Batch : PIONEER (CAT)

Subject : Quantitative Aptitude

Topic : Inequalities 3 (Higher Powers + (x+1/x)+Word Problems)

DPP - 09

1. If $8 \geq \frac{x+5}{3x+1} \geq 2$, then the number of integral values assumed by 115x will be _______.

2. If, a, b, c, and d are distinct natural numbers. Then, find the minimum positive integer value of x in $\frac{5x+3}{x+1} \geq k$, where 5k is the minimum possible value of (a + 2b + 3c + 4d).

3. Let $f = x^4 - 6x^3 + 12x^2 - 8x$. Determine the set of values of x for which the inequality f > 0 holds true:
(a) $(-\infty, 0) \cup (2, +\infty)$
(b) $(-\infty, 0) \cup (1, 3) \cup (3, +\infty)$
(c) $(-\infty, 1) \cup (2, 3) \cup (3, +\infty)$
(d) $(-\infty, 1) \cup (1, 2) \cup (3, +\infty)$

4. Let $g = x^5 - 10x^4 + 40x^3 - 80x^2 + 64x$. Determine the set of values of x for which the inequality g > 0 holds true:
(a) $(-\infty, 0) \cup (1, 4)$ (b) $(-\infty, 0) \cup (2, 4)$
(c) $(-\infty, 1) \cup (2, 4)$ (d) $(0, 2) \cup (4, +\infty)$

5. Consider the system of inequalities:
$2x - 3y \leq 6$
$x + y \geq 5$
Which of the following points is not in the solution set?
(a) (3, 2) (b) (2, 3)
(c) (1, 3) (d) (4, 1)

6. Given the following system of linear inequalities:
$x + 3y \leq 12$
$y - 2x \geq -3$
$2x + y \geq 2$
Which of the following points is in the solution set?
1. (1, 2) 2. (2, 3)
3. (3, 1) 4. (4, 2)
(a) Only 2 (b) Both 1 and 2
(c) Both1 and 3 (d) All of the above

7. Consider the following system of linear inequalities:
I) $x - y \geq 1$
II) $2x + y < 8$
III) $y > x^2 - 6x + 8$
Which of the following points is in the solution set?
1. (1, 0)
2. (3, 1)
3. (2, 3)
4. (4, 3)
(a) Only 2 (b) Both 1 and 2
(c) Both 1 and 3 (d) None of the above

8. Consider the following system of linear inequalities:
$5x - 3y \geq 15$
$2x + 4y \leq 12$
$x - y \geq 0$
$x \geq 0$
$y \geq 0$
Which of the following points (x, y) lies inside the feasible region of the system of inequalities?
1. (2, 1)
2. (5, 1)
3. (1, 1)
4. (3, 2)
(a) Only 2 (b) Both 1 and 2
(c) Both 1 and 3 (d) None of the above

9. If $2x + 5 < 5(y + 1)$ and $\frac{7y+6}{y+9} \leq 1$, then
(a) $x < \frac{5}{4}$
(b) $x > -\frac{5}{4}$
(c) $x < -\frac{5}{4}$
(d) $y > \frac{5}{4}$

10. Consider a polynomial function f = (x – 1)(x + 2)(x – 3)(x + 4). Determine the solution for the following inequality:
$f < 0$
(a) $(-\infty, -4) \cup (-2, 1) \cup (3, +\infty)$
(b) $(-4, -2) \cup (1, 3)$
(c) $(-\infty, -4) \cup (-2, 1) \cup (3, 5)$
(d) $(-\infty, -4) \cup (-2, 1) \cup (2, 3)$

11. If $9x + 2 < (6y + 2)$ and $\frac{5y+6}{y+1} \le 2$, then
(a) $x < \frac{2}{3}$
(b) $x > -\frac{2}{3}$
(c) $x < -\frac{2}{3}$
(d) $y > \frac{2}{3}$

12. If $(3x + 2y + 5z) \le 50$, where x, y, and z are distinct odd prime numbers, then how many distinct values z can assume?
(a) 1 (b) 4
(c) 3 (d) 2

13. If $\frac{2x+3}{4x+1} \ge 2$, then the number of integral values assumed by 24x will be _______.
(a) 10 (b) 12
(c) 13 (d) 14

14. Prasoon went to a shop to purchase a few pens, pencils and erasers. The number of pens purchased is more than the number of pencils purchased which in turn is more than the number of erasers purchased. It is also given that the rates of the pens, pencils and erasers are distinct prime numbers when expressed in $ and the sum of rates of unit pen and pencil is equal to that of one eraser. If the total number of items purchased is 6 then find the minimum possible cost of purchase of all the items in $.
(a) 15 (b) 16
(c) 17 (d) 18

15. If $\frac{2x+9}{x+1} \ge 8$, then the number positive of integral values assumed by 12x will be _______.
(a) 12
(b) 13
(c) 14
(d) 15

16. If, a, b, c, d, e and f are distinct natural numbers. Then, find the minimum positive integer value of x in
$\frac{8x+2}{x+2} \ge m$, where 8m is the minimum possible value of $(a + 2b + 3c + 4d + 5e + 6f)$.
(a) 10
(b) 11
(c) 12
(d) 13

17. If x4- 6x3+ 14x2- 15x + 6 > 0. Choose the correct option below:
(a) $(-\infty, 1) \cup (2, 3) \cup (4, \infty)$
(b) $(-\infty, 1) \cup (2, \infty)$
(c) $(1, 2) \cup (3, 4) \cup (4, \infty)$
(d) $(1, 2) \cup (4, \infty)$

18. Determine the intervals for which $x^3 - 5x^2 + 8x < 4$. Choose the correct option below:
(a) $(-\infty, 1)$
(b) $(1, 2) \cup (4, \infty)$
(c) $(1, 2)$
(d) $(-\infty, 1) \cup (4, \infty)$

19. If $(2x^3 - x^2 - 13x) > 6$ then x must be from
(a) $(-\infty,-2) \cup \left(-\frac{1}{2}, 3\right)$
(b) $(-\infty,-2) \cup \left(-\frac{1}{2}, 3\right) \cup (3,\infty)$
(c) $\left(-2,-\frac{1}{2}\right) \cup (3,\infty)$
(d) $(-\infty, -2) \cup (3, \infty)$

20. Let $h = x^3 - 4x - x^2 + 4$. Determine the intervals where h is positive or negative.

(a) $h > 0$ for $x \in (-\infty, -3) \cup (-1, 1) \cup (2, 4)$ and $h < 0$ for $x \in (-3, -1) \cup (1, 2) \cup (4, +\infty)$

(b) $h > 0$ for $x \in (-\infty, -3) \cup (-1, 1) \cup (3, 4)$ and $h < 0$ for $x \in (-3, -1) \cup (1, 3) \cup (4, +\infty)$

(c) $h > 0$ for $x \in (-2, 1) \cup (2, \infty)$ and $h < 0$ for $x \in$ $x \in (-\infty, -2) \cup (1, 2)$

(d) $h > 0$ for $x \in (-\infty, -1) \cup (1, 3)$ and $h < 0$ for $x \in (-1, 1) \cup (3, +\infty)$

21. If $x^6 + \left(\frac{1}{x^6}\right) + 6 \le 4\left(x^3 + \frac{1}{x^3}\right)$, determine the values of x for which the given inequality holds true.

22. If $(x^2 - 4)(x^2 - x - 6) < 0$. Then find the number of integral values that 3x can assume.

23. If $x^4 + \left(\frac{1}{x^4}\right) - 2\left(x^2 + \frac{1}{x^2}\right) + 2 \le 0$

Determine the values of x for which the given inequality holds true.

(a) 1
(b) 0
(c) -1
(d) Both A and C

24. A metal wire company sells metal wires in bulk to various customers. The company has a policy that the total sales of metal wires should not exceed \$12000 per day. The company charges \$200 for every kilogram of metal wire it sells. The company also has a policy that the weight of metal wire sold to a single customer should not exceed 20 kilograms. What is the minimum number of customers the company per day can sell metal wires to, as per the above constraints?

25. If $x^4 - 10x^3 + 35x^2 - 50x + 24 < 0$, then find the integral number of values that 4x can take.

26. A farmer wants to fence a rectangular field using 200 meters of fencing material. The farmer wants to maximize the area of the field. What is the maximum area of the field that can be fenced in square meters?

27. If $x^3 - 6x^2 + 11x < 6$ and $x^3 - 11x^2 + 34x < 24$, then x the value of

(a) $1 < x < 6$
(b) $1 < x < 3$
(c) $x > 1$
(d) $x < 1$

28. If $x^3 - 8x^2 + 11x + 20 > 0$. Then x can be from which of the below intervals

(a) $(3, 5)$ and $(-\infty, -1)$
(b) $(-1, 4)$ and $(5, \infty)$
(c) $(-1, 3)$ and $(5, \infty)$
(d) $(-\infty, -2)$ and $(3, 5)$

29. If $(x-3)^2(x-5)^5(x-7)^3(x-9)^7 < 0$, then how many positive integral values of x can be found?

(a) 5
(b) 4
(c) 3
(d) 2

30. Let $P = (x^2 - 9)(x^2 - 4)(x^2 - 1)(x^2 - 16)$. Find the intervals of x for which $P > 0$.

(a) $(-\infty, -4) \cup (-3, -2) \cup (-1, 1) \cup (2, 3) \cup (4, \infty)$
(b) $(-\infty, -3) \cup (-2, -1) \cup (1, 2) \cup (3, \infty)$
(c) $(-\infty, -4) \cup (-3, -2) \cup (1, 2) \cup (4, \infty)$
(d) $(-\infty, -3) \cup (-2, 1) \cup (2, 3) \cup (4, \infty)$

Answer Key

1.	(85)	11.	(c)	21.	(1)
2.	(1)	12.	(d)	22.	(2)
3.	(a)	13.	(a)	23.	(d)
4.	(d)	14.	(c)	24.	(3)
5.	(c)	15.	(c)	25.	(6)
6.	(b)	16.	(c)	26.	(2500)
7.	(a)	17.	(b)	27.	(d)
8.	(d)	18.	(a)	28.	(b)
9.	(a)	19.	(c)	29.	(b)
10.	(b)	20.	(c)	30.	(a)

Hints & Solutions

1. **(85)**

$\frac{x+5}{3x+1} \geq 2$

$\frac{x+5}{3x+1} - 2 \geq 0$

$\Rightarrow \frac{x+5-2(3x+1)}{3x+1} \geq 0$

$\Rightarrow \frac{-5x+3}{3x+1} \geq 0$

$\Rightarrow -5x + 3 \geq 0$ and $3x + 1 > 0$ [Since, at x = 0, the expression will not exist.]

$\Rightarrow 5x \leq 3$ and $3x > -1$

$\Rightarrow x \leq \frac{3}{5}$ and $x > -\frac{1}{3}$

$\Rightarrow -\frac{1}{3} < x \leq \frac{3}{5}$ …. (i)

Also,

$\frac{x+5}{3x+1} \leq 8$

$\frac{x+5}{3x+1} - 8 \leq 0$

$\Rightarrow \frac{x+5-8(3x+1)}{3x+1} \leq 0$

$\Rightarrow \frac{-23x-3}{3x+1} \leq 0$

$\Rightarrow 3x + 1 < 0$ or $-23x - 3 \leq 0$ [Since, at x = 0, the expression will not exist.]

$\Rightarrow x < -\frac{1}{3}$ or $x \geq -\frac{3}{23}$ …. (ii)

So, combining (i) and (ii), we have

$-\frac{3}{23} \leq x \leq \frac{3}{5}$

$-15 \leq 115x \leq 69$

So, 115x can assume all the values from – 15 to 69.

Thus, the integral numbers that can be assumed by 115x will be 85.

2. (1)

The minimum value of (a + 2b + 3c + 4d) can be obtained by making sure that the lower numbers are multiplied with the higher coefficients.

Also, a, b, c, and d can assume minimum values of 1, 2, 3, and 4 where d = 1, c = 2, b = 3, a = 4.

So, min (a + 2b + 3c + 4d) = 4 + (2 × 3) + (3 × 2) + (4 × 1)

$\Rightarrow 5k = 20$

$\Rightarrow k = 4$

So, $\frac{5x+3}{x+1} - 4 \geq 0$

$\frac{5x+3-4(x+1)}{x+1} \geq 0$

$\Rightarrow \frac{x-1}{x+1} \geq 0$

Now, x = 1 and x = -1 are the critical points.

Note that x cannot be –1, since at x = – 1, the expression will not exist.

Also, at x = 1, the inequality is satisfied.

Now, we need to determine whether x < –1 or –1 < x < 1 or x > 1.

<u>Case 1:</u> x < -1

Let x = – 2, then we have

$\frac{-2-1}{-2+1} = 3 \geq 0$

<u>Case 2:</u> -1 < x < 1

Let x = 0, then we have

$\frac{0-1}{0+1} = -1$, cannot be greater than equals 0.

<u>Case 3:</u> x > 1

Let x = 2, then we have

$\frac{2-1}{2+1} = \frac{1}{3} \geq 0$

So, we can conclude that,

x < -1 or x ≥ 1.

So, the minimum positive integer value of x is 1.

3. **(a)**

To solve the inequality f > 0, we first need to find the critical points, which are the values of x where f = 0. Then, we will use the wavy curve method to determine the intervals where f > 0.

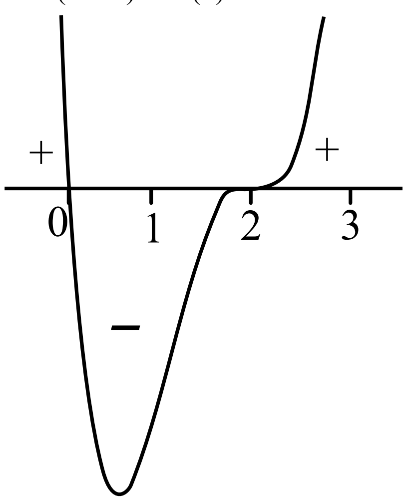
Step 1: Find the critical points

$f = x^4 - 6x^3 + 12x^2 - 8x$

$f = x(x^3 - 6x^2 + 12x - 8)$

Now, we need to find the roots of the cubic equation $x^3 - 6x^2 + 12x - 8 = 0$. Observe that $x = 2$ is a root:
$2^3 - 6(2)^2 + 12(2) - 8 = 0$
Using synthetic division or polynomial long division, we can factor the cubic equation:
$x^3 - 6x^2 + 12x - 8 = (x - 2)(x^2 - 4x + 4)$
Now, we can factor the quadratic equation as well:
$x^2 - 4x + 4 = (x - 2)(x - 2)$
So, the complete factorization of f(x) is:
$f = x(x - 2)^3$
The critical points are $x = 0$ and $x = 2$.

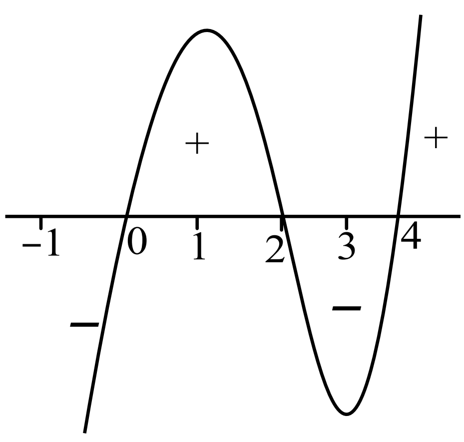
Step 2: Apply the wavy curve method
We can now apply the wavy curve method to determine the intervals where $f > 0$. On a number line, we mark the critical points 0 and 2. Between these critical points, we'll test the intervals $(-\infty, 0)$, $(0, 2)$, and $(2, +\infty)$.
Test the interval $(-\infty, 0)$: Choose $x = -1$
$f = -1(-1 - 2)^3 = -1(-3)^3 = 27 > 0$
Test the interval $(0, 2)$: Choose $x = 1$
$f = 1(1 - 2)^3 = 1(-1)^3 = -1 < 0$
Test the interval $(2, +\infty)$: Choose $x = 3$
$f = 3(3 - 2)^3 = 3(1)^3 = 3 > 0$



Based on the wavy curve method, $f > 0$ in the intervals $(-\infty, 0)$ and $(2, +\infty)$. Thus, the correct answer is:
(A) $(-\infty, 0) \cup (2, +\infty)$

4. (d)
To solve the inequality $g > 0$, we first need to find the critical points, which are the values of x where $g = 0$. Then, we will use the wavy curve method to determine the intervals where $g > 0$.

Step 1: Find the critical points
$$g = x^5 - 10x^4 + 40x^3 - 80x^2 + 64x$$
$$g = x\left(x^4 - 10x^3 + 40x^2 - 80x + 64\right)$$
Now, we need to find the roots of the quartic equation $x^4 - 10x^3 + 40x^2 - 80x + 64 = 0$. Notice that $x = 2$ and $x = 4$ are roots:
$$2^4 - 10(2)^3 + 40(2)^2 - 80(2) + 64 = 0$$
$$4^4 - 10(4)^3 + 40(4)^2 - 80(4) + 64 = 0$$
Using synthetic division or polynomial long division, we can factor the quadratic equation :
$$x^4 - 10x^3 + 40x^2 - 80x + 64 = (x - 2)(x - 4)\left(x^2 - 4x + 8\right)$$
Now, we have a quadratic equation $x^2 - 4x + 8$. Since its discriminant is negative, the quadratic equation has no real roots.
So, the complete factorization of g is:
$$g = x(x - 2)(x - 4)\left(x^2 - 4x + 8\right)$$
The critical points are $x = 0$, $x = 2$, and $x = 4$.
Step 2: Apply the wavy curve method
We can now apply the wavy curve method to determine the intervals where $g > 0$. On a number line, we mark the critical points 0, 2, and 4. Between these critical points, we'll test the intervals $(-\infty, 0)$, $(0, 2)$, $(2, 4)$, and $(4, +\infty)$.



For $(0, 2)$ & $(4, +\infty)$ we have positive value of the function. So, the answer is $(0, 2) \cup (4, +\infty)$.

5. **(c)**
To find which point is in the solution set, we need to test each option to see if they satisfy both inequalities.
A) (3, 2)
$2(3) - 3(2) \leq 6 \rightarrow 6 - 6 \leq 6 \rightarrow 0 \leq 6$ (True)
$3 + 2 \geq 5 \rightarrow 5 \geq 5$ (True)

B) (2, 3)
$2(2) - 3(3) \leq 6 \rightarrow 4 - 9 \leq 6 \rightarrow -5 \leq 6$ (True)
$2 + 3 \geq 5 \rightarrow 5 \geq 5$ (True)

C) (1, 3)
$2(1) - 3(3) \leq 6 \rightarrow 2 - 9 \leq 6 \rightarrow -7 \leq 6$ (True)
$1 + 3 \geq 5 \rightarrow 4 \geq 5$ (False)

D) (4, 1)
$2(4) - 3(1) \leq 6 \rightarrow 8 - 3 \leq 6 \rightarrow 5 \leq 6$ (True)
$4 + 1 \geq 5 \rightarrow 5 \geq 5$ (True)
Hence, option C is correct.

6. **(b)**
To find which point is in the solution set, we need to test each option to see if they satisfy all inequalities.
1) (1, 2)
$1 + 3(2) \leq 12 \rightarrow 1 + 6 \leq 12 \rightarrow 7 \leq 12$ (True)
$2 - 2(1) \geq -3 \rightarrow 2 - 2 \geq -3 \rightarrow 0 \geq -3$ (True)
$2(1) + 2 \geq 2 \rightarrow 2 + 2 \geq 2 \rightarrow 4 \geq 2$ (True)

2) (2, 3)
$2 + 3(3) \leq 12 \rightarrow 2 + 9 \leq 12 \rightarrow 11 \leq 12$ (True)
$3 - 2(2) \geq -3 \rightarrow 3 - 4 \geq -3 \rightarrow -1 \geq -3$ (True)
$2(2) + 3 \geq 2 \rightarrow 4 + 3 \geq 2 \rightarrow 7 \geq 2$ (True)

3) (3, 1)
$3 + 3(1) \leq 12 \rightarrow 3 + 3 \leq 12 \rightarrow 6 \leq 12$ (True)
$1 - 2(3) \geq -3 \rightarrow 1 - 6 \geq -3 \rightarrow -5 \geq -3$ (False)
$2(3) + 1 \geq 2 \rightarrow 6 + 1 \geq 2 \rightarrow 7 \geq 2$ (True)

4) (4, 2)
$4 + 3(2) \leq 12 \rightarrow 4 + 6 \leq 12 \rightarrow 10 \leq 12$ (True)
$2 - 2(4) \geq -3 \rightarrow 2 - 8 \geq -3 \rightarrow -6 \geq -3$ (False)
$2(4) + 2 \geq 2 \rightarrow 8 + 2 \geq 2 \rightarrow 10 \geq 2$ (True)
Hence, only 1 and 2 are correct.

7. **(a)**
To find which point is in the solution set, we need to test each option to see if they satisfy all inequalities.
1) (1, 0)
$1 - 0 \geq 1 \rightarrow 1 \geq 1$ (True)
$2(1) + 0 < 8 \rightarrow 2 < 8$ (True)
$0 > 1^2 - 6(1) + 8 \rightarrow 0 > 3$ (False)

2) (3, 1)
$3 - 1 \geq 1 \rightarrow 2 \geq 1$ (True)
$2(3) + 1 < 8 \rightarrow 7 < 8$ (True)
$1 > 3^2 - 6(3) + 8 \rightarrow 1 > -1$ (True)

3) (2, 3)
$2 - 3 \geq 1 \rightarrow -1 \geq 1$ (False)

4) (4, 3)
$4 - 3 \geq 1 \rightarrow 1 \geq 1$ (True)
$2(4) + 3 < 8 \rightarrow 11 < 8$ (False)
Option A is the correct answer.

8. (d)
We will examine the coordinates given in the answer choices:
1) (2, 1)
$5(2) - 3(1) = 7 \geq 15$ (False)

2) (5, 1)
$2(5) + 4(1) = 14 \leq 12$ (False)

3) (1, 1)
$5(1) - 3(1) = 2 \geq 15$ (False)

4) (3, 2)
$5(3) - 3(2) = 9 \geq 15$ (False)
The only point that lies inside the feasible region of the system of inequalities is the one that satisfies all the inequalities. From the analysis, none of the given points lies inside the feasible region.
Hence, option D is correct.

9. **(a)**
Given, $2x + 5 < 5(y + 1)$
$\Rightarrow 2x + 5 < 5y + 5$
$\Rightarrow 2x - 5y < 0$

$\Rightarrow 2x < 5y$ ….. (i)

Now, $\frac{7y+6}{y+9} \le 1$

$\Rightarrow \frac{7y+6}{y+9} - 1 \le 0$

$\Rightarrow \frac{7y+6-y-9}{y+9} \le 0$

$\Rightarrow \frac{6y-3}{y+9} \le 0$

Now, the critical points are $\frac{1}{2}$ and – 9.

Note that, since, at y = 9, the expression will not exist. Also, at $y = \frac{1}{2}$, the inequality will be satisfied.

Now, we need to determine, whether y < – 9, or $-9 < y < \frac{1}{2}$

<u>Case 1:</u> y < -9.

Let $y = -10$, then we have

$\frac{6(-10)-3}{-10+9} = 63$, cannot be less or equals 0.

<u>Case 1:</u> $-9 < y < \frac{1}{2}$

Let y = 0, then we have

$\frac{6(0)-3}{0+9} = -\frac{1}{3} \le 0$

So, we can conclude that,

$-9 < y \le \frac{1}{2}$

$\Rightarrow -45 < 5y \le \frac{5}{2}$ …… (ii)

Using (i) and (ii), we have

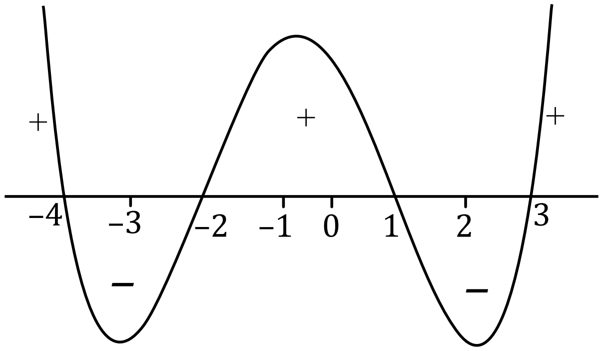
$2x < 5y \le \frac{5}{2}$

$\Rightarrow 2x < \frac{5}{2}$

$\Rightarrow x < \frac{5}{4}$

10. **(b)**

We have the polynomial f = (x – 1)(x + 2)(x – 3)(x + 4). The critical points for this function are x = 1, x = – 2, x = 3, and x = – 4. The wavy curve method can be applied to determine the intervals where the function is positive or negative.



The critical points divide the real line into 5 intervals:

1. (– ∞, –4)
2. (– 4, – 2)
3. (– 2, 1)
4. (1, 3)
5. (3, + ∞)

We will now determine the sign of f in each interval:

(– ∞, – 4): Since all factors are negative in this interval, f is positive, i.e., f > 0.

(– 4, – 2): Only the factor (x + 4) becomes positive, and the other factors are still negative. Therefore, f(x) is negative, i.e., f < 0.

(– 2, 1): Now, the factors (x + 4) and (x + 2) become positive, while (x – 1) and (x – 3) remain negative. Thus, f(x) is positive, i.e., f > 0.

(1, 3): In this interval, the factors (x + 4), (x + 2), and (x - 1) are positive, and only (x - 3) is negative. Hence, f(x) is negative, i.e.,
f < 0.

(3, + ∞): All factors are positive in this interval, and so f is positive, i.e., f > 0.

We are asked to find the intervals where f < 0. From our analysis, these intervals are:

(– 4, – 2) ∪ (1, 3)

Thus, the correct answer is option (B).

11. **(c)**

Given, $9x + 2 < (6y + 2)$

$\Rightarrow 3x < 2y$ …. (i)

Now, $\frac{5y+6}{y+1} \le 2$

$\Rightarrow \frac{5y+6}{y+1} - 2 \le 0$

$\Rightarrow \frac{5y+6-2(y+1)}{y+1} \le 0$

$\Rightarrow \frac{3y+4}{y+1} \le 0$

Now, the critical points are $-\frac{4}{3}$ and -1.

Note that, since, at y = – 1, the expression will not exist. Also, at $y = -\frac{4}{3}$, the inequality will be satisfied.

Now, we need to determine, whether y < $-\frac{4}{3}$, or $-\frac{4}{3} < y < -1$ or, $y > -1$.

Case 1: y < $-\frac{4}{3}$

Let y = -2, then we have

$\frac{3(-2)+4}{-2+1} = 2$, cannot be ≤ 0.

Case 2: $-\frac{4}{3} < y < -1$

Let $y = -\frac{6}{5}$, then we have

$\frac{3\left(-\frac{6}{5}\right)+4}{-\frac{6}{5}+1} = -2 \le 0$

Case 3: $y > -1$

Let y = 0, then we have

$\frac{3(0)+4}{0+1} = 4$, cannot be ≤ 0.

So, we can conclude that,

$-\frac{4}{3} \le y < -1$

$\Rightarrow -\frac{8}{3} \le 2y < -2$ ….. (ii)

Using (i) and (ii), we have

3x = 2y < – 2

⇒ 3x < – 2

$\Rightarrow x < -\frac{2}{3}$

12. **(d)**

Min (3x + 2y + 5z) can be obtained by multiplying lower numbers with the highest coefficient.

So, min

(3x + 2y + 5z) = (3 × 5) + (2 × 7) + (5 × 3) = 44

If we make z = 5, then y = 7, x = 3 satisfy

Then, (3x + 2y + 5z) = 3(3) + 2(7) + 5(5) = 48

So, z can assume only 2 values.

13. **(a)**

$\frac{2x+3}{4x+1} \ge 2$

$\frac{2x+3}{4x+1} - 2 \ge 0$

$\Rightarrow \frac{2x+3-2(4x+1)}{4x+1} \ge 0$

$\Rightarrow \frac{-6x+1}{4x+1} \ge 0$

Now, the critical points are $\frac{1}{6}$ and $-\frac{1}{4}$.

Note that, since, at x = $-\frac{1}{4}$, the expression will not exist. Also, at $x = \frac{1}{6}$, the inequality will be satisfied.

Now, we need to determine, whether x < $-\frac{1}{4}$, or $-\frac{1}{4} < x < \frac{1}{6}$ or, $x > \frac{1}{6}$.

Case 1: x < $-\frac{1}{4}$

Let x = – 1, then we have

$\frac{-6(-1)+1}{4(-1)+1} = -\frac{7}{3}$, cannot be ≥ 0.

Case 2: $-\frac{1}{4} < x < \frac{1}{6}$

Let $x = 0$, then we have

$\frac{-6(0)+1}{4(0)+1} = 1 \geq 0$

Case 3: $x > \frac{1}{6}$

Let x = 1, then we have

$\frac{-6(1)+1}{4(1)+1} = -1$, cannot be ≥ 0.

So, we can conclude that,

$-\frac{1}{4} < x \leq \frac{1}{6}$ …… (i)

$\Rightarrow -6 < 24x \leq 4$

So, 24x can assume all the values from – 5 to 4.

Thus, the integral numbers that can be assumed by 24x will be 10.

14. **(c)**

$\Rightarrow$ To minimize the total cost we need to minimize the cost of the items. As it is given that the rates of the pens, pencils and erasers are distinct prime numbers when expressed in \$ and the sum of rates of unit pen and pencil is equal to that of one eraser, so we can conclude that one of the rates between pen or pencil has to be \$2. If all three prime numbers are odd, then it will not hold the above condition true as the sum of two odds is even.

$\Rightarrow$ As the number of pens purchased is the most out of all the items, we can say that to minimize the total cost we need to make sure the minimum cost/unit is assigned to the item having the highest quantity.

$\Rightarrow$ So, the price of the pen = \$2/unit

$\Rightarrow$ It is also given that the total items purchased is 6. So, the number of pens purchased is 3, pencils purchased will be 2 and the eraser will be 1.

$\Rightarrow$ To minimize the total cost, the other costs will be \$3 and \$5. Also, to minimize the overall cost, we need to make sure the higher number is getting multiplied with the lower coefficients.

$\Rightarrow$ So, the price of a pencil is \$3/unit and that of eraser is \$5/unit.

$\Rightarrow$ Hence, the total cost is (\$2 ×3 + \$3×2 + \$5×1) = \$17

$\Rightarrow$ Thus the minimum total cost is \$17.

15. **(c)**

$\frac{2x+9}{x+1} - 8 \geq 0$

$\Rightarrow \frac{2x+9-8x-8}{x+1} \geq 0$

$\Rightarrow \frac{-6x+1}{x+1} \geq 0$

Now, the critical points are $\frac{1}{6}$ and -1.

Note that, since, at x = –1, the expression will not exist. Also, at $x = \frac{1}{6}$, the inequality will be satisfied.

Now, we need to determine, whether x < –1, or $-1 < x < \frac{1}{6}$ or, $x > \frac{1}{6}$.

Case 1: $x < -1$

Let x = – 2, then we have

$\frac{-6(-2)+1}{-2+1} = -13$, cannot be ≥ 0.

Case 2: $-1 < x < \frac{1}{6}$

Let x = 0, then we have

$\frac{-6(0)+1}{0+1} = 1 \geq 0$

Case 3: $x > \frac{1}{6}$

Let x =1, then we have

$\frac{-6(1)+1}{1+1} = -\frac{5}{2}$, cannot be ≥ 0.

So, we can conclude that,

$-1 < x \leq \frac{1}{6}$

$\Rightarrow -12 < 12x \leq 2$

So, 12x can assume all the values from – 11 to 2.

Thus, the positive integral numbers that can be assumed by 12x will be 14.

16. **(c)**

The minimum value of (a + 2b + 3c + 4d + 5e + 6f) can be obtained by making sure that the lower numbers are multiplied with the higher coefficients. Also, a, b, c, d, e and f can assume minimum values of 1, 2, 3, 4, 5, and 6, where f=1, e = 2, d = 3, c = 4, b = 5, a = 6.

So, min (a + 2b + 3c + 4d + 5e + 6f) = 6 + (2 × 5) + (3 × 4) + (4 × 3) + (5 × 2) + (6 × 1)

$\Rightarrow 8m = 56$

$\Rightarrow m = 7$

So, $\frac{8x+2}{x+2} - 7 \geq 0$

$\Rightarrow \frac{8x+2-7(x+2)}{x+2} \geq 0$

$\Rightarrow \frac{x-12}{x+2} \geq 0$

Now, x = 12 and x = – 2 are the critical points.

Note that x cannot be – 2, since at x = – 2, the expression will not exist.

Also, at x = 12, the inequality is satisfied.

Now, we need to determine whether x < – 2 or – 2 < x < 12 or x > 12.

<u>**Case 1:**</u> **x < -2**

Let x = -3, then we have

$\frac{-3-12}{-3+2} = 15 \geq 0$

<u>**Case 2:**</u> **-2 < x < 12**

Let x = 0, then we have

$\frac{0-12}{0+2} = -6$, cannot be greater than equals 0.

<u>**Case 3:**</u> **x > 12**

Let x = 13, then we have

$\frac{13-12}{13+2} = \frac{1}{15} \geq 0$

So, we can conclude that,

x < -2 or x ≥ 12.

Thus, x can assume a minimum positive integer value of 12.

17. **(b)**

To solve the inequality f(x) > 0 using the wavy curve method, we first need to find the critical points of f(x), which are the zeros of the function.

$f(x) = x^4 - 6x^3 + 14x^2 - 15x + 6$

The critical points can be found by setting f(x) to zero:

$x^4 - 6x^3 + 14x^2 - 15x + 6 = 0$

x = 1 and x = 2 are two roots of the equation.

This equation factors into:

$(x-1)(x-2)(x^2 - 3x + 3) = 0$

From this, we can identify the critical points as x1 = 1 and x2 = 2. For the quadratic term, $x^2 - 3x + 3$, we can use the discriminant to determine if there are any real roots:

$\Delta = b^2 - 4ac = (-3)^2 - 4(1)(3) = 9 - 12 = -3$

Since the discriminant is negative, the quadratic term has no real roots.

Now, we'll test intervals around the critical points to determine the sign of f(x):

x < 1: Choose x = 0, then f(x) = 6 > 0.

1 < x < 2: Choose x = 1.5, then f(x) = – 0.125 < 0.

x > 2: Choose x = 3, then f(x) = 6 > 0.

Considering the inequality f(x) > 0, the intervals that satisfy this inequality are:

(– ∞, 1) ∪ (2, ∞)

Thus, the correct answer is (B) (– ∞, 1) ∪ (2, ∞).

18. **(a)**

To factorize the cubic polynomial $x^3 - 5x^2 + 8x - 4$, we can try to find a common factor or use synthetic division to find one of its linear factors. In this case, we can use the rational root theorem to test potential rational roots.

The rational root theorem states that if a rational number p/q is a root of a polynomial with integer coefficients, then p must be a factor of the constant term and q must be a factor of the leading coefficient.

For the given polynomial, the possible factors of the constant term, – 4, are ± 1, ± 2, and ± 4, and the possible factors of the leading coefficient, 1, are ± 1. Therefore, the possible rational roots are ± 1, ± 2, and ± 4.

Now we can test these potential roots in the polynomial:

1: $(1)^3 - 5(1)^2 + 8(1) - 4 = 1 - 5 + 8 - 4 = 0$

We find that x = 1 is a root of the polynomial. The result of the synthetic division is the quadratic factor $x^2 - 4x + 4$. Now, we can factor this quadratic:

$x^2 - 4x + 4 = (x-2)^2$

So, the complete factorization of the cubic polynomial is:

$x^3 - 5x^2 + 8x - 4 = (x-1)(x-2)^2$

To solve the inequality f < 0 using the wavy curve method, we first need to find the critical points of f, which are the zeros of the function.

$f = (x-1)(x-2)^2$

The critical points can be found by setting f to zero:

$(x-1)(x-2)^2 = 0$

From this, we can identify the critical points as 1 and 2. Since 2 has an even multiplicity, it does not change the sign of the function.

Now, we'll test intervals around the critical points to determine the sign of f :

x < 1: Choose x = 0, then $f = (-1)(2)^2 = -4$, so f < 0.

Considering the inequality f < 0, the intervals that satisfy this inequality are $(-\infty, 1)$

19. (c)

To factorize the cubic polynomial $2x^3 - x^2 - 13x - 6$, we can try to find a common factor or use synthetic division to find one of its linear factors. In this case, we can use the rational root theorem to test potential rational roots.

The rational root theorem states that if a rational number $\frac{p}{q}$ is a root of a polynomial with integer coefficients, then p must be a factor of the constant term and q must be a factor of the leading coefficient.

For the given polynomial, the possible factors of the constant term, −6, are ±1, ±2, ±3, and ±6, and the possible factors of the leading coefficient, 2, are ±1 and ±2. Therefore, the possible rational roots are ±1, ±2, ±3, ±6, ±1/2, and ±3/2.

Now we can test these potential roots in the polynomial:

$1 : 2(-1)^3 - (-1)^2 - 13(-1) - 6 = 2 + 1 + 13 - 6 \neq 0$

$2 : 2(2)^3 - (2)^2 - 13(2) - 6 = 16 - 4 - 26 - 6 \neq 0$

$3 : 2(3)^3 - (3)^2 - 13(3) - 6 = 54 - 9 - 39 - 6 = 0$

We find that x = 3 is a root of the polynomial. The result of the synthetic division is the quadratic factor $2x^2 + 5x + 2$. Now, we can factor this quadratic:

$2x^2 + 5x + 2 = (2x+1)(x+2)$

So, the complete factorization of the cubic polynomial is:

$2x^3 - x^2 - 13x - 6 = (x-3)(2x+1)(x+2)$

To solve the inequality using the wavy curve method, we first identify the critical points, which are the points where the inequality is equal to 0. This occurs when any of the factors are equal to 0:

$x - 3 = 0 \Rightarrow x = 3$

$x + 2 = 0 \Rightarrow x = -2$

$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

Next, we create a number line including these critical points $(-2, -\frac{1}{2}$, and 3) and divide the number line into intervals based on these points:

Interval 1: $x < -2$

Interval 2 : $-2 < x < -\frac{1}{2}$

Interval 3: $-\frac{1}{2} < x < 3$

Interval 4: $x > 3$

Now, we check the sign of the inequality expression in each interval by selecting a test point from each interval:

Interval 1 ($x < -2$): Test point $x = -3$

$(-3-3)(-3+2)(2(-3)+1) = (-6)(-1)(-5) < 0$

Interval 2 ($-2 < x < -\frac{1}{2}$): Test point $x = -1$

$(-1-3)(-1+2)(2(-1)+1) = (-4)(1)(-1) > 0$

Interval 3 ($-1/2 < x < 3$): Test point x = 0

$(0-3)(0+2)(2(0)+1) = (-3)(2)(1) < 0$

Interval 4 ($x > 3$): Test point x = 4

$(4-3)(4+2)(2(4)+1) = (1)(6)(9) > 0$

Based on the signs obtained in each interval, the inequality $(x-3)(x+2)(2x+1) > 0$ is true for intervals 2 and 4. Therefore, the solution set for the inequality is:

$x \in (-2, -\frac{1}{2}) \cup (3, \infty)$

The correct answer is option C.

20. **(c)**

To find the intervals where h is positive or negative, we first need to find the critical points of the two factors. The critical points are the roots of the equations $x^3 - 4x - x^2 + 4 = 0$

$x^3 - 4x - x^2 + 4$

$= x^3 - x^2 - 4x + 4$

$= x^2(x-1) - 4(x-1)$

$= (x^2 - 4)(x-1)$

$= (x+2)(x-1)(x-2)$

Now, we will use the wavy curve method.

Thus, h is positive for $x \in (-2, 1) \cup (2, \infty)$

h is negative for $x \in (-\infty, -2) \cup (1, 2)$.

21. **(1)**

Let's rewrite the inequality by making a substitution: let $y = x^3$. Then $\frac{1}{x^3} = y^{-1}$. The inequality becomes:

$y^2 + y^{-2} - 4(y + y^{-1}) + 6 \le 0$

Now, let's multiply throughout by y^2 to get rid of the fractions:

$y^4 + 1 - 4y^3 - 4y + 6y^2 \le 0$

Rearrange the terms to form a polynomial:

$y^4 - 4y^3 + 6y^2 - 4y + 1 \le 0$

The above inequality can be written as,

$y^4 + y^2 - 2y^3 + y^2 + 1 - 2y - 2y^3 - 2y + 4y^2 \le 0$

$y^2(y^2 - 2y + 1) + 1(y^2 - 2y + 1) - 2y(y^2 - 2y + 1) \le 0$

$(y^2 - 2y + 1)(y^2 - 2y + 1) \le 0$

$(y-1)^2 (y-1)^2 \le 0$

Now, we want to find the critical points of this polynomial. Notice that the polynomial can be factored as:

$(y-1)^4 \le 0$

The critical point is $y = 1$. Since $y = x^3$, we have $x^3 = 1$, which gives us $x = 1$ (since the real cube root of 1 is 1). The inequality holds true at this critical point, as $(y-1)^4$ is always non-negative and equals zero only when $y = 1$.

22. **(2)**

To solve the inequality $(x^2 - 4)(x^2 - x - 6) < 0$ using the wavy curve method, we first need to find the critical points.

$(x^2 - 4)(x^2 - x - 6) < 0$

The critical points can be found by setting the inequality to zero:

$(x^2 - 4)(x^2 - x - 6) = 0$

This equation factors further into:

$(x-2)(x+2)(x-3)(x+2) = 0$

From this, we can identify the critical points as $x_1 = -2$, $x_2 = 2$, and $x_3 = 3$.

Now, we'll test intervals around the critical points to determine the sign :

$x < -2$: Choose $x = -3$, then $f(x) = -5 \times (-1)(-1)(-6) = 30 > 0$.

$-2 < x < 2$: Choose $x = 0$, then $f(x) = (-4)(-6) = 24 > 0$.

$2 < x < 3$: Choose $x = 2.5$, then $f(x) = (.5)(4.5)(-.5)(4.5) = -5.0625 < 0$.

$x > 3$: Choose $x = 4$, then $f(x) = 2 \times 6 \times 1 \times 6 = 72 > 0$.

Considering the inequality $f(x) < 0$, the intervals that satisfy this inequality are: (2, 3)

Thus, the number of integral values that 3x can assume is 2.

23. **(d)**

$x^4 + \left(\frac{1}{x^4}\right) - 2\left(x^2 + \frac{1}{x^2}\right) + 2 \le 0$

$\left(x^2 + \frac{1}{x^2}\right)^2 - 2\left(x^2 + \frac{1}{x^2}\right) \le 0$

$\left(x^2 + \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2} - 2\right) \le 0$

$$0 \le \left(x^2 + \frac{1}{x^2}\right) \le 2$$

$$\Rightarrow 0 + 2 \le \left(x^2 + \frac{1}{x^2} + 2\right) \le 2 + 2$$

$$\Rightarrow 2 \le \left(x^2 + \frac{1}{x^2} + 2\right) \le 4$$

$$\Rightarrow 2 \le \left(x + \frac{1}{x}\right)^2 \le 4$$

The above inequality holds true for x = 1 and -1
Hence, option D is correct.

24. **(3)**
Let x be the number of customers the company per day can sell metal wires to. If it needs to sell to the least number of customers, then it needs to make sure that it sells the maximum possible weight to all the customers.
Then, the total weight of metal wire sold to all the customers is 20x kilograms, and the total cost is 200 × 20x = \$4000x dollars. The constraints can be written as:
$4000x \le 12000$
$\Rightarrow x \le 3$.

25. **(6)**
To solve the inequality by using the wavy curve method, we first need to find the critical points.
$x^4 - 10x^3 + 35x^2 - 50x + 24 < 0$
The critical points can be found by setting the inequality to zero:
$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$
If we put x = 1, then we get the above expression equal to 0.

To divide the polynomial $x^4 - 10x^3 + 35x^2 - 50x + 24$ by (x - 1), we can use synthetic division or polynomial long division.
Therefore,

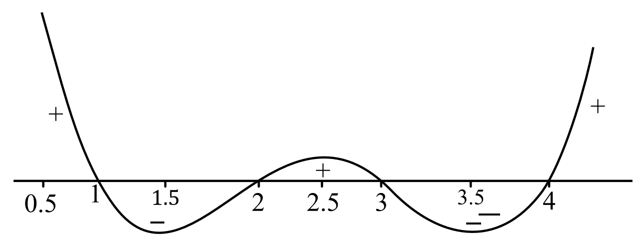
$$\frac{x^4 - 10x^3 + 35x^2 - 50x + 24}{x - 1} = x^3 - 9x^2 + 26x - 24$$

Similarly at x = 2 also it gives 0 for
$x^3 - 9x^2 + 26x - 24$

To divide the polynomial $x^3 - 9x^2 + 26x - 24$ by (x – 2), we can use synthetic division or polynomial long division.
Therefore,

$$\frac{x^3 - 9x^2 + 26x - 24}{x - 2} = x^2 - 7x + 12 = (x - 3)(x - 4)$$

This equation factors into:
(x – 1)(x – 2)(x – 3)(x - 4) = 0
Therefore, we have
$x^4 - 10x^3 + 35x^2 - 50x + 24 < 0$
$\Rightarrow (x - 1)(x - 2)(x - 3)(x - 4) < 0$
Now, let us draw the wavy curve:

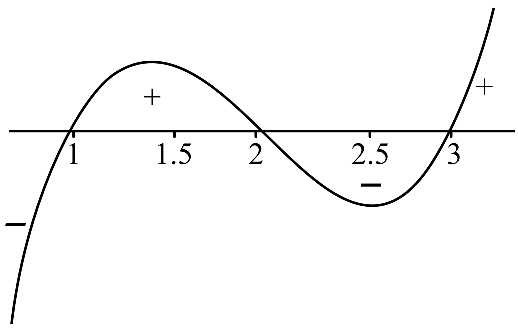


Considering the inequality is < 0, the intervals that satisfy this inequality are:
$x \in (1, 2) \cup (3, 4)$
So, 4x is in (4, 8) U (12, 16)
Thus, the number of integral values of 4x can be assumed is 6.

26. **(2500)**
Let x m be the width of the field, then the length of the field is (100 – x)
The area of the field which is fenced A
$= x(100 - x) = 100x - x^2$
So,
$A = 2500 + 100x - x^2 - 2500$
$\Rightarrow A = 2500 - (50 - x)^2$
A will be maximum when x = 50.
$\Rightarrow Max(A) = 2500 - (50 - 50)^2$
Thus, the maximum area which can be fenced is 2500 square meters.

27. **(d)**
Given that,
$x^3 - 6x^2 + 11x < 6$
$\Rightarrow x^3 - x^2 - 5x^2 + 6x + 5x - 6 < 0$
$\Rightarrow x^3 - 5x^2 + 6x - x^2 + 5x - 6 < 0$
$\Rightarrow (x^2 - 5x + 6)(x-1) < 0$
$\Rightarrow (x^2 - 2x - 3x + 6)(x - 1) < 0$

$\Rightarrow (x - 3)(x - 2)(x - 1) < 0$



i.e., $x < 1$ and $2 < x < 3$

Again, $x^3 - 11x^2 + 34x < 24$

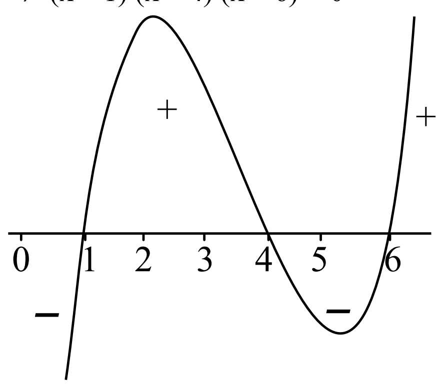
$\Rightarrow x^3 - x^2 - 10x^2 + 24x + 10x - 24 < 0$

$\Rightarrow x^3 - 10x^2 + 24x - x^2 + 10x - 24 < 0$

$\Rightarrow (x^2 - 10x + 24)(x - 1) < 0$

$\Rightarrow (x^2 - 4x - 6x + 24)(x - 1) < 0$

$\Rightarrow (x - 1)(x - 4)(x - 6) < 0$



Therefore, $x < 1$, $4 < x < 6$

So, combining the two conditions, we get $x < 1$.

28. **(b)**

To solve the inequality we will first find the critical points, which we can find by finding the roots of the equation,

$x^3 - 8x^2 + 11x + 20 > 0$

To find the roots, we can try to factor the polynomial:

$(x - 5)(x + 1)(x - 4) > 0$

Now we have three roots: $x = -1$, $x = 4$, and $x = 5$. These are our critical points.

Next, we will use the wavy curve method to determine the sign of the polynomial in each interval created by the critical points:

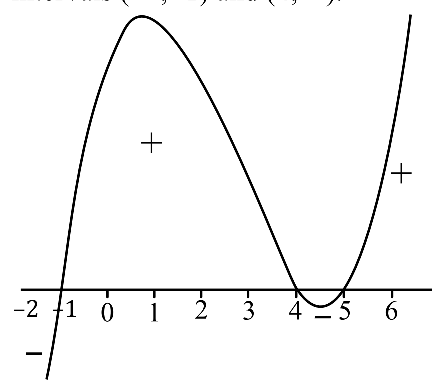
1. $(-\infty, -1)$: Choose a test point, say $x = -2$, the value is $= (-2 - 5)(-2 + 1)(-2 - 4) = (-)(-)(-) < 0$ (Not possible)

2. $(-1, 4)$: Choose a test point, say $x = 0$. The value is $= (0 - 5)(0 + 1)(0 - 4) = (-)(+)(-) > 0$.(Possible)

3. $(4, 5)$: Choose a test point, say $x = 4.5$. The value is $= (4.5 - 5)(4.5 + 1)(4.5 - 4) = (-)(+)(+) < 0$. (Not possible)

4. $(5, \infty)$: Choose a test point, say $x = 6$. The value is $= (6 - 5)(6 + 1)(6 - 4) = (+)(+)(+) > 0$ (Possible)

Now we know that the expression is positive in intervals $(-\infty, -1)$ and $(4, \infty)$.



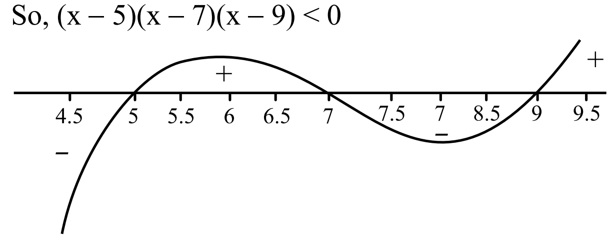
Therefore, the correct answer is: option B) $(-1, 4)$ and $(5, \infty)$

29. **(b)**

$(x - 3)^2(x - 5)^5(x - 7)^3(x - 9)^7 < 0$

As, $(x - 3)^2 \geq 0$, $(x - 5)^4 \geq 0$, $(x - 7)^2 \geq 0$ and $(x - 9)^6 \geq 0$ [Since, even powered x is always greater than equals zero]

So, $(x - 5)(x - 7)(x - 9) < 0$



So, $x < 5$ and $7 < x < 9$ and $x \neq 3$

Thus, the positive integer values that x can assume are 1, 2, 4, and 8.

30. **(a)**

To solve this inequality, we need to factorize the expression and determine the critical points. We will use the wavy curve method to determine the intervals of x where $P > 0$.

First, we factorize the given expression:

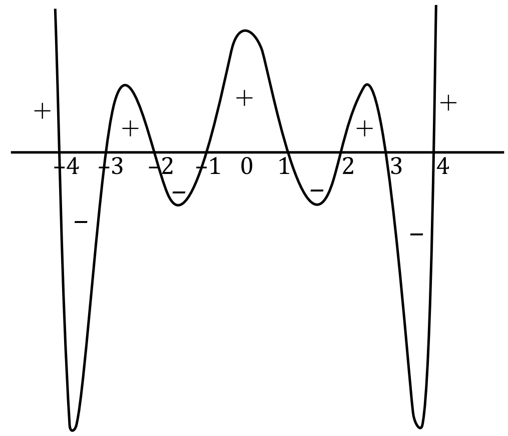
$$P = \left(x^2 - 9\right)\left(x^2 - 4\right)\left(x^2 - 1\right)\left(x^2 - 16\right)$$

The critical points occur where each factor is equal to zero:

1. $x^2 - 9 = 0 \Rightarrow x = \pm 3$
2. $x^2 - 4 = 0 \Rightarrow x = \pm 2$
3. $x^2 - 1 = 0 \Rightarrow x = \pm 1$
4. $x^2 - 16 = 0 \Rightarrow x = \pm 4$

Now, we will construct a wavy curve based on these critical points:

We start with a positive sign at the rightmost end and then alternate signs while moving towards the left side. The signs indicate the overall value of the expression in each interval. Since we are looking for the intervals where P > 0, we will select the intervals with a positive sign.



The intervals where $P > 0$ are: $(-\infty, -4) \cup (-3, -2) \cup (-1, 1) \cup (2, 3) \cup (4, \infty)$

The correct answer is option (A).