Assignment No.5

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Download all python codes from

https://github.com/Abhishek7008/Assignment−5.git

and latex-tikz codes from

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1 Linear Forms Exercise 2.5(a)

Find the coordinates of the foci, the vertices, the lengths of major and minor axes and the eccentricity of the ellipse.

Lemma 1.1. *For ellipse, Property:*

$$|\mathbf{V}| > 0 \tag{1.0.1}$$

$$\lambda_1 > 0, \lambda_2 < 0 \tag{1.0.2}$$

Standard Form:

$$\frac{\mathbf{x}^T\mathbf{D}\mathbf{x}}{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{1.0.3}$$

Centre:

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} \tag{1.0.4}$$

Axes:

$$\begin{cases} \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1}} \\ \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_2}} \end{cases} \tag{1.0.5}$$

Focus:

$$\mathbf{F} = \sqrt{\frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}} \tag{1.0.6}$$

2 Solution

The given matrix equation for ellipse

$$\mathbf{X}^T\begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} = 36 \tag{2.0.1}$$

It can be represent as

$$\mathbf{x}^T\begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} = 1 \tag{2.0.2}$$

From the general formula of ellipse we get

$$\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} = 1 \tag{2.0.3}$$

From equation 2.0.2, we can define the value of

$$\lambda_1 = \frac{1}{4} \tag{2.0.4}$$

$$\lambda_2 = \frac{1}{9} \tag{2.0.5}$$

So the major axis of ellipse is on Y-axis.
Focus

$$F = \begin{pmatrix} 0 \\ \pm c \end{pmatrix} \tag{2.0.6}$$

$$F = \sqrt{\frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_2\lambda_1}} \tag{2.0.7}$$

$$F = \sqrt{\frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_2\lambda_1}} \tag{2.0.8}$$

$$F = \sqrt{\frac{\lambda_2 - \lambda_1}{\lambda_2\lambda_1}} \tag{2.0.9}$$

$$F = \sqrt{\frac{\frac{1}{9} - \frac{1}{4}}{\frac{1}{9} \times \frac{1}{4}}} = \sqrt{5} \tag{2.0.10}$$

$$F = \begin{pmatrix} 0 \\ \pm\sqrt{5} \end{pmatrix} \tag{2.0.11}$$

Axis

$$\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1}} = \sqrt{\frac{1}{\frac{1}{4}}} = 2 \tag{2.0.12}$$

$$\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}}{\lambda_2}} = \sqrt{\frac{1}{\frac{1}{9}}} = 3 \tag{2.0.13}$$

Length of the Major Axis

$$2 \times 3 = 6 \tag{2.0.14}$$

Length of the Minor Axis

$$2 \times 2 = 4 \tag{2.0.15}$$

Vertices, $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$

Eccentisity of Major Y-axis

$$= C\sqrt{\lambda_2} \tag{2.0.16}$$

$$= \sqrt{5} \times \frac{1}{3} \tag{2.0.17}$$

$$= \frac{\sqrt{5}}{3} \tag{2.0.18}$$