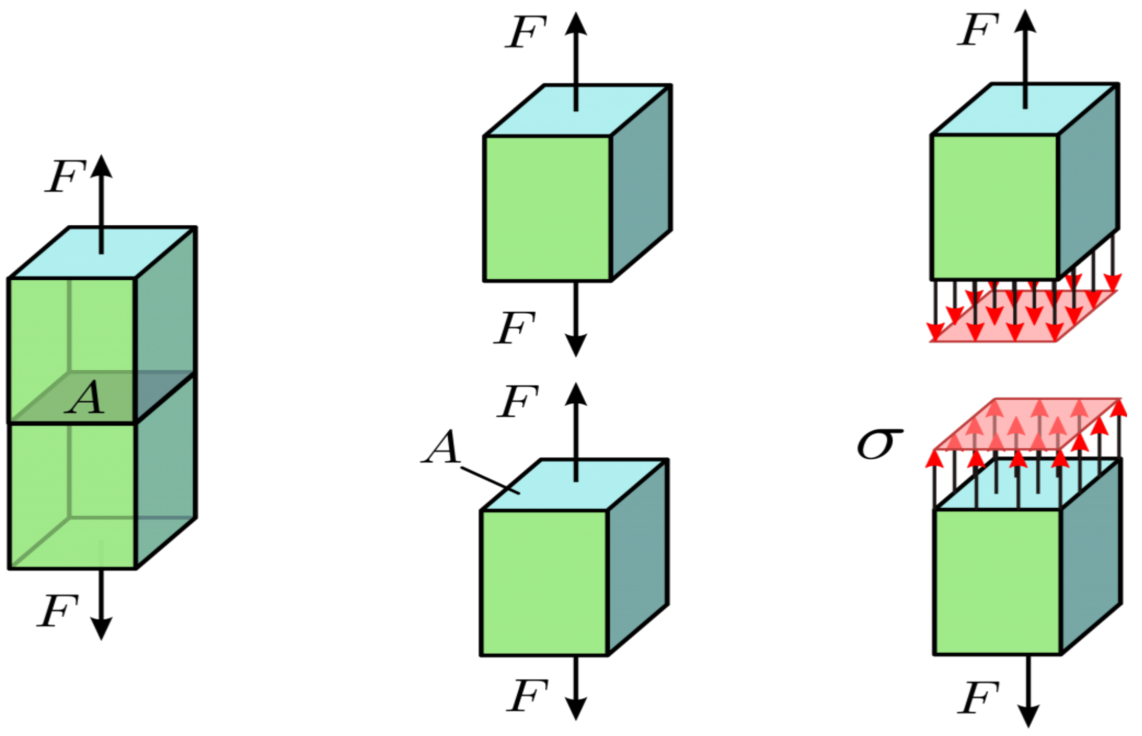
Stress & Strain

STRESS

- Stress is the force applied to a material, divided by the material's cross-sectional area
- Resistance offered by the material per unit cross- sectional area is called STRESS.

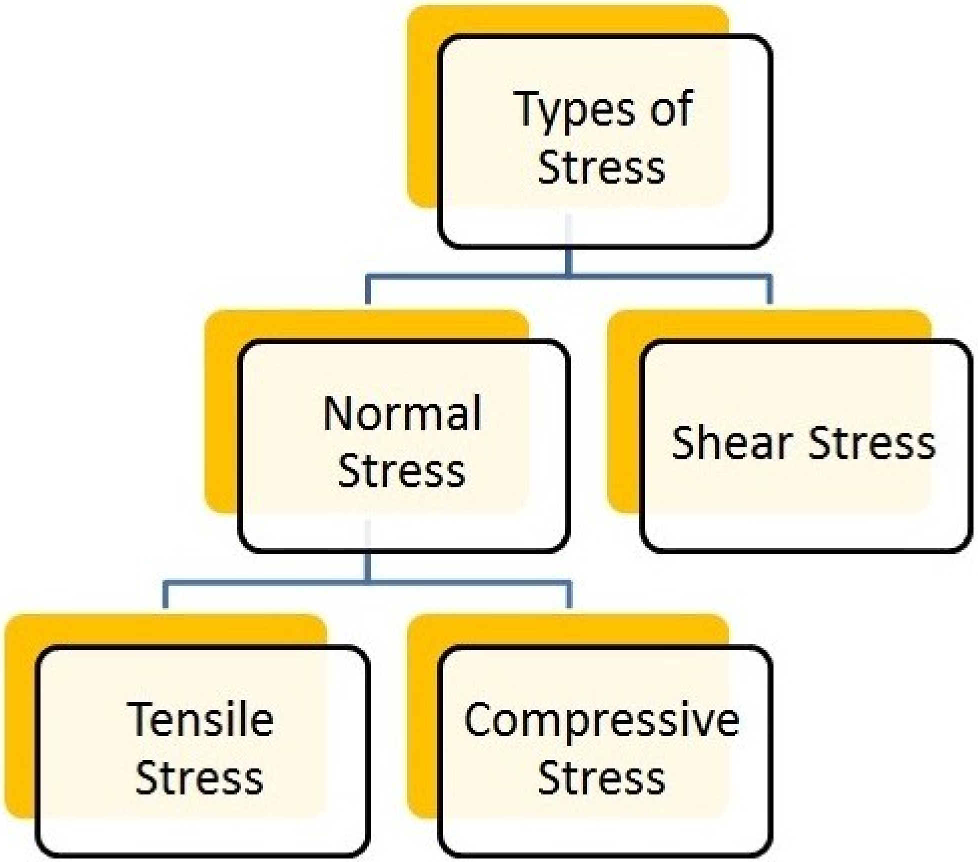
$$\sigma = \frac{F}{A_0}$$

σ = stress (N/m^2, Pa)
F = force (N)
A_0 = original cross-sectional area (m^2)



Types of Stress

A stress acts on a body may be normal stress or shear stress.



Normal Stress:

- Normal stress is a stress that acts perpendicular to the area.
- The formula for the normal stress is given by

$$\sigma = \frac{Resisting\ force}{Area} = \frac{R}{A}$$

❑ The normal stress is again subdivided into two parts.

1. Tensile Stress
2. Compressive Stress

Normal Stress:

Tensile Stress:

- The stress-induced in a body when it is subjected to two equal and opposite pulls as shown in the figure given below is called tensile stress.

- ❑ Due to the tensile stress there is an increase in the length of the body and decrease in the cross section area of the body.
- ❑ Tensile stress is a type of normal stress, so it acts at 90 degree to the area.
- ❑ The strain which is induced due to tensile stress is called tensile strain. It is equals to the ratio of increase in the length to the original length.

Normal Stress:

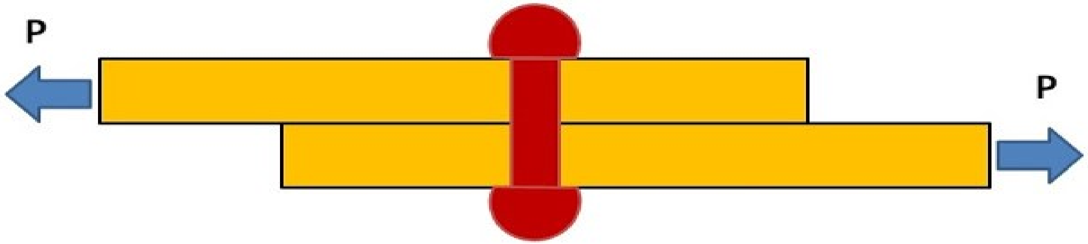
Compressive Stress:

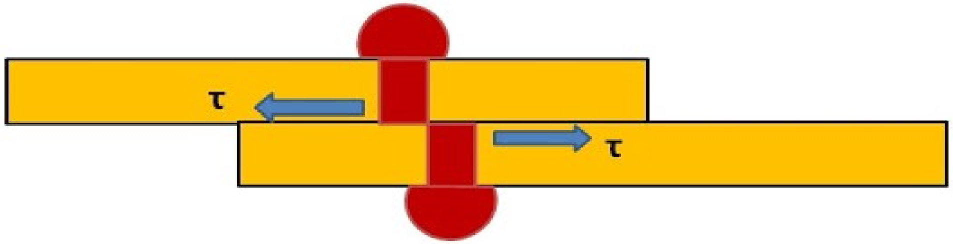
- The stress which induced in a body when it is subjected to two equal and opposite pushes as shown in the figure given below is called compressive stress.

- ❑ Due to the compressive stress, there is a decrease in the length and increase in the cross
  section area of the body.
- ❑ Compressive stress is also a type of normal stress and so it also acts at 90 degree to the area.
- ❑ The strain which is induced due to compressive stress is called compressive strain. It is equals to the ratio of decrease in the length to the original length.

Shear Stress

Shear stress induced in a body when it is subjected to two equal and opposite forces that acts tangential to the area





- ❑The strain produced due to the shear stress is called shear strain.
- ❑The shear stress is denoted by the symbol τ (tau). It is a Greek letter.
- ❑It is defined as ratio of shear resistance to the shear area.
- ❑The formula for the shear stress is given below.
- ❑Shear stress is responsible for the change in the shape of the body. It does on affect the volume of the body.

$$\tau = \frac{shear\ resistance}{Shear\ area}$$

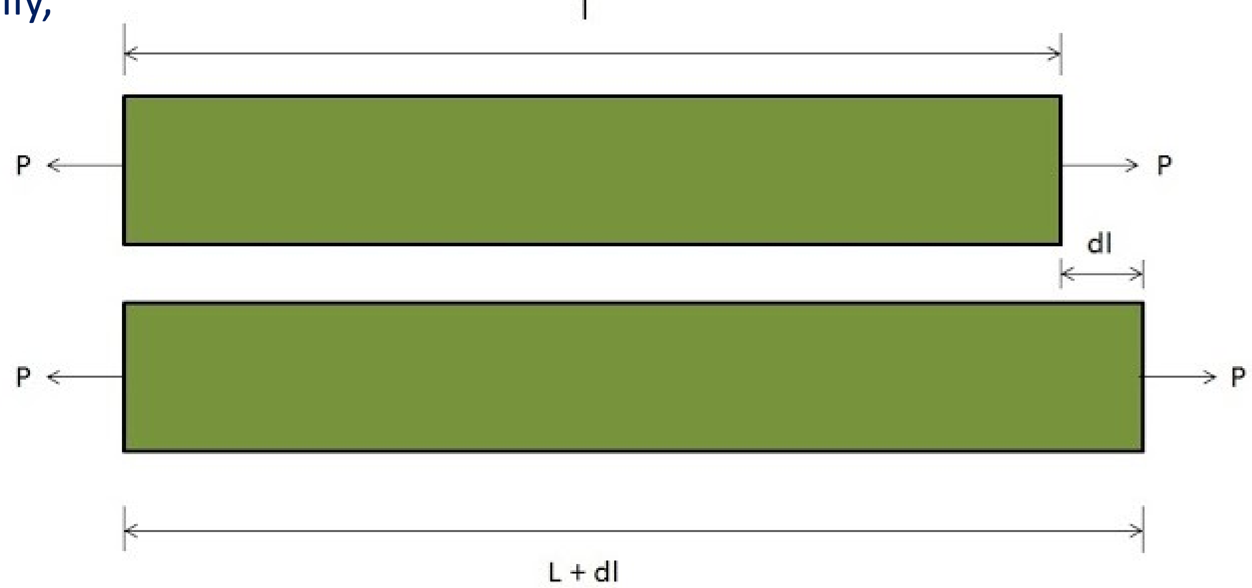
STRAIN

❑When an external force is applied on a body, there is some change occur in the dimension of the body.

❑The ratio of this change of dimension in the body to its actual dimension is called strain.

For example: if you have a bar of length l and an external force P is applied to the bar, then there is some change in the length of the bar. Let the change produced in the bar is dl. Then the strain is the ratio of this change in the length to the original (actual) length. The strain is a dimensionless quantity.

Mathematically,



strain= (change in length)/(original length)

e= dl/l

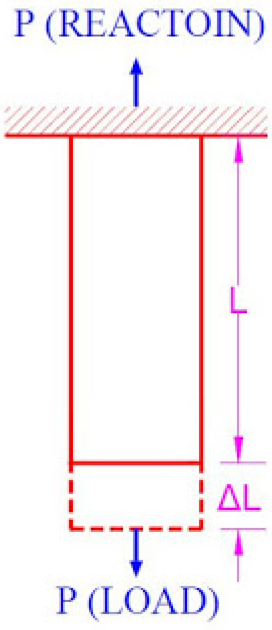
Types of strain

- Strain in mechanics is of four types and these are

1. Tensile strain
2. Compressive strain
3. Volumetric strain
4. Shear strain

1. Tensile strain:

The strain produced in a body due to tensile force is called the tensile strain. The tensile force always results in the increment of the length and decrease in the cross-section area of the body. In this case, the ratio of the increase in length to the original length is called tensile strain.



Tensile strain= (change in length)/(original length)

$$e = \Delta L / L$$

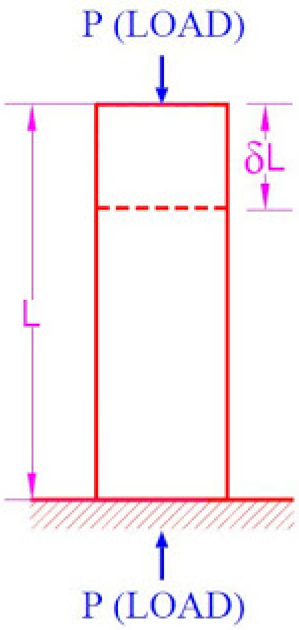
Types of strain

- **Strain in mechanics is of four types and these are**

1. **Tensile strain**
2. **Compressive strain**
3. **Volumetric strain**
4. **Shear strain**

2. Compressive strain:

The strain appears due to the compressive force is called compressive strain. In compressive force there is a decrease in the dimension of the body. So the ratio of the decrease in the length of the body to the original length is called compressive strain.



Compressive strain= (change in length)/(original length)

e= ΔL/L

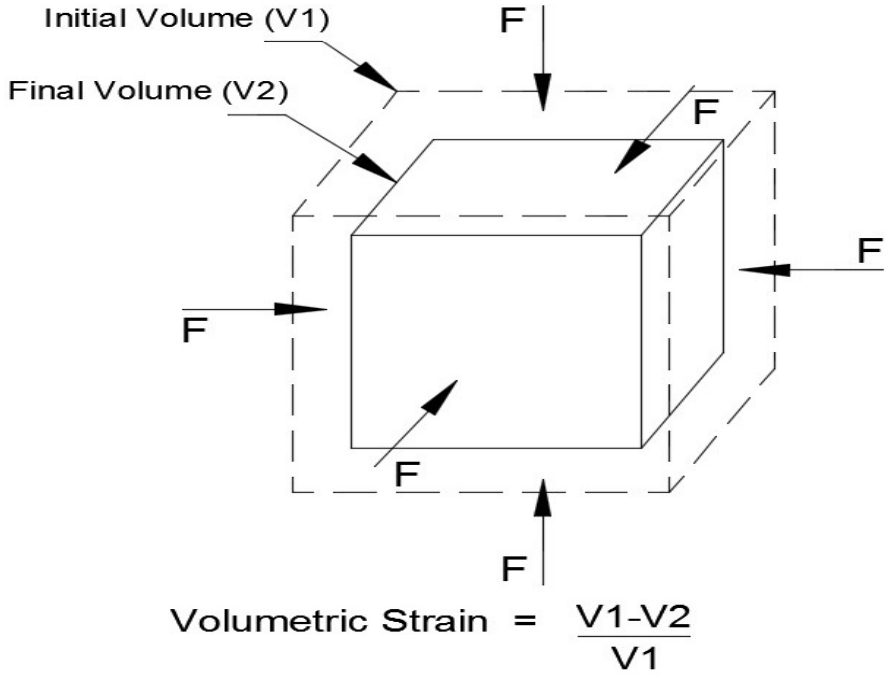
Types of strain

- Strain in mechanics is of four types and these are

1. Tensile strain
2. Compressive strain
3. Volumetric strain
4. Shear strain

3. Volumetric strain:

The ratio of the change in the volume of a body to the original volume is called the volumetric strain. In volumetric strain there is a change in the volume of the body due to application of the external forces.



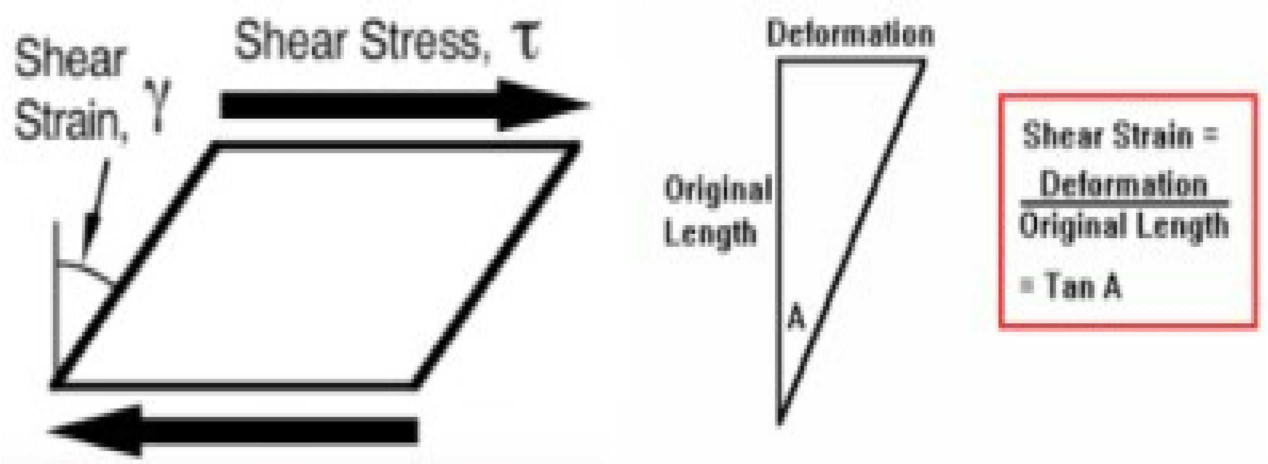
Types of strain

- Strain in mechanics is of four types and these are

1. Tensile strain
2. Compressive strain
3. Volumetric strain
4. Shear strain

4. Shear strain:

The strain which is produced in a body due to shear force is called shear strain. This is all about what is stress, types of stress and definition of each type of strain in strength of materials.



Difference Between Stress and Strain

S.no	Stress	Strain
1.	Stress is defined as the resisting force per unit area.	Strain is defined as the deformation per unit area.
2.	The ratio of resisting force (or applied load) to the cross section area of the body is called stress.	The ratio of change in dimension of the body to the original dimension is called strain.
3.	Stress is denoted by the Greek symbol 'σ' (sigma).	Strain is denoted by the symbol 'e'.
4.	The formula of stress is given by $\sigma = \frac{P}{A}$	The formula of strain is given by $e = \frac{dl}{l}$
5.	The unit of stress is N/m2 or N/mm2.	The strain is a unitless quantity.
6.	Stress can exist without strain.	Strain cannot exist without stress.
7.	The various types of stress are: tensile stress, compressive stress and shear stress.	The various types of strain are: tensile strain, compressive strain, shear strain and volumetric strain.

Stress and Strain curve

- we will Study About the stress strain curve relationship, diagram and explanation.
- It is the graphical representation of the stress against
- strain for a ductile material. A tensile test is conducted in order to get the stress strain diagram.

What is Stress-Strain Curve?

- tress strain curve is the plot of stress and strain of a material or metal on the graph.
- In this, the stress is plotted on the y-axis and its corresponding strain on the x-axis.
- After plotting the stress and its corresponding strain on the graph, we get a curve, and this curve is called stress strain curve or stress strain diagram.
- The stress-strain diagram for different material is different.
- It may vary due to the temperature and loading condition of the material

Stress and Strain curve

How to Draw Stress-Strain Curve or Diagram

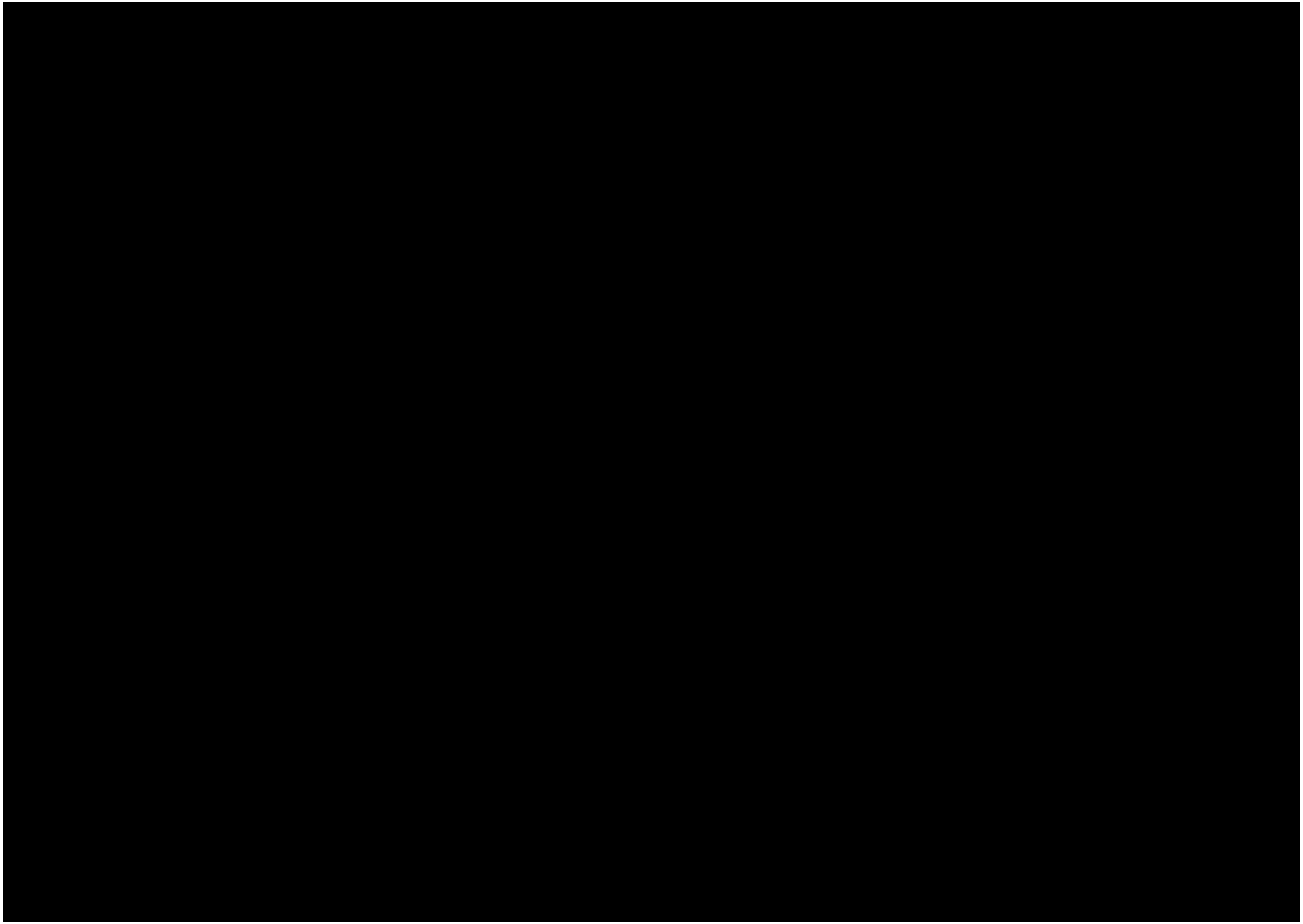
- A tensile test is done on the material for drawing the stress strain curve. A specimen of specific dimension is taken generally a circular rod. A tensile test is than conducted on this rod by the use of tensile testing machine.
- In this test, the specimen is fixed at one ends and tensile load is applied on the other end. The value of load and the extension in the rod is noted down. As we have noted down the load and extension, the stress and the corresponding strain can be easily calculated.
- The formula that is used for the calculation of stress and strain are
- We plot a graph between the stress and strain and a curve is obtained. This curve so obtained is called the stress strain curve or stress strain diagram.
- The stress strain diagram for the same material is different for different temperature and loading condition of the material.
- In the graph the slope represents the young's modulus of the material.

$$\sigma = \frac{P}{A}$$

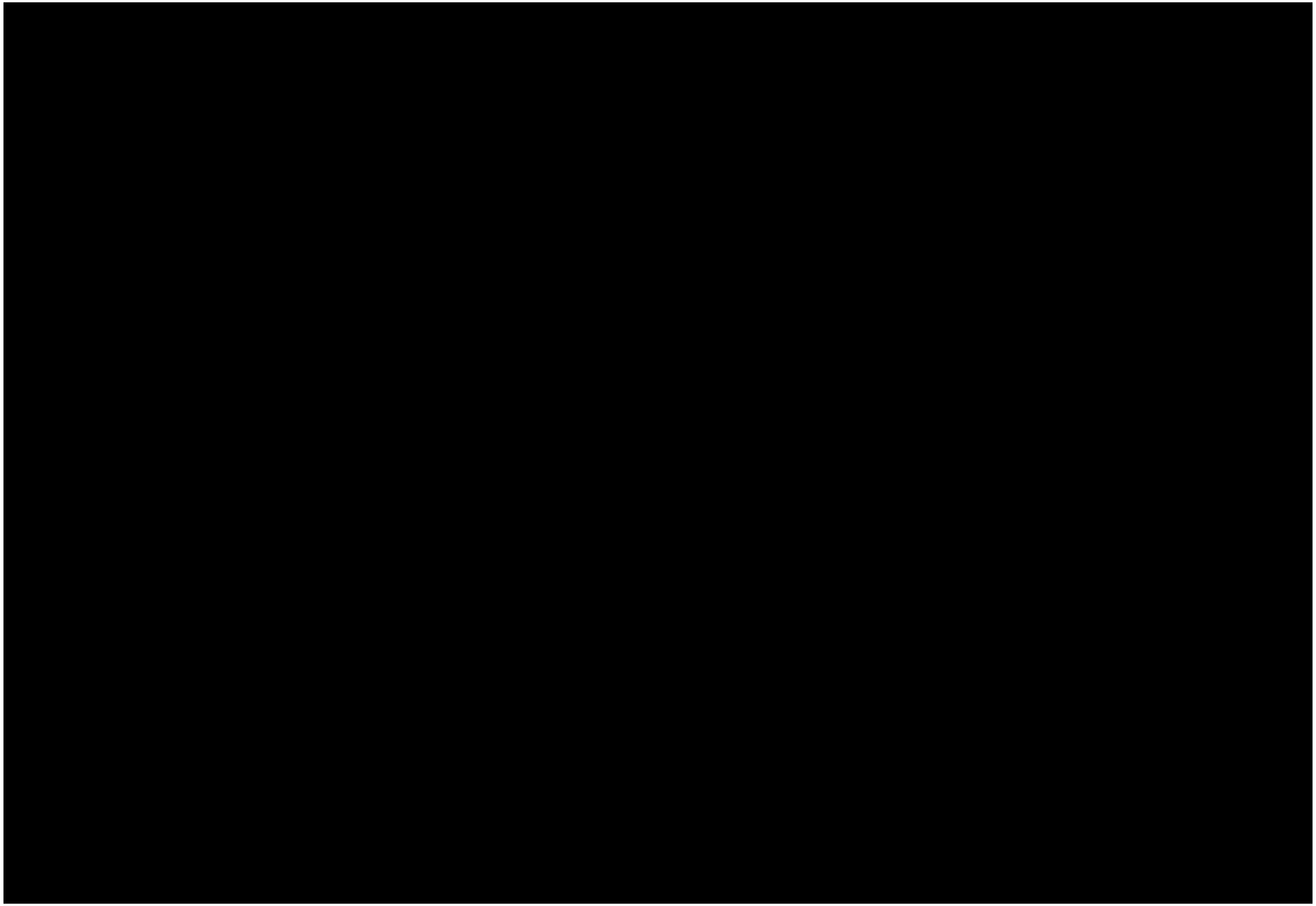
$$e = \frac{dL}{L}$$

Where,
σ = stress
P = Load
e = strain
dL = extension produced in the rod
L = original length
A = cross section area

Stress and Strain curve -1

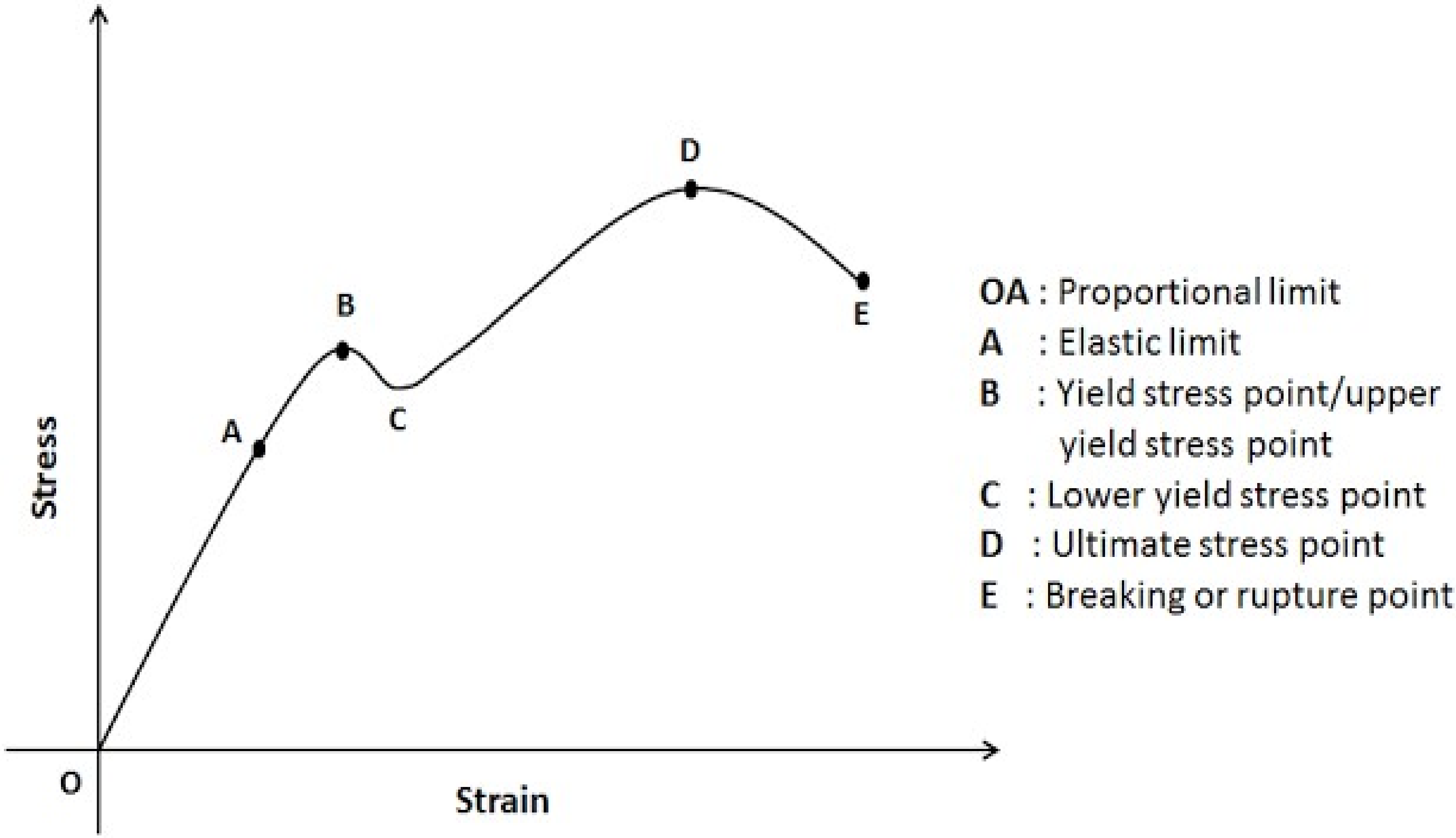


Stress and Strain curve -2



Stress and Strain curve

Explanation

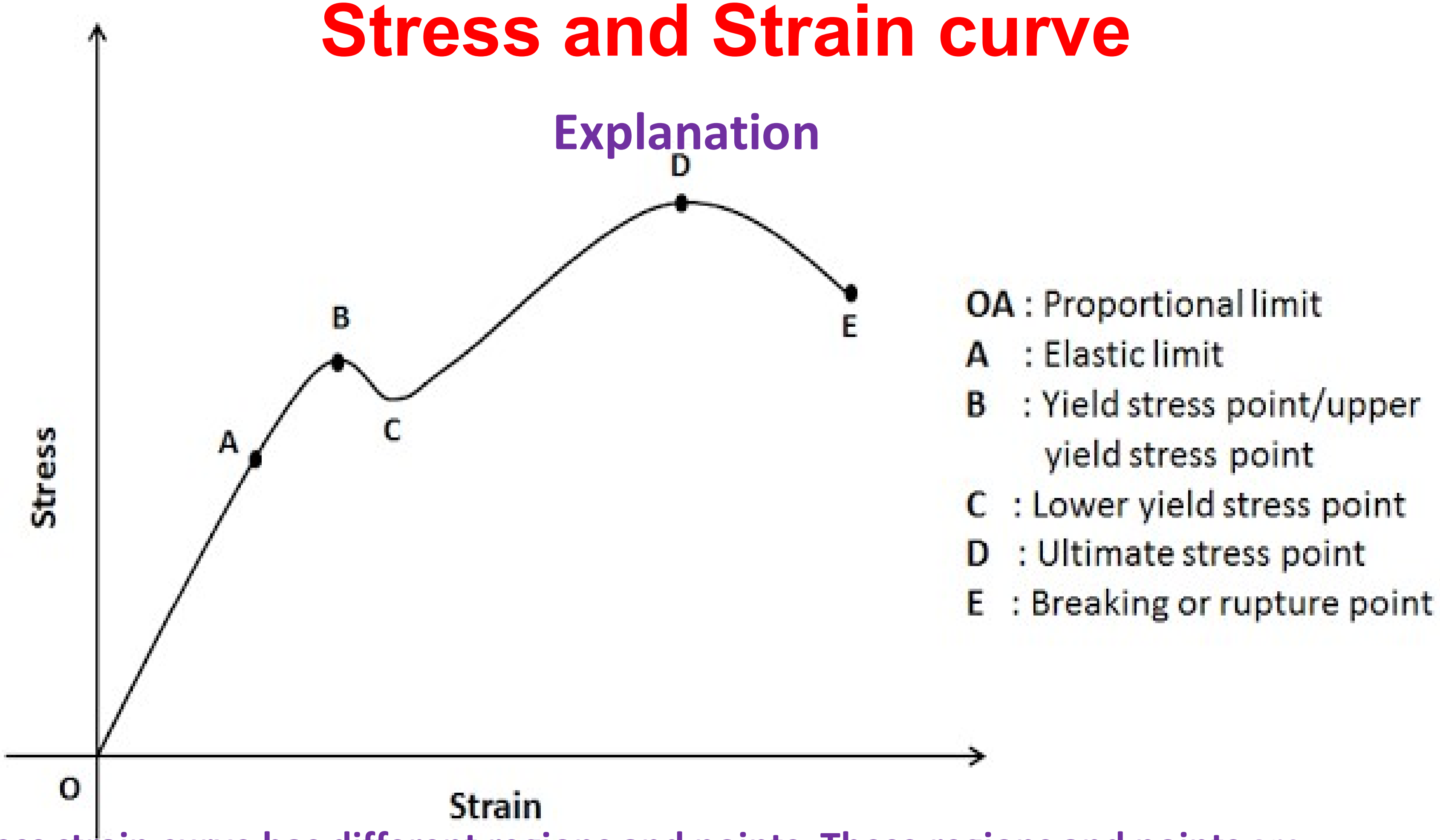


Stress and Strain curve

Explanation

Stress and Strain curve

Explanation

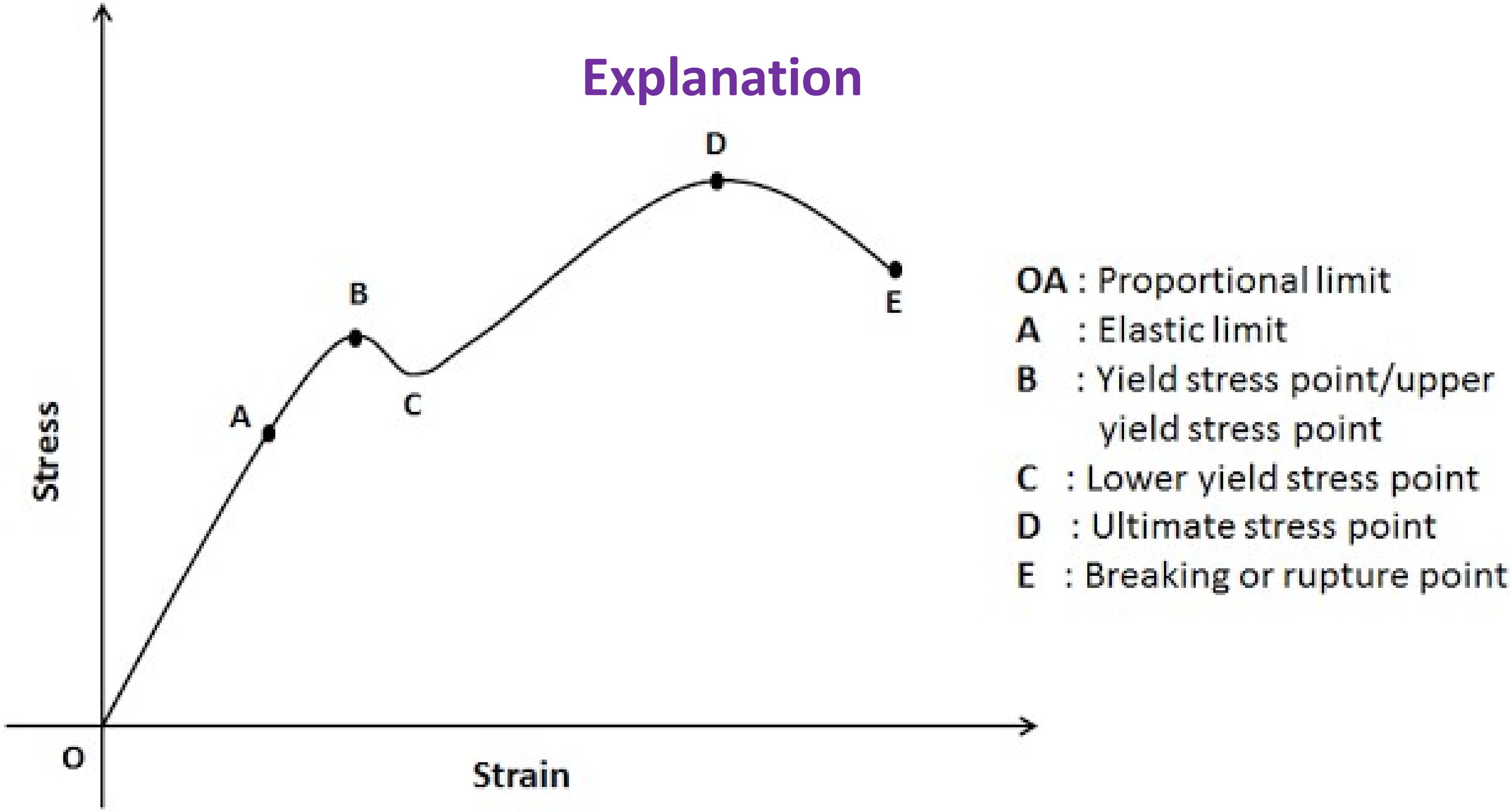


tress strain curve has different regions and points. These regions and points are:

- ❑ Proportional limit
- ❑ Elastic limit
- ❑ Yield point
- ❑ Ultimate stress point
- ❑ Fracture or breaking point

Stress and Strain curve

Explanation

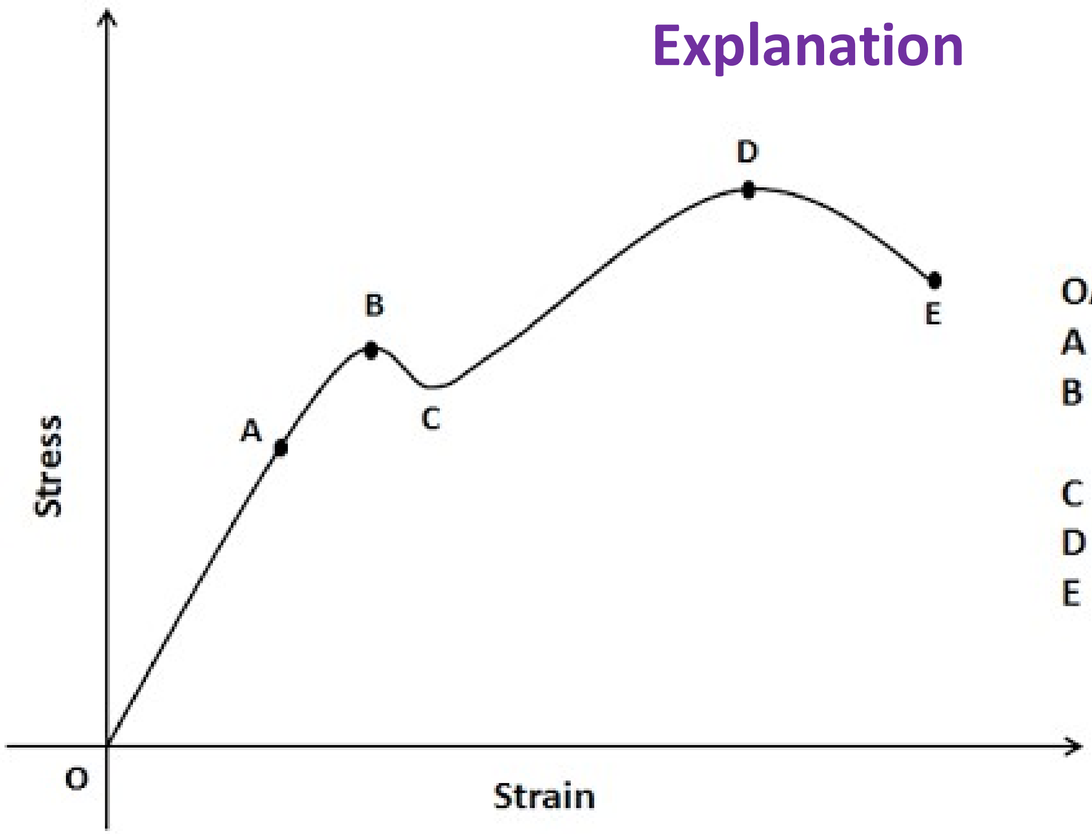


1. Proportional Limit:

It is the region in the strain curve which obeys hooke’s law i.e. within elastic limit the stress is directly proportional to the strain produced in the material. In this limit the ratio of stress with strain gives us proportionality constant known as young’s modulus. The point OA in the graph is called the proportional limit.

Stress and Strain curve

Explanation



OA : Proportional limit
A : Elastic limit
B : Yield stress point/upper yield stress point
C : Lower yield stress point
D : Ultimate stress point
E : Breaking or rupture point

What is Hooke's Law

In the Stress strain curve the proportionality limit indicates the Hooke's law. The curve shows linearity within elastic limit. The ratio of stress and corresponding strain in the stress strain curve gives us Young's Modulus

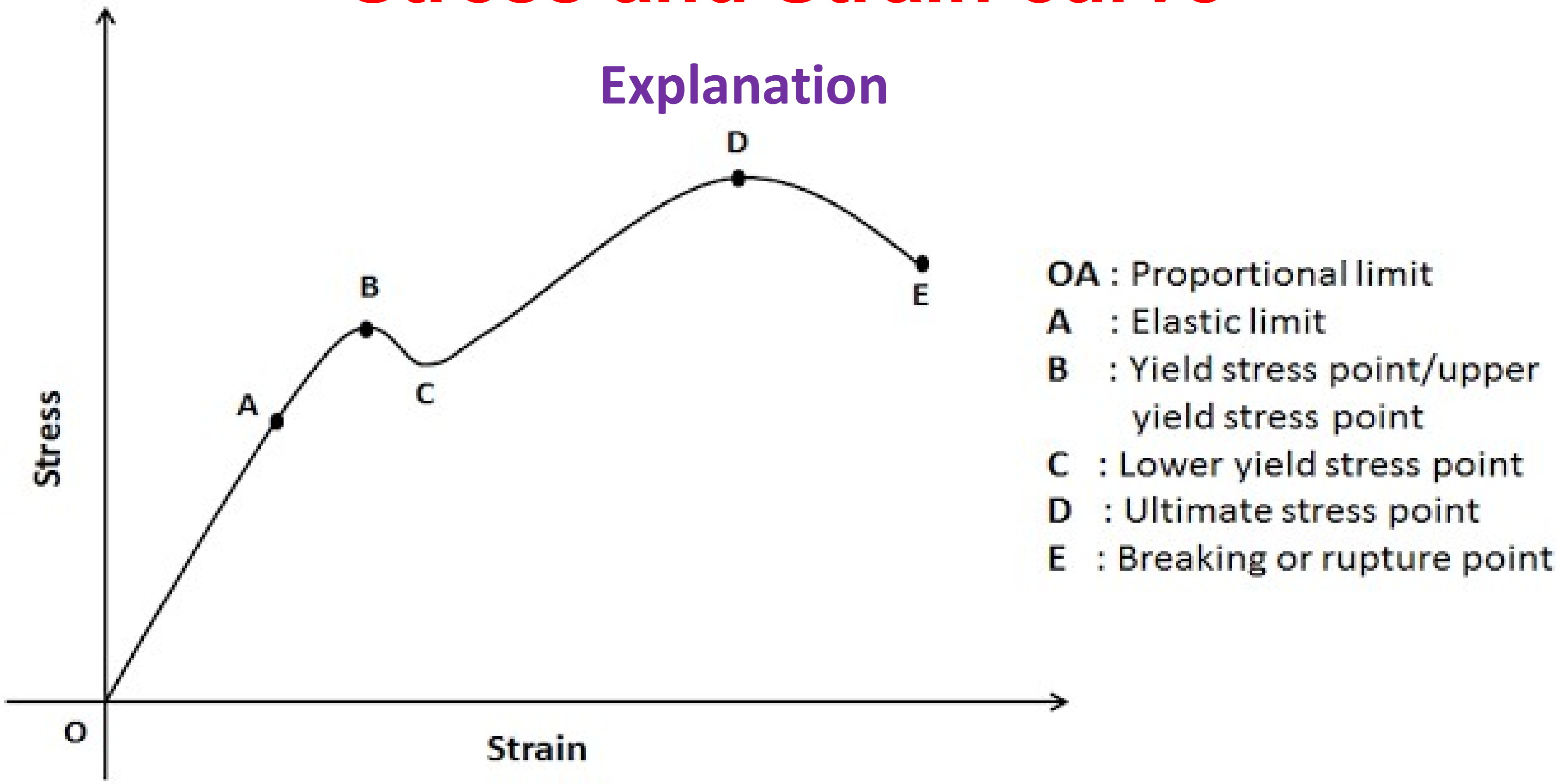
$$\sigma \propto e$$

$$\sigma = Ee$$

$$E = \frac{\sigma}{e} = \frac{P/A}{dl/l}$$

Stress and Strain curve

Explanation

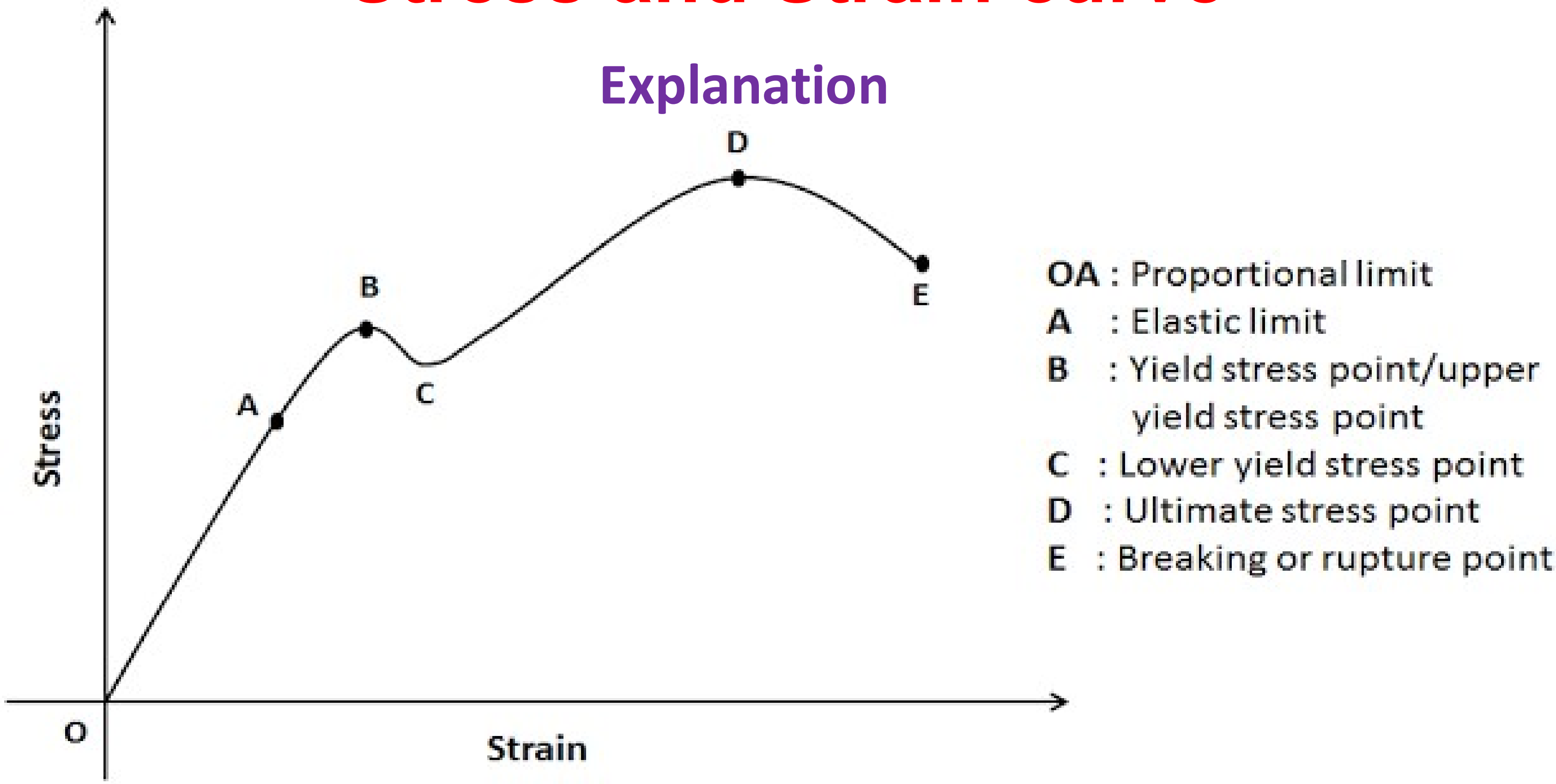


2. Elastic Limit:

It is the point in the graph upto which the material returns to its original position when the load acting on it is completely removed. Beyond this limit the material cannot return to its original position and a plastic deformation starts to appear in it. The point A is the Elastic limit in the graph.

Stress and Strain curve

Explanation



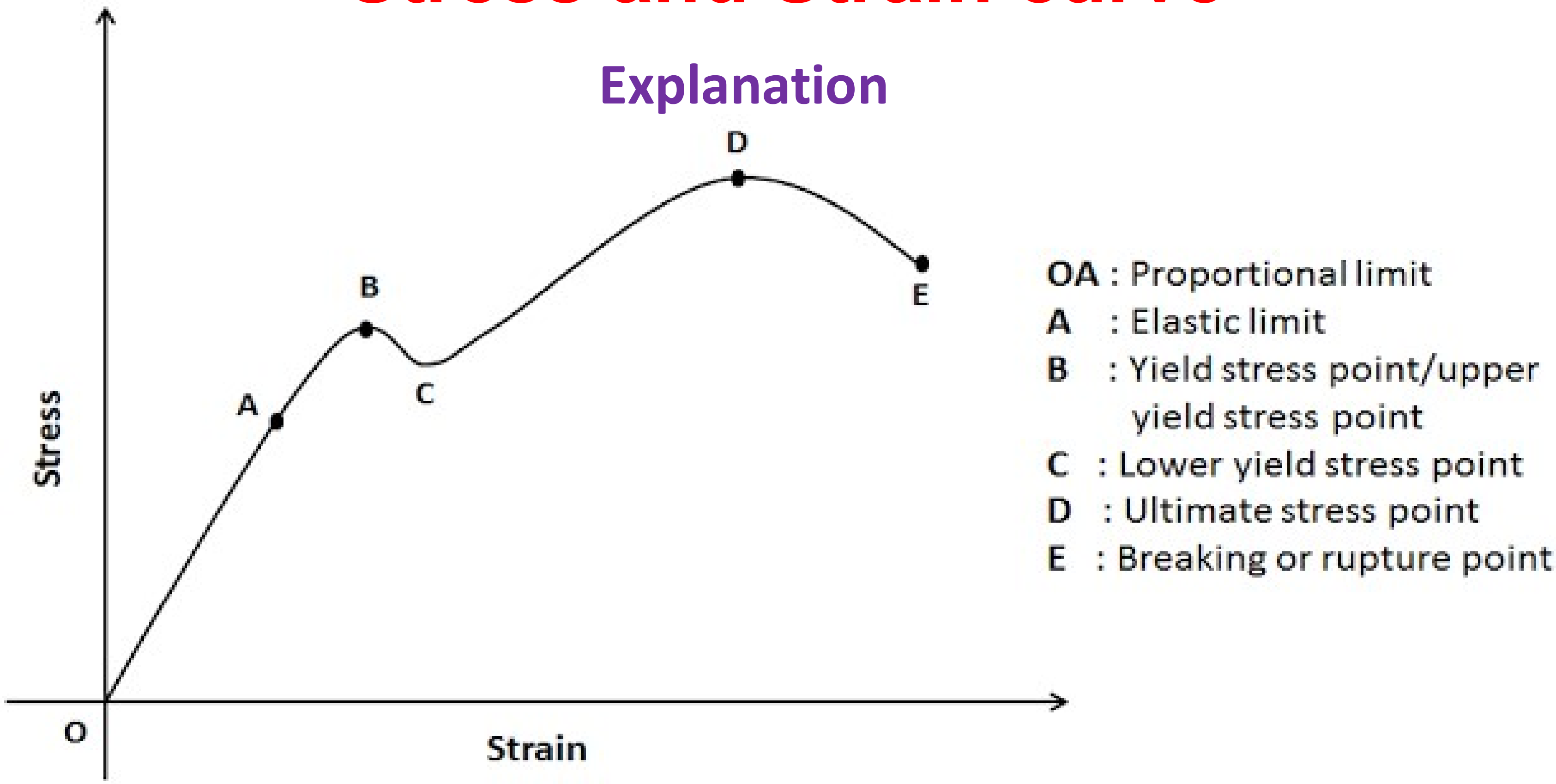
3. Yield Point or Yield Stress Point:

Yield point in a stress strain diagram is defined as the point at which the material starts to deform plastically. After the yield point is passed there is permanent deformation develops in the material and which is not reversible.

There are two yield points and it is upper yield point and lower yield point. The stress corresponding to the yield point is called yield point stress. The point B is the upper yield stress point and C is the lower yield stress point.

Stress and Strain curve

Explanation

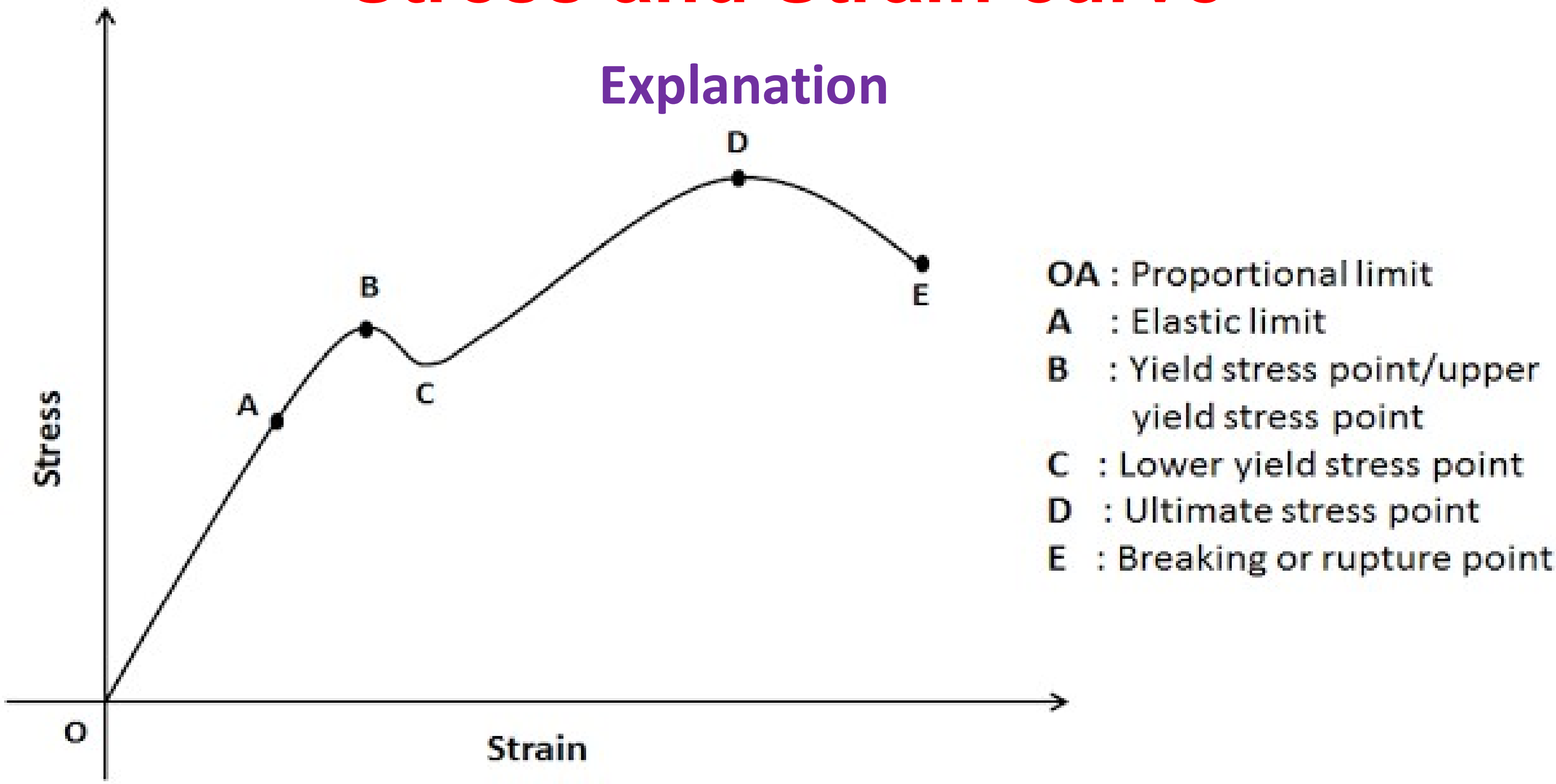


4. Ultimate Stress Point:

It is the point corresponding to the maximum stress that a material can handle before failure. It is the maximum strength point of the material that can handle the maximum load. Beyond this point the failure takes place. Point D in the graph is the ultimate stress point.

Stress and Strain curve

Explanation



5. Fracture or Breaking Point:

It is the point in the curve at which the failure of the material takes place. The fracture or breaking of material takes place at this point. The point e is the breaking point in the graph.

Elastic Constants

Young's Modulus, Modulus of Rigidity and Bulk Modulus

- ## Young's Modulus

- **Definition:** It is defined as the ratio of tensile stress or compressive stress to the corresponding strain within elastic limit. It is denoted by symbol E. It is also known as modulus of elasticitv or elastic modulus.

$$E = \frac{Tensile\ or\ compressive\ stress}{Tensile\ or\ compressive\ strain}$$

$$E = \frac{\sigma}{e}$$

Where

E = Young's Modulus

σ = Tensile or compressive stress

e = Tensile or compressive strain

The SI unit of Young's modulus is N/mm^2

Elastic Constants

- ## Modulus of Rigidity
- **Definition:** It is defined as the ratio of shear stress to corresponding shear strain within elastic limit. It is also known as shear modulus. It is represented by C or G or N.

$$C = \frac{Shear\ stress}{Shear\ strain}$$

$$C = \frac{\tau}{\Phi}$$

Where
τ = Shear stress
ϕ = Shear strain

The SI unit of C is N/mm²

Elastic Constants

- **Bulk Modulus**
- **Definition:** It is defined as the ratio of direct stress to the corresponding volumetric strain within the tic limit. It is denoted by K.

$$K = \frac{Direct\ stress}{Volumetric\ strain}$$

$$K = \frac{\sigma}{\left(\frac{dV}{V}\right)}$$

σ = Direct stress
(dV/V) = Volumetric strain

The SI unit of C is N/mm^2

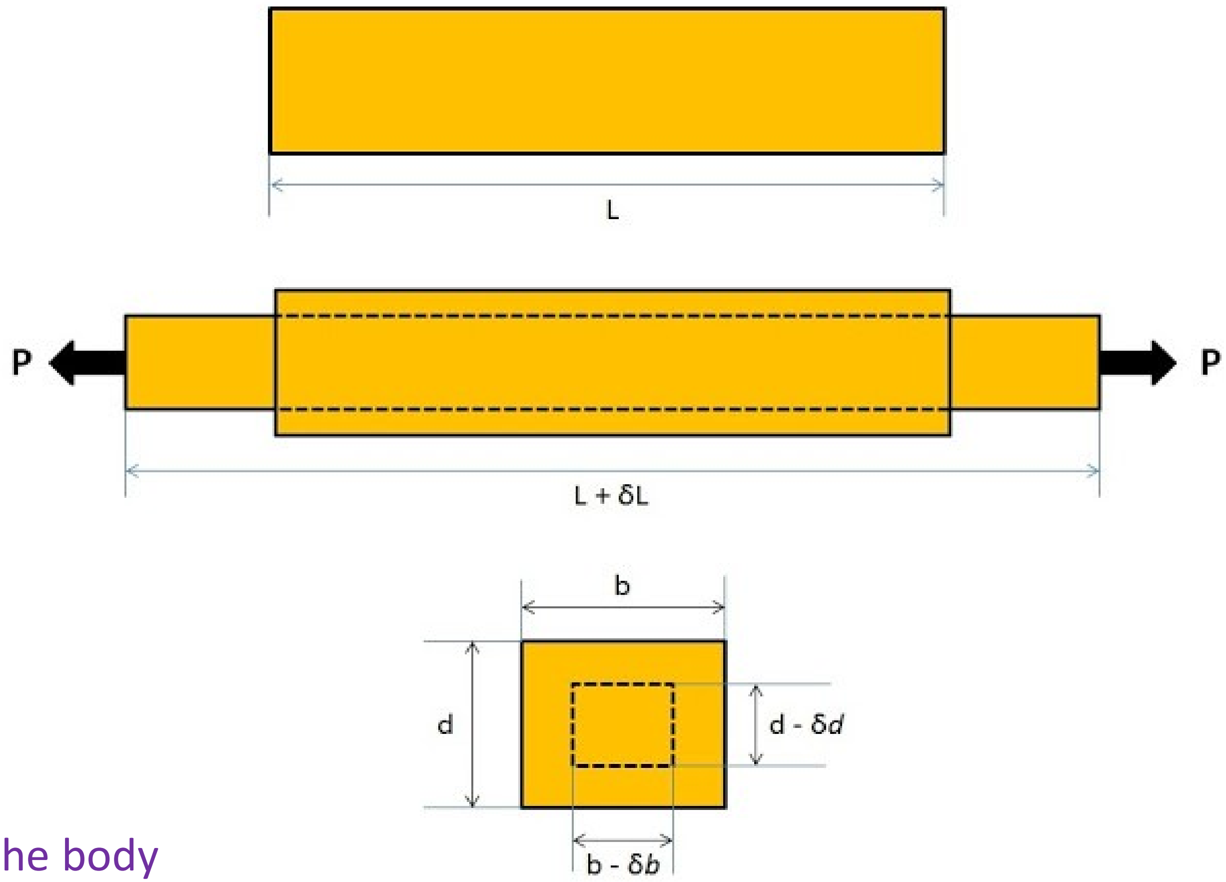
Difference Between
Young's Modulus, Modulus of Rigidity and Bulk Modulus

S.no	Young's Modulus	Modulus of Rigidity	Bulk Modulus
1.	It is the ratio of tensile or compressive stress to the corresponding strain within elastic limit.	It is the ratio of shear stress to the corresponding shear strain within elastic limit.	It is the ratio of direct stress to the corresponding volumetric strain within elastic limit.
2.	It is denoted by E.	It is denoted by C or G or N.	It is denoted by K.
3.	E = stress/ strain	C = shear stress/shear strain	K = direct stress/ volumetric strain.

Longitudinal Strain

- When an axial tensile or compressive load is applied on a body, then there is an axial deformation appears in the length of the body. The ratio of axial deformation to the original length of the body is called as longitudinal strain.
- It is also defined as the deformation per unit length in the direction of the applied load.
- Considered a body having length L and axial tensile load P is applied on it. There is an increase in the length of the body in the direction of the applied load as shown in the figure ‘a’ given below

Longitudinal Strain



Let

L = Length of the body

P = Tensile load acting on the body

δL = increase in the length of the body in the direction of P

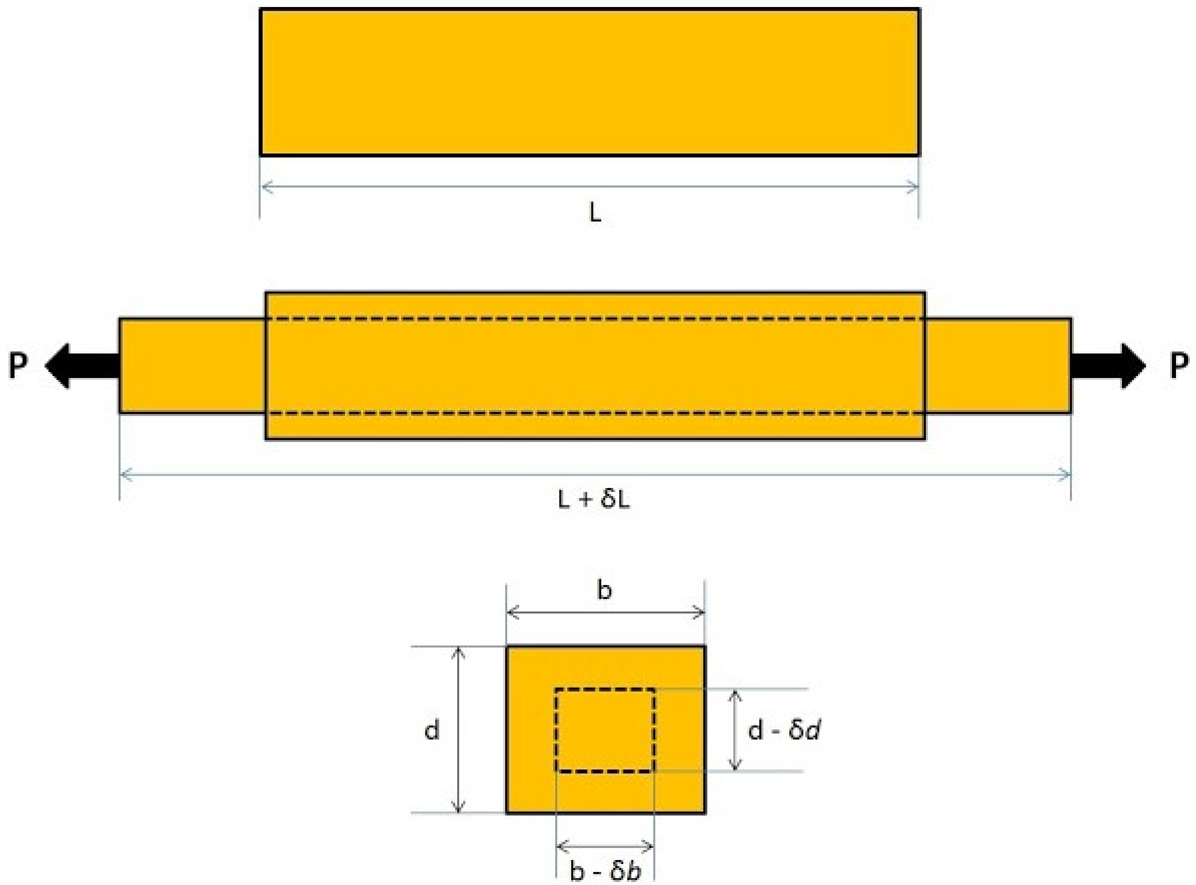
Then, the longitudinal strain is given by

$$Longitudinal\ strian = \frac{Increase\ in\ length}{Original\ length} = \frac{\delta L}{L}$$

Lateral Strain

- ❑When an axial tensile load is applied on a body, there is an increase in the length of the body. But at the same time, other dimensions which are at right angles to the line of action of the applied load decreases.
- ❑ The strain which produced at right angles to the direction of the applied load is known as lateral strain.
- ❑Considered a rectangular bar having length L, breadth b and depth d are subjected to an axial tensile load P as shown in the figure given below.

Lateral Strain



Let
δL = increase in length
δb = decrease in breadth, and
δd = decrease in depth.

$$Longitudinal\ strain = \frac{\delta L}{L}$$

$$Lateral\ Strain = \frac{\delta b}{b} = \frac{\delta d}{d}$$

the lateral strain is of opposite kind i.e. if the longitudinal strain is tensile (compressive), the lateral strain will be compressive (tensile). So from the formula of Poisson's ratio, algebraically lateral strain can also be expressed as

Lateral strain= -μ x Longitudinal strain

Here the minus sign is used to indicate the opposite nature of both the lateral and longitudinal strain.

Points to Remember

If the longitudinal strain is tensile, then the lateral strain will be compressive.

If the lateral strain is tensile, then the longitudinal strain will be compressive.

A longitudinal strain in the direction of applied load is every time accompanied by an equal and opposite lateral strain at right angle to the applied load

What is Poisson's Ratio

Poisson's Ratio:

- It is defined as the ratio of lateral strain to the longitudinal strain within elastic limit. It means that when a material is loaded within elastic limit than the ratio of lateral strain to the longitudinal strain gives us a constant called poisson's ratio.
- It is denoted by the symbol μ.
- The value of poisson's ratio varies from 0.25 to 0.33. For rubber its value varies from 0.45 to 0.50.

Mathematically

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

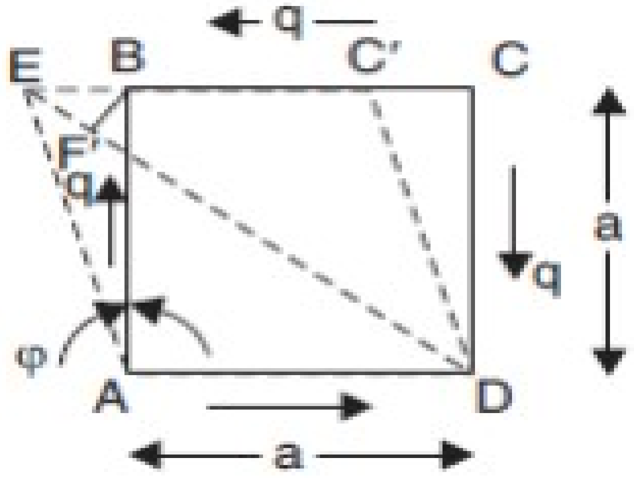
Relationship between Elastic Constants

RELATIONSHIP BETWEEN MODULUS OF ELASTICITY AND MODULUS OF RIGIDITY

Consider a square element ABCD of sides „a‟ subjected to pure shear „q‟ as shown in Fig. .

AEC'D shown is the deformed shape due to shear q. Drop perpendicular BF to diagonal DE.

Let υ be the shear strain and G



Now, strain in diagonal $BD = \dfrac{DE - DF}{DF}$

$$= \frac{EF}{DB}$$

$$= \frac{EF}{AB\sqrt{2}}$$

Relationship between Elastic Constants

Since angle of deformation is very small we can assume $\angle BEF = 45°$, hence $EF = BE \cos 45°$

$\therefore$ Strain in diagonal $\quad BD = \frac{EF}{BD} = \frac{BE \cos 45°}{AB\sqrt{2}}$

$$= \frac{a \tan \phi \cos 45°}{a\sqrt{2}}$$

$$= \frac{1}{2} \tan \phi = \frac{1}{2}\phi \qquad \text{(Since } \phi \text{ is very small)}$$

$$= \frac{1}{2} \times \frac{q}{G}, \text{ since } \phi = \frac{q}{G} \qquad ...(1)$$

Now, we know that the above pure shear gives rise to axial tensile stress q in the diagonal direction of DB and axial compression q at right angles to it. These two stresses cause tensile strain along the diagonal DB.

Tensile strain along the diagonal $DB = \frac{q}{E} + \mu\frac{q}{E} = \frac{q}{E}(1 + \mu)$ ...(2)

From equations (1) and (2), we get

$$\frac{1}{2} \times \frac{q}{G} = \frac{q}{E}(1 + \mu)$$

$$E = 2G(1 + \mu) \qquad ...(8.22)$$

RELATIONSHIP BETWEEN MODULUS OF ELASTICITY AND BULK MODULUS

Consider a cubic element subjected to stresses p in the three mutually perpendicular direction x, y, z as shown in Fig. 8.32.

Now the stress p in x direction causes tensile strain $\frac{p}{E}$ in x direction while the stress p in y and z direction cause compressive strains $\mu \frac{p}{E}$ in x direction.

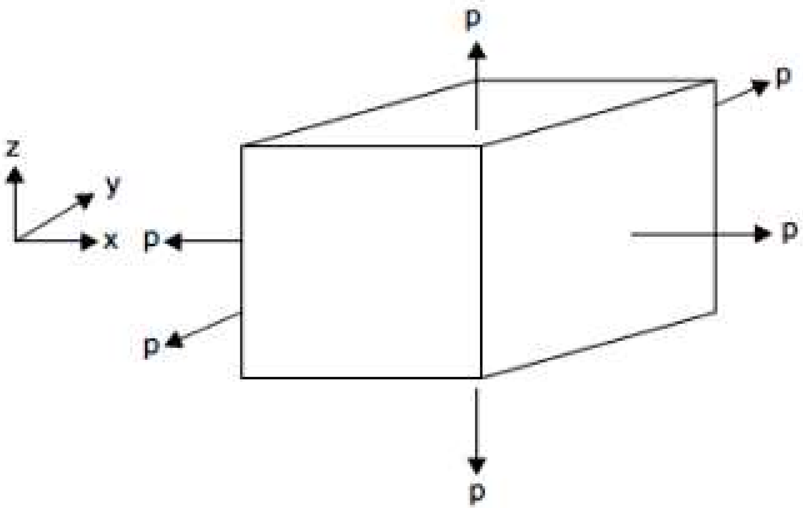


Fig. 8.32

Hence, $$e_x = \frac{p}{E} - \mu\frac{p}{E} - \mu\frac{p}{E}$$

$$= \frac{p}{E}(1-2\mu)$$

Similarly $$e_y = \frac{p}{E}(1-2\mu)$$

$$e_z = \frac{p}{E}(1-2\mu) \qquad \text{...(1)}$$

$\therefore$ Volumetric strain $$e_v = e_x + e_y + e_z = \frac{3p}{E}(1-2\mu)$$

From definition, bulk modulus K is given by

$$K = \frac{p}{e_v} = \frac{p}{\frac{3p(1-2\mu)}{E}}$$

or $$E = 3K(1-\mu) \qquad \text{...(2)}$$

RELATIONSHIP BETWEEN MODULUS OF ELASTICITY AND BULK MODULUS

Relationship between EGK:

We know $$E = 2G(1 + \mu) \qquad \text{...}(a)$$

and $$E = 3K(1 - 2\mu) \qquad \text{...}(b)$$

By eliminating μ between the above two equations we can get the relationship between E, G, K, free from the term μ.

From equation (a) $\mu = \dfrac{E}{2G} - 1$

Substituting it in equation (b), we get

$$E = 3K\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]$$

$$= 3K\left(1 - \frac{E}{G} + 2\right) = 3K\left(3 - \frac{E}{G}\right)$$

$$= 9K - \frac{3KE}{G}$$

$$\therefore \qquad E\left(1 + \frac{3K}{G}\right) = 9K$$

or $$E\left(\frac{G + 3K}{G}\right) = 9K \qquad \text{...}(c)$$

or $$E = \frac{9KG}{G + 3K} \qquad \text{...}(8.23a)$$

RELATIONSHIP BETWEEN MODULUS OF ELASTICITY AND BULK MODULUS

Equation (c) may be expressed as

$$\frac{9}{E} = \frac{G + 3K}{KG}$$

i.e.,

$$\frac{9}{E} = \frac{3}{G} + \frac{1}{K} \qquad \text{...(8.23}b\text{)}$$

$$e_z = \frac{p}{E}(1 - 2\mu) \qquad \text{...(1)}$$

$\therefore$ Volumetric strain $\quad e_v = e_x + e_y + e_z = \frac{3p}{E}(1 - 2\mu)$

From definition, bulk modulus K is given by

$$K = \frac{p}{e_v} = \frac{p}{\frac{3p(1 - 2\mu)}{E}}$$

or

$$E = 3K(1 - \mu) \qquad \text{...(2)}$$

Relationship between EGK:

We know $\quad E = 2G(1 + \mu) \qquad \text{...(}a\text{)}$

and $\quad E = 3K(1 - 2\mu) \qquad \text{...(}b\text{)}$

RELATIONSHIP BETWEEN ELASTIC CONSTANTS

Elastic constant formula

$$E = \frac{9KG}{G+3K}$$

Where,

- K is the Bulk modulus
- G is shear modulus or modulus of rigidity.
- E is Young's modulus or modulus of Elasticity.

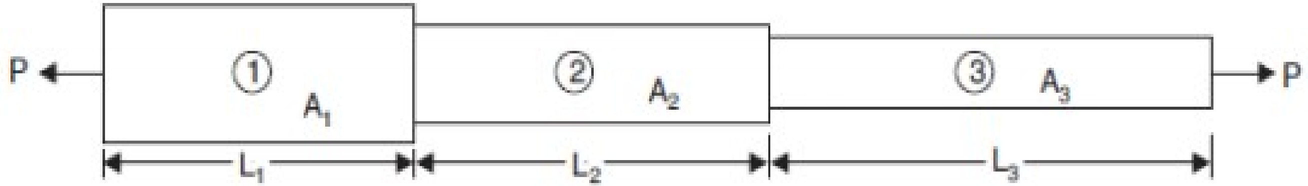
Individually Young's modulus and bulk modulus and modulus of rigidity are related as-

	Formula	SI Units
The relation between modulus of elasticity and modulus of rigidity	$E = 2G(1+\mu)$	N/m^2 or pascal(Pa)
The relation between Young's modulus and bulk modulus	$E = 3K(1-2\mu)$	N/m^2 or pascal(Pa)

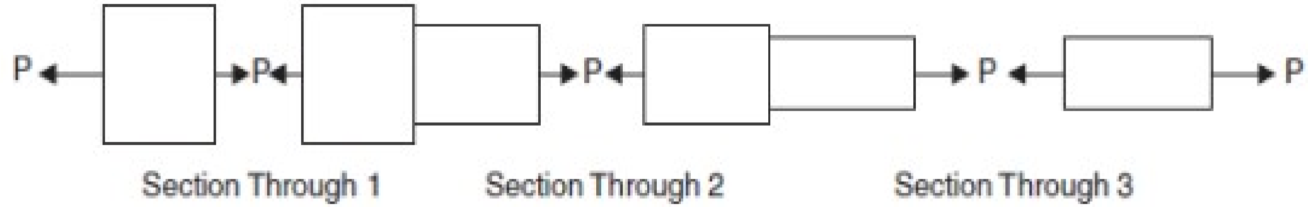
BARS WITH CROSS-SECTIONS VARYING IN STEPS

A typical bar with cross-sections varying in steps and subjected to axial load is as shown in Fig. 7(*a*). Let the length of three portions be L_1, L_2 and L_3 and the respective cross-sectional areas of the portion be A_1, A_2, A_3 and E be the Young's modulus of the material and P be the applied axial load. \

Figure 7 (*b*) shows the forces acting on the cross-sections of the three portions. It is obvious that to maintain equilibrium the load acting on each portion is P only. Hence stress, strain and extension of each of these portions are as listed below:



(a)



(b)

BARS WITH CROSS-SECTIONS VARYING IN STEPS

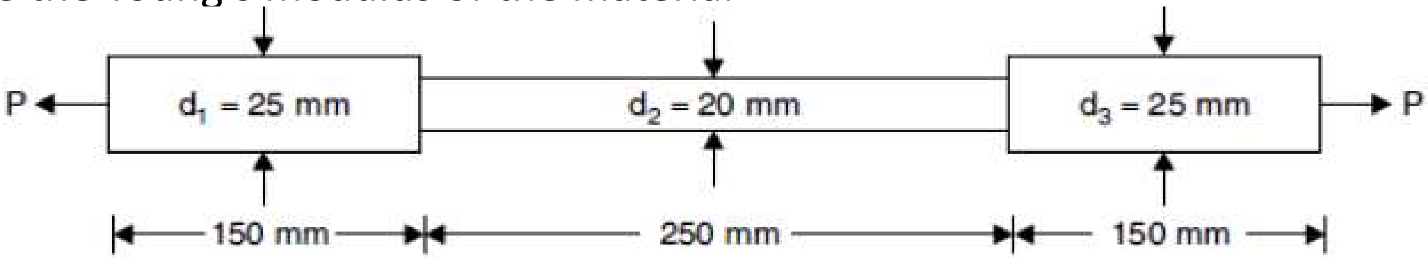
Portion	Stress	Strain	Extension
1	$p_1 = \dfrac{P}{A_1}$	$e_1 = \dfrac{p_1}{E} = \dfrac{P}{A_1 E}$	$\Delta_1 = \dfrac{PL_1}{A_1 E}$
2	$p_2 = \dfrac{P}{A_2}$	$e_2 = \dfrac{p_2}{E} = \dfrac{P}{A_2 E}$	$\Delta_2 = \dfrac{PL_2}{A_2 E}$
3	$p_3 = \dfrac{P}{A_3}$	$e_3 = \dfrac{p_3}{E} = \dfrac{P}{A_3 E}$	$\Delta_3 = \dfrac{PL_3}{A_3 E}$

Hence total change in length of the bar

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3 = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$$

BARS WITH CROSS-SECTIONS VARYING IN STEPS

Example 1. The bar shown in below Fig. is tested in universal testing machine. It is observed that at a load of 40 kN the total extension of the bar is 0.280 mm. Determine the Young's modulus of the material



Solution: Extension of portion 1, $\frac{PL_1}{A_1E} = \frac{40 \times 10^3 \times 150}{\frac{\pi}{4} \times 25^2 E}$

Extension of portion 2, $\frac{PL_2}{A_2E} = \frac{40 \times 10^3 \times 250}{\frac{\pi}{4} \times 20^2 E}$

Extension of portion 3, $\frac{PL_3}{A_3E} = \frac{40 \times 10^3 \times 150}{\frac{\pi}{4} \times 25^2 E}$

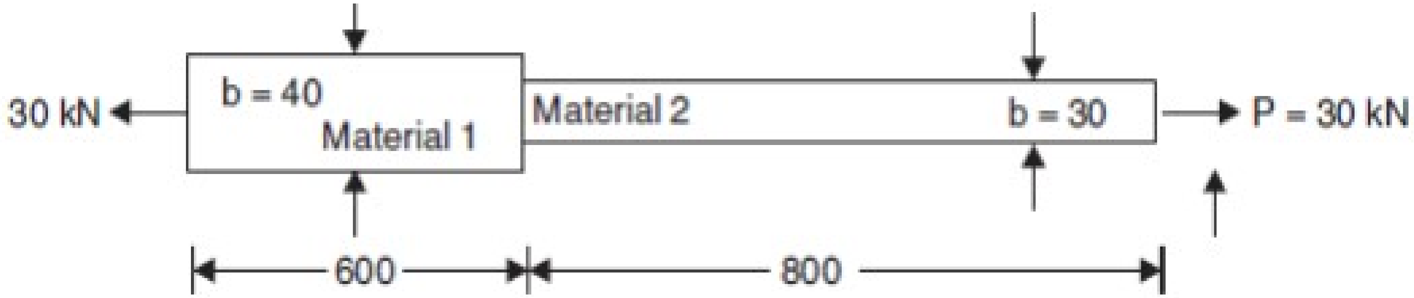
$$\text{Total extension} = \frac{40 \times 10^3}{E} \times \frac{4}{\pi}\left\{\frac{150}{625} + \frac{250}{400} + \frac{150}{625}\right\}$$

$$0.280 = \frac{40 \times 10^3}{E} \times \frac{4}{\pi} \times \frac{1.112}{E}$$

$$\boldsymbol{E = 200990 \text{ N/mm}^2}$$

BARS WITH CROSS-SECTIONS VARYING IN STEPS

Example(2).The stepped bar shown in below figure is made up of two different materials. The material 1 has Young's modulus = 2 × 105 N/mm, while that of material 2 is 1 × 105 N/mm2 . Find the extension of the bar under a pull of 30 kN if both the portions are 20 mm in thickness



Solution:

$$A_1 = 40 \times 20 = 800 \text{ mm}^2$$
$$A_2 = 30 \times 20 = 600 \text{ mm}^2$$

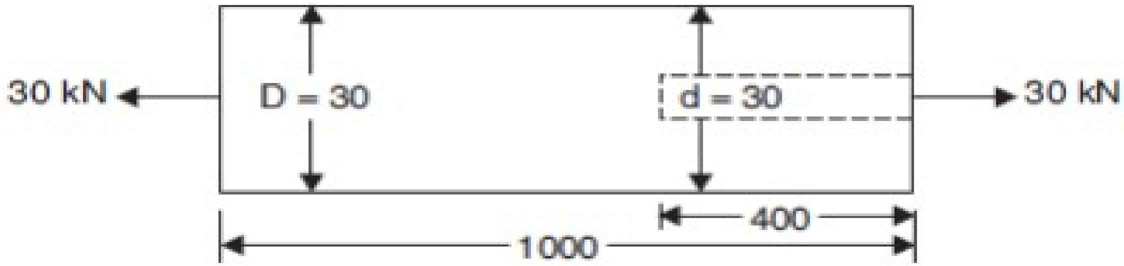
Extension of portion 1, $\dfrac{PL_1}{A_1E_1} = \dfrac{30 \times 10^3 \times 600}{800 \times 2 \times 10^5} = 0.1125$ mm.

Extension of portion 2, $\dfrac{PL_2}{A_2E_2} = \dfrac{30 \times 10^3 \times 800}{600 \times 1 \times 10^5} = 0.4000$ mm.

$\therefore$ **Total extension of the bar** = 0.1125 + 0.4000 = **0.5125 mm.**

BARS WITH CROSS-SECTIONS VARYING IN STEPS

Example(3). A bar of length 1000 mm and diameter 30 mm is centrally bored for 400 mm, the bore diameter being 10 mm as shown in Fig.. Under a load of 30kN, if the extension of the bar is 0.222 mm, what is the modulus of elasticity of the bar?



Solution: Now

$$L_1 = 1000 - 400 = 600 \text{ mm}$$

$$L_2 = 400 \text{ mm}$$

$$A_1 = \frac{\pi}{4} \times 30^2 = 225\ \pi$$

$$A_2 = \frac{\pi}{4} \times (30^2 - 10^2) = 200\ \pi$$

$$\Delta_1 = \frac{PL_1}{A_1 E}$$

$$\Delta_2 = \frac{PL_2}{A_2 E}$$

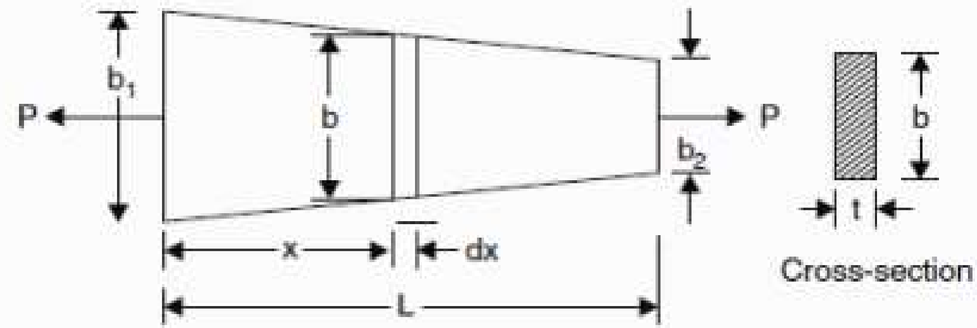
$\therefore$
$$\Delta = \Delta_1 + \Delta_2 = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2}\right)$$

i.e.,
$$0.222 = \frac{30 \times 10^3}{E}\left(\frac{600}{225\ \pi} + \frac{400}{200\ \pi}\right)$$

$\therefore$
$$E = 200736 \text{ N/mm}^2.$$

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

Example (4). A bar of uniform thickness‘t’ tapers uniformly from a width of b1 at one end to b2 at other end in a length ‘L’ as shown in figure. Find the expression for the change in length of the bar when subjected to an axial force P



Solution: Consider an elemental length dx at a distance x from larger end. Rate of change of breadth is $\frac{b_1 - b_2}{L}$.

Hence, width at section x is $b = b_1 - \frac{b_1 - b_2}{L} x = b_1 - kx$

where $$k = \frac{b_1 - b_2}{L}$$

$\therefore$ Cross-section area of the element $= A = t(b_1 - kx)$

Since force acting at all sections is P only,

Extension of element $= \frac{Pdx}{AE}$ [where length $= dx$]

$$= \frac{Pdx}{(b_1 - kx)tE}$$

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

Total extension of the bar $= \int_0^L \frac{Pdx}{(b_1 - kx)tE} = \frac{P}{tE}\int_0^L \frac{dx}{(b_1 - kx)}$

$$= \frac{P}{tE}\left(\frac{1}{-k}\right)\Big[\log(b_1 - kx)\Big]_0^L$$

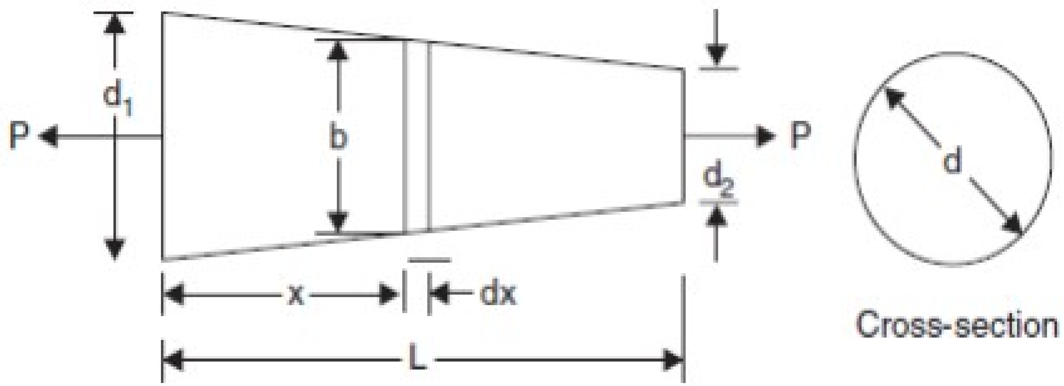
$$= \frac{P}{tEk}\left[-\log\left(b_1 - \frac{b_1 - b_2}{L}x\right)\right]_0^L$$

$$= \frac{P}{tEk}[-\log b_2 + \log b_1] = \frac{P}{tEk}\log\frac{b_1}{b_2}$$

$$= \frac{PL}{tE(b_1 - b_2)}\log\frac{b_1}{b_2}.$$

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

Example(5). A tapering rod has diameter d1 at one end and it tapers uniformly to a diameter d2 at the other end in a length L as shown in Fig. If modulus of elasticity of the material is E, find its change in length when subjected to an axial force P.



Solution: Change in diameter in length L is $d_1 - d_2$

$\therefore$ Rate of change of diameter, $k = \dfrac{d_1 - d_2}{L}$

Consider an elemental length of bar dx at a distance x from larger end. The diameter of the bar at this section is

$$d = d_1 - kx.$$

Cross-sectional area $\quad A = \dfrac{\pi d^2}{4} = \dfrac{\pi}{4}(d_1 - kx)^2$

$\therefore$ Extension of the element $\quad = \dfrac{P\,dx}{\dfrac{\pi}{4}(d_1 - kx)^2 E}$

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

Extension of the entire bar
$$\Delta = \int_0^L \frac{P\,dx}{\frac{\pi}{4}(d_1 - kx)^2 E}$$

$$= \frac{4P}{\pi E}\int_0^L \frac{dx}{(d_1 - kx)^2}$$

$$= \frac{4P}{\pi E k}\left(\frac{1}{d_1 - kx}\right)_0^L$$

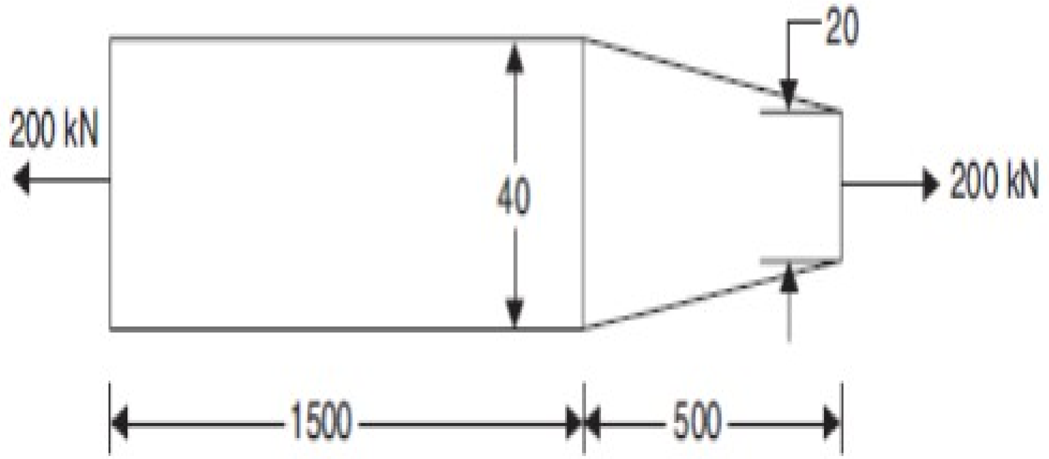
$$= \frac{4P}{\frac{\pi E(d_1 - d_2)}{L}}\left(\frac{1}{d_2} - \frac{1}{d_1}\right), \text{ since } d_1 - kL = d_2$$

$$\therefore \quad \Delta = \frac{4PL}{\pi E(d_1 - d_2)} \times \frac{(d_1 - d_2)}{d_1 d_2} = \frac{\mathbf{4PL}}{\mathbf{\pi E d_1 d_2}}.$$

Note: For bar of uniform diameter extension is $\dfrac{PL}{\dfrac{\pi d^2}{4}E}$ and for tapering rod it is $\dfrac{PL}{\dfrac{\pi}{4}d_1 d_2 E}$.

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

Example(6). A 2.0 m long steel bar is having uniform diameter of 40 mm for a length of 1 m and in the next 0.5 m its diameter gradually reduces from 40 mm to 20 mm as shown in Fig. Determine the elongation of this bar when subjected to an axial tensile load of 200 kN. Given E = 200 GN/m2



Solution: Now, $P = 200 \times 10^3$ N

$$E = 200 \text{ GN/m}^2 = \frac{200 \times 10^9}{(1000)^2} \text{ N/mm}^2$$

$$= 200 \times 1000 \text{ N/mm}^2$$

$$= 2 \times 10^5 \text{ N/mm}^2$$

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

Extensions of uniform portion and tapering portion are worked out separately and then added to get extension of the given bar.

Extension of uniform portion

$$\Delta_1 = \frac{PL}{AE} = \frac{200 \times 10^3 \times 1500}{\frac{\pi}{4} \times 40^2 \times 2 \times 10^5} = 1.194 \text{ mm.}$$

Extension of tapering portion

$$\Delta_2 = \frac{4\,PL}{E\pi d_1 d_2} = \frac{4 \times 200 \times 10^3 \times 500}{2 \times 10^5 \times \pi \times 60 \times 40}$$

$$= 0.265 \text{ mm}$$

Total extension $= \Delta_1 + \Delta_2 = 1.194 + 0.265 = \mathbf{1.459\ mm}$

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

Example (7). A bar of 25 mm diameter is tested in tension. It is observed that when a load of 60 kN is applied, the extension measured over a gauge length of 200 mm is 0.12 mm and contraction in diameter is 0.0045 mm. Find Poisson's ratio and elastic constants E, G, K

Solution: Now,

$$P = 60 \text{ kN} = 60000 \text{ N}$$

$$\text{Area } A = \frac{\pi}{4} \times 25^2 = 156.25\pi \text{ mm}^2$$

$$\text{Guage length } L = 200 \text{ mm}$$

$$\Delta = 0.12 \text{ mm}$$

$$\Delta d = 0.0045 \text{ mm}$$

$$\text{Linear strain} = \frac{\Delta}{L} = \frac{0.12}{200} = 0.0006$$

$$\text{Lateral strain} = \frac{\Delta d}{d} = \frac{0.0045}{25} = 0.00018$$

$$\therefore \quad \text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{0.00018}{0.0006}$$

$$\mu = 0.3$$

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

Now, $$\Delta = \frac{PL}{AE}$$

$$0.12 = \frac{60000 \times 200}{156.25\pi \times E}$$

or $$E = 203718.3 \text{ N/mm}^2$$

Using the relation $$E = 2G(1 + \mu)$$

We get $$G = \frac{E}{2(1+\mu)} = \frac{203718.3}{2(1+0.3)} = 78353.2 \text{ N/mm}^2$$

From the relation, $E = 3K(1 - 2\mu)$, we get

$$K = \frac{E}{3(1-2\mu)} = \frac{203718.3}{3(1-2\times 0.3)} = 169765.25 \text{ N/mm}^2$$

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

Example (8). A circular rod of 25 mm diameter and 500 mm long is subjected to a tensile force of 60 kN. Determine modulus of rigidity, bulk modulus and change in volume if Poisson's ratio = 0.3 and Young's modulus E = 2 × 105 N/mm2 .

Solution: From the relationship

$$E = 2G(1 + \mu) = 3k(1 - 2\mu)$$

We get,

$$G = \frac{E}{2(1+\mu)} = \frac{2 \times 10^5}{2(1+0.3)} = 0.7692 \times 10^5 \text{ N/mm}^2$$

and

$$K = \frac{E}{3(1+2\mu)} = \frac{2 \times 10^5}{3(1-2\times 0.3)} = 1.667 \times 10^5 \text{ N/mm}^2$$

$$\text{Longitudinal stress} = \frac{P}{A} = \frac{60 \times 10^3}{\frac{\pi}{4} \times 25^2} = 122.23 \text{ N/mm}^2$$

$$\text{Linear strain} = \frac{\text{Stress}}{E} = \frac{122.23}{2 \times 10^5} = 61.115 \times 10^{-5}$$

BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

$$\text{Linear strain} = \frac{\text{Stress}}{E} = \frac{122.23}{2 \times 10^5} = 61.115 \times 10^{-5}$$

$$\text{Lateral strain} = e_y = -\mu e_x \quad \text{and} \quad e_z = -\mu e_x$$

$$\text{Volumetric strain} \quad e_v = e_x + e_y + e_z$$

$$= e_x(1 - 2\mu)$$

$$= 61.115 \times 10^{-5} \, (1 - 2 \times 0.3)$$

$$= 24.446 \times 10^{-5}$$

but $\dfrac{\text{Change in volume}}{v} = e_v$

$$\therefore \quad \text{Change in volume} = e_v \times v$$

$$= 24.446 \times 10^{-5} \times \frac{\pi}{4} \times (25^2) \times 500$$

$$= 60 \text{ mm}^3$$