1

Assignment No.4

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Download all python codes from

https://github.com/Abhishek7008/Assignment-4.git

and latex-tikz codes from

https://github.com/Abhishek7008/Assignment-4.git

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/ linear_forms/gvv_ncert_linear_forms.pdf

1 Linear Forms Exercise 2.5(a)

Find out whether the following pair of linear equations are consistent, or inconsistent.

$$(3 \quad 2)\mathbf{x} = 5 \tag{1.0.1}$$

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = 7 \tag{1.0.2}$$

2 Solution

$$(3 \quad 2)\mathbf{x} = 5 \tag{2.0.1}$$

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = 7 \tag{2.0.2}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} \tag{2.0.3}$$

The augmented matrix for the above equation is row reduced as follows:

$$\begin{pmatrix} 3 & 2 & 5 \\ 2 & -3 & 7 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{2}{3} & \frac{5}{3} \\ 2 & -3 & 7 \end{pmatrix}$$
 (2.0.4)

$$\stackrel{R_2 \leftarrow -2R_1 + R_2}{\longleftrightarrow} \begin{pmatrix} 1 & \frac{2}{3} & \frac{5}{3} \\ 0 & \frac{-13}{3} & \frac{11}{3} \end{pmatrix} \tag{2.0.5}$$

$$\stackrel{R_2 \leftarrow \frac{-3}{13}R_2}{\longleftrightarrow} \begin{pmatrix} 1 & \frac{2}{3} & \frac{5}{3} \\ 0 & 1 & \frac{-11}{13} \end{pmatrix}$$
 (2.0.6)

$$\stackrel{R_1 \leftarrow \frac{-2}{3}R_2 + R_1}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & \frac{29}{13} \\ 0 & 1 & \frac{-11}{13} \end{pmatrix}$$
 (2.0.7)

So reduction of the 2×3 matrix

$$\begin{pmatrix} 3 & 2 & 5 \\ 2 & -3 & 7 \end{pmatrix} \tag{2.0.8}$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} \tag{2.0.9}$$

is also 2.

:. lines are Consistent and gives unique solution.

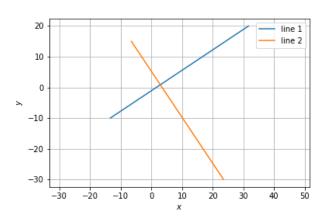


Fig. 2.1: Graphical solution

... This figure verifies that two lines are intersecting at one point.