D’Alemberts Principle

Newton’s second law is F = ma

For the system of forces acting on a body the second law can be write as

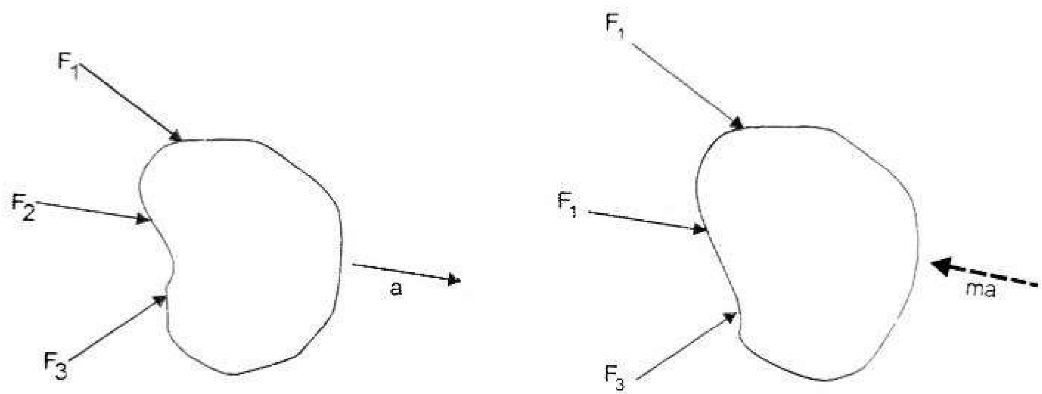
F + (-ma) = 0

**D’Alemberts principle** states that the system of forces acting on a body in motion is in dynamic equilibrium with the inertia force of the body.

F + (-ma) = 0

F + Inertia force = 0

Inertia force always acts in opposite direction to the motion of the body.



A man weighing W newtons entered a lift which moves with an acceleration of 'a' m/sec2. Find the force exerted by the man on the floor of lift when

i. Lift is moving downward
ii. Lift is moving upward

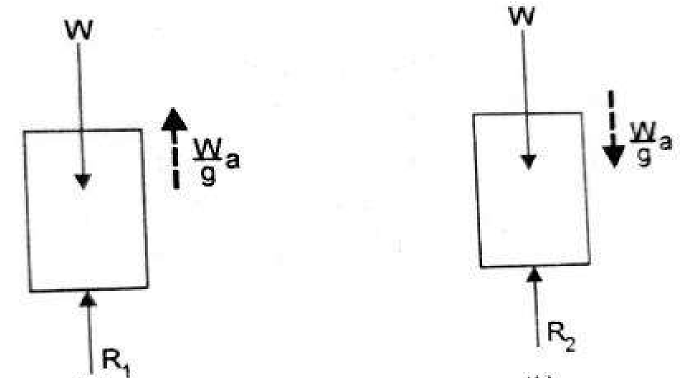
**Case 1.** Lift is moving downwards

When the lift is moving downwards inertia force will acts upwards.

$\Sigma F_y = 0; R_1 - W + (W/g)\,a = 0$

$R_1 = W - (W/g)\,a$

$R_1 = W\,(1 - a/g)$




(a) Lift Moving Downward

(b) Lift Moving Upward

**Case 2**. Lift is moving upwards

When the lift is moving upwards inertia force will acts downwards.

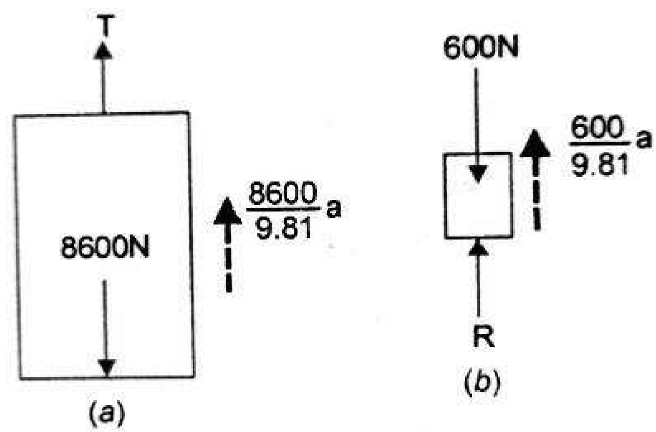
$\Sigma Fy = 0; W + (W/g)\ a - R2 = 0$

$R2 = W + (W/g)\ a$

$R2 = W\ (1 + a/g)$

Thus, when lift is moving downwards, the man exerts less force on the floor of the lift and while moving upwards he will exerts more force on the floor.

An elevator cage of a mine shaft weighing 8KN, when empty is lifted or lowered by means of a wire rope. Once a man weighing 600N, exerted it and lowered with uniform acceleration such that when a distance of 187.5m was covered, the velocity of the case was 25m/sec. Determine the tension in the rope and the force exerted by the man on the floor of the case.




Initial velocity = u = 0
Final velocity v = 25 m/sec
Distance s = 187.5 m
From, $v^2$-$u^2$ = 2as
$25^2 - 0^2 = 2 \times a \times 187.5$
a = 1.664 m/sec2

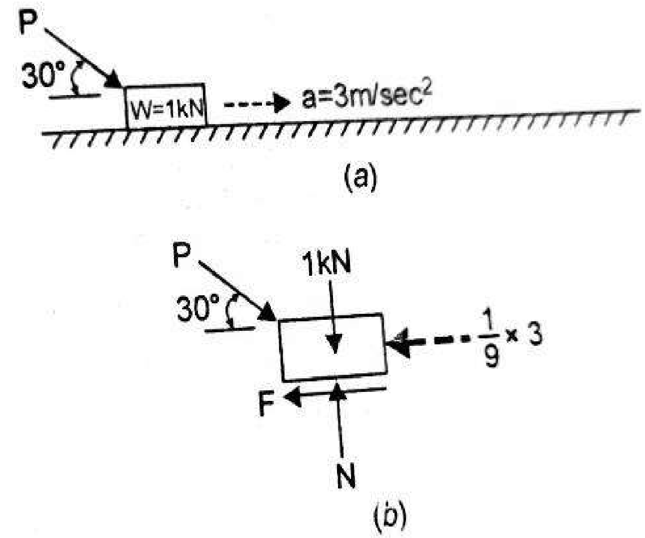
**Case1**. When the lift is lowered

T + (W/g) a = 8000 + 600
T = 8600 – (8600/9.81) x 1.667
T = 7138.90N

**Case2**. Force exerted by the man on the floor

600 – R = (W/g) a
R = 600 – (600/9.81) x 1.667
R = 498.06N

A block weighing 1KN rests on a horizontal plane as shown in figure. Find the magnitude of the force P required to give the block an acceleration of 3 m/sec2 to the right. Assume $\mu = 0.25$




By applying equilibrium conditions

$\Sigma Fy = 0;\ N - 1 - P\sin 30^\circ = 0$

$N = 1 + P/2$

$F = \mu N = \mu\ (1 + P/2)$

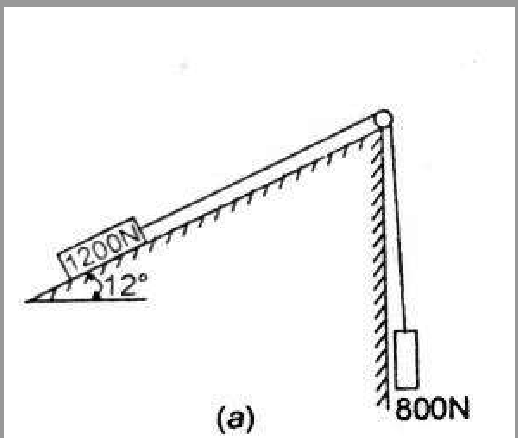
$\Sigma Fx = 0;\ P\cos 30^\circ - F - (W/g)\ a = 0$

$P\ (\sqrt{3}/2) = \mu(1 + P/2) + (W/g)a$

$P\ (\sqrt{3}/2 - 0.25/2) = 0.25 + (3/9.81)$

$P = (0.25 + (3/9.81))\ /\ (\sqrt{3}/2 - 0.125)$

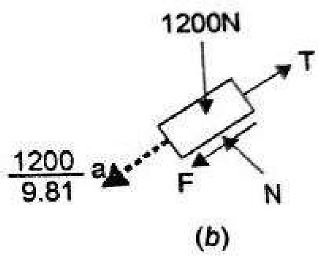
$P = 0.561KN$



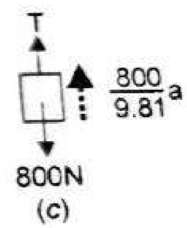

A body weighing 1200N rests on a rough plane inclined at 12° to the horizontal. It is pulled up the plane by means of a light flexible rope running parallel to the plane and passing over a light frictionless pulley at the top of the plane as shown in figure. The portion of the rope beyond the pulley hangs vertically down and carries a weight of 800n at its end. μ = 0.2. find

i. Tension in the rope
ii. Acceleration with which the body moves up the plane
iii. The distance moved by the body in 3sec after starting from rest.

$\Sigma Fy = 0$; $N = 1200 \cos 12^\circ$
$N = 1173.77N$
$F = \mu N = 0.2 \times 1173.77 = 234.76N$




$\Sigma Fx = 0$; $T = F + (W/g)a + 1200 \sin 12^\circ$
$T = 234.76 + (1200/9.81)\,a + 1200 \sin 12^\circ$
$T = 122.32a + 484.25$................(1)




$\Sigma Fy = 0$; $T + (W/g)a = 800$
$T = 800 - (800/9.81)\,a$ .................(2)

From (1) and (2)

$122.32\ a + 484.25 = 800 - (800/9.81)\ a$

$(122.32 + (800/9.81))a = 800 - 484.25$

$203.86\ a = 315.75$

$a = 1.549$ m/sec2

from (2)

$T = 800 - (800/9.81)\ a$

$T = 800 - (800/9.81)\ 1.549$

$T = 673.68$N

Distance travelled in 3 seconds

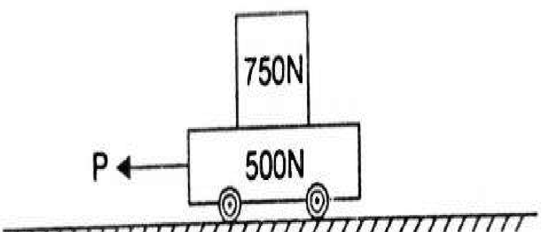
Initial velocity $u = 0$

$a = 1.549\ m/sec^2$, $t = 3$ sec

From, $s = ut + ½\ at^2$

$s = 0 + ½ \times 1.549 \times (3)^2$

$s = 6.9871$m




(a)

A 750N crate rests on a 500N cart. μ between the crate and cart is 0.3 and between cart and the road is 0.2. if the cart is to be pulled by a force P as shown in figure such that the crate does not slip. Determine

i. The maximum allowable magnitude of P
ii. Corresponding acceleration of the cart

$\Sigma Fy = 0$; $N = W = 750N$

$F = \mu N = \mu \times 750 = 0.3 \times 750 = 225N$

$\Sigma Fx = 0$; $F = (W/g)\, a$

$225 = (750/9.81)\, a$

$a = 2.943$ m/sec2

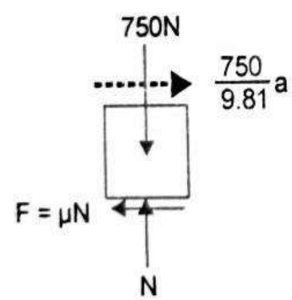
$\Sigma Fy = 0$; $N = W = 1250N$

$F = \mu N = 0.2 \times 1250 = 250N$

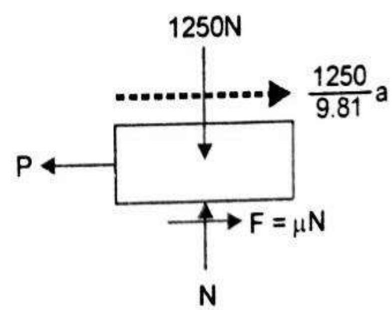
$\Sigma Fx = 0$; $P - (W/g)a - F = 0$

$P = (W/g)a + F$

$P = (1250/9.81) \times 2.943 + 250 = 625N$




(b)




(c)

Two rough planes inclined at 30° and 60° to horizontal are placed back to back as shown in figure. The blocks of weights 50N and 100N are placed on the forces and are connected by a string running parallel to the planes and passing over a friction less pulley. μ=1/3. Find the resulting acceleration and tension in the string.

$\Sigma Fy = 0$; $N1 = 100\cos\theta 1$

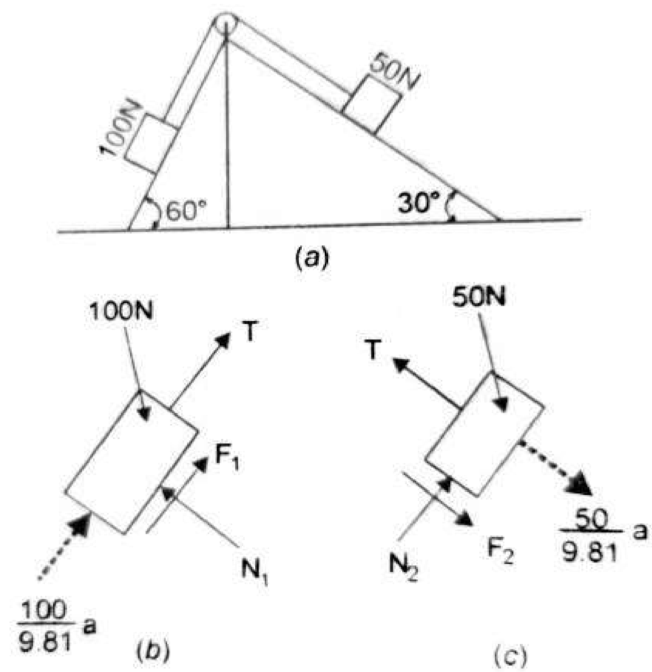
$100 \cos 60° = 50N$

$F1 = \mu N1 = 1/3 \times 50 = 16.67N$

$\Sigma Fx = 0$; $T + (W/g)\,a = 100\sin\theta 1 - F1$

$T = 100 \sin 60° - 16.67 - (100/9.81)a$

$T = 69.93 - (100/9.81)a$.............(1)



$\Sigma Fy = 0$; $N2 = 50\cos\theta 2$

$N2 = 50 \cos 30° = 43.30N$

$F2 = \mu N2 = (1/3) \times 43.30 = 14.43N$

$\Sigma Fx = 0$; $T - 50 \cos\theta 2 - (W/g)a - F2 = 0$

$T = 50 \sin 30° + (50/9.81)a + 14.43$

$T = (50/9.81)\ a + 39.43$..........(2)

From, (1) and (2)

$69.93 - (100/9.81)a = (50/9.81)a + 39.43$

$69.93 - 39.43 = (150/9.81)\ a$

$30.50 = 15.29\ a$

$a = 1.9947$

From, (1)

$T = 69.93 - (100/9.81) \times 1.9947$

$T = 49.6N$

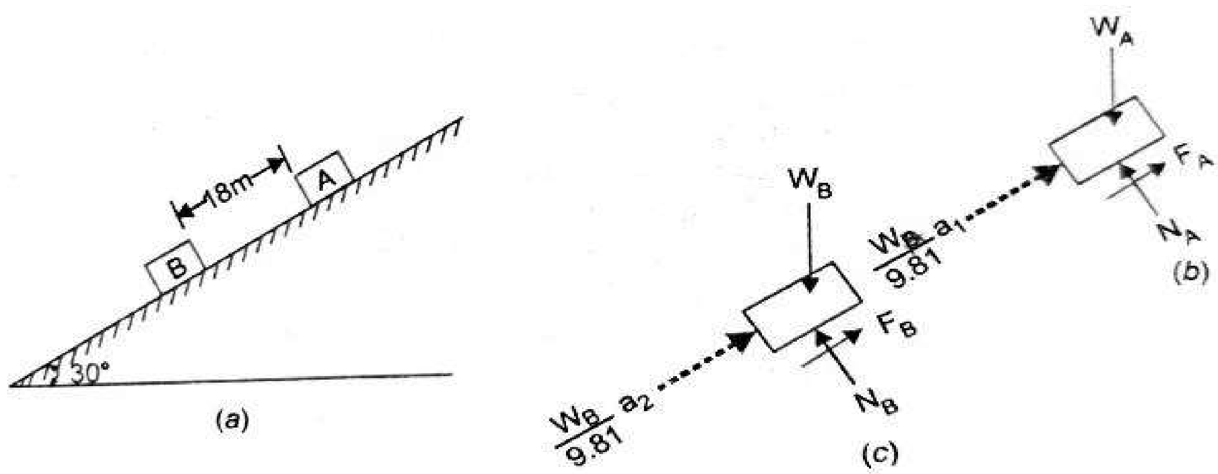
Two blocks A and B are released from rest on a 30 degree incline, when they are 18 m apart. The coefficient of friction under the upper block A Is 0.2 and that under the block B is 0.4. In what time block A reaches the block B? After they touch and move as a single unit, what will be the contact force between them? Weights of the block A and B are 100 N and 80 N respectively.

**Solution.** Let block $A$ move with an acceleration $a_1$ and block $B$ with an acceleration $a_2$. The free body diagrams of the blocks $A$ and $B$ along with inertia forces are shown in Fig. 11.11($b$) and 11.11($c$) respectively.

Consider block $A$.

$\Sigma$ Forces normal to the plane = 0, gives

$$N_A = W_A \cos\theta = W_A \cos 30° \quad ...(1)$$



$$\text{From the law of friction,} \qquad F_A = \mu N_A$$
$$= 0.2\ W_A \cos 30° \qquad \text{...(2)}$$

$$\Sigma \text{ Forces parallel to the plane} = 0, \text{ gives}$$

$$\frac{W_A}{9.81} a_1 + F_A - W_A \sin 30° = 0$$

$$\therefore \frac{W_A}{9.81} a_1 + 0.2 W_A \cos 30° - W_A \sin 30° = 0$$

$$\frac{a_1}{9.81} + 0.2 \cos 30° - \sin 30° = 0$$

$$a_1 = 3.2058 \text{ m/sec}^2 \qquad \text{...(3)}$$

*Consider block B.*

$$\Sigma \text{ Forces normal to the plane} = 0, \text{ gives}$$

$$N_B = W_B \cos 30° \qquad \text{...(4)}$$

$$\text{From the law of friction,} \qquad F_B = \mu N_B$$

*i.e.,*
$$F_B = 0.4\ W_B \cos 30° \qquad \text{...(5)}$$

$$\Sigma \text{ Forces parallel to the plane} = 0, \text{ gives}$$

$$\frac{W_B}{9.81} a_2 + F_B - W_B \sin 30° = 0$$

$$\frac{W_B}{9.81} a_2 + 0.4 W_B \cos 30° - W_B \sin 30° = 0$$

$$\therefore \qquad a_2 = 1.5067 \text{ m/sec}^2$$

Let $t$ be the time elapsed until the blocks touch each other. Displacement of block A in this period

$$s_1 = u_1 t + \frac{1}{2} a_1 t^2 = \frac{1}{2} \times 3.2058\, t^2$$

since initial velocity $u_1 = 0$

Displacement of block $B$ in this time

$$s_2 = u_2 t + \frac{1}{2} a_2 t^2 = \frac{1}{2} \times 1.5067\, t^2$$

When the two blocks touch each other

$$S_1 = S_2 + 18$$

$$\frac{1}{2} \times 3.2058\, t^2 = \frac{1}{2} \times 1.5067\, t^2 + 18$$

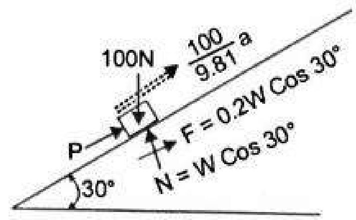
$\therefore$ $\quad$ $\boldsymbol{t = 4.60}$ **sec** $\quad$ **Ans.**

After the blocks touch each other, let the common acceleration be $a$. Summing up the forces including inertia forces along the inclined plane.

$$\frac{100}{9.81}a + 0.2 \times 100 \cos 30° - 100 \sin 30 + \frac{80}{9.81}a + 0.4 \times 80 \cos 30° - 80 \sin 30° = 0$$

$$a = 2.45 \text{ m/sec}^2$$

Considering the free body diagram of any one of the blocks, contact force $P$ can be obtained. Free body diagram of block $A$ along with inertia force is shown in Fig. 11.11($d$).




($d$)

**Fig. 11.11**

Now $\Sigma$ forces parallel to plane = 0, gives

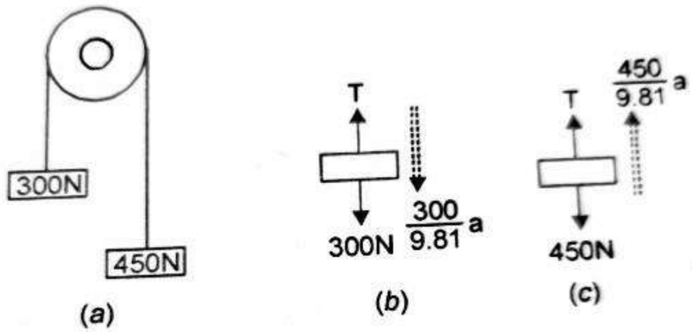
$$P - 100 \sin 30° + F + \frac{100}{9.81}a = 0$$

$$P - 100 \sin 30° + 0.2 \times 100 \cos 30° + \frac{100}{9.81} \times 2.45 = 0$$

$$P = 7.7 \text{ N}$$ **Ans.**

Two bodies weighing 300N and 450N are hung to the ends of a rope passing over an ideal pulley as shown in figure. With what acceleration the heavier body comes down? What is the tension in the string?

Let 'a' be the acceleration and t be the tension in the string.




ΣFy = 0; T − W − (W/g)a = 0

T = 300 + (300/9.81)a .............(1)

ΣFy = 0; T − 450 + (W/g)a = 0

T = 450 − (450/9.81) a .............(2)

From, (1) and (2)

300 + (300/9.81) a = 450 − (450/9.81) a

(750/9.81) a = 450-300 = 150

a = (150 x 9.81) / 750 = 1.962 m/sec2

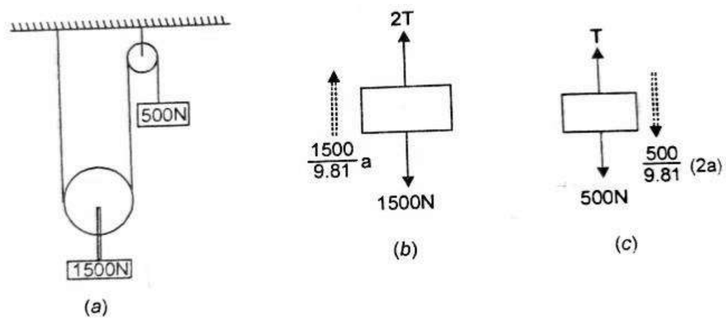
from (1), T = 300 + (300/9.81)a

T = 300 + (300/9.81) x 1.962

T = 360N

Determine the tension in the string and acceleration of blocks A and B weighing 1500N and 500N connected by an inextensible string as shown in figure. Assume pulleys as frictionless and weightless.

**Note**: in the above pulley system it is observed that if 1500N block moves downward by a distance 'x' then the 500N block moves up by '2x'. Similarly if 1500N block acceleration is 'a' that of 500N block is '2a'.




ΣFy = 0; 2T + (W/g)a = W
2T = W − (W/g) a
2T = 1500 − (1500/9.81) a
T = 750 − (750/9.81)a ..........(1)

ΣFy = 0; T = 500 + (W/g) (2a)
T = 500 + (500/9.81) x (2a)............(2)
From (1) and (2)
750 − (750/9.81) a = 500 + (500/9.81) (2a)
250 = a (1000/9.81 + 750/9.81)
250 = a (1750/9.81)
a = (250 x 9.81) /1750 = 1.401 m/sec2
from (1), T = 750 − (750/9.81) a
T = 750 − (750/9.81) 1.401
T = 642.89N