

# ASSIGNMENT 5

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Download all python codes from

<https://github.com/Abhishek7008/Assignment-5.git>

and latex-tikz codes from

<https://github.com/Abhishek7008/Assignment-5.git>

## 1 QUADRATIC FORMS 2.27

Find the coordinates of the foci, the vertices, the lengths of major and minor axes and the eccentricity of the ellipse

$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 36 \quad (1.0.1)$$

## 2 SOLUTION

Given ellipse is

$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 36 \quad (2.0.1)$$

On comparing it with standard form we have,

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}, \mathbf{u} = 0, f = -36 \quad (2.0.2)$$

$$\Rightarrow \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 36 \quad (2.0.3)$$

$$\Rightarrow \mathbf{c} = -\mathbf{V}^{-1} \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$(2.0.5)$$

The eigen vector decomposition of

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \quad (2.0.6)$$

is given by

$$\mathbf{D} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \Rightarrow \lambda_1 = 9, \lambda_2 = 4 \quad (2.0.7)$$

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.8)$$

Since

$$\lambda_1 > \lambda_2 \quad (2.0.9)$$

Eccentricity of the ellipse is,

$$e = \sqrt{1 - \frac{\lambda_2}{\lambda_1}} = \frac{\sqrt{5}}{3} \quad (2.0.10)$$

Semi major and minor axes of ellipse are,

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = 3 \quad (2.0.11)$$

$$b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} = 2 \quad (2.0.12)$$

The co-ordinates of vertices are,

$$\pm \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (2.0.13)$$

The co-ordinates of foci are given by,

$$\mathbf{F} = \frac{ce^2 \mathbf{n} - \mathbf{u}}{\lambda_1} \quad (2.0.14)$$

Where,

$$\mathbf{n} = \sqrt{\lambda_1} \mathbf{p}_2 \quad (2.0.15)$$

$$c = \frac{e \mathbf{u}^T \mathbf{n} \pm \sqrt{e^2 (\mathbf{u}^T \mathbf{n})^2 - \lambda_2 (e^2 - 1) (\|\mathbf{u}\|^2 - \lambda_2 f)}}{\lambda_2 e (e^2 - 1)} \quad (2.0.16)$$

Substituting we have,

$$\mathbf{n} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (2.0.17)$$

$$c = \pm \frac{27}{\sqrt{5}} \quad (2.0.18)$$

$$\mathbf{F} = \pm \begin{pmatrix} 0 \\ \sqrt{5} \end{pmatrix}. \quad (2.0.19)$$

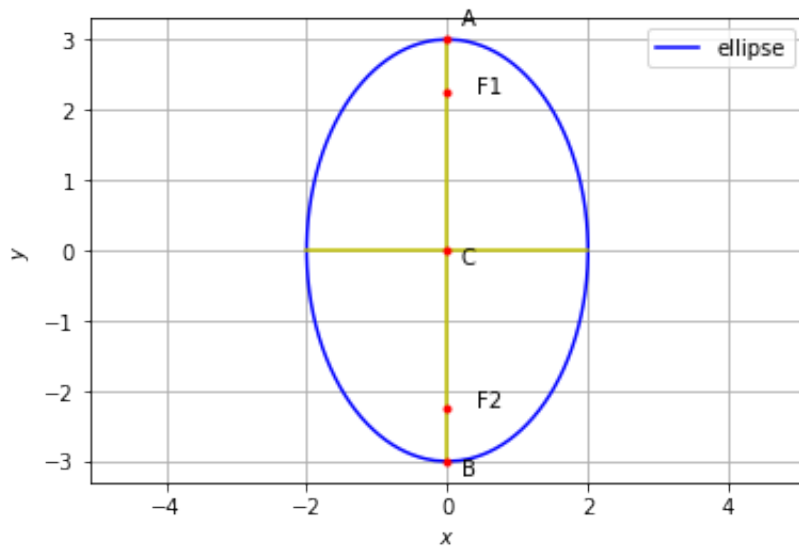


Fig. 0: Plot of the ellipse