PART - 2

Friction :-

Whenever the surfaces of two bodies are in contact, there is a limited amount of resistance to sliding b/w them which is called friction

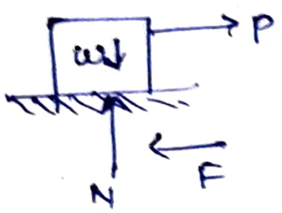
* It is a force distribution at the surface of contact & acts tangential to surface.

Dry friction & fluid friction:-

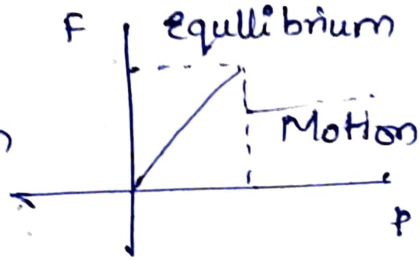
⇒ Friction b/w dry surfaces in contact is called dry friction / coloumb friction

⇒ Major cause of this friction is interlocking of microscopic protuberances which oppose the relative motion

⇒ Friction b/w two surfaces in presents of fluid is called fluid friction



→ direction of impedance motion



Limiting friction:-

Consider a body of mass 'm' leaving corresponding weight 'w' resting on a horizontal surface. let a continous increasing force 'p' is applied to a body as shown.

This force 'p' will be opposed / resisted by frictional force 'f', the force f shall goes on increasing to balance the increasing the applied 'p' the bodies remains rest. Then they comes a limit beyond this the frictional force 'F' can't increase then body begins to move. The frictional force at this moment is called limiting friction.

Causes of dry friction:-

⇒ total friction that can be developed is independent of magnitude of area in contact

⇒ Total friction that can be developed is proportional to the normal force transmitted across the surface of contact

$F \propto N \quad \Rightarrow F = \mu N$

⇒ For low velocities the total amount of frictional force that can be developed pratically independent of velocities But the force neccesary to motion is greater than that neccessary to maintain the motion

Angle of friction & resultant reaction:-

* Normal reaction "N" and the limiting frictional force "F" acting at a surface of contact can combined into a single resultant (R) called the resultant reaction. At the point of contact. the angle b/w R makes with normal reaction N is called angle of friction and is denoted by ϕ

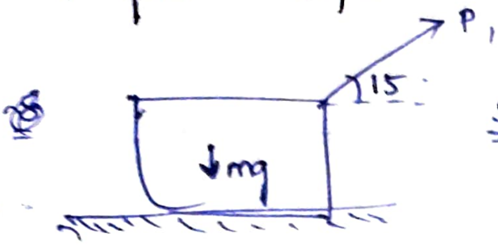
$$\tan\phi = \frac{\mu \cancel{N}}{\cancel{N}}$$

$$\mu = \tan\phi$$

$$\phi = \tan^{-1}(\mu)$$

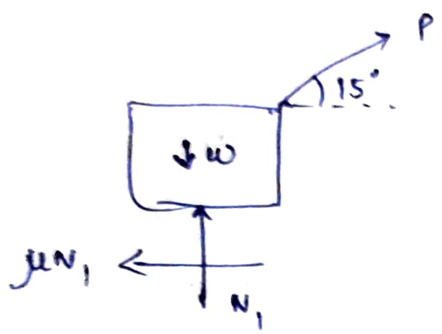
A wooden block rests on a horizontal plane. Determine the force required to ; Assuming mass of block (m) = 5 kg, coff of friction $\mu = 0.4$.

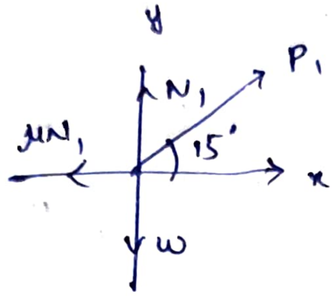
i) pull it ii) push it. Take $g = 9.81 \, m/s^2$



S $\quad w = mg$

$= 5 \times 9.81 = 49.05 \, N$





i) $\Sigma f_x = 0 \Rightarrow P_1 \cos 15 - 0.4 N_1 = 0 \quad - (1)$ $\quad \dfrac{P_1 \cos 15}{0.4} = N_1$

$\Sigma f_y = 0 \Rightarrow P_1 \sin 15 + N_1 - w = 0$

$$P_1 \sin 15 + \frac{P_1 \cos 15}{0.4} = 49.05$$

$$0.4 P_1 \sin 15 + P_1 \cos 15 = 49.05 \times 0.4$$

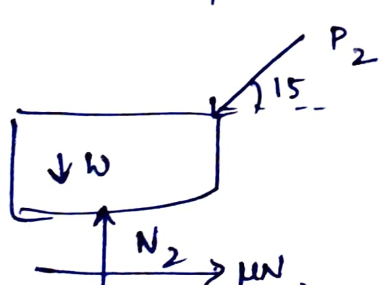
$$0.4 P_1 \sin 15 + P_1 \cos 15 = 19.62$$

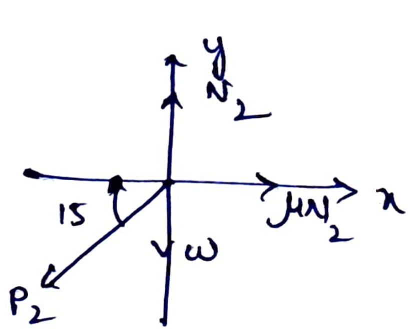
$$0.4 \times 0.258 \sin 15 P_1 + P_1 \cdot 0.965 = 19.62$$

$$0.103 P_1 + 0.965 P_1 = 19.62$$

$$P_1 (1.0694) = 19.62$$

$$P_1 = 18.34 \, N.$$





$\mu N_2 - P_2 \cos 15 = 0$

$N_2 = \dfrac{P_2 \cos 15^\circ}{0.4}$ - (1)

$N_2 - w - P_2 \sin 15^\circ = 0$

$P_2 \cos 15 - 0.4 P_2 \sin 15 = 19.62$

$P_2 (0.258) - 0.1035 P_2 = 19.62$

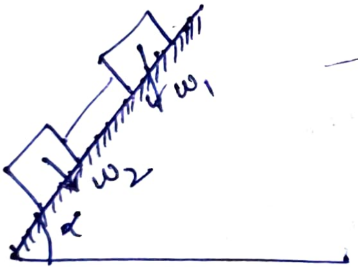
$P_2 (0.8623) = 19.02$

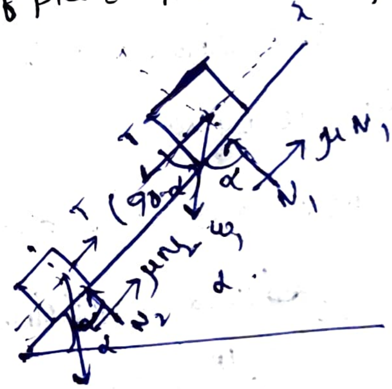
$P_2 = 22.75 N$

$\approx 23 N$

Two blocks of weight $w_1 = 50N$ & $w_2 = 50N$ rest on a rough inclined planed & are connected by a string as shown in figure. The coff of friction b/w the inclined friction & w_1, w_2 are $\mu_1 = 0.3$, $\mu_2 = 0.20$ respectively. Find the inclination of plane for which slipping will impend.

S

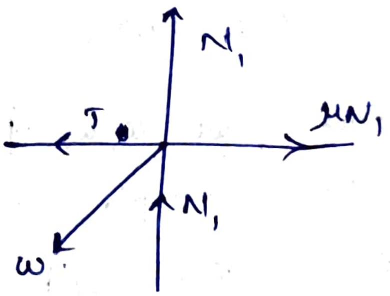




S

For w_1,

$\Sigma f_x = 0$



$\mu_1 N_1 - w_1 \cos(90-\alpha) - T = 0$

$\mu_1 N_1 - 50 \cos(90-\alpha) - T = 0.$

$0.3 N_1 - 50 \cos(90-\alpha) - T = 0.$ - (1)

$\Sigma f_y = 0$

~~0.3x~~

$N_1 - w_1 \sin(90-\alpha) = 0.$

$N_1 = 50 \sin(90-\alpha)$

- (2)

$N_1 = 50 \cos\alpha$

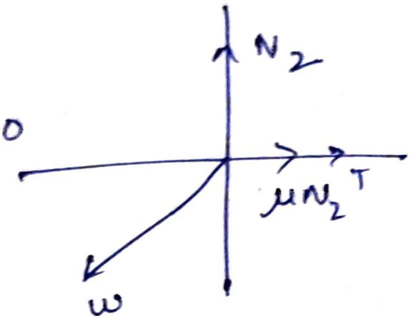
$0.3 N_1 - 50 \sin\alpha - T = 0$

$0.3 \times (50 \cos\alpha) - 50 \sin\alpha - T = 0.$

$15 \cos\alpha - 50 \sin\alpha = T \quad - (3)$

For block 2,

$\Sigma f_x = 0$



$T + \mu_2 N_2 - w_2 \cos(90-\alpha) = 0$

$- (4)$

$\Sigma f_y = 0.$

$N_2 - w_2 \sin(90-\alpha) = 0$

$N_2 = w_2 \cos\alpha$

$T + 0.2 \times 50 \cos\alpha - 50 \sin\alpha = 0.$

$T + 25 \cos\alpha - 50 \sin\alpha = 0.$

$\cancel{25}\ 50 \sin\alpha - 25 \cos\alpha = -T \quad (6)$

(3) & (6) are equal

$15 \cos\alpha - 50 \sin\alpha = 50 \sin\alpha - 10 \cos\alpha$

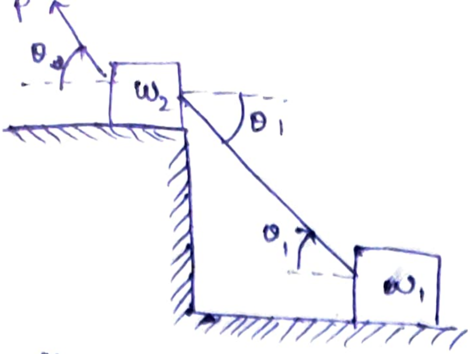
$25\,\cancel{40} \cos\alpha = 10\cancel{0} \sin\alpha$

$\frac{1}{4}\frac{\cancel{10}}{\cancel{10}} = \tan\alpha$

$\alpha = 14.036$

Two blocks of wt. w_1, w_2 ~~(pla)~~ are connected by a string and rest on a horizontal plane. find the magnitude & direction of least force P that should be applied to induce sliding. coeff of friction is taken to be same.

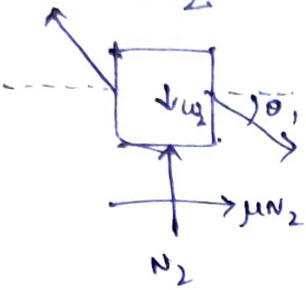
(2) ni (1)



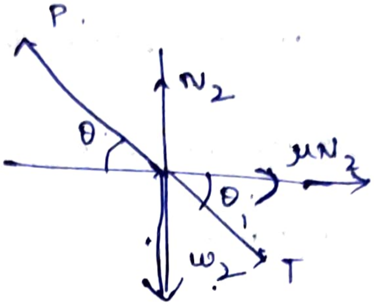
$\mu_1 = \mu_2 = \mu$

S

Block w_2



⇒



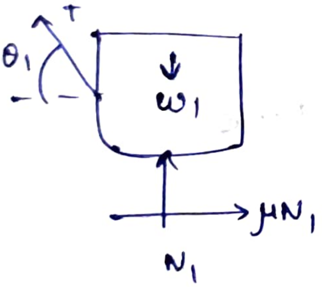
$\Sigma f_x = 0 \quad \mu N_2 - P\cos\theta + T\cos\theta_1 = 0 \quad -(4)$

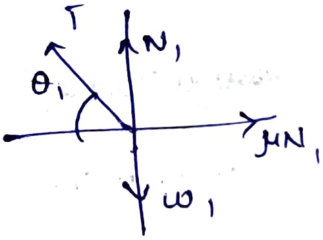
$$N_2 = \frac{T\cos\theta_1 - P\cos\theta}{\mu}$$

$$\Rightarrow \frac{T\cos\theta_1 - P\cos\theta}{\mu} - w_2 + P\sin\theta - T\sin\theta_1 = 0$$

$\Sigma f_y = 0 \Rightarrow N_2 - w_2 + P\sin\theta - T\sin\theta_1 = 0 \quad -(5)$

Block w_1





$\Sigma f_x = 0 \Rightarrow \mu N_1 - T\cos\theta_1 = 0$

$$N_1 = \frac{T\cos\theta_1}{\mu} \quad -(1)$$

$\Sigma f_y = 0 \Rightarrow N_1 + T\sin\theta_1 - w_1 = 0 \quad -(2)$

$$\frac{T\cos\theta_1}{\mu} + T\sin\theta_1 = w_1$$

$$T\cos\theta_1 + \mu T\sin\theta_1 = \mu w_1$$

$$T = \frac{\mu w_1}{\cos\theta_1 + \mu\sin\theta_1} \quad -(3)$$

(4) in (5)

$$\frac{P\cos\theta - T\cos\theta_1}{\mu} + P\sin\theta - T\sin\theta_1 - \omega_2 = 0$$

$$P\cos\theta - T\cos\theta_1 + \mu P\sin\theta + \mu T\sin\theta - \mu\omega_2 = 0$$

$$P(\cos\theta + \mu\sin\theta) - T(\cos\theta_1 + \mu\sin\theta_1) - \mu\omega_2 = 0$$

$$\frac{P(\cos\theta + \mu\sin\theta) - \mu\omega_2}{\cos\theta_1 + \mu\sin\theta_1} = T \quad - (6)$$

(3) & (6) are same

$$\frac{\mu\omega_1}{\cos\theta_1 + \mu\sin\theta_1} = \frac{P(\cos\theta + \mu\sin\theta) - \mu\omega_2}{\cos\theta_1 + \mu\sin\theta_1}$$

$$\mu(\omega_1 + \omega_2) = P(\cos\theta + \mu\sin\theta)$$

$$P = \frac{\mu(\omega_1 + \omega_2)}{\cos\theta + \mu\sin\theta} \qquad \boxed{\mu = \tan\phi}$$

$$P = \frac{\frac{\sin\phi}{\cos\phi}(\omega_1 + \omega_2)}{\cos\theta + \frac{\sin\phi}{\cos\phi}\sin\theta}$$

$$= \frac{\sin\phi(\omega_1 + \omega_2)}{\cos\theta\cos\phi + \sin\phi\sin\theta}$$

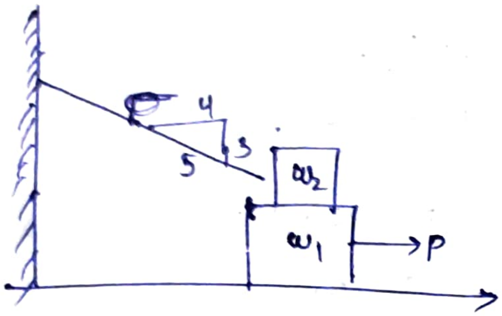
$$P = \frac{\sin\phi(\omega_1 + \omega_2)}{\cos(\theta - \phi)}$$

for P_{min} $\cos(\theta - \phi)$ should be max

$$(\theta - \phi) = 0 \Rightarrow \boxed{\theta = \phi}$$

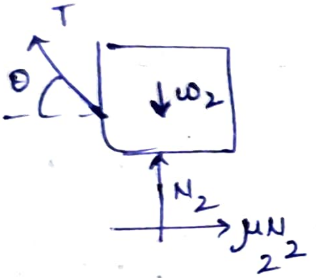
$P_{min} = (\omega_1 + \omega_2) \sin\phi$ inclined at $\theta = \phi$

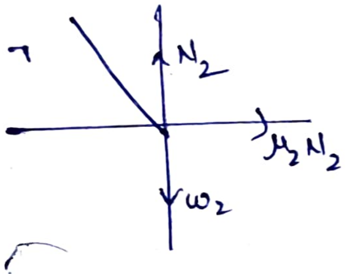
~~A~~ block of wt $\omega_1 = 200N$ rests on a horizontal surfaces & top, on it another block of wt $\omega_2 = 50N$ the block ω_2 is attached to vertical wall by a inclined string AB find magnitude of horizontal force P applied to lower block as shown ; ie lesser to in cause slipping to impend coff of static friction for all contact surfaces $\mu = 0.3$



$\tan\theta = 3/4$.

$\theta = 36.86°$





$\Sigma f_x = 0$

$\mu_2 N_2 - T\cos\theta = 0$.

$\mu_2 N_2 - T(\cancel{53.13}) = 0$. (0.8)

$N_2 = \dfrac{T(\cancel{53.13})}{0.3}$ (0.8)

$N_2 = \cancel{177.17}$ 2.66T

$\Sigma f_y = 0$

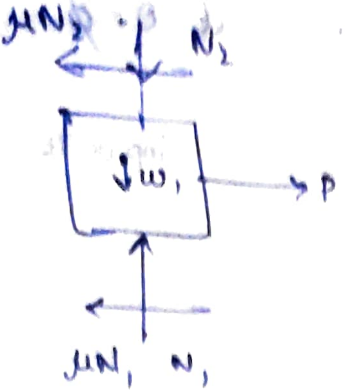
$N_2 + T\sin\theta - \omega_2 = 0$

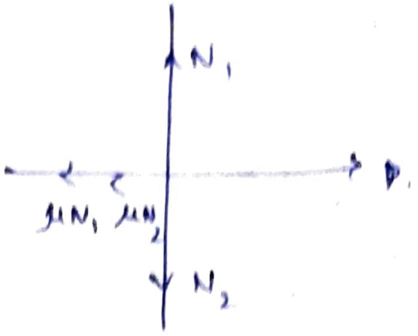
$\cancel{177.17} + \cancel{53.13}T = 50$ (2.66 T + 0.60 T)

$\cancel{230.25}T = 50$ (3.259)

$T = 15.33N$

$N_2 = 41N$





$\Sigma f_x = 0$

$P - \mu N_1 - \mu N_2 = 0$

$P = \mu N_1 + \mu N_2$

$P = \cancel{0.3(41)}\ 0.3(241) + 0.3(41)$

$P = 84.3$ N

$\Sigma f_x = 0$

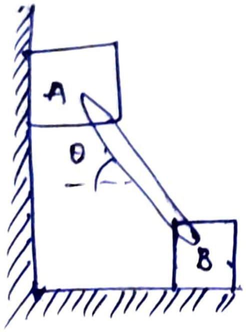
$N_1 - N_2 - W_1 = 0$

$N_1 = N_2 + W_1$

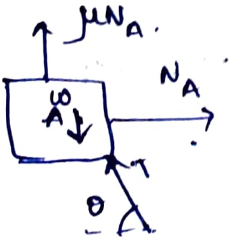
$= 200 + 41$

$= 241$

26-10-22

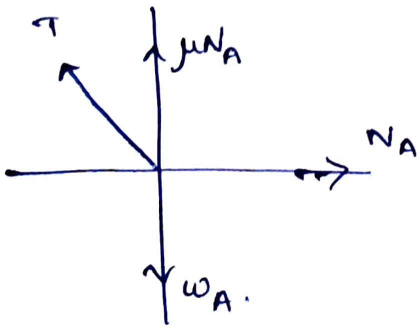


Two indentical blocks. A & B are connected by a rod & rest aganist vertical & horizontal planes respectively as shown in fig. If sliding impends when θ = 45° determine the coff of friction μ assumin it to be same both floor and wall



$\mu_A = \mu_B = \mu$

$W_A = W_B$



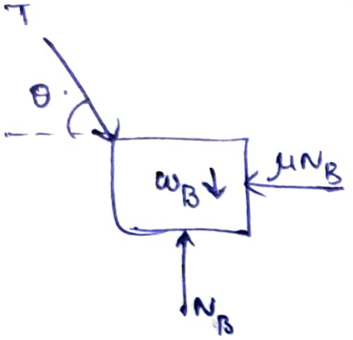
$N_A - T\cos\theta = 0 \qquad \Sigma f_x = 0$

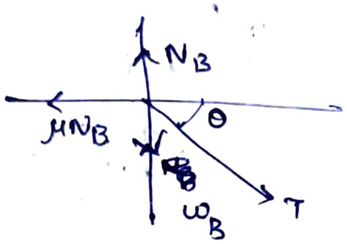
$-T\sin\theta + \mu N_A - w_A = 0 \qquad \Sigma f_y = 0$

$N_A = T/\sqrt{2} \quad -(1) \qquad T/\sqrt{2} + \mu\left(\frac{T}{\sqrt{2}}\right) = w_A.$

$$\frac{T(\mu+1)}{\sqrt{2}} = w_A.$$

$$T = \frac{\sqrt{2}\,w_A}{(\mu+1)} \quad -(3) \qquad , T = \frac{w_A}{\sin\theta + \mu\cos\theta}$$





$T\cos\theta - \mu N_B = 0 \qquad \Rightarrow N_B = \frac{T\cos\theta}{\mu} = \frac{T}{\sqrt{2}\mu}$

$-T\sin\theta + N_B - w_B = 0$

$$\frac{-T}{\sqrt{2}} + \frac{T}{\mu\sqrt{2}} = w_B.$$

$$\frac{-\mu T + T}{\sqrt{2}\mu} = w_B.$$

$$\frac{T(1-\mu)}{\sqrt{2}\mu} = w_B. \qquad \frac{-2 \pm \sqrt{8}}{2}$$

$$= -1 \pm$$

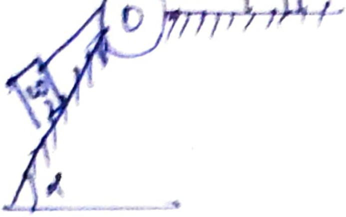
$$T = \frac{\sqrt{2}\mu\, w_B}{1-\mu}.$$

$$\frac{\cancel{\sqrt{2}}\,\cancel{w_A}}{\mu+1} = \frac{\cancel{\sqrt{2}}\mu\,\cancel{w_B}}{1-\mu}$$

$$\frac{1}{\mu+1} = \frac{\mu}{1-\mu}. \qquad 1-\mu = \mu^2+\mu.$$

$$\mu^2+2\mu-1 = 0$$

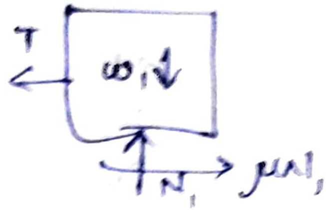
$$\Rightarrow \boxed{\mu = 0.414}$$



a inclined & horizontal planes. If cord passes over a pulley as shown in fig. In the particular case where $w_1 = w_2$ & coff of static friction μ is same for all contact find α of inclination of plane at force which motion of system is impend. neglect friction in pulley.

$w_1 = w_2$; $\mu_1 = \mu_2 = \mu$

For w_1 :-



$\Sigma f_x = 0$

$\mu N_1 - T = 0$

$T = \mu N_1$ - (1)

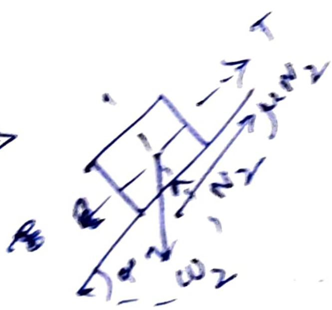
$\Sigma f_y = 0$

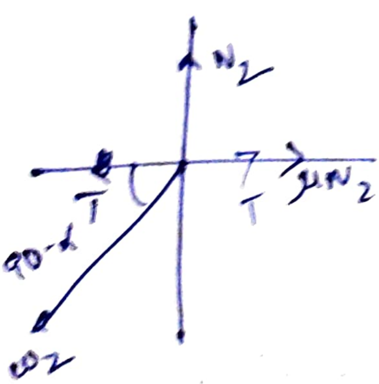
$N_1 - w_1 = 0$

$N_1 = w_1$ - (2)

$$\boxed{T = \mu w_1 \quad - (3)}$$

For w_2 :-





$\Sigma f_x = 0$

$\mu N_2 + T - w_2 \cos(90 - \alpha) = 0$

$\mu N_2 + T - w_2 \sin\alpha = 0$.

Σf_y $N_2 - w_2 \sin(90 - \alpha) = 0$

$N_2 = w_2 \cos\alpha$

$N_2 = w_2 \cos\alpha$

$$\mu^2\cos\alpha - \mu = \sin\alpha.$$

$$N_2 - w_2\cos\alpha = 0$$

$$\mu w_2\cos\alpha + T - w_2\sin\alpha = 0$$

$$w_2(\mu\cos\alpha - \sin\alpha) = -T$$

$$-\cancel{w_2}(\mu\cos\alpha - \sin\alpha) = \mu\cancel{w_1}$$

$$-\mu\cos\alpha + \sin\alpha = \mu$$

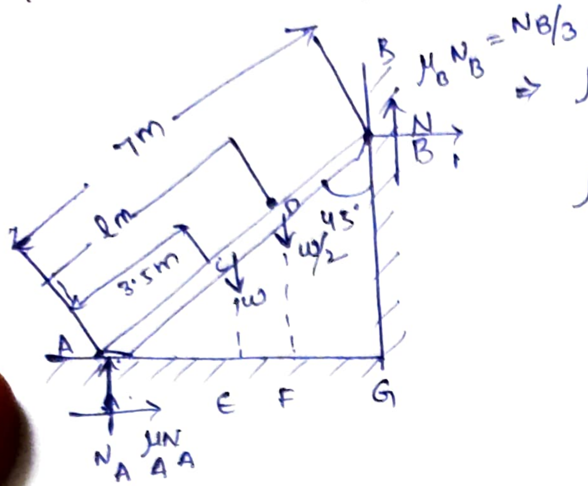
$$\mu + \mu\cos\alpha - \sin\alpha = 0.$$

$$\cancel{\mu(1+} \quad \mu = \frac{\sin\alpha}{1+\cos\alpha} = \frac{\cancel{2}\sin\alpha/2\,\cancel{\cos\alpha/2}}{\cancel{2}\cos^2\alpha/2\ \cos\alpha/2}$$

$$\mu = \tan\alpha/2$$

$$\boxed{\alpha = 2\tan^{-1}\mu}$$

A 7m long ladder rests x vertical wall with which it makes a angle of 45° and on a floor. If a man whose weight is 1/2 of that of ladder, climbes it at what (along the) distance along the ladder will he be when ladder is about to slip, the coff of friction b/w ladder and wall is 1/3 & b/w ladder and floor is 1/2

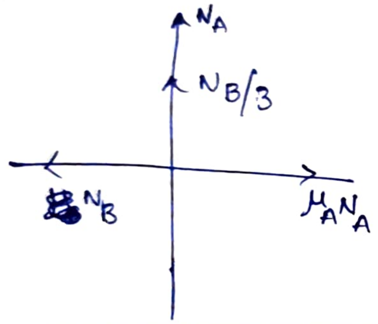


$\Rightarrow \mu_{lw} = 1/3 = \mu_A$

$\mu_{lf} = 1/2 = \mu_B$

$\mu_A N_A = N_A/2 \qquad \mu_B N_B = N_B/3$

$\Sigma f_x = 0$



$\frac{1}{2} N_A - N_B = 0$

$\boxed{N_A = 2N_B}$ -(1)

$\Sigma f_y = 0$

$N_A + \frac{N_B}{3} - w - w/2 = 0$

$2N_B + \frac{N_B}{3} = \frac{3w}{2}$

$\frac{7N_B}{3} = \frac{3w}{2}$

$14N_B = 9w$

$\boxed{N_B = \frac{9w}{14}}$

$\boxed{N_A = \frac{9w}{7}}$

$\Sigma M_A = 0$

$- w \times AE - \frac{w}{2} \times AF + \frac{1}{3} N_B \times AG$

$+ N_B \times BG$

$= - w \times 3.5 \cos 45° - \frac{w \times l}{2} \cos 45° + \frac{1}{3} \times \frac{9w}{14} \times 7\cos 45°$

$+ \frac{9}{14} w \times 7 \sin 45°$

$\Sigma m_A = 0$

$-\omega \times AE - \frac{\omega}{2} \times AF + \frac{1}{3} N_B \times AG + N_B \times BG = 0$

$-\cancel{\omega} \times 3.5 \cos 45 - \frac{\cancel{\omega}}{2} \times l \cos 45 + \frac{1}{\cancel{3}} \times \frac{\cancel{9}^{3}}{14} \times \cancel{\omega} \times 7 \cos 45$

$+ \frac{9}{14} \cancel{\omega} \times \cancel{7} \sin 45 = 0$

$-3.5 - \frac{l}{2} + \frac{21}{14} + \frac{9}{2} = 0$

$-\frac{7}{2} - \frac{l}{2} + \frac{3}{2} + \frac{9}{2} = 0$

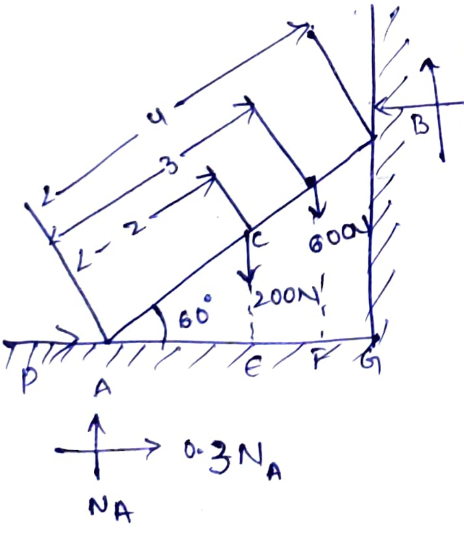
$\frac{12}{2} \cdot \frac{9}{2}$

$-\frac{7}{2} + \frac{12}{2} = \frac{l}{2}$

$\frac{17}{14} = $

$\boxed{l = 5m}$

A ladder of length 4m weighing 200N placed aganist a vertical wall as shown in fig. The coff of friction b/w wall & ladder is 0.2 & that floor & ladder is 0.3. In addition to self weight the ladder has to support a man weighing 600N at a distance of 3 m from A. Calculate the min horizontal force to be applied to prevent slipping



$\Sigma m_A = 0$.

$-200 \times 2\cos 60° - 600 \times 3\cos 60°$

$+ 0.2 N_B \times 4\cos 60° + N_B \times 4\sin 60°$

$= 0$.

$-200 \times \cancel{2} \times \frac{1}{\cancel{2}} - \cancel{600}^{300} \times 3 \times \frac{1}{\cancel{2}}$

$+ 0.2 N_B \times \frac{\cancel{4}^{2}}{\cancel{2}} + N_B \times \cancel{4}^{2} \times \frac{\sqrt{3}}{\cancel{2}} = 0$.

$-200 - 900 + 0.4 N_B + 2\sqrt{3} N_B = 0$.

$1100 = N_B(0.4 + 2\sqrt{3})$

$1100 = N_B(3.864)$

$N_B = 284.68$ N

$$\Sigma f_x = 0$$

$$P + 0.3N_A - N_B = 0$$

$$P = N_B - 0.3N_A \quad - (2)$$

$$\boxed{P = 81.76N}$$

$$\Sigma f_y = 0$$

$$N_A + 0.2N_B - 200 - 600 = 0$$

$$N_A + 56.93 - 800 = 0$$

$$N_A = 743.064\,N$$