^3 - 18x^2 + 99x - 162$ can be written as $x^3 - 18x^2 + 99x - 162$

$= x^3 - 9x^2 + 18x - 9x^2 + 81x - 162$

$= (x^2 - 9x + 18)(x - 9)$

$= (x - 3)(x - 6)(x - 9)$

Also, $x^3 - 12x^2 + 39x - 28$

$= x^3 - 5x^2 + 4x - 7x^2 + 35x - 28$

$= (x^2 - 5x + 4)(x - 7)$

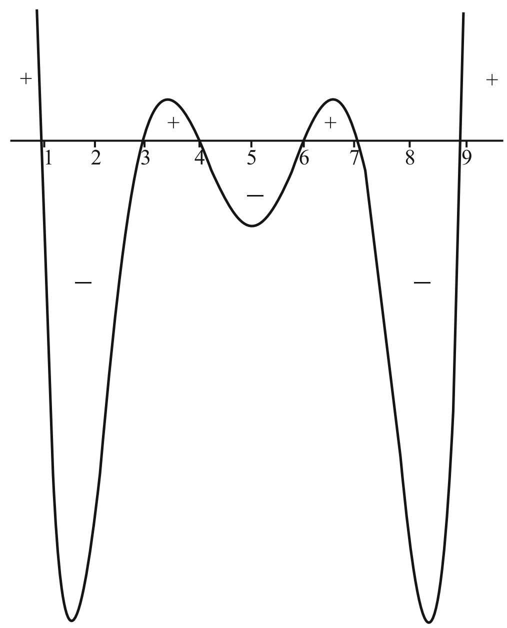
$= (x - 1)(x - 4)(x - 7)$

Now,

$$\frac{x^3-12x^2+39x-28}{x^3-18x^2+99x-162}>0$$

$$\frac{(x-1)(x-4)(x-7)}{(x-3)(x-6)(x-9)}>0$$

$$\Rightarrow \frac{(x-1)(x-3)(x-4)(x-6)(x-7)(x-9)}{(x-3)^2(x-6)^2(x-9)^2}>0$$




The values of x which satisfies the inequation,

$x > 9 \cup (6,7) \cup (3,4) \cup x < 1$