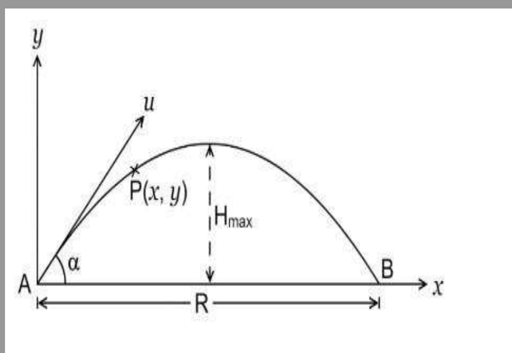


Projectile: If any object thrown up with some velocity, and during its subsequent motion it is subjected to only the acceleration due to gravity is called as projectile. The path traced out by the projectile is called as trajectory.

Velocity of projection (u): The velocity with which the particle is projected is called as velocity of projection (u).

Angle of projection (α): The angle between the direction of projection and horizontal direction is called as angle of projection.

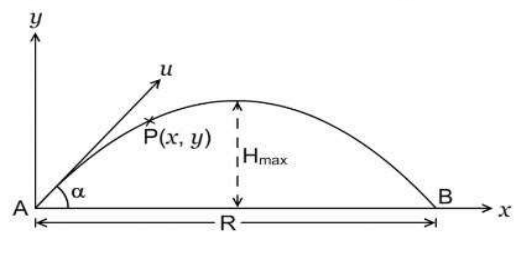
Range (R): the horizontal distance covered by the projectile during its motion is called range.

Time of flight (t_f): the time interval during which the projectile is in motion is called the time of flight. It is the sum of time of ascent and descent.

$\mathbf{t_f = t_a + t_d}$

Show that path of a projectile is a parabola

Consider a body is projected with a velocity u m/sec and at an angle of α (with horizontal)



In vertical direction

Initial velocity = u sinα

a = -g

In horizontal direction

Initial velocity = u cosα

let P(x,y) be the point of projectile after t seconds.

For vertical motion

From, $s = ut + ½ at^2$

$y = (u \sin\alpha)\, t - ½ gt^2$ ....(1)

For horizontal motion

From, s= ut + ½ gt2

$x = (u\cos\alpha)\, t + 0$

$x = u \cos\alpha \times t$

$t = x/(u \cos\alpha)$ ....(2)

Substituting (2) in (1)

$y = (u\sin\alpha)(x/u\cos\alpha) - ½ g (x/u\cos\alpha)^2$

$y = x \tan\alpha - (g/ 2u^2\cos^2\alpha)\, x^2$

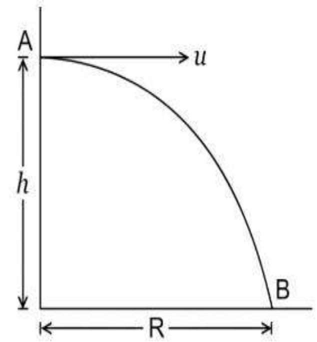
$y = A x \pm B x^2$

Where, $A = \tan\alpha$ and $B = g/(2u^2 \cos^2\alpha)$

The above equation is in the form of parabola. Hence the path of a projectile is a parabola.

Derive an expression for the maximum time and range for a body projected horizontally?

Consider a particle is projected horizontally with a velocity u m/sec from a height 'h'.



From, $s = ut + \frac{1}{2} gt^2$

$h = ut + \frac{1}{2} gt^2$

In vertical motion, $u = 0$; $a = +g$

$h = 0 + \frac{1}{2} gt^2$

$t = \sqrt{(2h/g)}$

Horizontal range (R) = velocity x time of flight

$R = u \times t$

$R = u \times \sqrt{(2h/g)}$

Derive the expression for maximum height, time required to reach maximum height, time of flight, horizontal range, maximum range, angle of projection for the range of a projectile projected with an inclination of α with horizontal and with a velocity of u m/sec.

Maximum height

In vertical direction, initial velocity = u sinα

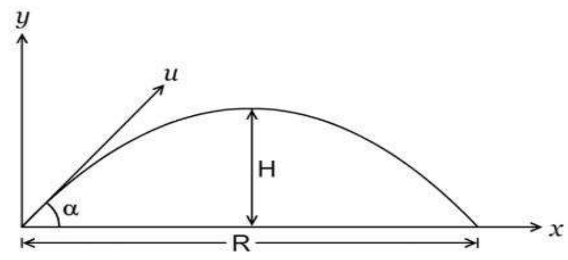
Final velocity = 0

a = -g

from, $v^2 - u^2 = 2as$

$0^2 - (u\sin\alpha)^2 = 2(-g)H$

$$\mathbf{H = (u^2 \sin^2\alpha) / 2g}$$



Time required reaching the maximum height (t_a)

Initial velocity = usinα

Final velocity = 0; a = -g

From, v = u + at

$0 = u\sin\alpha - gt$

$$\mathbf{t = (u\sin\alpha) / g}$$

Time required to reach the maximum height = time required to reach the ground = (usinα) / g

Time of flight

Time of flight = time of ascent + time of descent

$t_f = t_a + t_d$

$t_f = (u\sin\alpha/g) + (u\sin\alpha/g)$

$$\mathbf{t_f = (2u\sin\alpha)/g}$$

Horizontal range

R = velocity x time of flight

$R = u \cos\alpha \times (2u\sin\alpha)/g$

$\mathbf{R = (u^2/g) \sin 2\alpha}$

Maximum range

We have range $R = (u^2/g) \sin 2\alpha$

For maximum range $\sin 2\alpha = 1$

$\sin 2\alpha = \sin 90^\circ$

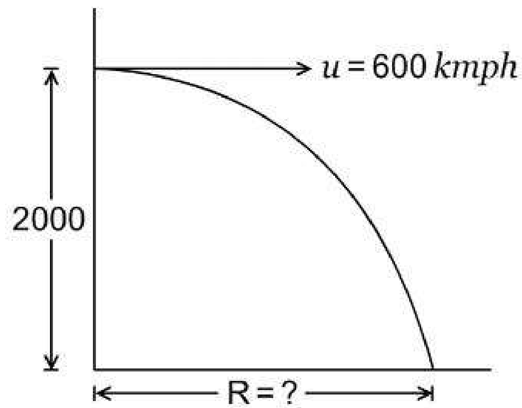
$2\alpha = 90^\circ$

$\mathbf{\alpha = 45^\circ}$

$R_{max} = (u^2/g) \sin 2\alpha = (u^2/g) \sin 2(45^\circ)$

$\mathbf{R_{max} = (u^2/g)}$

Find the horizontal range?



u = 600Kmph = (600 x 1000)/ (60 x 60) = 166.67 m/sec

from, $h = ut + ½ at^2$

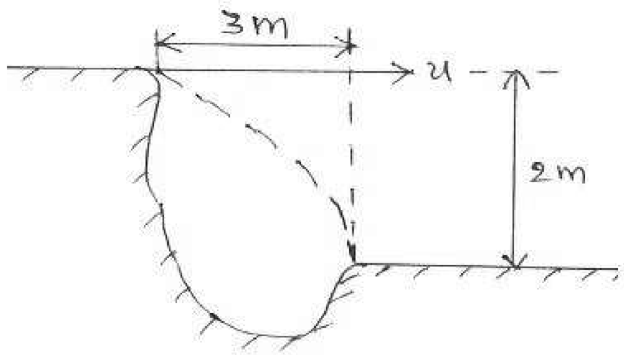
2000 = 0 x t + ½ g t2

t^2 = (2000 x 2)/9.81

t= 20.19 sec

Horizontal range = R = u x t = 166.67 x 20.19 = 3365.46m

Find the velocity with which a man has to jump to cross a ditch?



Given, $h = 2m$, $R = 3m$

$H = ut + \frac{1}{2} at^2$

$2 = 0 + \frac{1}{2} \times 9.81 \times t^2$

$t = 0.6386$ sec

$R = 3 = u \times t$

$3 = u \times 0.6386$

$u = 4.698$ m/sec

Body A is thrown with a velocity of 10 m/sec at an angle of 60° to the horizontal. If another body B is thrown at an angle of 45° to the horizontal. Find its velocity if it has the same (a) horizontal range (b) maximum height (c) time of flight as the body A.

u_A = 10 m/sec, $\theta_A = 60°$

$\theta_B = 45°$, u_B =?

i. $R_A = R_B$

$(u^2 \sin 2\alpha/g)_A = (u^2 \sin 2\alpha/g)_B$

$10^2 \sin 2(60°) / g = u_B^2 \sin 2(45°) /g$

$(100 \sin 120°) / \sin 90° = u_B^2$

u_B = 9.3 m/sec

ii. $H_A = H_B$

$(u^2 \sin^2\alpha / 2g)_A = (u^2 \sin^2\alpha / 2g)_B$

$10^2 \sin^2 60° = u_B^2 \sin^2 45°$

$10^2 \times (0.866)^2 = u_B^2 \times (0.707)^2$

u_B = 12.25 m/sec

iii. $tA = tB$

$(2u \sin\alpha / g)_A = (2u \sin\alpha / g)_B$

$10 \times \sin 60° = u_B \times \sin 45°$

u_B = 12.25 m/sec

The horizontal component of the velocity of a projectile is twice its initial vertical component. Find range on the horizontal plane, if the projectile passes through a point 18m horizontally and 3m vertically above the point of projection?

Let u be the initial velocity and α is angle of projection

Horizontal component of velocity = $u \cos\alpha$

Vertical component of velocity = $u \sin\alpha$

Given, $u \cos\alpha = 2 u \sin\alpha$

$\tan\alpha = ½$

$\alpha = 26.565^o$

given point = P(18,3)

from equation of projectile $y = x \tan\alpha - ½ (gx^2 / u^2\cos^2\alpha)$

$3 = 18 (1/2) - ½ (9.81 \times (18)^2) / (u^2 \cos^2 26.565)$

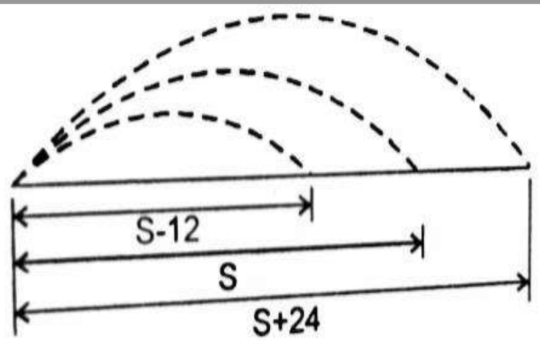
$6 = ½ (9.81 \times (18)^2) / (u^2 \cos^2 26.565)$

u = 18.196 m/sec

horizontal range $R = (u^2 \sin 2\alpha) / g$

$R = (18.196)^2 \times \sin (2 \times 26.565) / 9.81$

R = 27m



A projectile is aimed at a target on the horizontal plane and falls 12m short when the angle of projection is 15º while it over shoots by 24m when the angle is 45º. Find the angle of projection to hit the target.

Let s be the target distance and u be the angle of projection

$R = u^2 \sin 2\alpha / g$

$S - 12 = (u^2/g) \sin(2 \times 15^\circ) = (u^2/g)(1/2) = u^2 / 2g$ ....(1)

$S + 24 = (u^2/g) \sin(2 \times 45^\circ) = (u^2/g)(1) = u^2 / g$ ....(2)

From, (1) and (2)

$S + 24 = 2(S-12)$

$S + 24 = 2S - 24$

$S = 48$

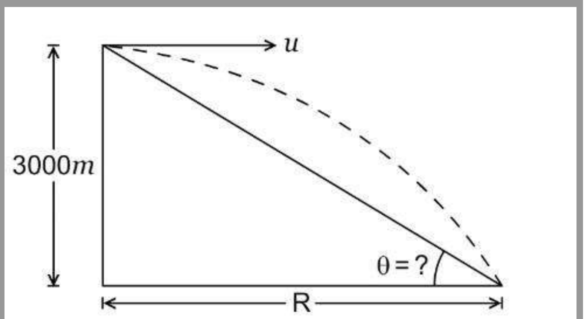
$48 = (u^2/g)(\sin 2\alpha)$

From (2) $u^2/g = S + 24 = 48 + 24 = 72$

$48 = 72 \sin 2\alpha$

$2\alpha = 41.81$

$\alpha = 20.905^\circ$



A rocket is released from a jet fighter flying horizontally at 1200 Kmph at an altitude of 3000m above its target. The rocket thrust gives it a constant horizontal acceleration of 6m/sec2. at what angle below the horizontal should pilot see the target at the instant of releasing the rocket in order to score a hit?

given, H = 3000m

in vertical direction, u = 0; s = 3000

from, $s = ut + ½ at^2$

$3000 = 0 + ½ \times 9.81 \times t^2$

t = 24.73 sec.

In horizontal motion, u= 1200 Kmph

= (1200 x 1000) / (60 x 60) = 333.33 m/sec

a= 6 m/sec^2

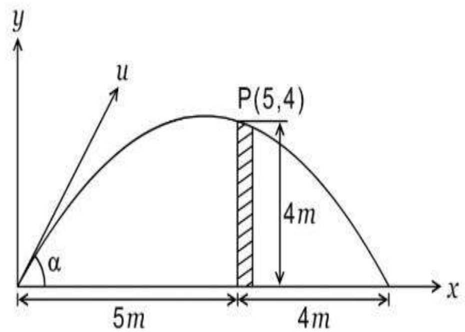
From, $s = ut + ½ at^2$

$R = 333.33 (24.73) + ½ \times 6 \times (24.73)^2$

R = 10078.5m

Tanθ = H / R = 3000 / 10078.5

θ= 16.576



Find the least velocity with which a projectile is to be projected so that it clears a wall 4m height located at a distance of 5m and strikes the ground at a distance 4m beyond the wall as shown in figure. The point of projection is at the same level as the foot of the wall?

Let u be the initial velocity

α is the angle of projection

given, R = 5 + 4 = 9m

$9 = u^2 \sin 2\alpha / g$

$u^2 = 9g / \sin 2\alpha$ ....(1)

From equation of trajectory

$Y = x \tan\alpha - ½ (gx^2 / u^2\cos^2\alpha)$

Let P(5,4) be a point on the trajectory

$4 = 5 \tan\alpha - ½ \times (g (5)^2 / u^2 \cos^2\alpha)$

$4 = 5 \tan\alpha - ½ \times (g (25) / (9g/\sin 2\alpha) \cos^2\alpha)$

$4 = 5 \tan\alpha - (25/2) (g / (9g \cos^2\alpha / 2\sin\alpha\cos\alpha))$

$4 = 5 \tan\alpha - (50/ 18) \tan\alpha$

$4 = 2.222 \tan\alpha$

$\tan\alpha = 4/2.222 = 1.8$

$\alpha = 60.95$

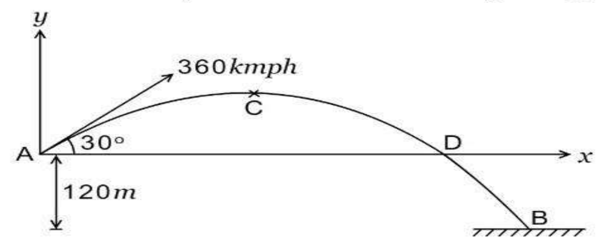
From, $u^2 = 9g / \sin 2\alpha$

$u^2 = (9 \times 9.81) / \sin (2 \times 60.95)$

$u = 10.20$ m/sec

A bullet is fired from a height of 120m at a velocity of 360Kmph at an angle of 30° upwards. Neglecting air resistance find

i. total time of flight
ii. horizontal range of the bullet
iii. maximum height reached by the bullet
iv. final velocity of the bullet just before touching the ground.



i. u = 360 Kmph = (360 x 1000) / (60 x 60) = 100 m/sec
Total time of flight = time for A to D + time for D to B
Time for A to D
$t_1 = (2u/g) \sin\alpha$
$= (2 \times 100 \times \sin 30°) / 9.81$
t_1 = 10.19 sec
Time for D to B
Initial velocity u = 100 sin30° = 50 m/sec
g = 9.81 m/sec2
s = 120 m
$s = ut + ½ at^2$
$120 = 50 \times t + ½ \times 9.81 \times t^2$
$t^2 + 10.1937\, t - 24.4648 = 0$
t_2 = 2.01 sec
Total time = A to D + D to B = 10.19 + 2.01 = 12.20 sec

ii. **maximum height reached by bullet**

$h = (u^2/2g) \times \sin^2\alpha = (100^2 \times \sin^2 30°) / (2 \times 9.81)$

h = 127.42 m (above point A) = 127.42 + 120 = 247.42 m (above the ground)

iii. **horizontal range**

R = velocity x time of flight

$R = u \cos\alpha \times t$

$R = 100 \cos 30° \times 12.2$

R = 1056.55m

iv. **velocity of bullet just before striking the ground**

Horizontal component of velocity = $v_x = u + at$

$v_x = u \cos\alpha + 0$

$v_x = 100 \cos 30° = 86.603$ m/sec

Vertical component of velocity $v_y = u + at$

$v_y = u \sin\alpha - gt = 100 \sin 30° - 9.81 \times 12.2$

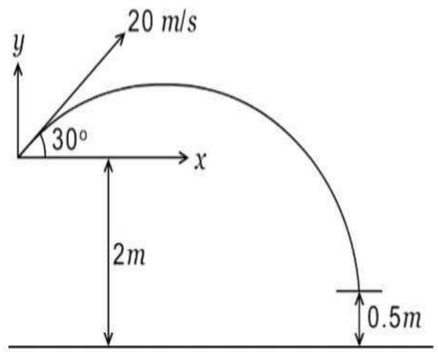
$v_y = - 69.682$ m/sec

$v = \sqrt{(v_x^2 + v_y^2)} = \sqrt{((69.682)^2 + (86.603)^2)}$

v = = 111.16 m/sec

$\tan\theta = v_y / v_x = 69.682 / 86.603$

θ= 38.82



A cricket ball is thrown by a fielder from a height of 2m, at an angle of 30° to the horizontal with an initial velocity of 20 m/sec hits the wickets at a height of 0.5m from the ground. How far was the fielder from the wickets?

Initial velocity u = 20 m/sec; α = 30°

$y_0 = -(2 - 0.5) = -1.5$m

From, $S = ut + \frac{1}{2} at^2$

$-1.5 = 20 \sin 30° - \frac{1}{2} \times 9.81 \times t^2$

t = 2.179 sec

Range R = velocity x time of flight

R = ucosα x t

R = 20 cos30° x 2.179 = 37.742 m