Assignment No.1

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Download all python codes from

```
https://github.com/Abhishek7008/Assignment_1.git
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and latex-tikz codes from

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https://github.com/Abhishek7008/Assignment_1.git
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1 Question No.1

The sum of the digits of a two-digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number ? [CBSE/MATH/10/2006 set2- Q1(b)]

2 Solution

Let the tens digit of the required number be $\mathbf{x}$ and the units digit be $\mathbf{y}$. Then,

$$\mathbf{a_1} + \mathbf{a_0} = 12 \tag{2.0.1}$$

Required Number is

$$(10\mathbf{a_1} + \mathbf{a_0}) \tag{2.0.2}$$

Which can be written in vector form as

$$\begin{pmatrix} 10 & 1 \end{pmatrix} \mathbf{x} \tag{2.0.3}$$

where

$$\mathbf{x} = \begin{pmatrix} a_1 \\ a_0 \end{pmatrix} \tag{2.0.4}$$

Number obtained on reversing the digits=$(10\mathbf{a_1}+\mathbf{a_0})$

Therefore,

$$\Rightarrow (10\mathbf{a_0} + \mathbf{a_1}) - (10\mathbf{a_1} + \mathbf{a_0}) = 18 \tag{2.0.5}$$

$$\Rightarrow 9\mathbf{a_0} - 9\mathbf{a_1} = 18 \tag{2.0.6}$$

$$\Rightarrow \mathbf{a_0} - \mathbf{a_1} = 2 \tag{2.0.7}$$

Solving 2.0.1 and 2.0.7 , can be expressed as a Matrix Equation

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 12 \\ 2 \end{pmatrix} \tag{2.0.8}$$

The augmented matrix for the above equation is row reduced as follows:

$$\begin{pmatrix} 1 & 1 & 12 \\ -1 & 1 & 2 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & 1 & 12 \\ 0 & 2 & 14 \end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{2} R_2} \begin{pmatrix} 1 & 1 & 12 \\ 0 & 1 & 7 \end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \end{pmatrix} \tag{2.0.11}$$

$$\Rightarrow \mathbf{a_1} = 5 \tag{2.0.12}$$

$$\Rightarrow \mathbf{a_0} = 7 \tag{2.0.13}$$

As Required Number

$$= 10\mathbf{a_1} + \mathbf{a_0} \tag{2.0.14}$$

$$= 10(5) + 7 \tag{2.0.15}$$

$$= 57 \tag{2.0.16}$$

Hence, the required number is 57.