

Assignment No.1

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Download all python codes from

https://github.com/Abhishek7008/Assignment_1.git

and latex-tikz codes from

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1 QUESTION No.1

The sum of the digits of a two-digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number ? [CBSE/MATH/10/2006 set2- Q1(b)]

2 SOLUTION

Let the tens digit of the required number be a_1 and the units digit be a_0 . Then

$$a_1 + a_0 = 12 \quad (2.0.1)$$

Which can be expressed as vector form,

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 12 \quad (2.0.2)$$

Where

$$\mathbf{x} = \begin{pmatrix} a_1 \\ a_0 \end{pmatrix} \quad (2.0.3)$$

Required Number is

$$(10a_1 + a_0) \quad (2.0.4)$$

It can be written in vector form as

$$\begin{pmatrix} 10 & 1 \end{pmatrix} \mathbf{x} \quad (2.0.5)$$

The Number obtained by reversing the digits

$$(a_1 + 10a_0) \quad (2.0.6)$$

It can be expressed in vector form

$$\begin{pmatrix} 1 & 10 \end{pmatrix} \mathbf{x} \quad (2.0.7)$$

As mentioned in question the number obtained by interchanging the two digits exceed by 18
Therefore

$$\Rightarrow \begin{pmatrix} 1 & 10 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 10 & 1 \end{pmatrix} \mathbf{x} = 18 \quad (2.0.8)$$

$$\Rightarrow \begin{pmatrix} 1 & -9 \end{pmatrix} \mathbf{x} = 18 \quad (2.0.9)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 2 \quad (2.0.10)$$

Solving (2.0.2) and (2.0.10), can be expressed as a Matrix Equation

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 12 \\ 2 \end{pmatrix} \quad (2.0.11)$$

The augmented matrix for the above equation is row reduced as follows:

$$\left(\begin{array}{cc|c} 1 & 1 & 12 \\ -1 & 1 & 2 \end{array} \right) \xrightarrow{R_2 \leftarrow R_2 + R_1} \left(\begin{array}{cc|c} 1 & 1 & 12 \\ 0 & 2 & 14 \end{array} \right) \quad (2.0.12)$$

$$\xrightarrow{R_2 \leftarrow \frac{1}{2} R_2} \left(\begin{array}{cc|c} 1 & 1 & 12 \\ 0 & 1 & 7 \end{array} \right) \quad (2.0.13)$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 7 \end{array} \right) \quad (2.0.14)$$

$$\Rightarrow a_1 = 5 \quad (2.0.15)$$

$$\Rightarrow a_0 = 7 \quad (2.0.16)$$

As Required Number

$$= 10a_1 + a_0 \quad (2.0.17)$$

$$= 10(5) + 7 \quad (2.0.18)$$

$$= 57 \quad (2.0.19)$$

Hence, the required number is 57.

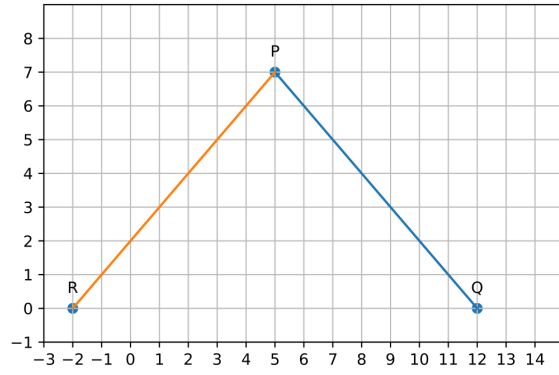


Fig. 2.1: Graphical solution

\therefore This figure verifies that two lines are intersecting at one point.