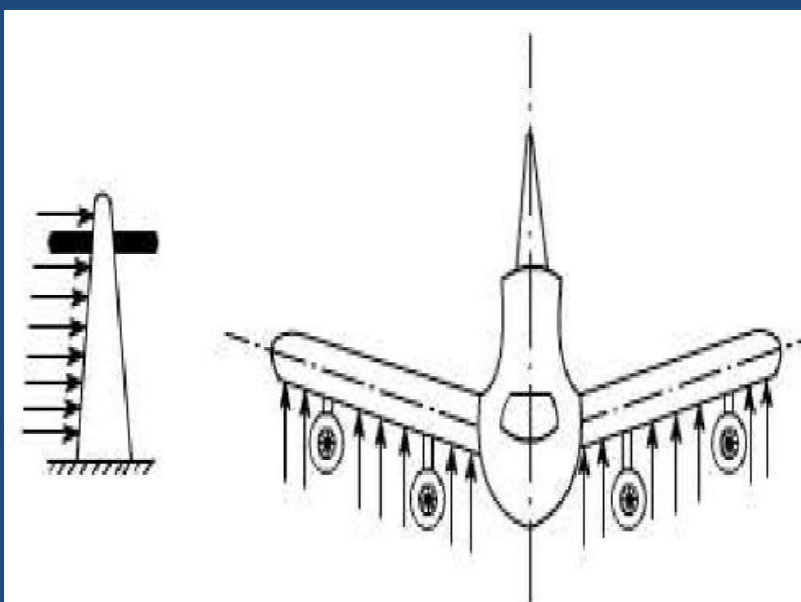
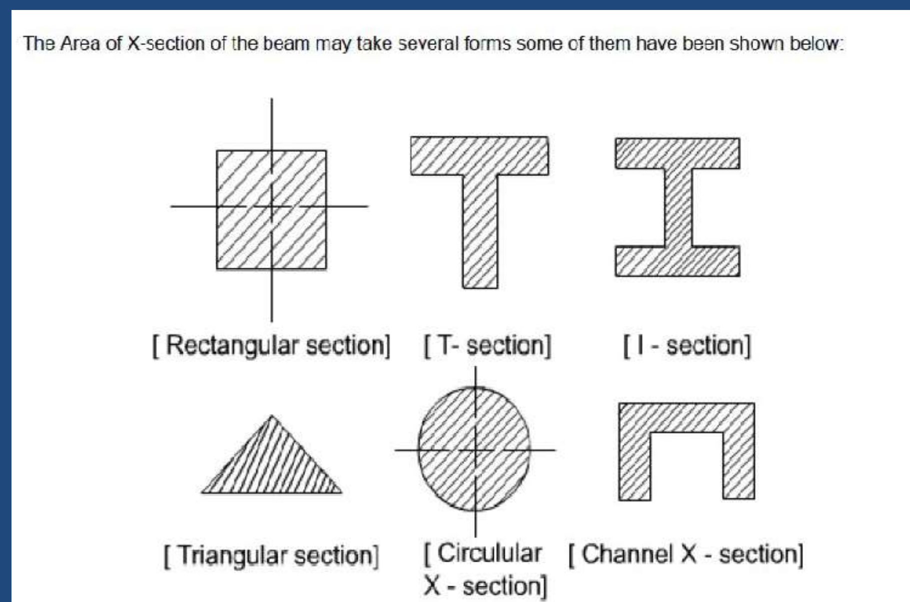
UNIT-II

Introduction to Beams

- In many engineering structures members are required to resist forces that are applied laterally or transversely to their axes. These type of members are termed as beam.
- **Definition** : A beam is a structural member subjected to a system of external forces at right angles to its axis.

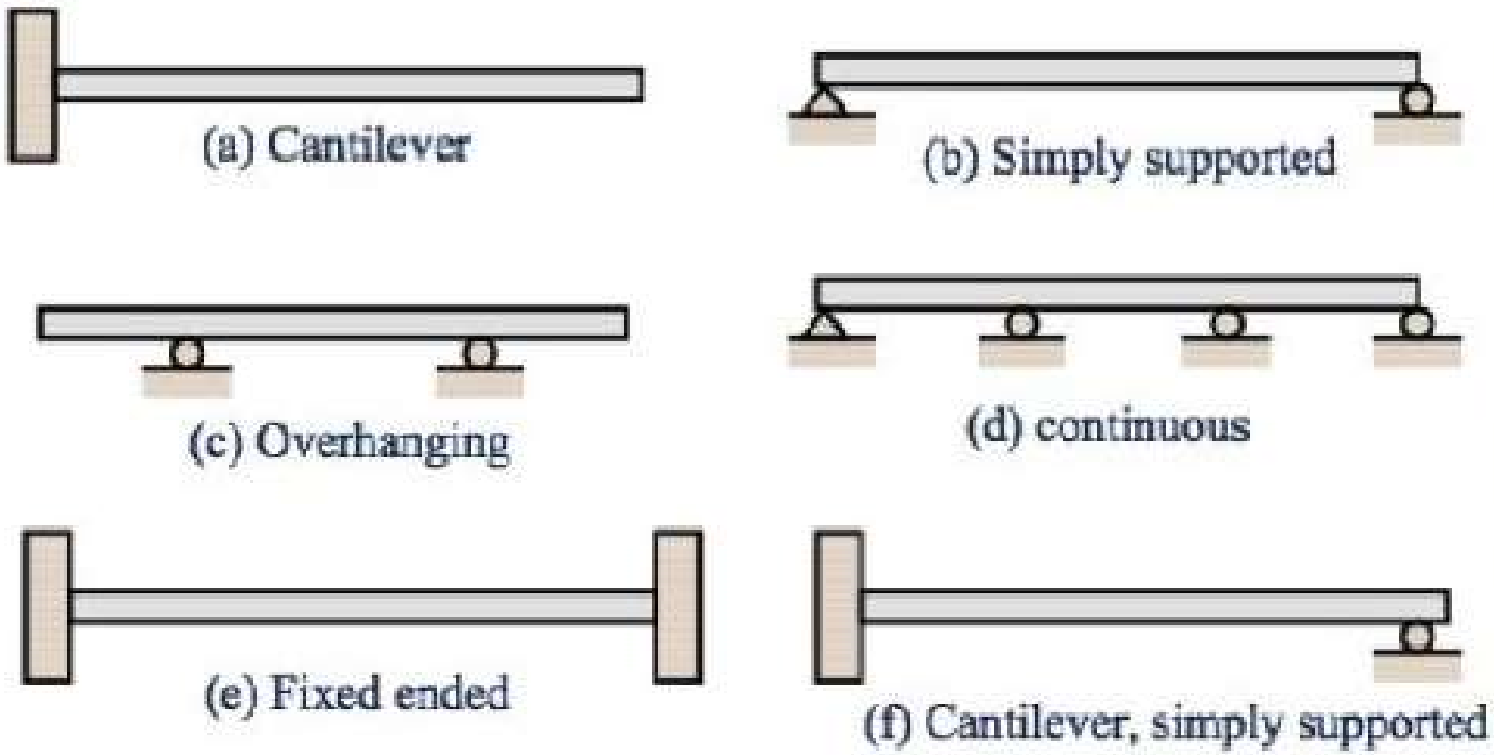




UNIT-II

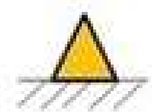
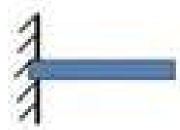
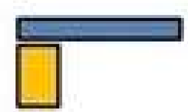
Classification of Beams

- Types of beams- depending on how they are supported.



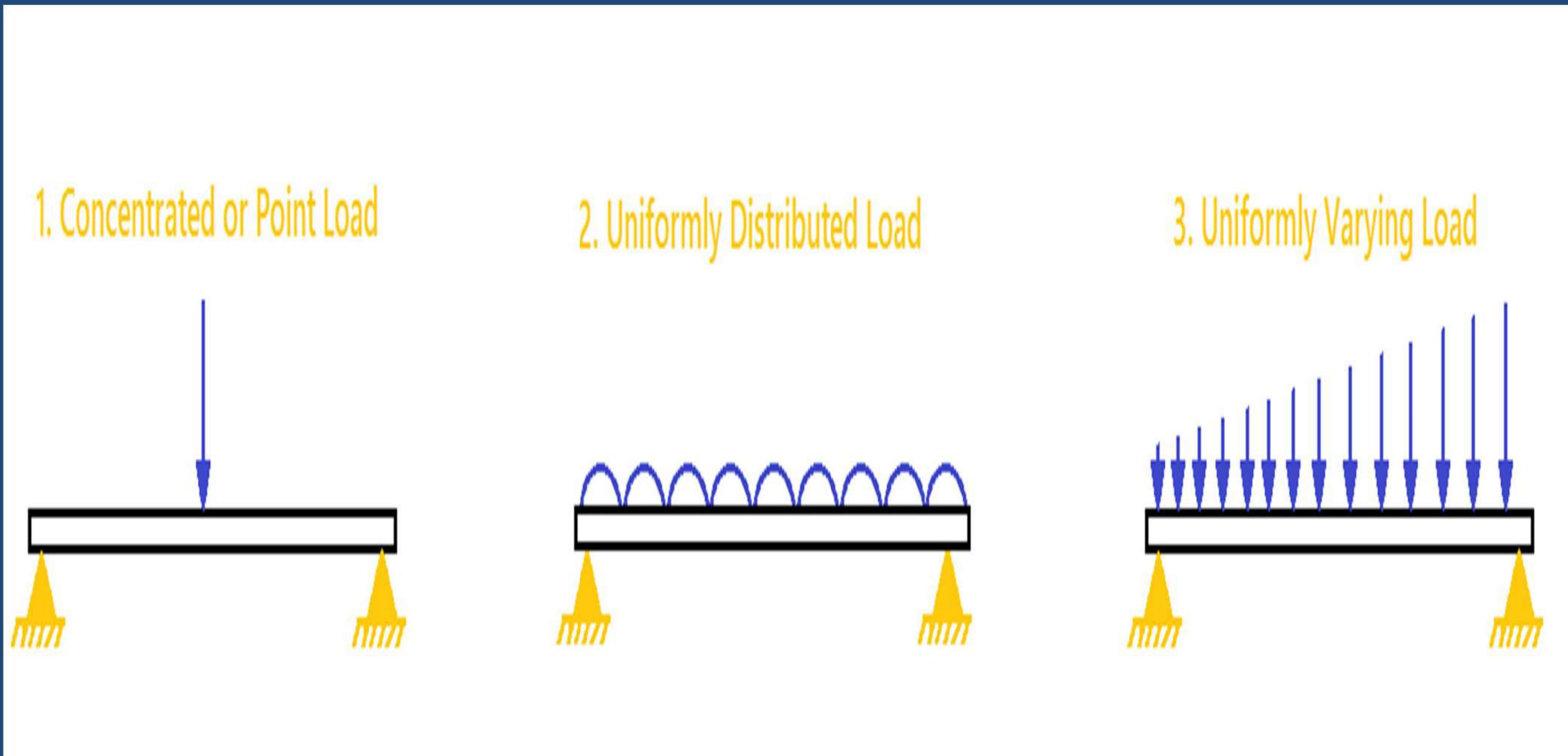
UNIT-II

Types of support

S.no	Types of Support	Representation by	Reaction Force	Resisting Load
1.	**Roller Support**		Vertical	Vertical loads
2.	**Pinned Support**		Horizontal and vertical	Vertical and horizontal loads
3.	**Fixed Support**		Horizontal, vertical and moments	All types of loads Horizontal, vertical and Moments
4.	**Simple Support**		Vertical	Vertical loads

UNIT-II

Types of Loads on Beams



Shear Force Diagram

Bending Moment Diagram

6.7. SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A CANTILEVER WITH A POINT LOAD AT THE FREE END

Fig. 6.14 shows a cantilever AB of length L fixed at A and free at B and carrying a point load W at the free end B.

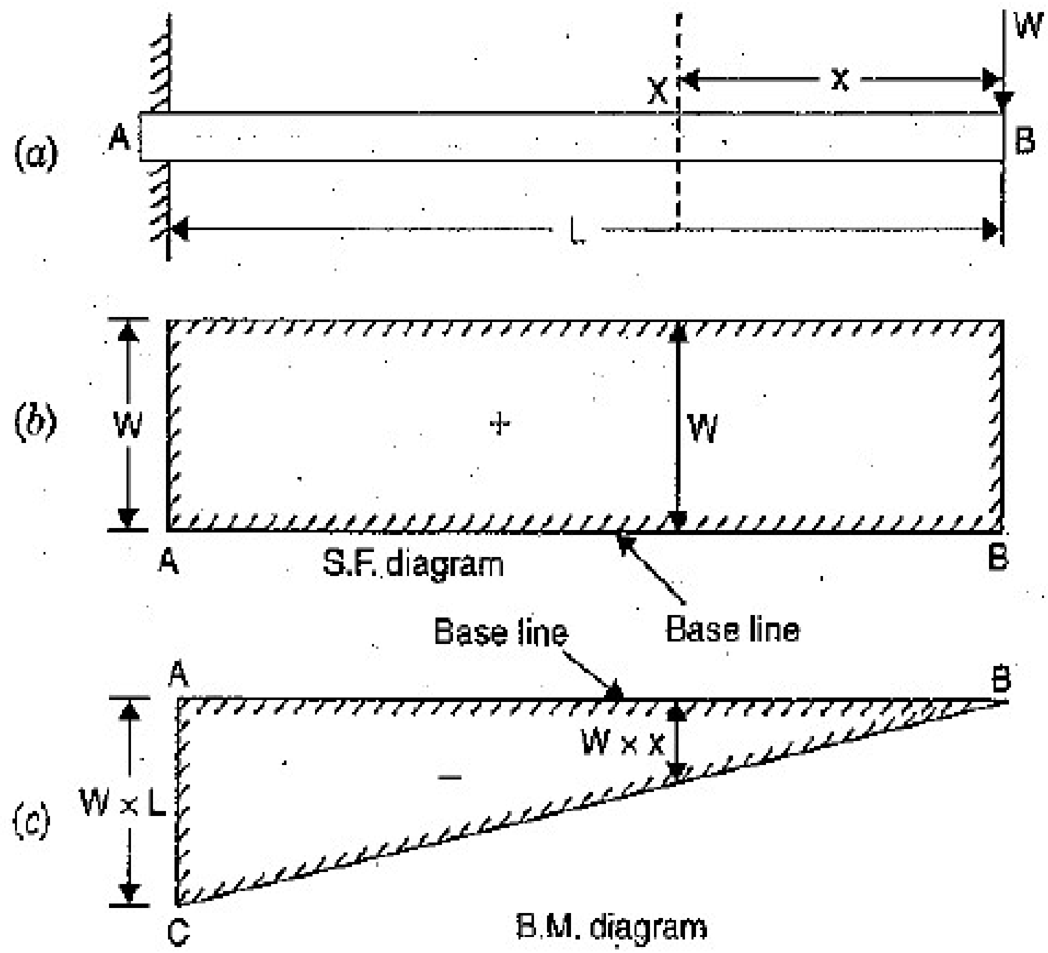


Fig. 6.14

Let F_x = Shear force at X, and

M_x = Bending moment at X.

Take a section X at a distance x from the free end. Consider the right portion of the section.

The shear force at this section is equal to the resultant force acting on the right portion at the given section. But the resultant force acting on the right portion at the section X is W and acting in the downward direction. But a force on the right portion acting downwards is considered positive. Hence shear force at X is positive.

$$\therefore \qquad F_x = +W$$

The shear force will be constant at all sections of the cantilever between A and B as there is no other load between A and B. The shear force diagram is shown in Fig. 6.14 (*b*).

Bending Moment Diagram

The bending moment at the section X is given by

$$M_x = -W \times x \qquad \ldots(i)$$

(Bending moment will be negative as for the right portion of the section, the moment of W at X is clockwise. Also the bending of cantilever will take place in such a manner that convexity will be at the top of the beam).

From equation (i), it is clear that B.M. at any section is proportional to the distance of the section from the free end.

At $x = 0$ *i.e.*, at B, B.M. $= 0$

At $x = L$ *i.e.*, at A, B.M. $= W \times L$

Hence B.M. follows the straight line law. The B.M. diagram is shown in Fig. 6.14 (c). At point A, take $AC = W \times L$ in the downward direction. Join point B to C.

The shear force and bending moment diagrams for several concentrated loads acting on a cantilever, will be drawn in the similar manner.

Problem 6.1. *A cantilever beam of length 2 m carries the point loads as shown in Fig. 6.15. Draw the shear force and B.M. diagrams for the cantilever beam.*

Sol. Given :

Refer to Fig. 6.15.

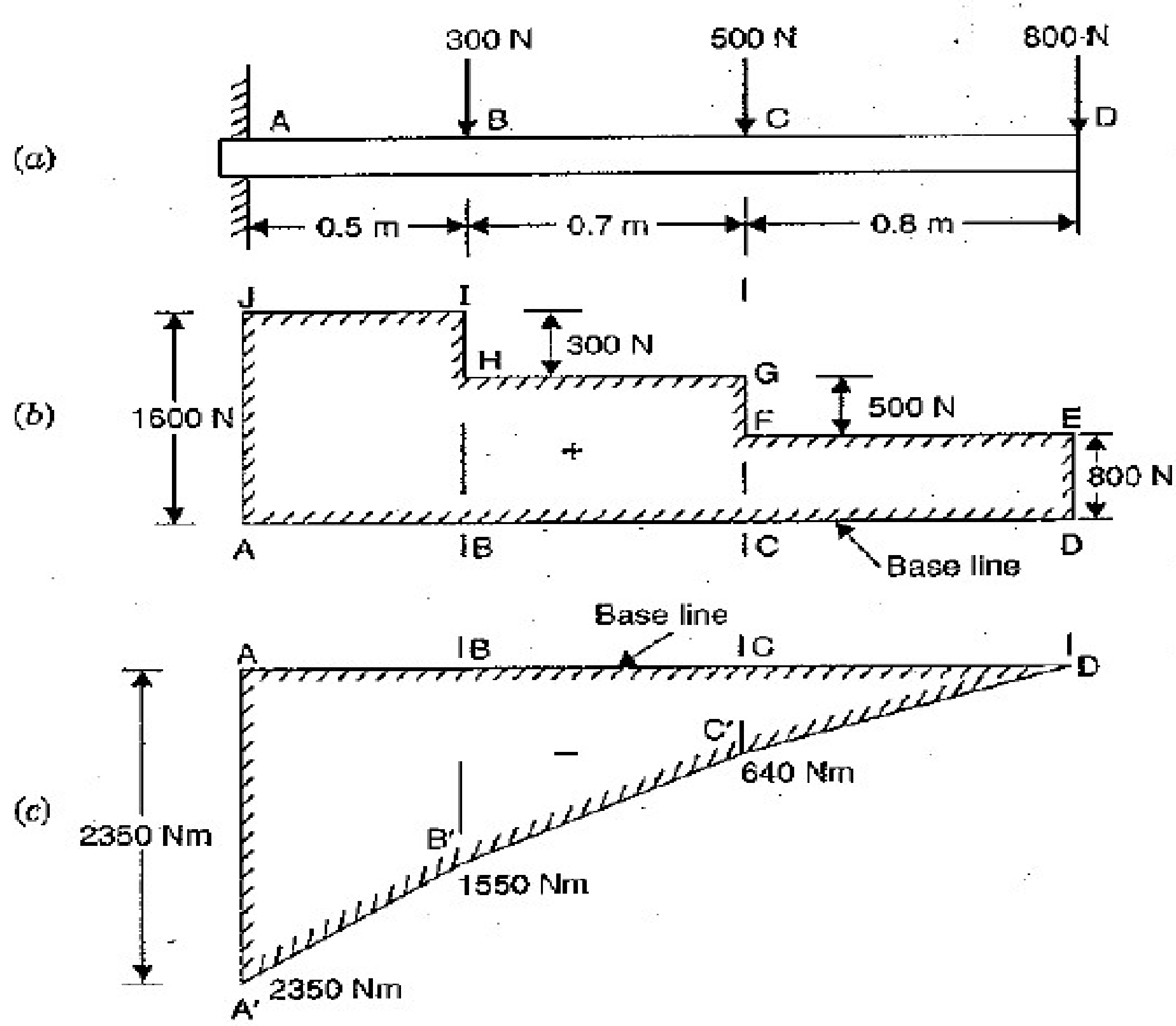


Fig. 6.15

Shear Force Diagram

The shear force at D is + 800 N. This shear force remains constant between D and C. At C, due to point load, the shear force becomes (800 + 500) = 1300 N. Between C and B, the shear force remains 1300 N. At B again, the shear force becomes (1300 + 300) = 1600 N. The shear force between B and A remains constant and equal to 1600 N. Hence the shear force at different points will be as given below :

S.F. at D, $F_D = +800$ N

S.F. at C, $F_C = +800 + 500 = +1300$ N

S.F. at B, $F_B = +800 + 500 + 300 = 1600$ N

S.F. at A, $F_A = +1600$ N.

The shear force, diagram is shown in Fig. 6.15 (*b*) which is drawn as :

Draw a horizontal line AD as base line. On the base line mark the points B and C below the point loads. Take the ordinate DE = 800 N in the upward direction. Draw a line EF parallel to AD. The point F is vertically above C. Take vertical line FG = 500 N. Through G, draw a horizontal line GH in which point H is vertically above B. Draw vertical line HI = 300 N. From I, draw a horizontal line IJ. The point J is vertically above A. This completes the shear force diagram.

Bending Moment Diagram

The bending moment at D is zero :

(*i*) The bending moment at any section between C and D at a distance x and D is given by,

$$M_x = -800 \times x \text{ which follows a straight line law.}$$

At C, the value of $x = 0.8$ m.

$\therefore$ B.M. at C, $\quad M_C = -800 \times 0.8 = -640$ Nm.

(*ii*) The B.M. at any section between B and C at a distance x from D is given by (At C, $x = 0.8$ and at B, $x = 0.8 + 0.7 = 1.5$ m. Hence here x varies from 0.8 to 1.5).

$$M_x = -800\,x - 500\,(x - 0.8) \quad \ldots(i)$$

Bending moment between B and C also varies by a straight line law.

B.M. at B is obtained by substituting $x = 1.5$ m in equation (*i*),

$$\therefore \quad M_B = -800 \times 1.5 - 500\,(1.5 - 0.8)$$

$$= -1200 - 350 = -1550 \text{ Nm.}$$

(*iii*) The B.M. at any section between A and B at a distance x from D is given by (At B, $x = 1.5$ and at A, $x = 2.0$ m. Hence here x varies from 1.5 m to 2.0 m)

$$M_x = -800\,x - 500\,(x - 0.8) - 300\,(x - 1.5) \qquad \text{...(ii)}$$

Bending moment between A and B varies by a straight line law.

B.M. at A is obtained by substituting $x = 2.0$ m in equation (*ii*),

$$\therefore \quad M_A = -800 \times 2 - 500\,(2 - 0.8) - 300\,(2 - 1.5)$$
$$= -800 \times 2 - 500 \times 1.2 - 300 \times 0.5$$
$$= -1600 - 600 - 150 = -2350 \text{ Nm.}$$

Hence the bending moments at different points will be as given below :

$$M_D = 0$$
$$M_C = -640 \text{ Nm}$$
$$M_B = -1550 \text{ Nm}$$
$$\text{and} \quad M_A = -2350 \text{ Nm.}$$

The bending moment diagram is shown in Fig. 6.15 (*c*) which is drawn as.

Draw a horizontal line AD as a base line and mark the points B and C on this line. Take vertical lines $CC' = 640$ Nm, $BB' = 1550$ Nm and $AA' = 2350$ Nm in the downward direction. Join points D, C', B' and A' by straight lines. This completes the bending moment diagram.

6.8. SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A CANTILEVER WITH A UNIFORMLY DISTRIBUTED LOAD

Fig. 6.16 shows a cantilever of length L fixed at A and carrying a uniformly distributed load of w per unit length over the entire length of the cantilever.

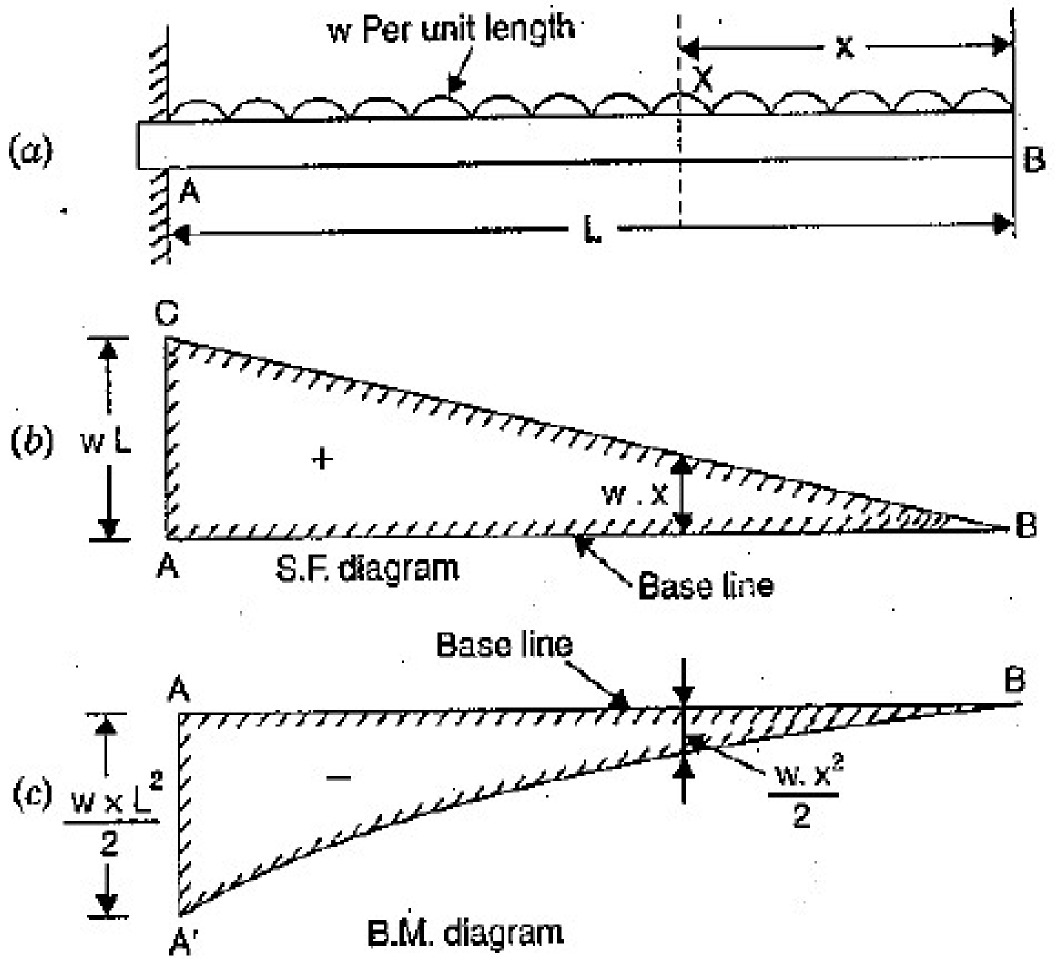


Fig. 6.16

Take a section X at a distance of x from the free end B.

Let F_x = Shear force at X, and

M_x = Bending moment at X.

Here we have considered the right portion of the section. The shear force at the section X will be equal to the resultant force acting on the right portion of the section. But the resultant force on the right portion = $w \times$ Length of right portion = $w.x$.

This resultant force is acting downwards. But the resultant force on the right portion acting downwards is considered positive. Hence shear force at X is positive.

$$\therefore \quad F_x = + w.x$$

The above equation shows that the shear force follows a straight line law.

At B, $x = 0$ and hence $F_x = 0$

At A, $x = L$ and hence $F_x = w.L$

The shear force diagram is shown in Fig. 6.16 (b).

Bending Moment Diagram

It is mentioned in Art. 6.4.3 that the uniformly distributed load over a section is converted into point load acting at the C.G. of the section.

The bending moment at the section X is given by

M_x = – (Total load on right portion) × Distance of C.G. of right portion from X

$$= -(w \cdot x) \cdot \frac{x}{2} = -w \cdot x \cdot \frac{x}{2} = -w \cdot \frac{x^2}{2} \qquad \ldots(i)$$

(The bending moment will be negative as for the right portion of the section, the moment of the load at x is clockwise. Also the bending of cantilever will take place in such a manner that convexity will be at the top of the cantilever).

From equation (*i*), it is clear that B.M. at any section is proportional to the square of the distance of the section from the free end. This follows a parabolic law.

At B, $x = 0$ hence $\quad M_x = 0$

At A, $x = L$ hence $\quad M_x = -w \cdot \dfrac{L^2}{2}$.

The bending moment diagram is shown in Fig. 6.16 (*c*).

Problem 6.2. *A cantilever of length 2.0 m carries a uniformly distributed load of 1 kN/m run over a length of 1.5 m from the free end. Draw the shear force and bending moment diagrams for the cantilever.*

Sol. Given :

U.D.L., w = **1** kN/m run

Refer to Fig. 6.17.

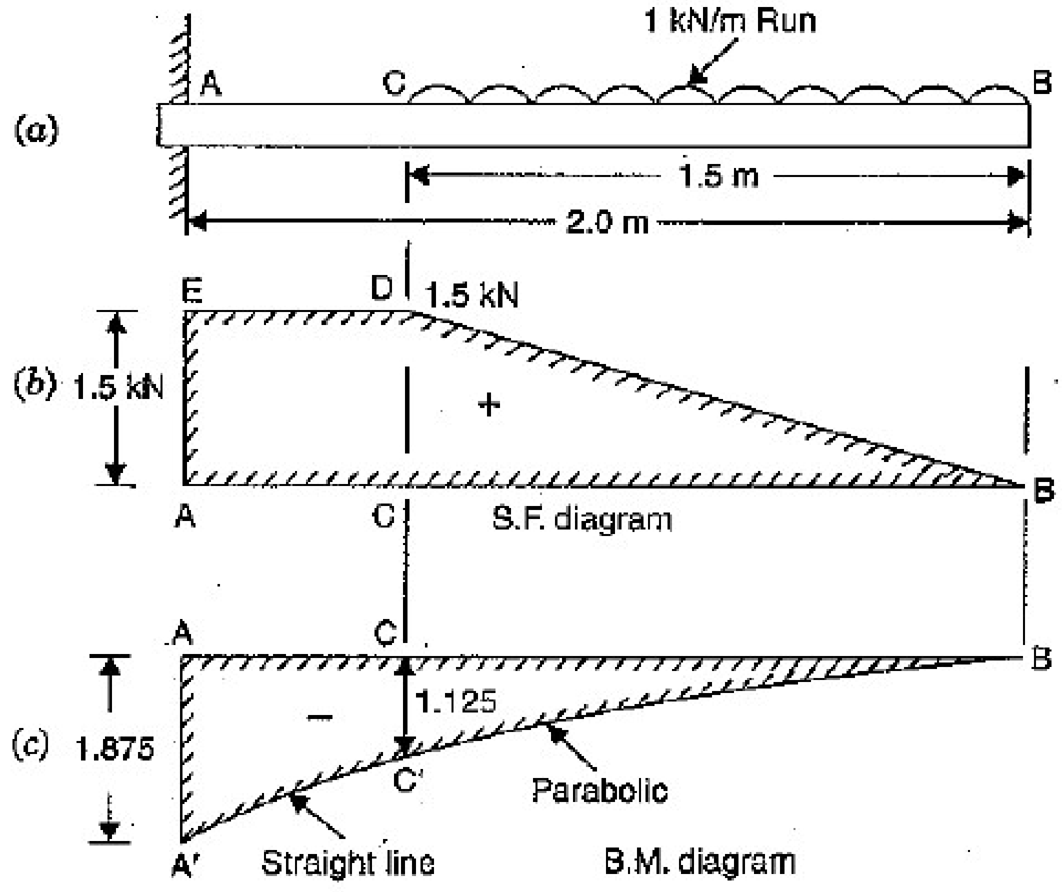


Fig. 6.17

Shear Force Diagram

Consider any section between C and B a distance of x from the free end B. The shear force at the section is given by

$F_x = w.x$ (+ve sign is due to downward force on right portion of the section)

$= 1.0 \times x$ ($\because$ $w = 1.0$ kN/m run)

At B, $x = 0$ hence $F_x = 0$

At C, $x = 1.5$ hence $F_x = 1.0 \times 1.5 = 1.5$ kN.

The shear force follows a straight line law between C and B. As between A and C there is no load, the shear force will remain constant. Hence shear force between A and C will be represented by a horizontal line.

The shear force diagram is shown in Fig. 6.17 (b) in which

$$F_B = 0, F_C = 1.5 \text{ kN and } F_A = F_C = 1.5 \text{ kN.}$$

Bending Moment Diagram

(i) The bending moment at any section between C and B at a distance x from the free end B is given by

$$M_x = -(w.x.).\frac{x}{2} = -\left(1.\frac{x^2}{2}\right) = -\frac{x^2}{2} \quad \ldots(i)$$

(The bending moment will be negative as for the right portion of the section the moment of load at x is clockwise).

At B, $x = 0$ hence $M_B = -\frac{0^2}{2} = 0$

At C, $x = 1.5$ hence $M_C = -\frac{1.5^2}{2} = -1.125$ Nm

From equation (i) it is clear that the bending moment varies according to parabolic law between C and B.

(*ii*) The bending moment at any section between A and C at a distance x from the free end B is obtained as : (here x varies from 1.5 m to 2.0 m)

Total load due to U.D.L. $= w \times 1.5 = 1.5$ kN.

This load is acting at a distance of $\frac{1.5}{2} = 0.75$ m from the free end B or at a distance of $(x - 0.75)$ from any section between A and C.

$\therefore$ Moment of this load at any section between A and C at a distance x from free end

$$= (\text{Load due to U.D.L.}) \times (x - 0.75)$$

$$\therefore \quad M_x = -1.5 \times (x - 0.75) \qquad \ldots(ii)$$

(– ve sign is due to clockwise moment for right portion)

From equation (*ii*) it is clear that the bending moment follows straight line law between A and C.

At C, $x = 1.5$ m hence $\quad M_C = -1.5\,(1.5 - 0.75) = -1.125$ Nm

At A, $x = 2.0$ m hence $\quad M_A = -1.5\,(2 - 0.75) = -1.875$ Nm.

Now the bending moment diagram is drawn as shown in Fig. 6.17 (*c*). In this diagram line $CC' = 1.125$ Nm and $AA' = 1.875$ Nm. The points B and C' are on a parabolic curve whereas the points A' and C' are joined by a straight line.

Problem 6.6. *A cantilever of length 5.0 m is loaded as shown in Fig. 6.21. Draw the S.F. and B.M. diagrams for the cantilever.*

Sol. The shear force at B is 2.5 kN and remains constant between B and C.

The shear force increases by a straight line law to $2.5 + 2 \times 1 = 4.5$ kN at D. The shear force remains constant between D and E. At point E, the shear force suddenly increases to $4.5 + 3 = 7.5$ kN due to point load at E. Again the shear force remains constant between A and E. Now the shear force diagram is drawn as shown in Fig. 6.21 (b).

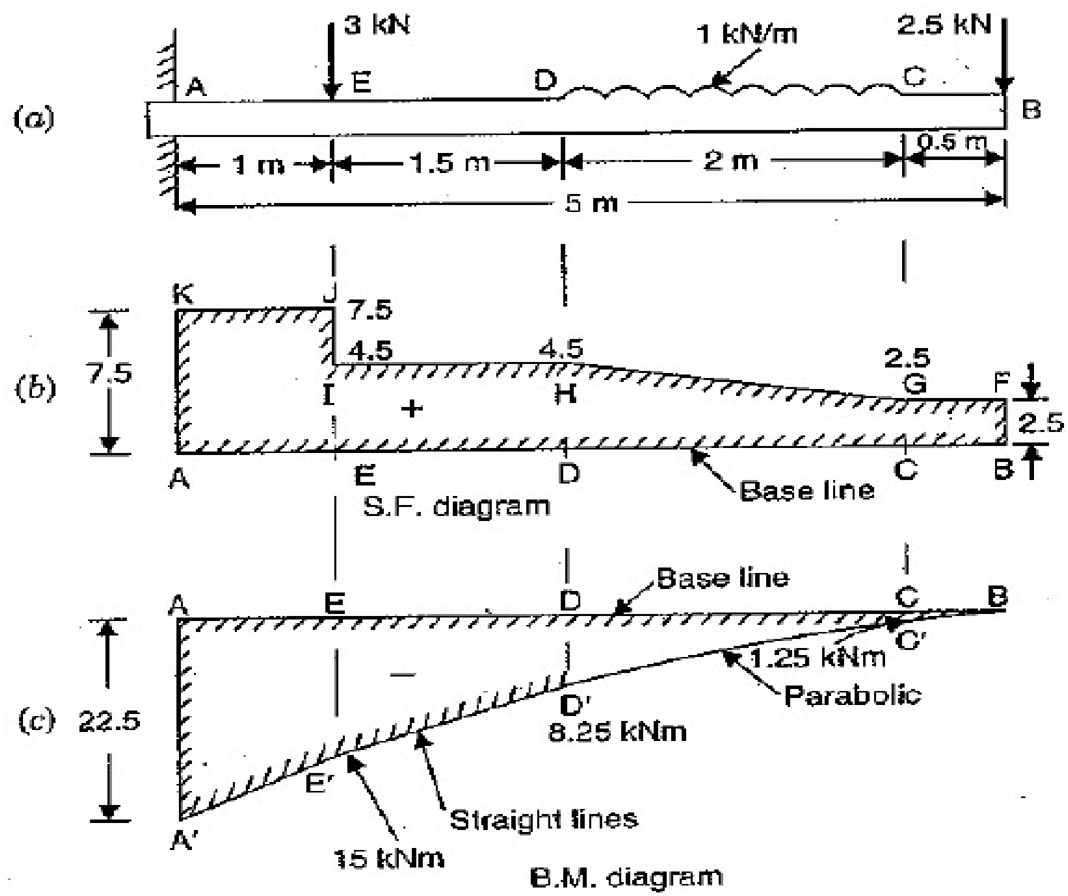


Fig. 6.21

Bending Moment Diagram

$$\text{B.M. at } B = 0$$

$$\text{B.M. at } C = -2.5 \times 0.5 = -1.25 \text{ kNm}$$

$$\text{B.M. at } D = -2.5 \times 2.5 - 2 \times 1 \times 1 = -8.25 \text{ kNm}$$

$$\text{B.M. at } E = -2.5 \times 4 - 2 \times 1 \times (1.5 + 1.0) = -10 - 5 = -15 \text{ kNm}$$

$$\text{B.M. at } A = -2.5 \times 5 - 2 \times 1 \times (1 + 1.5 + 1.0) - 3 \times 1$$
$$= -12.5 - 7.0 - 3 = -22.5 \text{ kNm.}$$

Now the bending moment diagram is drawn as shown in Fig. 6.21 (*c*). In this diagram, the B.M. varies according to parabolic law between points C and D only. Between other points B.M. varies according to straight line law.

6.10. SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A SIMPLY SUPPORTED BEAM WITH A POINT LOAD AT MID-POINT

Fig. 6.24 shows a beam AB of length L simply supported at the ends A and B and carrying a point load W at its middle point C.

The reactions at the support will be equal to $\frac{W}{2}$ as the load is acting at the middle point of the beam. Hence $R_A = R_B = \frac{W}{2}$.

Take a section X at a distance x from the end A between A and C.

Let F_x = Shear force at X,

and M_x = Bending moment at X.

Here we have considered the *left portion* of the section. The shear force at X will be equal to the resultant force acting on the left portion of the section. But the resultant force on the left portion is $\frac{W}{2}$ acting upwards. But according to the sign convention, the resultant force on the *left portion* acting upwards is considered positive. Hence shear force at X is positive and its magnitude is $\frac{W}{2}$.

$$\therefore \qquad F_x = +\frac{W}{2}$$

Hence the shear force between A and C is constant and equal to $+\frac{W}{2}$.

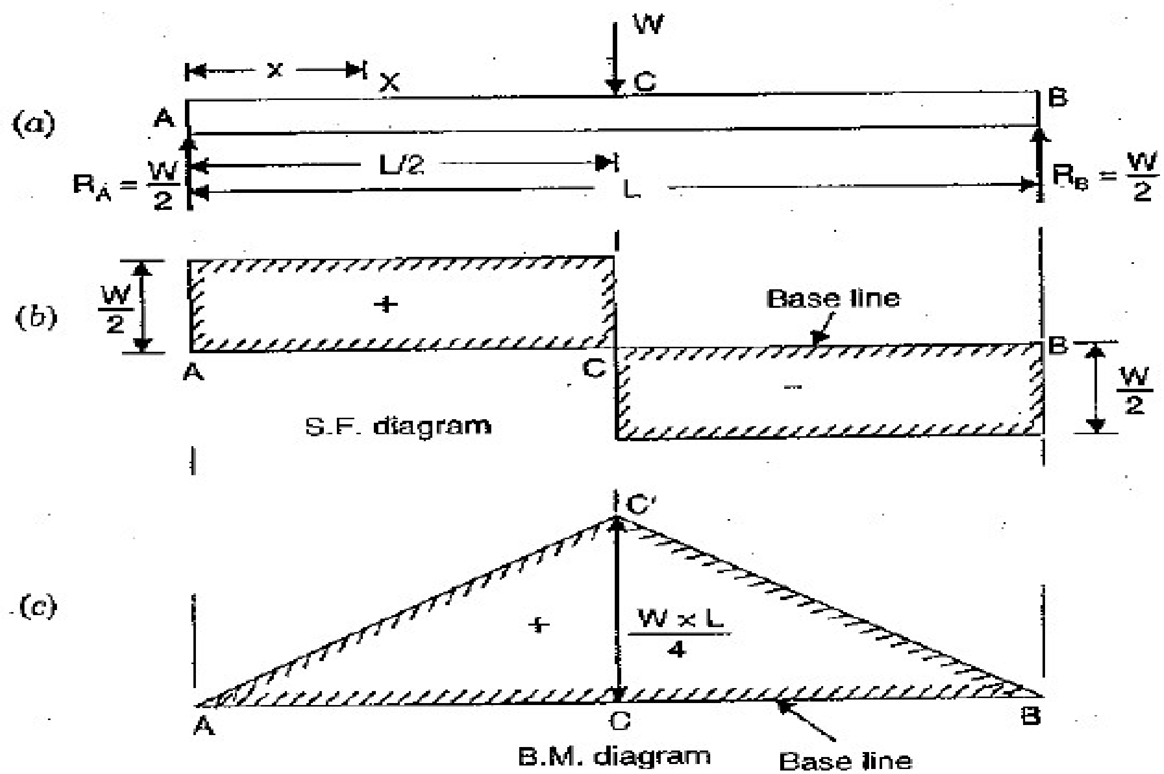


Fig. 6.24

Now consider any section between C and B at distance x from end A. The resultant force on the left portion will be

$$\left(\frac{W}{2} - W\right) = -\frac{W}{2}.$$

This force will also remain constant between C and B. Hence shear force between C and B is equal to $-\frac{W}{2}$.

At the section C the shear force changes from $+\frac{W}{2}$ to $-\frac{W}{2}$.

The shear force diagram is shown in Fig. 6.24 (b).

Bending Moment Diagram

(*i*) The bending moment at any section between A and C at a distance of x from the end A, is given by

$$M_x = R_A . x \quad \text{or} \quad M_x = +\frac{W}{2} . x \qquad \text{...}(i)$$

(B.M. will be positive as for the *left portion* of the section, the moment of all forces at X is clockwise. Moreover, the bending of beam takes place in such a manner that concavity is at the top of the beam).

At A, $x = 0$ hence $\qquad M_A = \frac{W}{2} \times 0 = 0$

At C, $x = \frac{L}{2}$ hence $\qquad M_C = \frac{W}{2} \times \frac{L}{2} = \frac{W \times L}{4}$.

From equation (*i*), it is clear that B.M. varies according to straight line law between A and C. B.M. is zero at A and it increases to $\frac{W \times L}{4}$ at C.

(*ii*) The bending moment at any section between C and B at a distance x from the end A, is given by

$$M_x = R_A . x - W \times \left(x - \frac{L}{2}\right) = \frac{W}{2} . x - Wx + W \times \frac{L}{2} = \frac{WL}{2} - \frac{2x}{2}$$

At C, $x = \frac{L}{2}$ hence $\quad M_C = \frac{WL}{2} - \frac{W}{2} \times \frac{L}{2} = \frac{W \times L}{4}$

At B, $x = L$ hence $\quad M_B = \frac{WL}{2} - \frac{W}{2} \times L = 0.$

Hence bending moment at C is $\frac{WL}{4}$ and it decreases to zero at B. Now the B.M. diagram can be completed as shown in Fig. 6.24 (c).

Note. The bending moment is maximum at the middle point C, where the shear force changes its sign.

Problem 6.8. *A simply supported beam of length 6 m, carries point load of 3 kN and 6 kN at distances of 2 m and 4 m from the left end. Draw the shear force and bending moment diagrams for the beam.*

Sol. First calculate the reactions R_A and R_B.

Taking moments of the force about A, we get

$$R_B \times 6 = 3 \times 2 + 6 \times 4 = 30$$

$$\therefore \qquad R_B = \frac{30}{6} = 5 \text{ kN}$$

$$\therefore \qquad R_A = \text{Total load on beam} - R_B = (3 + 6) - 5 = 4 \text{ kN}$$

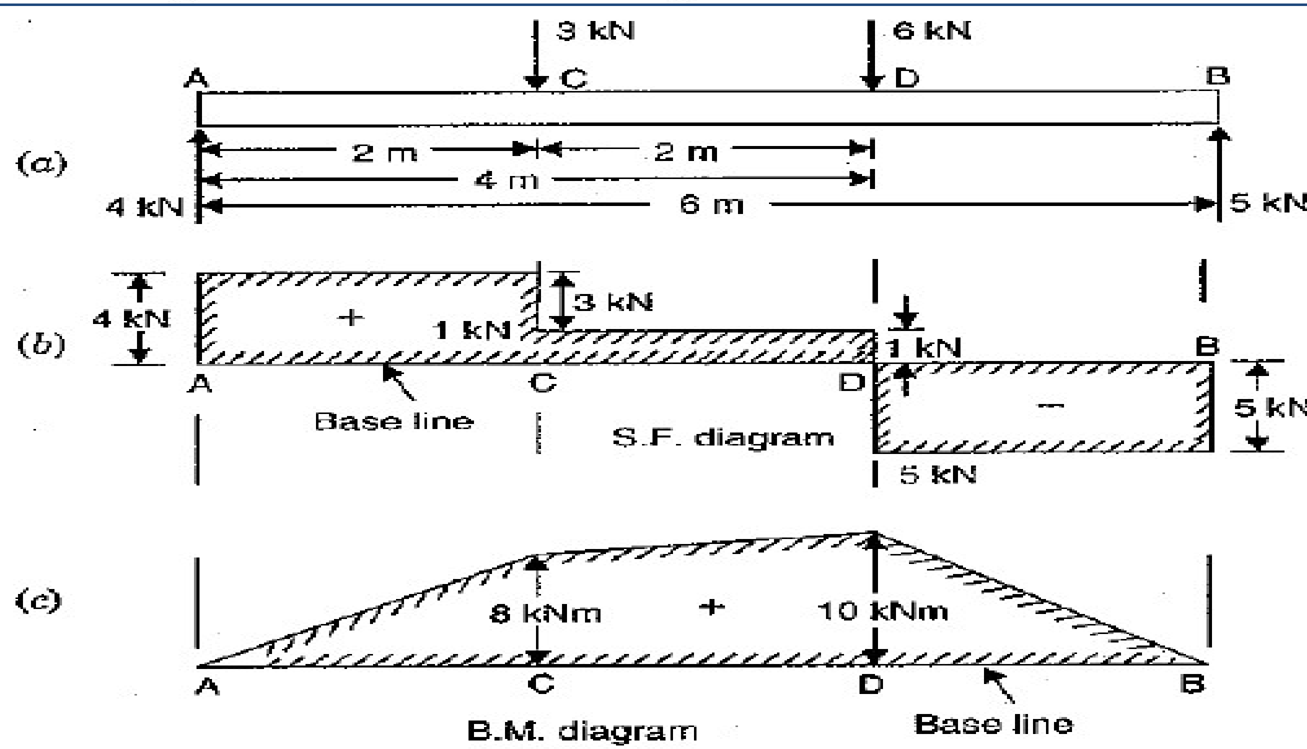


Fig. 6.26

Shear Force Diagram

Shear force at A, $F_A = +R_A = +4$ kN

Shear force between A and C is constant and equal to + 4 kN

Shear force at C, $F_C = +4 - 3.0 = +1$ kN

Shear force between C and D is constant and equal to + 1 kN.

Shear force at D, $F_D = +1 - 6 = -5$ kN

The shear force between D and B is constant and equal to − 5 kN.

Shear force at B, $F_B = -5$ kN

The shear force diagram is drawn as shown in Fig. 6.26 (*b*).

Bending Moment Diagram

B.M. at A, $M_A = 0$

B.M. at C, $M_C = R_A \times 2 = 4 \times 2 = +8$ kNm

B.M. at D, $M_D = R_A \times 4 - 3 \times 2 = 4 \times 4 - 3 \times 2 = +10$ kNm

B.M. at B, $M_B = 0$

The bending moment diagram is drawn as shown in Fig. 6.26 (*c*).

6.12. SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A SIMPLY SUPPORTED BEAM CARRYING A UNIFORMLY DISTRIBUTED LOAD

Fig. 6.27 shows a beam AB of length L simply supported at the ends A and B and carrying a uniformly distributed load of w per unit length over the entire length. The reactions at the supports will be equal and their magnitude will be half the total load on the entire length.

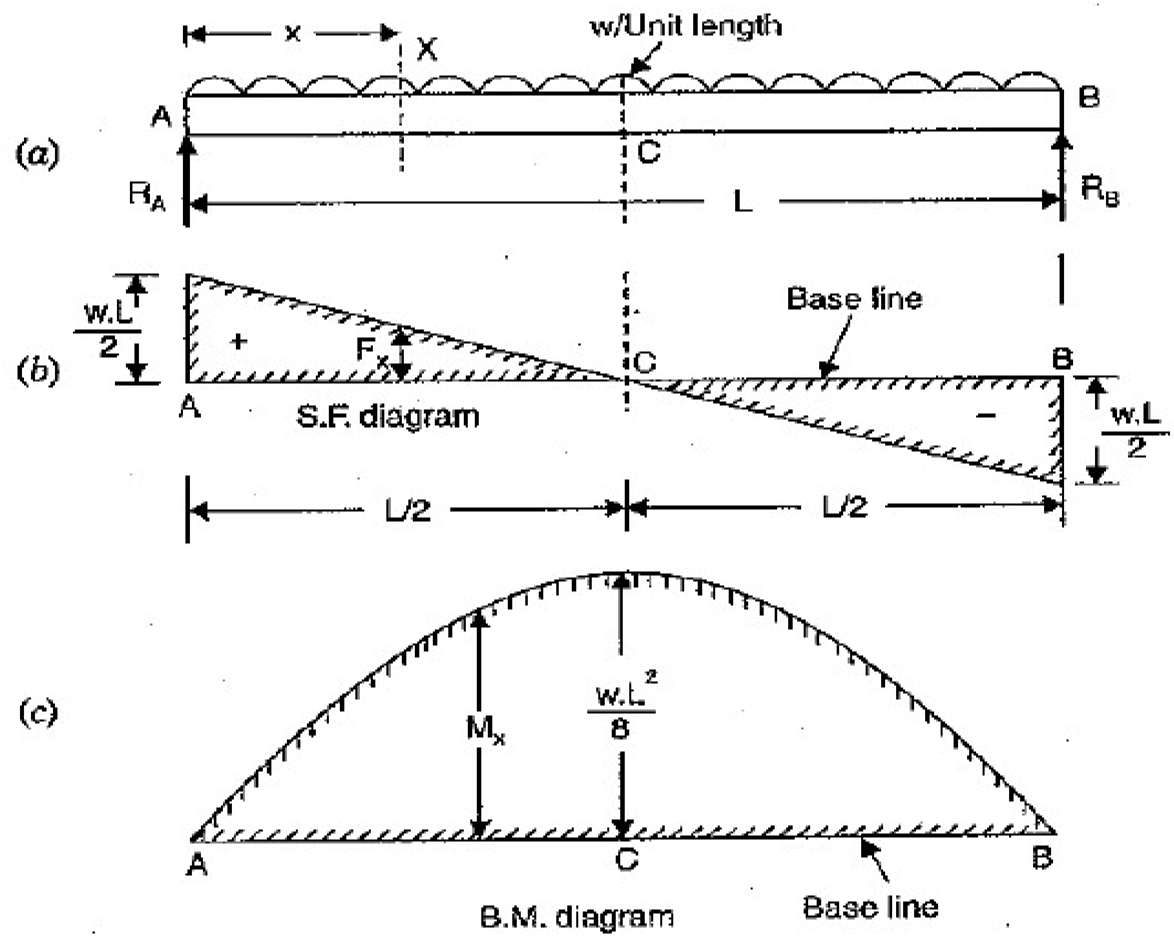


Fig. 6.27

Let $\quad R_A$ = Reaction at A, and

R_B = Reaction at B

$$\therefore \quad R_A = R_B = \frac{w.L}{2}$$

Consider any section X at a distance x from the *left end A*. The shear force at the section (*i.e.*, F_x) is given by,

$$F_x = +R_A - w\,.\,x = +\frac{w.L}{2} - w\,.\,x \qquad ...(i)$$

From equation (*i*), it is clear that the shear force varies according to straight line law. The values of shear force at different points are :

At A, $x = 0$ hence $\quad F_A = +\frac{w.L}{2} - \frac{w.0}{2} = +\frac{w.L}{2}$

At B, $x = L$ hence $\quad F_B = +\frac{w.L}{2} - w.L = -\frac{w.L}{2}$

At C, $x = \frac{L}{2}$ hence $\quad F_C = +\frac{w.L}{2} - w.\frac{L}{2} = 0$

The shear force diagram is drawn as shown in Fig. 6.27 (*b*).

The bending moment at the section X at a distance x from left end A is given by,

$$M_x = +R_A\,.\,x - w\,.\,x\,.\,\frac{x}{2}$$

$$= \frac{w.L}{2}\,.\,x - \frac{w.x^2}{2} \qquad \left(\because\ R_A = \frac{w.L}{2}\right) ...(ii)$$

From equation (*ii*), it is clear that B.M. varies according to parabolic law.

The values of B.M. at different points are :

At A, $x = 0$ hence $\quad M_A = \frac{w.L}{2} . 0 - \frac{w.0}{2} = 0$

At B, $x = L$ hence $\quad M_B = \frac{w.L}{2} . L - \frac{w}{2} . L^2 = 0$

At C, $x = \frac{L}{2}$ hence $\quad M_C = \frac{w.L}{2} . \frac{L}{2} - \frac{w}{2} . \left(\frac{L}{2}\right)^2 = \frac{w.L^2}{4} - \frac{w.L^2}{8} = + \frac{w.L^2}{8}$.

Thus the B.M. increases according to parabolic law from zero at A to $+\frac{w.L^2}{8}$ at the middle point of the beam and from this value the B.M. decreases to zero at B according to the parabolic law.

Now the B.M. diagram is drawn as shown in Fig. 6.27 (c).

Problem 6.9. *Draw the shear force and bending moment diagram for a simply supported beam of length 9 m and carrying a uniformly distributed load of 10 kN/m for a distance of 6 m from the left end. Also calculate the maximum B.M. on the section.*

Sol. First calculate reactions R_A and R_B.

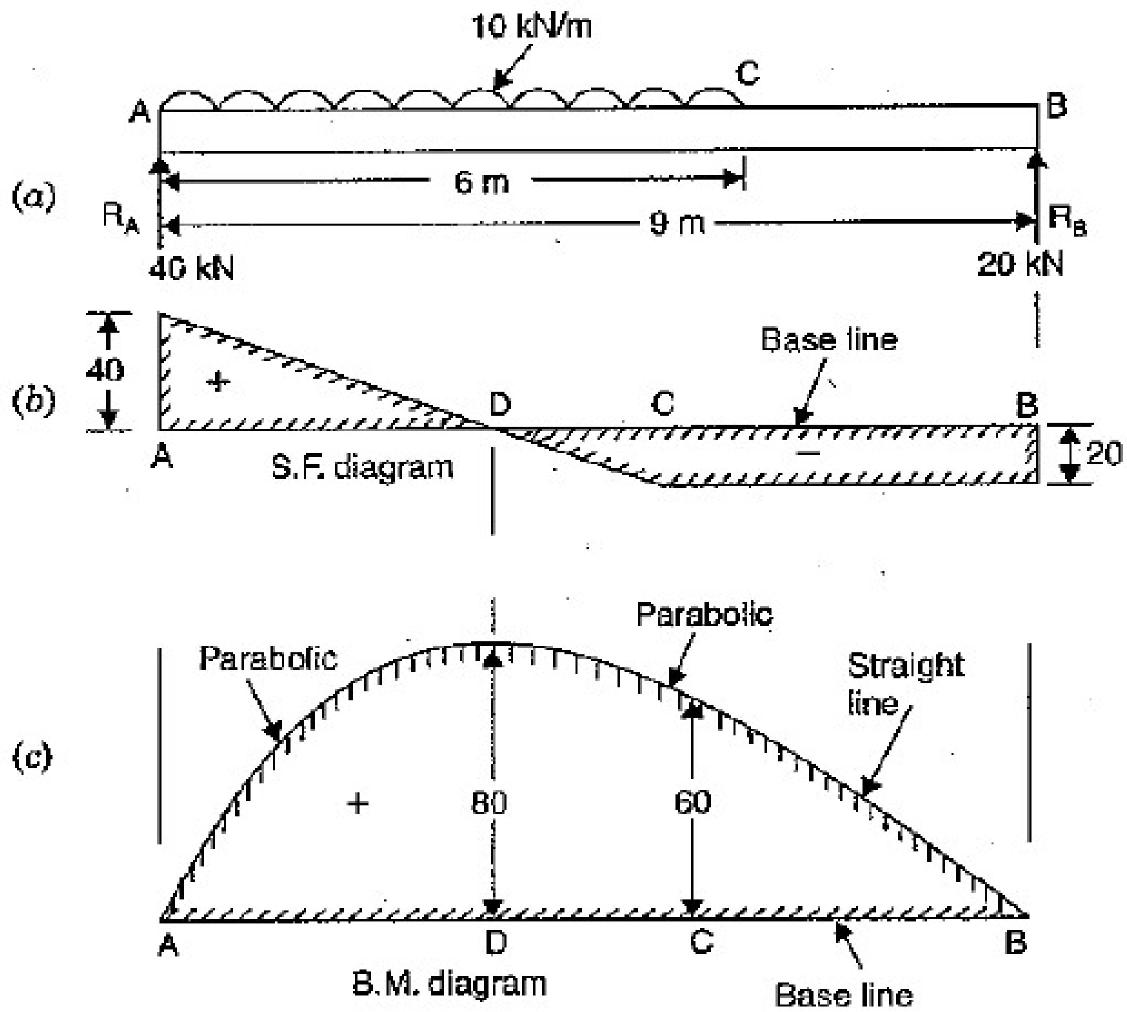


Fig. 6.28

Taking moments of the forces about A, we get

$$R_B \times 9 = 10 \times 6 \times \frac{6}{2} = 180$$

$$\therefore \quad R_B = \frac{180}{9} = 20 \text{ kN}$$

$$\therefore \quad R_A = \text{Total load on beam} - R_B = 10 \times 6 - 20 = 40 \text{ kN.}$$

Shear Force Diagram

Consider any section at a distance x from A between A and C. The shear force at the section is given by,

$$F_x = +R_A - 10\,x = +40 - 10\,x \qquad \ldots(i)$$

Equation (i) shows that shear force varies by a straight line law between A and C.

At A, $x = 0$ hence $\quad F_A = +40 - 0 = 40$ kN

At C, $x = 6$ m hence $\quad F_C = +40 - 10 \times 6 = -20$ kN

The shear force at A is + 40 kN and at C is – 20 kN. Also shear force between A and C varies by a straight line. This means that somewhere between A and C, the shear force is zero. Let the S.F. is zero at x metre from A. Then substituting the value of S.F. (*i.e.*, F_x) equal to zero in equation (i), we get

$$0 = 40 - 10x$$

$$\therefore \quad x = \frac{40}{10} = 4 \text{ m}$$

Hence shear force is zero at a distance 4 m from A.

The shear force is constant between C and B. This equal to – 20 kN.

Now the shear force diagram is drawn as shown in Fig. 6.28 (b). In the shear force diagram, distance AD = 4 m. The point D is at a distance 4 m from A.

B.M. Diagram

The B.M. at any section between A and C at a distance x from A is given by,

$$M_x = R_A \times x - 10 \cdot x \cdot \frac{x}{2} = 40x - 5x^2 \qquad \text{...(ii)}$$

Equation (*ii*) shows that B.M. varies according to parabolic law between A and C.

At A, $x = 0$ hence $\quad M_A = 40 \times 0 - 5 \times 0 = 0$

At C, $x = 6$ m hence $\quad M_C = 40 \times 6 - 5 \times 6^2 = 240 - 180 = + 60$ kNm

At D, $x = 4$ m hence $\quad M_D = 40 \times 4 - 5 \times 4^2 = 160 - 80 = + 80$ kNm

The bending moment between C and B varies according to linear law.

B.M. at B is zero whereas at C is 60 kNm.

The bending moment diagram is drawn as shown in Fig. 6.28 (*c*).

Maximum Bending Moment

The B.M. is maximum at a point where shear force changes sign. This means that the point where shear force becomes zero from positive value to the negative or *vice-versa*, the B.M. at that point will be maximum. From the shear force diagram, we know that at point D, the shear force is zero after changing its sign. Hence B.M. is maximum at point D. But the B.M. at D is + 80 kNm.

$\therefore \qquad$ Max. B.M. = **+ 80 kN. Ans.**

Problem 6.12. *A simply supported beam of length 10 m, carries the uniformly distributed load and two point loads as shown in Fig. 6.31. Draw the S.F. and B.M. diagram for the beam. Also calculate the maximum bending moment.*

Sol. First calculate the reactions R_A and R_B.

Taking moments of all forces about A, we get

$$R_B \times 10 = 50 \times 2 + 10 \times 4 \times \left(2 + \frac{4}{2}\right) + 40(2 + 4)$$

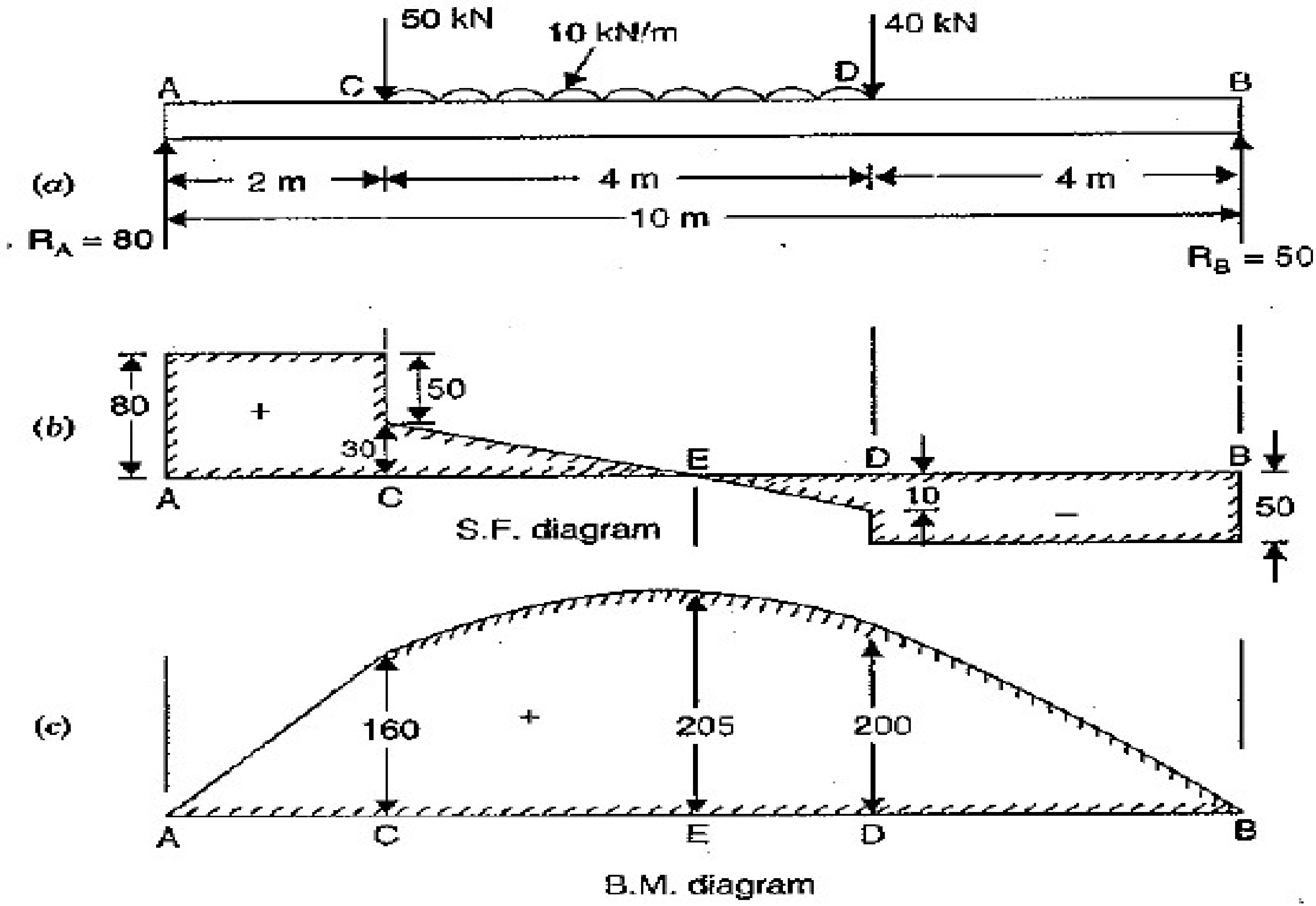
$$= 100 + 160 + 240 = 500$$

$$\therefore \quad R_B = \frac{500}{10} = 50 \text{ kN}$$

and

$$R_A = \text{Total load on beam} - R_B$$

$$= (50 + 10 \times 4 + 40) - 50 = 130 - 50 = 80 \text{ kN}$$



50 kN
10 kN/m
40 kN
A
C
D
B
(a)
2 m
4 m
4 m
10 m
R_A = 80
R_B = 50
(b)
80
50
30
+
E
D
B
10
50
A
C
S.F. diagram
(c)
160
+
205
200
A
C
E
D
B
B.M. diagram

Fig. 6.31

S.F. Diagram

The S.F. at A, $F_A = R_A = +80$ kN

The S.F. will remain constant between A and C and equal to + 80 kN

The S.F. just on R.H.S. of $C = R_A - 50 = 80 - 50 = 30$ kN

The S.F. just on L.H.S. of $D = R_A - 50 - 10 \times 4 = 80 - 50 - 40 = -10$ kN

The S.F. between C and D varies according to straight line law.

The S.F. just on R.H.S. of $D = R_A - 50 - 10 \times 4 - 40 = 80 - 50 - 40 - 40 = -50$ kN

The S.F. at $B = -50$ kN

The S.F. remains constant between D and B and equal to -50 kN

The shear force diagram is drawn as shown in Fig. 6.31 (*b*).

The shear force is zero at point E between C and D.

Let the distance of E from point A is x.

Now shear force at $E = R_A - 50 - 10 \times (x - 2)$

$$= 80 - 50 - 10x + 20 = 50 - 10x$$

But shear force at $E = 0$

$$\therefore \quad 50 - 10x = 0 \quad \text{or} \quad x = \frac{50}{10} = 5 \text{ m}$$

B.M. Diagram

B.M. at A is zero

B.M. at B is zero

B.M. at C, $\quad M_C = R_A \times 2 = 80 \times 2 = 160$ kNm

B.M. at D, $\quad M_D = R_A \times 6 - 50 \times 4 - 10 \times 4 \times \frac{4}{2}$

$$= 80 \times 6 - 200 - 80 = 480 - 200 - 80 = 200 \text{ kNm}$$

At E, $x = 5$ m and hence B.M. at E,

$$M_E = F_A \times 5 - 50(5-2) - 10 \times (5-2) \times \left(\frac{5-2}{2}\right)$$

$$= 80 \times 5 - 50 \times 3 - 10 \times 3 \times \tfrac{3}{2} = 400 - 150 - 45 = 205 \text{ kNm}$$

The B.M. between C and D varies according to parabolic law reaching a maximum value at E. The B.M. between A and C and also between B and D varies according to linear law. The B.M. diagram is shown in Fig. 6.31 (c).

Maximum B.M.

The maximum B.M. is at E, where S.F. becomes zero after changing its sign.

$\therefore$ Max. B.M. $= M_E =$ **205 kNm. Ans.**

Problem 6.22. *A beam 10 m long and simply supported at each end, has a uniformly distributed load of 1000 N/m extending from the left end upto the centre of the beam. There is also an anti-clockwise couple of 15 kNm at a distance of 2.5 m from the right end. Draw the S.F. and B.M. diagrams.*

Sol. The reaction at A will be upwards. To find whether the reaction at B is upwards or downwards, take the moments about A.

The following are the moments at A :

(*i*) Moment due to U.D.L. $= 1000 \times 5 \times \dfrac{5}{2} = 12500$ Nm (clockwise)

(*ii*) Moment of couple $= 15000$ Nm (Anti-clockwise)

$\therefore$ Net moment $= 15000 - 12500$

$= 2500$ Nm (Anti-clockwise)

This moment must be balanced by the moments due to reaction at B. Hence the moment about A due to reaction at B should be equal to 2500 Nm (clockwise). This is only possible when R_B is acting downwards. This is shown in Fig. 6.44 (*b*).

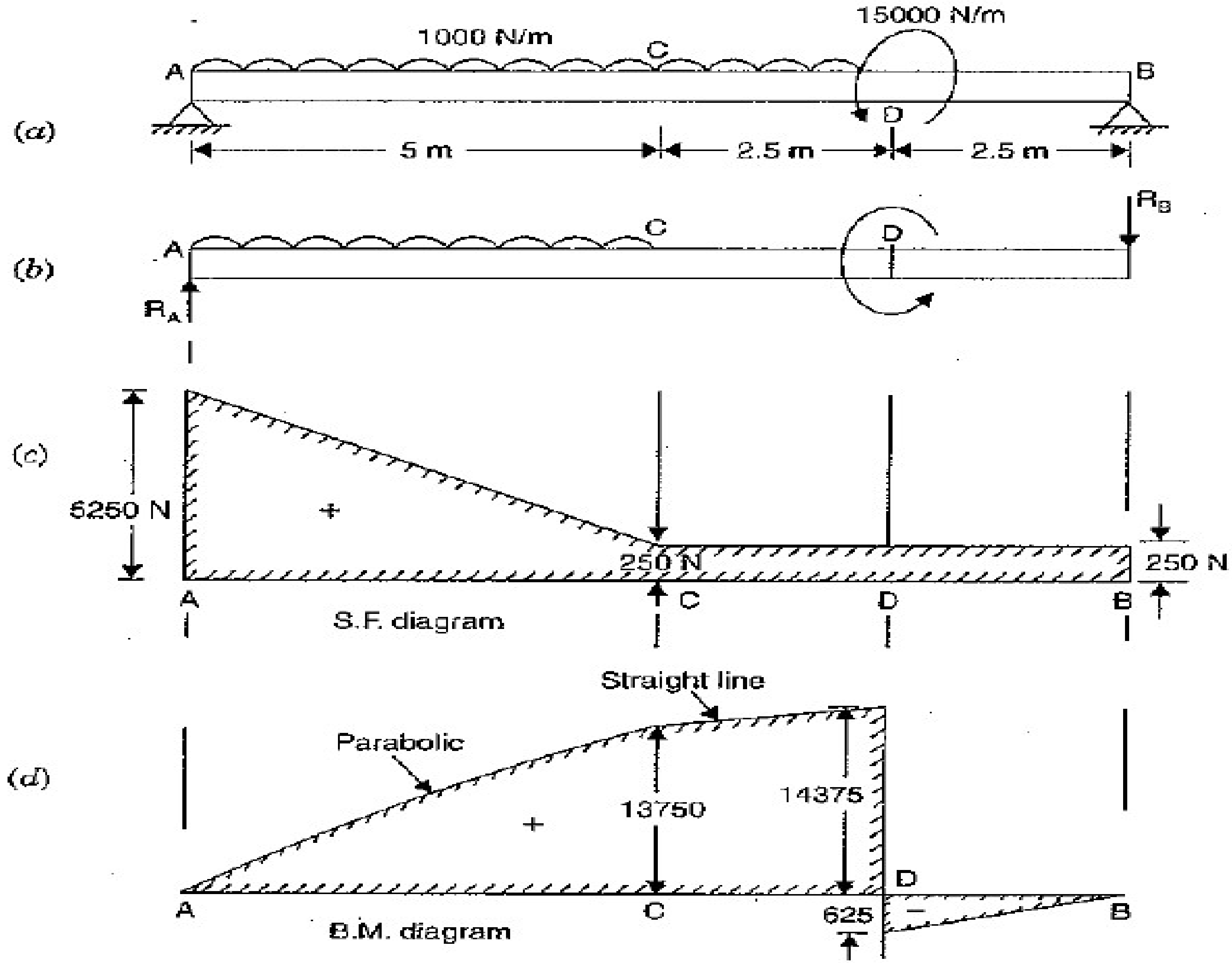

(a)
1000 N/m
15000 N/m
A
C
B
D
5 m
2.5 m
2.5 m
(b)
A
C
D
R_B
R_A
(c)
5250 N
+
250 N
250 N
A
C
D
B
S.F. diagram
(d)
Straight line
Parabolic
13750
14375
+
D
−
625
A
C
B
B.M. diagram

Fig. 6.44

$$\therefore \qquad R_B \times 10 = 2500$$

$$\therefore \qquad R_B = \frac{2500}{10} = 250 \text{ N}$$

and $\qquad R_A = \text{Total load on beam} + R_B$

(Here R_B is +ve as acting downwards)

$$= 1000 \times 5 + 250 = 5250 \text{ N}.$$

S.F. Diagram

S.F. at A $= + R_A = 5250$ N

S.F. at C $= 5250 - 5 \times 1000 = + 250$ N

S.F. between A and C varies according to straight line law.

S.F between C and B remains constant and equal to + 250 N

S.F. diagram is shown in Fig. 6.44 (*c*).

B.M. Diagram

B.M at A $= 0$

B.M. at C $= R_A \times 5 - 1000 \times 5 \times \dfrac{5}{2}$

$= 5250 \times 5 - 12500 = 13750$ Nm

B.M. just on the left hand side of D

$$= 5250 \times 7.5 - 1000 \times 5 \times \left(\frac{5}{2} + 2.5\right)$$

$= 39375 - 25000 = 14375$ Nm

B.M. just on the right hand side of D

$= - R_B \times 2.5 = - 250 \times 2.5 = - 625$ Nm

B.M. at B $= 0$

The B.M. diagram is shown in Fig. 6.44 (*d*).