16/04/2020

Cauchy's II^{nd} Theorem on limit

$$\boxed{\lim_{n\to\infty} (a_n)^{1/n} = \lim \frac{a_{n+1}}{a_n}}$$

Steps to solve question

We have to given, solve

$$\boxed{\lim_{n\to\infty} a_n^{1/n}}$$

Step 1: Take a_n

Step 2: Find a_{n+1} by putting $n = n+1$

Step 3: Find $\frac{a_{n+1}}{a_n}$

Step 4: Solve $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

Step 5: By Cauchy's II^{nd} theorem on limit

$$\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \text{write answer}$$

16/04/2020

Example

Show that the sequence $\{n^{1/n}\}$, converges to the limit 1

Solution

To prove that sequence $\{n^{1/n}\}$ is convergent, we have to solve $\lim_{n\to\infty} n^{1/n}$

Now, As we know that, by cauchy's second theorem on limit

$$\lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} \quad \text{---(1)}$$

Here let $a_n = n$

$\therefore a_{n+1} = n+1$

$\therefore \frac{a_{n+1}}{a_n} = \frac{n+1}{n} = 1 + \frac{1}{n}$

$$\therefore \lim_{n\to\infty} n^{1/n} = \lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)$$

$$= 1$$

i.e, $\boxed{\lim_{n\to\infty} n^{1/n} = 1}$

$\Rightarrow$ given sequence is convergent to 1.

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Assignment 16/04/2020

Q.1 Prove that $\lim_{n\to\infty} \sqrt[n]{n} = 1$

Hint $\left\{ \begin{array}{l} \sqrt[n]{n} = n^{1/n} \\ \text{Q.4 question example में solve है।} \end{array} \right.$

Q.2 Find $\lim_{n\to\infty} \left(\underline{\lfloor n}\right)^{1/n}$

Ans: ∞

Hint: take $a_n = \underline{\lfloor n}$

Q.3 Prove that

$$\lim_{n\to\infty} \left(\frac{n^n}{\underline{\lfloor n}}\right)^{1/n} = e$$

$\left\{ \begin{array}{l} \text{Hint: take } a_n = \dfrac{n^n}{\underline{\lfloor n}} \\ \text{and use } \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \end{array} \right.$

Q.4. Show that

$$\lim_{n\to\infty} (n^2+n)^{1/n} = 1$$

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$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}$$

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$$\boxed{\lim_{n\to\infty}(a_n)^{1/n} = \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \text{write answer}}$$

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ie, $\boxed{\lim_{n\to\infty} n^{1/n} = 1}$

$\Rightarrow$ given sequence is convergent to 1.