Dynamics is the branch of mechanics which deals with the behaviour of the bodies which are under motion

Dynamics is divided into two they are 1. Kinematics, 2. Kinetics

1. **Kinematics:** It is the branch of dynamics which deals with the motion of the bodies without considering the forces causing the motion of the body.
2. **Kinetics:** It is the branch of dynamics which deals with the motion of the bodies by considering the forces causing the motion of the body.

Motion: A body is said to be in motion if it is changing its position with respect to a reference point.

Distance: Distance moved by the body in time is the distance measured along the travelling path. It is a scalar quantity

Displacement: Displacement of a body in a time interval is the linear distance between the two positions of the body in the beginning and at the end of the time interval. It is a vector quantity.

Speed: the rate of change of distance with respect to time is called speed. It is a scalar quantity.

Velocity: the rate of change of displacement with respect to time is called velocity. It is a vector quantity.

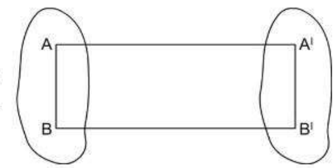
$v = ds/dt$; m/sec

Acceleration and deceleration: Rate of change of velocity with respect to time is called acceleration.

$a = dv/dt = ds^2/dt^2$; m^2/sec

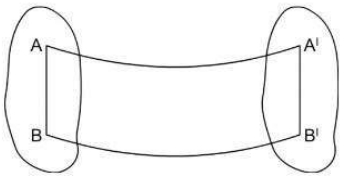
Negative acceleration is called as deceleration or retardation.

Translation: A motion is said to be in translation if a straight line drawn on the moving body remains parallel to its original position at any time.

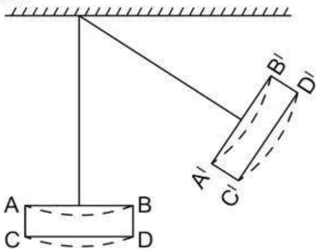


During the translation if the path travelled by a point is a straight line, it is called as **rectilinear translation**.

Similarly, if the path travelled by a point is a curved, it is called as **curvilinear translation**.



Rotation: A motion is said to be in rotation, if all particles of a rigid body move in a concentric circle.



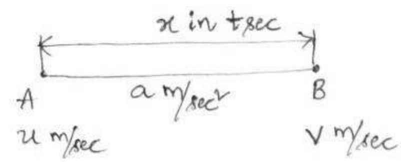
Derive linear motion conditions for a uniform accelerated body.

Ans. Consider the motion of a body with uniform acceleration 'a'.

Let u – initial velocity

v- Final velocity

t- Time taken



i. $a = dv/dt$

$dv = a\,dt$

On integrating on both sides

$\int_u^v dv = \int_0^t a\,dt$

$v - u = at$

$\mathbf{v = u + at}$

ii. $v = dx/dt$

$dx = v\,dt$

$dx = (u+at)\,dt$

Integrating on both sides

$\int_0^x dx = \int_0^t (u+at)\,dt$

$\mathbf{x = ut + (1/2)\,at^2}$

iii. $a = v\,(dv/dx)$

$a\,dx = v\,dv$

On integrating

$\int_0^x a\,dx = \int_u^v v\,dv$

$ax = [v^2/2]_u^v$

$ax = (v^2/2) - (u^2/2)$

$2ax = v^2 - u2$

$\mathbf{v^2 - u^2 = 2ax}$

A particle is projected vertically upwards from the ground with an initial velocity of u m/sec find

i. the time taken to reach the maximum height
ii. the maximum height reached
iii. time required for descending
iv. velocity with which it strikes the ground

i. initial velocity = u m/sec

$a = -g = -9.81\ m/sec^2$

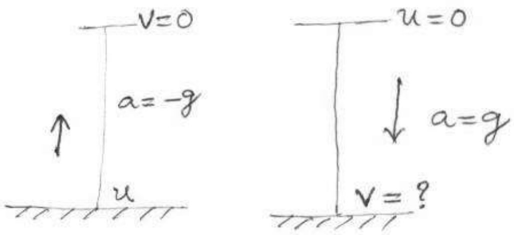
from $v = u + at$

$v = u - gt$

for vertical motion $v = 0$

$0 = u - gt$

$\mathbf{t = u/g}$



ii. from $v^2 - u^2 = 2as$

$v=0$

$o^2 - u^2 = -2gh$

$\mathbf{h = u^2/2g}$

iii. for down ward motion, u=0

From, $v^2 - u^2 = 2as$

$v^2 - 0^2 = 2gH$

$v^2 = 2g\,(u^2/2g)$

$\mathbf{v=u}$

iv. from, $v = u + at$

For down ward motion, $u = 0$, $v = u$

$u = 0 + gt$

$\mathbf{t = u/g}$

A stone is dropped into a well is heard to strike the water in 4 seconds. Find depth of the well assume velocity of sound to be 336 m/sec?

Let h is depth of well

t_1= time taken by stone to reach water

t_2 = time taken by sound to travel the height h

Total time = $t_1 + t_2 = 4$

For down ward motion, $S = ut + (1/2)\, at^2$

$h= 0 + (1/2)\, g\, t_1^2$..............(1)

For up ward motion velocity = 335 m/sec

v= s/t

s = v x t

$h = 335 \times t_2$.................(2)

But (1) = (2)

$(1/2)\, gt_1^2 = 335\, t_2$..........(3)

$t_1 + t_2 = 4$

$t_2 = 4 - t_1$

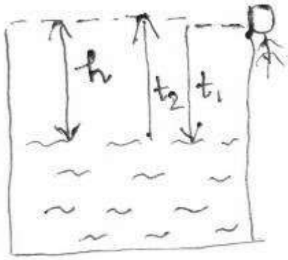
$(1/2)\, g\, t_1^2 = 335\, (4 - t_1)$

$t_1^2 + 68.30\, t_1 - 273.19 = 0$

$t_1 = 3.79$ sec

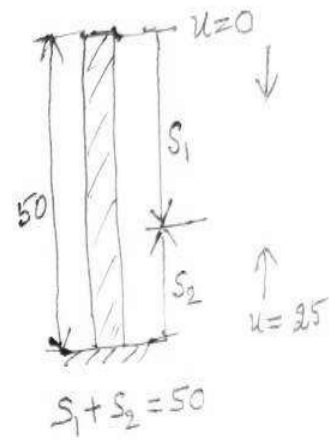
$h = (1/2)\, g\, t_1^2 = (1/2) \times 9.81 \times (3.79)^2$

h = 70.44m



$t_1 + t_2 = 4$ sec

A stone is dropped from the top of a tower 50m high. At the same time another stone is thrown up from the foot of the tower with a velocity of 25 m/sec. At what distance from the top and after how much time the two stones cross each other?



Given, $s1 + s2 = 50m$

For the down ward stone, $S = ut + (1/2)\ at^2$

$u= 0$

$s_1 = 0 + (1/2)\ at^2$

for upward motion stone

$S = ut + (1/2)\ at2$

$S_2 = 25 \times t - (1/2)\ gt^2$

From, $s1 + s2 = 50$

$(1/2)\ gt^2 + 25\ t - (1/2)\ gt^2 = 50$

$25t = 50$

$t= 50/25 = 2$ sec

$S_1 = (1/2)\ gt^2 = (1/2)\ 9.81\ (2)^2$

$S_1 = 19.6$ (from the top)

The motion of a particle moving in a straight line is given by $s = t^3 - 3t^2 + 2t + 5$ determine

i. Velocity and acceleration after 4 sec.

ii. maximum and minimum velocity & corresponding displacement

iii.time at which velocity is zero.

$s = t^3 - 3t^2 + 2t + 5$

$v = ds/dt = 3t^2 - 6t + 2$

$A = d^2s/dt^2 = 6t - 6$

i. after 4 seconds, $v = 3t^2 - 6t + 2 = 3(4)^2 - 6(4) + 2 = 26$m/sec

$a = 6t - 6 = 6(4) - 6 = 18$ m/sec2

ii. for maximum and minimum velocity $dv/dt = 0$

$6t - 6 = 0$

$t = 1$ sec

Displacement $= t^3 - 3t^2 + 2t + 5$

$(1)^3 - 3(1)^2 + 2(1) + 5 = 5$m

iii. $v = 0$

$3t^2 - 6t + 2 = 0$

On solving t = 1.577 & 0.423sec

The velocity of a particle moving in a straight line is given by the expression

$v = t^3 - t^2 - 2t + 2$

The particle is found to be at a distance of 4m from a station after 2 seconds. Determine

i. acceleration and displacement after 4 seconds.

ii. Maximum/minimum acceleration.

$v = t^3 - t^2 - 2t + 2$

$a = dv/dt = d/dt(t^3 - t^2 - 2t + 2)$

$a = 3t2 - 2t - 2$

$v = ds/dt$; $s = \int v dt$

$= \int (t^3 - t^2 - 2t + 2) dt$

$s = t^4/4 - t^3/3 - 2 \times t^2/2 + 2t + c$

s= 4m when t = 2 sec

$4 = 2^4/4 - 2^3/3 - 2^2 + 2(2) + c$

$4 = 16/4 - 8/3 - 4 + 4 + c$

$C = 8/3$

$s = t^4/4 - t^3/3 - t^2 + 2t + (8/3)$

i. When t= 4

$s = 4^4/4 - 4^3/3 - 4^2 + 2 \times 4 + (8/3)$

$= 256/4 - 64/3 - 16 + 8 + 8/3$

$s = 112/3$

$a = 3t^2 - 2t - 2$

$= 3(4)^2 - 2(4) - 2$

$= 38 \text{ m/sec}^2$

ii. For maximum and minimum acceleration, da/dt = 0

$d/dt(3t^2 - 2t - 2) = 0$

$6t - 2 = 0$

$t = 1/3$ sec

At $t = 1/3$, $a = 3t^2 - 2t - 2$

$= 3(1/3)^2 - 2(1/3) - 2$

$a = -7/3 = -2.33 \text{ m/sec}^2$

A body moves along a straight line and its acceleration varies with time is given by $a = 2 - 3t$. Five seconds after starting, its velocity is 20 m/sec. Ten seconds after the body is at 85m from starting position. Determine

i. Its acceleration, velocity and distance from the origin
ii. Determine time in which the velocity is zero and corresponding distance.

$a = 2 - 3t$

$a = dv/dt$

$v = \int a\, dt = \int (2\text{-}3t)\, dt$

$v = 2t - 3\,(t^2/2) + c1$

$v = 20$ at $t = 5$

$20 = 2(5) - (3/2)\,(5)^2 + c1$

$20 - 10 + (3/2)\,(25) = c1$

$C1 = 47.5$

$v = 2t - (3/2)\,t^2 + 47.5$

$v = ds/dt$

$s = \int v\, dt$

$s = \int (2t - (3/2)\,t^2 + 47.5)\, dt$

$s = 2\,(t^2/2) - 1.5\,(t^3/3) + 47.5\,t + c2$

$s = t^2 - 0.5\,t^3 + 47.5\,t + c2$

$s = 85$ at $t = 10$

$85 = 10^2 - 0.5\,(10)^3 + 47.5 \times 10 + c2$

$C2 = 10$

$s = t^2 - 0.5\,t^3 + 47.5t + 10$

i. When $t = 0$; $2t - 1.5\,t^2 + 47.5 = 0$

$t = 6.33$ sec

$s = (6.33)^2 - 0.5\,(6.33)^3 + 47.5\,(6.33) + 10$

$s = 223.926$m