1. Teaching Material : Directional Derivative

2. Subject Name & Code : BCA-405N

3. Unit : 3 [Vector]

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[Directional Derivative]

Date 28/03/2020

Steps to find directional derivative:

In question we have to given

⇒ A scalar function ϕ
⇒ A vector $\vec{a}$
and ⇒ A point (x, y, z)

and we have to find directional derivative of ϕ at the point (x, y, z) and in the direction of given vector $\vec{a}$
Then we follow the following step

Step1: Find grad ϕ

Step2: put given point (x, y, z) in grad ϕ to find $[\text{grad}\,\phi]_{(x,y,z)}$

Step3: Find unit vector $\hat{a} = \frac{\vec{a}}{|a|}$ from given vector $\vec{a}$

Step4: Then use the formula ——

The Directional derivative of given scalar function ϕ in the direction of $\vec{a}$ at the point (x, y, z)

$$DD = [\text{grad}\,\phi] \cdot \hat{a}$$

Example

Find directional derivative of $x^2+y^2+z^2=9$ in the direction of vector $2\hat{i}+3\hat{j}+4\hat{k}$ at the point $(1,-1,2)$

Solution.

Here we have —

⇒ given Scalar $\phi(x,y,z) = x^2+y^2+z^2-9$

⇒ given vector $\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}$

⇒ given point $= (1,-1,2)$

Now,

$$\text{grad}\,\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

$$= \hat{i}\frac{\partial}{\partial x}(x^2+y^2+z^2-9) + \hat{j}\frac{\partial}{\partial y}(x^2+y^2+z^2-9) + \hat{k}\frac{\partial}{\partial z}(x^2+y^2+z^2-9)$$

$$= \hat{i}(2x) + \hat{j}(2y) + \hat{k}(2z)$$

$$\text{grad}\,\phi = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

Now $[\text{grad}\,\phi]_{(1,-1,2)} = (2\times1)\hat{i} + (2\times-1)\hat{j} + (2\times2)\hat{k}$

$$\Rightarrow \boxed{[\text{grad}\,\phi]_{(1,-1,2)} = 2\hat{i} - 2\hat{j} + 4\hat{k}}$$

Now, we have to find unit vector

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i}+3\hat{j}+4\hat{k}}{\sqrt{(2)^2+(3)^2+(4)^2}}$$

$$\text{or } \hat{a} = \frac{2\hat{i}+3\hat{j}+4\hat{k}}{\sqrt{4+9+16}} = \frac{2\hat{i}+3\hat{j}+4\hat{k}}{\sqrt{29}}$$

$$\boxed{\hat{a} = \frac{1}{\sqrt{29}}(2\hat{i}+3\hat{j}+4\hat{k})}$$

⇒ Directional derivative

$$= (\text{grad}\,\phi)_{(1,-1,2)} \cdot \hat{a} = (2\hat{i}-2\hat{j}+4\hat{k}) \cdot \frac{1}{\sqrt{29}}(2\hat{i}+3\hat{j}+4\hat{k})$$

$$= \frac{1}{\sqrt{29}}(4-6+16) = \frac{14}{\sqrt{29}}$$ Ans

Assignment

Date 28/3/2024

(Q.1) Find grad ϕ when ϕ is given by $\phi = 3x^2y - y^3z^3$ at the point $(1,-1,-1)$

Ans $\Rightarrow$ $\boxed{-12\hat{i} - 2\hat{j} - 16\hat{k}}$

(Q2) Find a unit vector normal to the surface $x^2y + 2xz = 4$ at the point $(2,-2,3)$

Ans $\Rightarrow$ $\boxed{\frac{1}{3}[-\hat{i} + 2\hat{j} + 2\hat{k}]}$

(Q3) Find the directional derivative of the function $f(x,y,z) = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$

Ans $\Rightarrow$ $\boxed{-\frac{11}{3}}$

Ans