

Bootstrap assignment

There will be some functions that start with the word "grader" ex: grader_samples(), grader_30().. etc, you should not change those function definition.

Every Grader function has to return True.

Importing packages

```
In [49]: import numpy as np # importing numpy for numerical computation
from sklearn.datasets import load_boston # here we are using sklearn's boston dataset
from sklearn.metrics import mean_squared_error # importing mean_squared_error metric
```

```
In [50]: boston = load_boston()
x=boston.data #independent variables
y=boston.target #target variable
```

```
In [51]: print(x[:5])
print(y[:5])
```

```
[[6.3200e-03 1.8000e+01 2.3100e+00 0.0000e+00 5.3800e-01 6.5750e+00
 6.5200e+01 4.0900e+00 1.0000e+00 2.9600e+02 1.5300e+01 3.9690e+02
 4.9800e+00]
 [2.7310e-02 0.0000e+00 7.0700e+00 0.0000e+00 4.6900e-01 6.4210e+00
 7.8900e+01 4.9671e+00 2.0000e+00 2.4200e+02 1.7800e+01 3.9690e+02
 9.1400e+00]
 [2.7290e-02 0.0000e+00 7.0700e+00 0.0000e+00 4.6900e-01 7.1850e+00
 6.1100e+01 4.9671e+00 2.0000e+00 2.4200e+02 1.7800e+01 3.9283e+02
 4.0300e+00]
 [3.2370e-02 0.0000e+00 2.1800e+00 0.0000e+00 4.5800e-01 6.9980e+00
 4.5800e+01 6.0622e+00 3.0000e+00 2.2200e+02 1.8700e+01 3.9463e+02
 2.9400e+00]
 [6.9050e-02 0.0000e+00 2.1800e+00 0.0000e+00 4.5800e-01 7.1470e+00
 5.4200e+01 6.0622e+00 3.0000e+00 2.2200e+02 1.8700e+01 3.9690e+02
 5.3300e+00]]
[24. 21.6 34.7 33.4 36.2]
```

Task 1

Step - 1

- **Creating samples**

Randomly create 30 samples from the whole boston data points

- **Creating each sample:** Consider any random 303(60% of 506) data points from whole data set and then replicate any 203 points from the sampled points

For better understanding of this procedure lets check this examples, assume we have 10 data points [1,2,3,4,5,6,7,8,9,10], first we take 6 data points randomly , consider we have selected [4, 5, 7, 8, 9, 3] now we will replicate 4 points from [4, 5, 7, 8, 9, 3], consider they are [5, 8, 3,7] so our final sample will be [4, 5, 7, 8, 9, 3, 5, 8, 3,7]

- **Create 30 samples**

- **Note that as a part of the Bagging when you are taking the random samples make sure each of the sample will have different set of columns**
Ex: Assume we have 10 columns[1 ,2 ,3 ,4 ,5 ,6 ,7 ,8 ,9 ,10] for the first sample we will select [3, 4, 5, 9, 1, 2] and for the second sample [7, 9, 1, 4, 5, 6, 2] and so on... Make sure each sample will have atleast 3 feautres/columns/attributes

Step - 2

Building High Variance Models on each of the sample and finding train MSE value

- **Build a regression trees on each of 30 samples.**
- **Computed the predicted values of each data point(506 data points) in your corpus.**
- **Predicted house price of i^{th} data point**

$$y_{pred}^i = \frac{1}{30} \sum_{k=1}^{30} (\text{predicted value of } x^i \text{ with } k^{th} \text{ model})$$

- **Now calculate the $MSE = \frac{1}{506} \sum_{i=1}^{506} (y^i - y_{pred}^i)^2$**

Step - 3

- **Calculating the OOB score**
- **Predicted house price of i^{th} data point**
 $y_{pred}^i = \frac{1}{k} \sum_{k=\text{model which was buit on samples not included } x^i} (\text{predicted value of } x^i \text{ with } k^{th} \text{ model})$
- **Now calculate the $OOBScore = \frac{1}{506} \sum_{i=1}^{506} (y^i - y_{pred}^i)^2$.**

Task 2

- **Computing CI of OOB Score and Train MSE**
 - **Repeat Task 1 for 35 times, and for each iteration store the Train MSE and OOB score**
 - **After this we will have 35 Train MSE values and 35 OOB scores**
 - **using these 35 values (assume like a sample) find the confidence intravels of MSE and OOB Score**
 - **you need to report CI of MSE and CI of OOB Score**
 - **Note: Refer the Central_Limit_theorem.ipynb to check how to find the confidence intravel**

Task 3

- **Given a single query point predict the price of house.**

Consider $x_q = [0.18, 20.0, 5.00, 0.0, 0.421, 5.60, 72.2, 7.95, 7.0, 30.0, 19.1, 372.13, 18.60]$ Predict the house price for this point as mentioned in the step 2 of Task 1.

Task - 1

Step - 1

- **Creating samples**

Algorithm

Pesudo Code for generating Sample

```
def generating_samples(input_data, target_data):

    Selecting_rows <--- Getting 303 random row indices from the input_data

    Replaing_rows <--- Extracting 206 random row indices from the "Selecting_rows"

    Selecting_columns<--- Getting from 3 to 13 random column indices

    sample_data<--- input_data[Selecting_rows[:,None],Selecting_columns]

    target_of_sample_data <--- target_data[Selecting_rows]

    #Replicating Data

    Replicated_sample_data <--- sample_data [Replacing_rows]

    target_of_Replicated_sample_data<--- target_data[Replacing_rows]

    # Concatinating data

    final_sample_data <--- perform vertical stack on sample_data, Replicated_sample_data

    final_target_data<--- perform vertical stack on target_of_sample_data.reshape(-1,1), target_of_Replicated_sample_data.reshape(-1,1)

    return final_sample_data, final_target_data, Selecting_rows, Selecting_columns
```

- **Write code for generating samples**

```
In [52]: # '''In this function, we will write code for generating 30 samples '''
        # you can use random.choice to generate random indices without replacement
        # Please have a Look at this link https://docs.scipy.org/doc/numpy-1.16.0/reference
        # Please follow above pseudo code for generating samples

        # return sampled_input_data , sampled_target_data,selected_rows,selected_columns
        #note please return as Lists

def generating_samples(input_data, target_data):

    Selecting_rows = np.random.choice(506 ,303 ,replace=False)
    Selecting_columns = np.random.choice([0,1,2,3,4,5,6,7,8,9,10,11,12], np.random.choi
    sample_data = input_data[ Selecting_rows[:,None] , Selecting_columns ]
    target_of_sample_data = target_data[Selecting_rows]

    #Replicating data
    Replacing_rows = np.random.choice(303, 203)
    Replicated_sample_data = sample_data[Replacing_rows]
    target_of_Replicated_sample_data = target_data[Replacing_rows]
```

```
#Concatenation
final_sample_data = np.vstack((sample_data,Replicated_sample_data))
final_target_data = np.vstack((target_of_sample_data.reshape(-1,1),target_of_Replic

return final_sample_data,final_target_data,Selecting_rows,Selecting_columns
```

Grader function - 1

```
In [53]: def grader_samples(a,b,c,d):
length = (len(a)==506 and len(b)==506)
sampled = (len(a)-len(set([str(i) for i in a]))==203)
rows_length = (len(c)==303)
column_length= (len(d)>=3)
assert(length and sampled and rows_length and column_length)
return True
a,b,c,d = generating_samples(x, y)
grader_samples(a,b,c,d)
```

Out[53]: True

- Create 30 samples

Run this code 30 times, so that you will 30 samples, and store them in a lists as shown below:

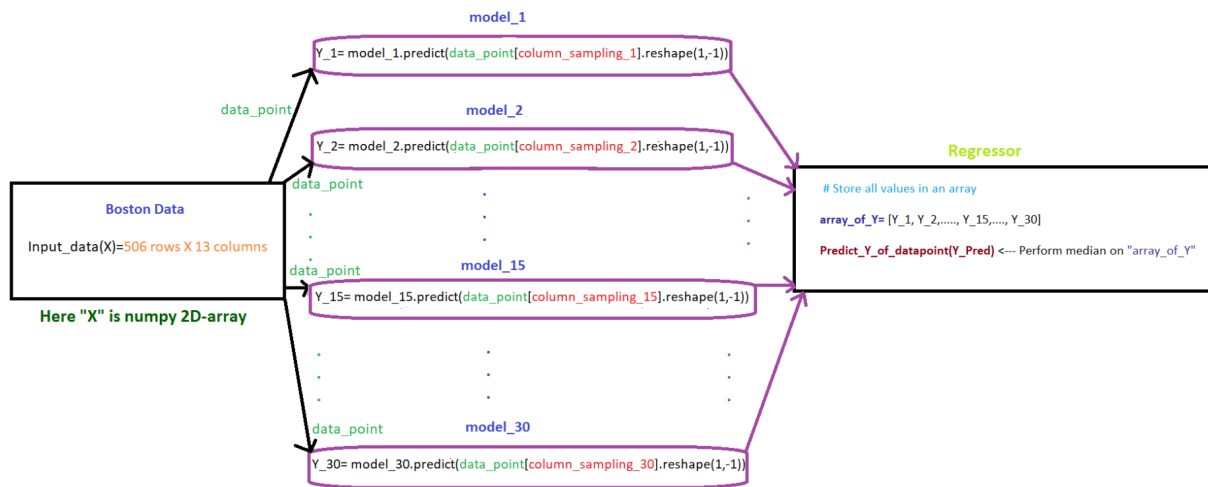
```
list_input_data=[]
list_output_data=[]
list_selected_row=[]
list_selected_columns=[]

for i in range(0,30):
    a,b,c,d=generating_sample(input_data,target_data)
    list_input_data.append(a)
    list_output_data.append(b)
    list_selected_row.append(c)
    list_selected_columns.append(d)
```

```
In [54]: # Use generating_samples function to create 30 samples
# store these created samples in a list
list_input_data = []
list_output_data = []
list_selected_row= []
list_selected_columns=[]

for i in range(0,30):
    a,b,c,d = generating_samples(x,y)
    list_input_data.append(a)
    list_output_data.append(b)
    list_selected_row.append(c)
    list_selected_columns.append(d)
```

Grader function - 2



After getting predicted_y for each data point, we can use sklearn's mean_squared_error to calculate the MSE between predicted_y and actual_y.

- Write code for calculating MSE

```

In [57]: from sklearn.metrics import mean_squared_error

array_of_Y = []

for k in range(len(list_of_all_models)):

    y_predicted = list_of_all_models[k].predict(list_input_data[k])

    array_of_Y.append(y_predicted)

array_of_Y = np.array(array_of_Y)  #array of predicted Y all data points for each samp
predicted_y = []

for j in range(array_of_Y.shape[1]):

    sorted_array_of_Y = np.sort(array_of_Y[:,j])

    median = ( (sorted_array_of_Y[14] + sorted_array_of_Y[15]) / 2)

    predicted_y.append(median)  #median of prediction of each data point from all the

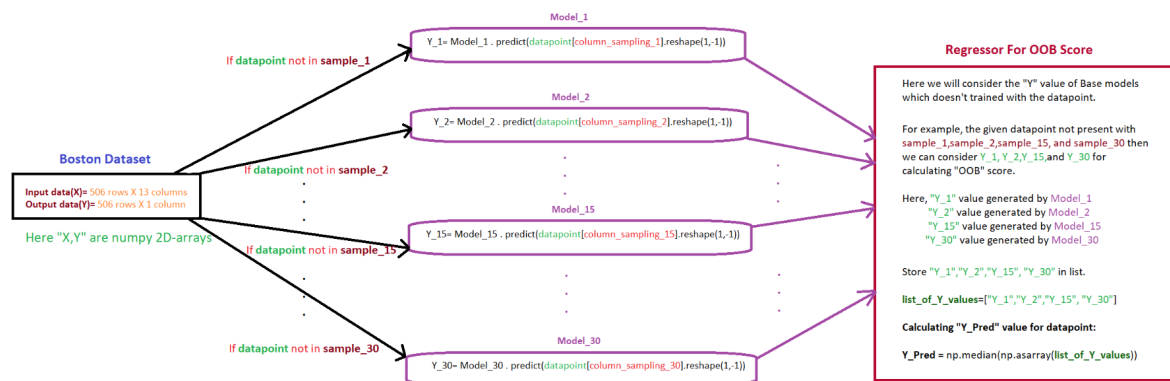
MSE = mean_squared_error(y , predicted_y )

print(MSE)
  
```

63.69688280789163

Step - 3

Flowchart for calculating OOB score



Now calculate the $OOBScore = \frac{1}{506} \sum_{i=1}^{506} (y^i - y_{pred}^i)^2$.

- Write code for calculating OOB score

```
In [58]: final_model_list = []    #for ith data point not in which samples/models

for i in range(len(x)):
    temp = []
    for j in range(len(list_selected_row)):
        if (i not in list_selected_row[j]): #for each data point , storing the models
            temp.append(j)
    final_model_list.append(temp)

print(final_model_list[:5])    #Length of 506

[[4, 11, 16, 21, 23, 24, 25, 26, 28], [4, 6, 7, 9, 10, 11, 13, 19, 20, 22, 25, 27], [0,
2, 4, 5, 13, 14, 15, 17, 19, 21, 24, 26, 27], [1, 7, 10, 11, 13, 14, 18, 19, 21, 22, 2
7], [2, 3, 4, 5, 8, 10, 11, 15, 17, 18, 20, 22, 23, 24, 26, 29]]
```

```
In [59]: #array_of_Y

predicted_y = []

for j in range(array_of_Y.shape[1]):

    sorted_array_of_Y = np.sort(array_of_Y[final_model_list[j],j]) #from array of all

    median = np.median(sorted_array_of_Y)

    predicted_y.append(median)

OOB = mean_squared_error(y , predicted_y )

print(OOB)
```

71.79590533935607

Task 2

```

In [60]: MSE_35 = []      #runinng above code for 35 times and storing MSE of each run
        OOB_35 = []      #runinng above code for 35 times and storing OOB of each run

for z in range(35):

    def generating_samples(input_data, target_data):

        Selecting_rows = np.random.choice(506 ,303 ,replace=False)
        Selecting_columns = np.random.choice([0,1,2,3,4,5,6,7,8,9,10,11,12], np.random.
        sample_data = input_data[ Selecting_rows[:,None] , Selecting_columns ]
        target_of_sample_data = target_data[Selecting_rows]

        #Replicating data
        Replacing_rows = np.random.choice(303, 203)
        Replicated_sample_data = sample_data[Replacing_rows]
        target_of_Replicated_sample_data = target_data[Replacing_rows]

        #Concatenation
        final_sample_data = np.vstack((sample_data,Replicated_sample_data))
        final_target_data = np.vstack((target_of_sample_data.reshape(-1,1),target_of_Re

        return final_sample_data,final_target_data,Selecting_rows,Selecting_columns

    list_input_data =[]
    list_output_data =[]
    list_selected_row= []
    list_selected_columns=[]

    for i in range(0,30):
        a,b,c,d = generating_samples(x,y)
        list_input_data.append(a)
        list_output_data.append(b)
        list_selected_row.append(c)
        list_selected_columns.append(d)

    from sklearn.tree import DecisionTreeRegressor

    list_of_all_models = []

    for i in range(len(list_input_data)):
        dt = DecisionTreeRegressor(max_depth = None)
        trained_dt = dt.fit(list_input_data[i],list_output_data[i])
        list_of_all_models.append(trained_dt)

    from sklearn.metrics import mean_squared_error

    array_of_Y = []

    for k in range(len(list_of_all_models)):

        y_predicted = list_of_all_models[k].predict(list_input_data[k])

        array_of_Y.append(y_predicted)

    array_of_Y = np.array(array_of_Y)

    predicted_y = []

    for j in range(array_of_Y.shape[1]):

        sorted_array_of_Y = np.sort(array_of_Y[:,j])

```



```

median = ( (sorted_array_of_Y[14] + sorted_array_of_Y[15]) / 2)

predicted_y.append(median)

MSE_35.append(mean_squared_error(y , predicted_y ))

predicted_y = []

for j in range(array_of_Y.shape[1]):

    sorted_array_of_Y = np.sort(array_of_Y[final_model_list[j],j]) #from array of

    median = np.median(sorted_array_of_Y)

    predicted_y.append(median)

OOB_35.append(mean_squared_error(y , predicted_y ))

print("35 MSE Scores : \n" , MSE_35)
print("\n35 OOB Scores : \n" , OOB_35)

```

35 MSE Scores :

```
[62.87163587690403, 60.35236882548309, 59.79737173501318, 58.55734358805446, 61.6072886
4832737, 59.13110365597571, 60.85119559956448, 59.462603955314, 59.58577976538483, 60.28
5602635432085, 60.992275470510286, 59.55276001376097, 58.57498569938515, 60.222766533541
936, 62.57789076654909, 58.5964568190851, 60.062999633563905, 60.34425025664251, 57.7855
2229501027, 57.590621983118, 60.17272204744875, 59.28634695734519, 58.93810888489843, 6
0.61029242589493, 60.9463873836524, 60.401829611989996, 60.686077669356614, 59.259083845
24593, 62.27745807502021, 57.702811970703415, 59.317539797682905, 61.60275844962491, 58.
1902653101392, 61.75259881327696, 60.40014180075935]
```

35 OOB Scores :

```
[69.43917390594916, 63.79730142594422, 61.169187136028654, 61.05299072655906, 64.721161
85161234, 66.25589068979342, 64.18830108970135, 64.81108712532937, 65.78733633069828, 6
6.12273521674413, 63.23785816631881, 63.15940151715171, 64.07090561182477, 64.7759007232
6527, 69.89834648065349, 64.8960658338669, 65.25904114102987, 61.71226485918972, 64.5052
3379442906, 61.24813392075373, 64.79482293946563, 66.41368525609354, 63.329238909263246,
66.49039194000227, 64.5371852912443, 65.28065537495262, 63.45298277898775, 63.5508875452
8986, 64.17475977921364, 60.815683701751816, 64.84481757109134, 65.22717109552853, 62.64
6836398117976, 67.39895942989679, 67.54913715928875]
```

In [61]: *#Confidence Interval using the python notebook on central limit theorem*

```

s_MSE_std = np.std(np.array(MSE_35))
x_MSE_mean = np.round(np.mean(np.array(MSE_35)), 3)
size_MSE = len(MSE_35)

left_limit = np.round(x_MSE_mean - 2*(s_MSE_std/np.sqrt(size_MSE)), 3)
right_limit = np.round(x_MSE_mean + 2*(s_MSE_std/np.sqrt(size_MSE)), 3)

print("95% CI of MSE : " , [left_limit,right_limit])

s_OOB_std = np.std(np.array(OOB_35))
x_OOB_mean = np.round(np.mean(np.array(OOB_35)), 3)
size_OOB = len(OOB_35)

left_limit = np.round(x_OOB_mean - 2*(s_OOB_std/np.sqrt(size_OOB)), 3)

```

```
right_limit = np.round(x_OOB_mean + 2*(s_OOB_std/np.sqrt(size_OOB)), 3)

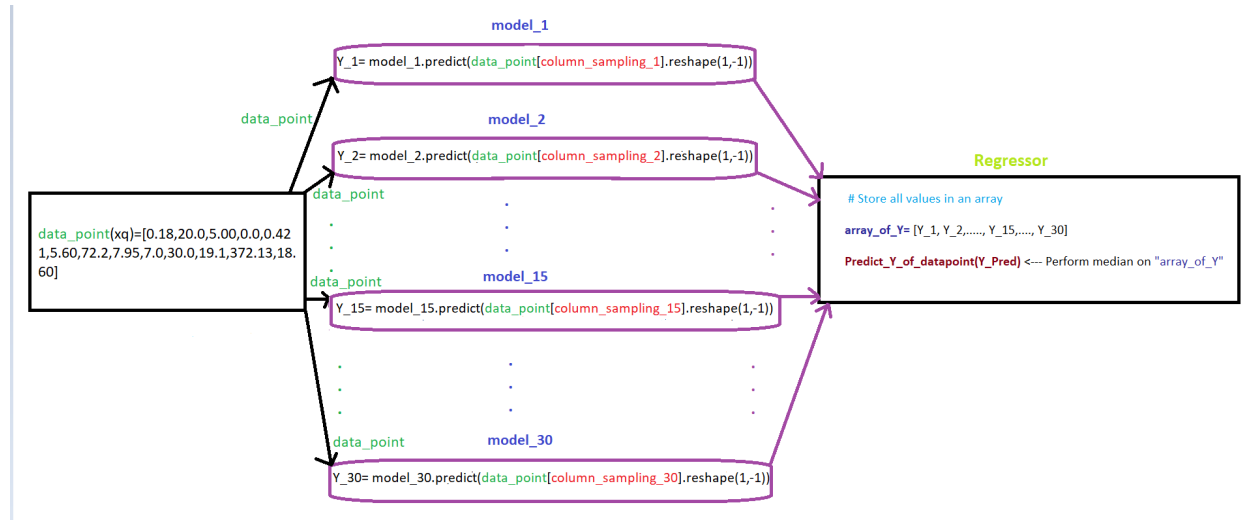
print("95% CI of OOB : " , [left_limit,right_limit])
```

95% CI of MSE : [59.56, 60.46]
 95% CI of OOB : [63.879, 65.299]

Task 3

Flowchart for Task 3

Hint: We created 30 models by using 30 samples in TASK-1. Here, we need send query point "xq" to 30 models and perform the regression on the output generated by 30 models.



- **Write code for TASK 3**

```
In [62]: def generating_samples(input_data, target_data):

    Selecting_rows = np.random.choice(506 ,303 ,replace=False)
    Selecting_columns = np.random.choice([0,1,2,3,4,5,6,7,8,9,10,11,12], np.random.choi
    sample_data = input_data[ Selecting_rows[:,None] , Selecting_columns ]
    target_of_sample_data = target_data[Selecting_rows]

    #Replicating data
    Replacing_rows = np.random.choice(303, 203)
    Replicated_sample_data = sample_data[Replacing_rows]
    target_of_Replicated_sample_data = target_data[Replacing_rows]

    #Concatenation
    final_sample_data = np.vstack((sample_data,Replicated_sample_data))
    final_target_data = np.vstack((target_of_sample_data.reshape(-1,1),target_of_Replic

    return final_sample_data,final_target_data,Selecting_rows,Selecting_columns

list_input_data = []
list_output_data = []
list_selected_row= []
list_selected_columns=[]

for i in range(0,30):
    a,b,c,d = generating_samples(x,y)
    list_input_data.append(a)
```

```

list_output_data.append(b)
list_selected_row.append(c)
list_selected_columns.append(d)

from sklearn.tree import DecisionTreeRegressor

list_of_all_models = []

for i in range(len(list_input_data)):
    dt = DecisionTreeRegressor(max_depth = None)
    trained_dt = dt.fit(list_input_data[i], list_output_data[i])
    list_of_all_models.append(trained_dt)

from sklearn.metrics import mean_squared_error

array_of_Y = []

for k in range(len(list_of_all_models)):

    xq = [[0.18,20.0,5.00,0.0,0.421,5.60,72.2,7.95,7.0,30.0,19.1,372.13,18.60],]

    xq = np.array(xq)

    nfeat = list_of_all_models[k].n_features_

    y_predicted = list_of_all_models[k].predict( xq[:, :nfeat] )

    array_of_Y.append(y_predicted)

array_of_Y = np.array(array_of_Y)

predicted_y = []

for j in range(array_of_Y.shape[1]):

    sorted_array_of_Y = np.sort(array_of_Y[:,j])

    median = ( (sorted_array_of_Y[14] + sorted_array_of_Y[15]) / 2)

    predicted_y.append(median)

print("Price of the house for given Xq: " , predicted_y)

Price of the house for given Xq: [23.049999999999997]

```

Write observations for task 1, task 2, task 3 in detail

Task 1 Observations :

MSE is 63.697 OOB is 71.796

Task 2 Observations :

35 MSE Scores : [62.87163587690403, 60.35236882548309, 59.79737173501318,
58.55734358805446, 61.60728864832737, 59.13110365597571, 60.85119559956448,
59.462603955314, 59.58577976538483, 60.285602635432085, 60.992275470510286,

59.55276001376097, 58.57498569938515, 60.222766533541936, 62.57789076654909, 58.5964568190851, 60.062999633563905, 60.34425025664251, 57.78552229501027, 57.590621983118, 60.17272204744875, 59.28634695734519, 58.93810888489843, 60.61029242589493, 60.9463873836524, 60.401829611989996, 60.686077669356614, 59.25908384524593, 62.27745807502021, 57.702811970703415, 59.317539797682905, 61.60275844962491, 58.1902653101392, 61.75259881327696, 60.40014180075935]

35 OOB Scores : [69.43917390594916, 63.79730142594422, 61.169187136028654, 61.05299072655906, 64.72116185161234, 66.25589068979342, 64.18830108970135, 64.81108712532937, 65.78733633069828, 66.12273521674413, 63.23785816631881, 63.15940151715171, 64.07090561182477, 64.77590072326527, 69.89834648065349, 64.8960658338669, 65.25904114102987, 61.71226485918972, 64.50523379442906, 61.24813392075373, 64.79482293946563, 66.41368525609354, 63.329238909263246, 66.49039194000227, 64.5371852912443, 65.28065537495262, 63.45298277898775, 63.55088754528986, 64.17475977921364, 60.815683701751816, 64.84481757109134, 65.22717109552853, 62.646836398117976, 67.39895942989679, 67.54913715928875]

Task 3 Observations :

Price of the house for given X_q : [23.05]