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D'Alembert's Ratio test

Statement:

If $\sum u_n$ is a series of positive terms such that

$$\boxed{\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = l} \quad \text{Then}$$

(I) $\sum u_n$ will be convergent if $l>1$

(II) $\sum u_n$ will be divergent if $l<1$

(III) The test fails if $l=1$

Steps to solve question

if we have to test convergence of $\sum u_n$ then

Step I: Write u_n

Step 2: put $n=n+1$ in u_n to get u_{n+1}

Step 3: Find $\boxed{\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = l}$ and then discuss result according to the statement of D'Alembert's test

Example: Test the convergence of series

$$1+\frac{\lfloor 2}{2^2}+\frac{\lfloor 3}{3^3}+\frac{\lfloor 4}{4^4}+\cdots$$

Solution-

Here n^{th} term will be

$$u_n = \frac{\lfloor n}{n^n}$$

$$\therefore u_{n+1} = \frac{\lfloor n+1}{(n+1)^{n+1}} \quad (\text{putting } n=n+1)$$

Now $$\frac{u_n}{u_{n+1}} = \frac{\lfloor n}{n^n}\times\frac{(n+1)^{n+1}}{\lfloor n+1} = \frac{\cancel{\lfloor n}}{n^n}\times\frac{(n+1)^{n+1}}{\cancel{(n+1)}\cancel{\lfloor n}}$$

$$= \frac{(n+1)^n}{n^n} = \left(\frac{n+1}{n}\right)^n$$

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Now

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} \left[\frac{n+1}{n}\right]$$

$$= \lim_{n\to\infty} \left[1+\frac{1}{n}\right]^n = e > 1$$

$\because \lim_{n\to\infty} \frac{u_n}{u_{n+1}} = e > 1$

$\therefore$ By D' Alembert's ratio test given series will be convergent

Assignment

Q. Test the convergence of series

(a) $\frac{2}{1^2+1} + \frac{2^2}{2^2+1} + \frac{2^3}{3^2+1} + \cdots$

divergent

Hint: $\boxed{\text{Here } u_n = \frac{2^n}{n^2+1}}$

(b) $\frac{\lfloor 2}{3} + \frac{\lfloor 3}{3^2} + \frac{\lfloor 4}{3^3} + \cdots$

Hint: $u_n = \frac{\lfloor n+1}{3^n}$

Divergent

(c) $\frac{1}{5} + \frac{\lfloor 2}{5^2} + \frac{\lfloor 3}{5^3} + \cdots$

Divergent

(d) $1 + \frac{\lfloor 2}{2^2} + \frac{\lfloor 3}{3^2} + \frac{\lfloor 4}{4^4} + \cdots$

Convergent

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