Assignment No.1

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Download all python codes from

https://github.com/Abhishek7008/Assignment_1.git

and latex-tikz codes from

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1 Question No.1

The sum of the digits of a two-digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number ? [CBSE/MATH/10/2006 set2- Q1(b)]

2 Solution

Let the tens digit of the required number be a_1 and the units digit be a_0. Then

$$a_1 + a_0 = 12 \tag{2.0.1}$$

Which can be expressed as vector form,

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 12 \tag{2.0.2}$$

Where

$$\mathbf{x} = \begin{pmatrix}a_1\\a_0\end{pmatrix} \tag{2.0.3}$$

Required Number is

$$(10a_1 + a_0) \tag{2.0.4}$$

It can be written in vector form as

$$\begin{pmatrix}10 & 1\end{pmatrix}\mathbf{x} \tag{2.0.5}$$

The Number obtained by reversing the digits

$$(a_1 + 10a_0) \tag{2.0.6}$$

It can be expressed in vector form

$$\begin{pmatrix}1 & 10\end{pmatrix}\mathbf{x} \tag{2.0.7}$$

As mentioned in question the number obtained by interchanging the two digits exceed by 18 Therefore

$$\implies \begin{pmatrix}10 & 1\end{pmatrix}\mathbf{x} - \begin{pmatrix}1 & 10\end{pmatrix}\mathbf{x} = 18 \tag{2.0.8}$$

$$\implies \begin{pmatrix}9 & -9\end{pmatrix}\mathbf{x} = 18 \tag{2.0.9}$$

$$\implies \begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 2 \tag{2.0.10}$$

Solving (2.0.2)and (2.0.10) , can be expressed as a Matrix Equation

$$\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}12\\2\end{pmatrix} \tag{2.0.11}$$

The augmented matrix for the above equation is row reduced as follows:

$$\begin{pmatrix}1 & 1 & 12\\-1 & 1 & 2\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix}1 & 1 & 12\\0 & 2 & 14\end{pmatrix} \tag{2.0.12}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{2}R_2} \begin{pmatrix}1 & 1 & 12\\0 & 1 & 7\end{pmatrix} \tag{2.0.13}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix}1 & 0 & 5\\0 & 1 & 7\end{pmatrix} \tag{2.0.14}$$

$$\implies a_1 = 5 \tag{2.0.15}$$

$$\implies a_0 = 7 \tag{2.0.16}$$

As Required Number

$$= 10a_1 + a_0 \tag{2.0.17}$$

$$= 10(5) + 7 \tag{2.0.18}$$

$$= 57 \tag{2.0.19}$$

Hence, the required number is 57.

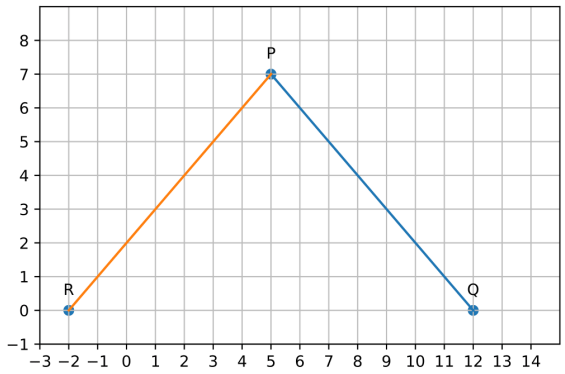

Fig. 2.1: Graphical solution

∴ This figure verifies that two lines are intersecting at one point.