Convergence of Infinite Series

PAGE NO.:
DATE: 19/04/2020

[Helping Series]

$\sum \frac{1}{n^p}$ will be convergent if $p>1$

& will be divergent if $p \leq 1$

Example (i) $\sum \frac{1}{n^2}$ will be Convergent $\because p=2>1$

$\sum \frac{1}{n}$ will be divergent $\because p=1$

$\sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$ divergent $\because p=\frac{1}{2}<1$

$\boxed{\text{Comparison Test}}$

If $\sum u_n$ and $\sum v_n$ be two positive term series and If

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \text{Non-zero finite number}$$

then $\sum u_n$ and $\sum v_n$ both converge or diverge together.

$\boxed{\text{Method to choose } v_n \text{ from } u_n}$

We choose v_n from u_n as

$$\boxed{v_n = \frac{\text{Highest power of } n \text{ from Numerator of } u_n}{\text{Highest power of } n \text{ from Denominator of } u_n}}$$

Example. If $u_n = \frac{n^2+n}{n^3+1}$

then we take $v_n = \frac{\text{Highest power of } n}{\text{Highest power of } n} = \frac{n^2}{n^3}$

$\Rightarrow v_n = \frac{1}{n}$ होगा

19/04/2020

PAGE NO.:
DATE: / /

Steps to Solve problem.

Suppose we have to test Convergence of $\sum u_n$ then we have to follow ———

Step 1: Write u_n

Step 2: Choose $v_n = \dfrac{\text{Highest power of } n \text{ from Numer.}}{\text{Highest power of } n \text{ in Denomi.}}$

Step 3: Find $\lim_{n \to \infty} \dfrac{u_n}{v_n}$ = Non-zero finite.

Step 4: $\sum u_n$ and $\sum v_n$ behave alike

Step 5: By helping series check $\sum v_n$ is Convergent or divergent: ($p>1$ or $p \le 1$)

Step 6: Write answer
If $\sum v_n$ Convergent $\Rightarrow \sum u_n$ will be Convergent
and if $\sum v_n$ divergent $\Rightarrow \sum u_n$ will be divergent

Conclusion ⇒ जो भी u_n question मे दिया है। उसके Numerator और Denominator मे से highest power लेके v_n find करो उसके बाद $\lim_{n \to \infty} \dfrac{u_n}{v_n}$ Find करो अगर Answer non-zero finite आयेगा तो $\sum u_n$ और $\sum v_n$ का behavior एक जैसा होगा। v_n के बारे मे helping series से पता करके $\sum u_n$ के बारे मे बताओ।

19/04/2020

Example

PAGE NO.:
DATE: / /

Examine the convergence of the series
$$\sum \frac{\sqrt{n^2-1}}{n^3+1}$$

Solution

Here we have: given series $\sum \dfrac{\sqrt{n^2-1}}{n^3+1}$

Now let, $u_n = \dfrac{\sqrt{n^2-1}}{n^3+1}$

Now we choose v_n such that ———

$$v_n = \frac{\text{Highest power from Numerator of } u_n}{\text{Highest power from Denominator of } u_n}$$

$$= \frac{n}{n^3} = \frac{1}{n^2}$$

$\sqrt{n^2-1}$ मे Square root होने से n highest power होगा

So let $\boxed{v_n = \dfrac{1}{n^2}}$

Now,
$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{\sqrt{n^2-1}}{n^3+1} \times \frac{n^2}{1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\sqrt{1-\frac{1}{n^2}}}{n^{\cancel{3}}\left(1+\frac{1}{n^3}\right)} \times \frac{\cancel{n^2}}{1}$$

$$= \frac{\sqrt{1-\frac{1}{\infty}}}{1+\frac{1}{\infty}} = 1 = \text{Non-zero finite}$$

$\therefore \sum u_n$ and $\sum v_n$ behave alike

Now By helping series $\sum \dfrac{1}{n^2} = \sum v_n$ will be Convergent ($\because p = 2 > 1$)

$\therefore$ By Comparison test given series $\sum u_n$ will be convergent

19/04/2020

Assignment

Q.1. Examine the convergence of the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{2n+1}$$

Ans: Divergent

Q.2. Examine the convergence of the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

Ans: Convergent

Q.3. Examine the convergence of the series

$$\sum_{n=1}^{\infty} \sqrt{\frac{n}{2+3n^3}}$$

Ans: Divergent

Q.4. Examine the Convergence of the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+1}$$

(example में solve किया है) अपने से solve करो)

Ans: Convergent