Cauchy's First theorem on limit

If $\{a_n\}$ converges to l, then the sequence $\{x_n\}$ where $x_n = \frac{a_1+a_2+\cdots+a_n}{n}$ also converges to l

$$\boxed{\text{ie, if } \lim_{n\to\infty} a_n = l \Rightarrow \lim_{n\to\infty} \frac{a_1+a_2+\cdots+a_n}{n} = l}$$

step to use this formula

We have to solve $\lim_{n\to\infty} \frac{a_1+a_2+\cdots+a_n}{n}$

Focus only on a_n

Step 1. take the sequence $a_1+a_2+a_3+\cdots+a_n$ and write $a_n =$ take $a_n = ?$

Step 2. Find the limit $\longrightarrow \lim_{n\to\infty} a_n = L$ (say)

Step 3. Write the answer of given question

as

$\therefore$ By Cauchy's first theorem on limit

$$\lim_{n\to\infty} \frac{a_1+a_2+\cdots+a_n}{n} = \lim_{n\to\infty} a_n = L$$

Note जो limit a_n $(n \to \infty)$ का Answer होगा। वही $\lim\limits_{n\to\infty} \frac{a_1 + a_2 + \cdots + a_n}{n}$ का होगा।

Example

Show that

$$\lim_{n\to\infty} \frac{1}{n}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right) = 0$$

Solution

Here we have to solve

$$\lim_{n\to\infty} \frac{1}{n}\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]$$

$$= \lim_{n\to\infty} \frac{\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}\right]}{n}$$

$$\boxed{\text{Let } a_n = \frac{1}{n}}$$

$$\therefore \lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$$

$\therefore$ By Cauchy's first theorem on limit

$$\lim_{n\to\infty} \frac{\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]}{n} = \lim_{n\to\infty} \frac{1}{n} = 0$$

$$\therefore \boxed{\lim_{n\to\infty} \frac{1}{n}\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right] = 0}$$

Proved Ans

Assignment 14/04/2020

Q.1. Show that

$$\lim_{n\to\infty} \frac{1}{n}\left[\frac{2}{1}+\frac{3}{2}+\frac{4}{3}+\cdots+\frac{n+1}{n}\right]=1$$

Hint: $\boxed{\text{take } a_n=\frac{n+1}{n}}$

Q.2. Show that

$$\lim_{n\to\infty} \frac{1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}}{n}=0$$

Hint $\boxed{\text{take } a_n=\frac{1}{n^2}}$

Q.3. Show that

$$\lim_{n\to\infty} \frac{1+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}}{n}=0$$

Hint $\boxed{\text{take } a_n=\frac{1}{2n-1}}$

Solve and submit tomorrow