

Study material. : Scalar Potential

Subject Name. : Mathematics III

Subecct Code. : BCA 405 N

Unit. : 3 (Vector Calculus)

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Scalar Potential function

• हमें Question में एक vector given होगा और हमसे उस vector का Scalar Potential Find करना होगा। तो हम लोग इन steps को follow करेंगे।

Steps:

मान लो Question में vector है given है। तो
Step 1: Let ϕ be scalar potential of \vec{A}

Step 2: Find $\vec{A} \cdot d\vec{r}$

Example:

if $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$
हमेशा लेते हैं

Then $\vec{A} \cdot d\vec{r} = [A_1\hat{i} + A_2\hat{j} + A_3\hat{k}] \cdot [dx\hat{i} + dy\hat{j} + dz\hat{k}]$

∴ $\boxed{\vec{A} \cdot d\vec{r} = A_1 dx + A_2 dy + A_3 dz}$ होगा

Step 3: Now, ~~come~~ to find scalar potential
we have to write.

$$d\phi = \vec{A} \cdot d\vec{r}$$

Step 4: on integrating both side

$$\int d\phi = \int \vec{A} \cdot d\vec{r} = \int (A_1 dx + A_2 dy + A_3 dz) + C$$

Step 5: Use the formula -

$$\phi = \int_{y=\text{constant}} A_1 dx + \int [\text{only those term of } A_2 \text{ which do not}]$$

$$\text{contain } x] dy + \int [\text{only those term of } A_3 \text{ which do not contain } x \text{ and } y] dz$$

Example: Find the scalar potential function for vector $\vec{A} = y^2 \hat{i} + 2xy \hat{j} - z^2 \hat{k}$.

Soln — Let ϕ be the scalar potential of \vec{A}

Note

$$\vec{A} \cdot d\vec{r} = (y^2 \hat{i} + 2xy \hat{j} - z^2 \hat{k}) \cdot (dx \hat{i} + dy \hat{j} + dz \hat{k})$$

$$\boxed{\vec{A} \cdot d\vec{r} = y^2 dx + 2xy dy - z^2 dz}$$

Now, if ϕ be the scalar potential then

$$d\phi = \vec{A} \cdot d\vec{r}$$

$$\therefore d\phi = y^2 dx + 2xy dy - z^2 dz$$

on, integrating both side

$$\int d\phi = \int (y^2 dx + 2xy dy - z^2 dz)$$

According to steps please note

$$\phi = \int_{y \text{ constant}} y^2 dx + \int \text{term which is free from } x \text{ } 2xy dy \neq \int z^2 dz + \text{term which is free from } x \text{ and } y$$

$$= y^2 \int_{y \text{ constant}} dx + \text{Since these is no term free from } x \text{ } 0 \neq \int z^2 dz + \text{इसमें कोई भी चीज नहीं है।}$$

$$= y^2 \cdot x + 0 \neq \frac{z^3}{3} + C$$

$$\boxed{\phi = xy^2 + \frac{z^3}{3} + C} \text{ Answer}$$

Assignment

Q.1. A vector field is given by

$\vec{A} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$. Show that the field is irrotational and find scalar potential

Hint

$$\text{Curl } \vec{A} = 0$$

Prove that it is

irrotational

Find scalar potential

Ans.

$$\phi = \frac{x^3}{3} + \frac{y^3}{3} + \frac{x^2y^2}{2} + C$$

Irrotational if $\text{Curl } \vec{A} = 0$

Q.2. Find scalar potential of

$$\vec{V} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$$

Ans.

$$\phi = xy + yz + zx + C$$

Q.3. Find scalar potential function of vector

$$\vec{A} = y^2\hat{i} + 2xy\hat{j} - z^2\hat{k}$$

$$\text{Ans: } \phi = xy^2 + \frac{z^2}{2} + C$$

this question, I have solved but you all have to also solve

Ans