Scope of Mechanics.

The Subject of Mechanics occupies unique position in the physical Sciences because it is fundamental to So many fields of Study. In its broadest sinse, Mechanics may be defined as the Science which describes and predicts. The conditions of rest or motion of body under the action of forces.

We consider only the mechanics of rigid bodies. which is divided into two parts. Statics and dynamics. In Statics we consider the effects and distribution of forces on rigid bodies which are at rest. In dynamics we consider the various motions of rigid bodies and the correlation of these motions with the forces passing them

ENGG MECHANICS.

- Mechanics of Solids
  - MECHANICS OF RIGID BODIES
    - STATICS
    - DYNAMICS
      - KINEMATICS
      - KINETICS.
  - MECHANICS OF DEFORMABLE BODIES.
    - THEORY OF ELASTICITY
    - THEORY OF PLASTICITY
    - STRENGTH OF MATERIALS
- Mechanics of Fluids.

* A rigid body is defied as a around of matics the Parts of which are fixed in position relative to one another.

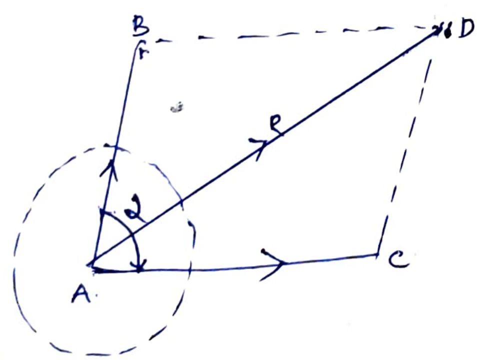
Actually solid bodies are never rigid, they deform under the action of applied forces. In many cases, the deformed as negligible compared to the size of the body and body may be assumed rigid

Bodies made of steel or cast iron

Force:

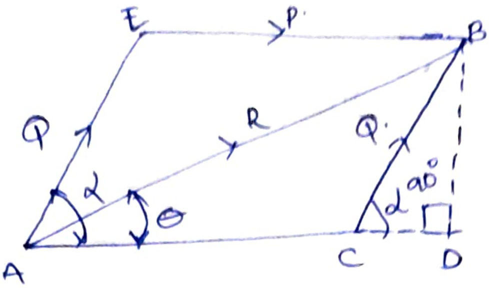
It may be defined as any action that tends to change the state of rest of a body to which it is obtained. (S.I - Newton)

Parallelogram Law:



Two forces Represented by vectors $\bar{AB}$, $\bar{AC}$ acting under an angle α are applied to a body at point A, This action is equivalent to the action of one force, represented by the vector $\bar{AD}$. obtained as the diagonal of the parallelogram constructed on the vectors $\bar{AB}$ and $\bar{AC}$ and directed as shown in fig.

The force $\bar{AD}$ is called the resultent of ~~two~~ forces. $\bar{AB}$, $\bar{AC}$ are called components of the forces $\bar{AD}$. This law is called as Parallelogram law.



$$AB^2 = AD^2 + BD^2$$

$$R^2 = (AC+CD)^2 + BD^2$$

$$= (P+Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

$$= P^2 + 2PQ\cos\alpha + Q^2\cos^2\alpha + Q^2\sin^2\alpha$$

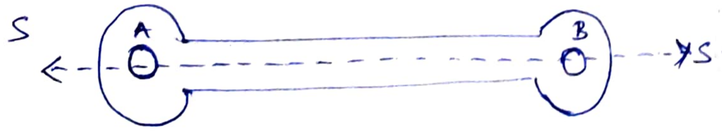
$$R^2 = P^2 + 2PQ\cos\alpha + Q^2$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\tan\theta = \frac{BD}{AD} = \frac{BD}{AC+CD} = \frac{Q\sin\alpha}{P+Q\cos\alpha}.$$

Equillibrium law:

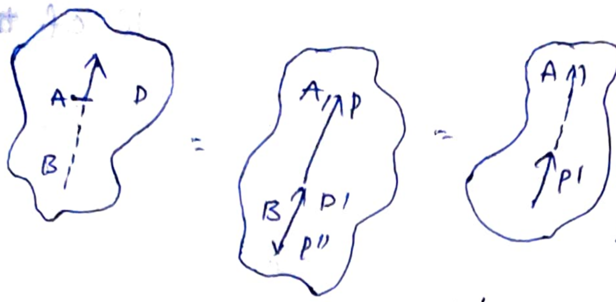
Two forces can be in equillibrium only if they are equal in magnitude, opp in direction and collinear in action.



Law of Superposition:

The action of a given System of forces on a rigid body will in no way be changed to be add to or Subtract from them another System of

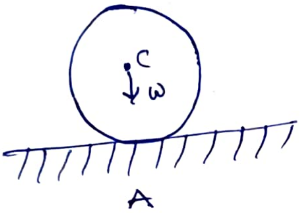
~~Feat to~~ forces in equillibrium.



$$\boxed{P = P' = P''}$$

Law of action and reactions:

Any pressure on a Support Causes on equal and opposite pressure from the Support. So that $\boxed{\text{opposite}}$ pressure from the $\boxed{\text{Support}}$ action and reaction are two equillalent & opposite forces.



FREE BODY DIAGRAMS

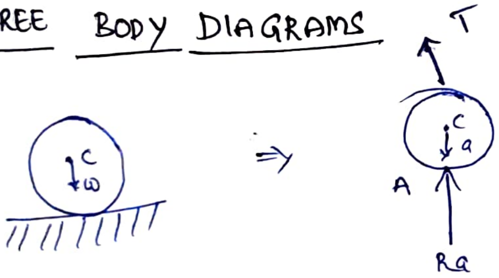


Fig-1

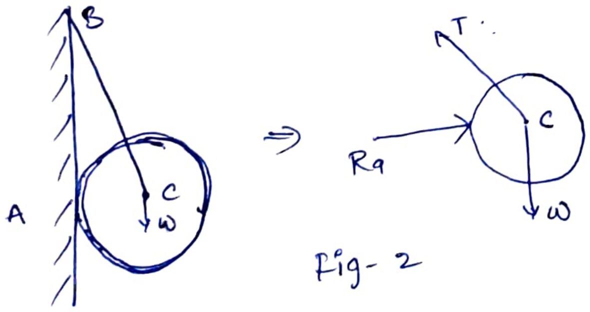
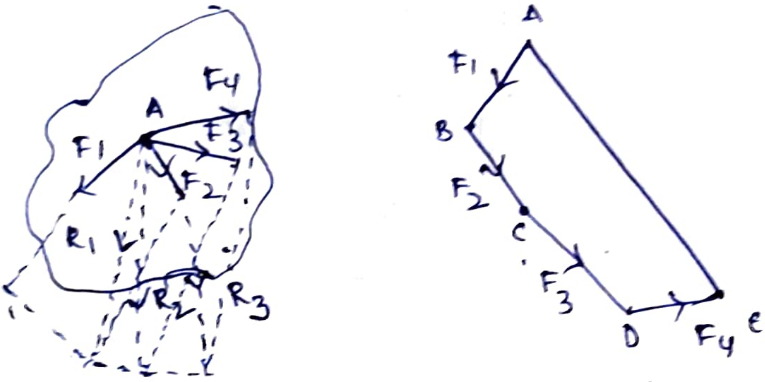


Fig-2

To investigate the equillibrium of the constrain body shall always remains in that to remove the Supports and replace them by the reaction which they exert on the body. Thus in the case of ball in fig 1 we remove the Supporting Surface and replace it by the reaction R_a that is exerts on that ball. In fig 2 the ball completely isolated from its Support and inwhich all forces acting on it are shown by vectors is called a free body diagram.

11-10-22

Composition & Resolution of forces :-

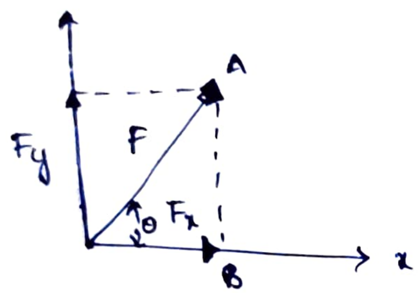


⇒ Composistion : The reduction of a given system of forces to the simplest system that will be its equivalent is called the problem of composistion of forces. If several forces F_1, F_2, F_3 and so on... applied to a body at one point, All acting in same place; they represent a system of forces that can be reduced to a single resultant force. If then becomes to find the resultant by sucessive applications of parallelogram law of the law of polygon of forces.

Law of polygon of forces:

It may be stated as if no. of co-planar forces are acting at a point such that they can represent in magnitude & direction by the sides of a polygon taken in an order, their resultant is represented in both magnitude & direction by the closing side of a polygon taken in opposite order

Resolving a force into rectangular components:-



$$\cos\theta = \frac{OB}{OA} = \frac{F_x}{F}$$

$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

$$\tan\theta = F_y / F_x$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

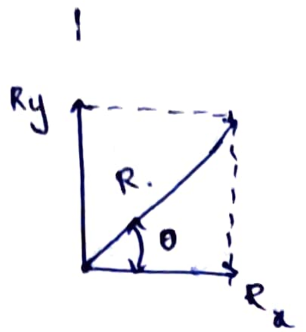
i.e,

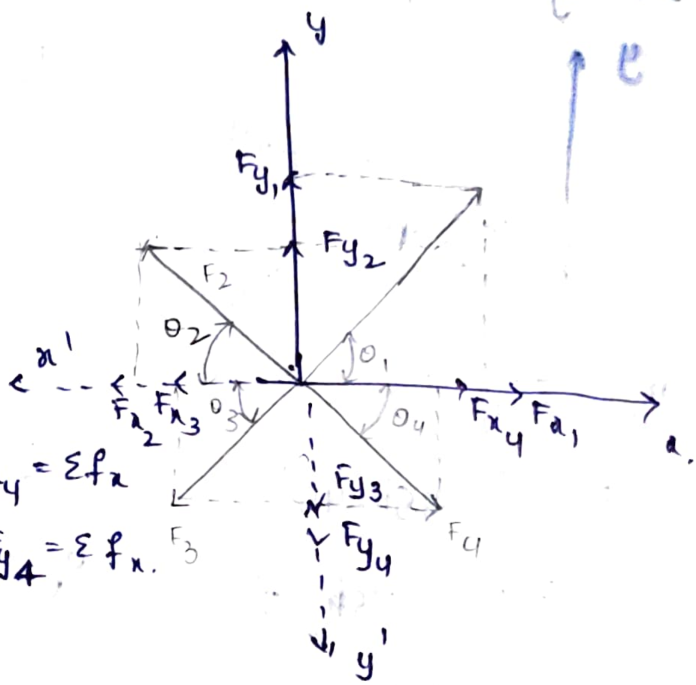
$$F_x^2 + F_y^2 = F^2\cos^2\theta + F^2\sin^2\theta$$

$$F_x^2 + F_y^2 = F^2$$

$$F = \sqrt{F_x^2 + F_y^2}$$

Resultant of a several concurrent coplanar forces by summation of rectangular components :-





$R_x = F_{x_1} + F_{x_2} + F_{x_3} + F_{x_4} = \Sigma f_x$

$R_y = F_{y_1} + F_{y_2} + F_{y_3} + F_{y_4} = \Sigma f_x.$

$\Rightarrow$

$R = \sqrt{R_x^2 + R_y^2}$

$\tan\theta = \dfrac{R_y}{R_x}$, $\theta = \tan^{-1}\left(\dfrac{R_y}{R_x}\right)$

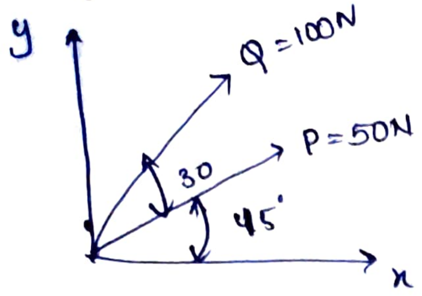
Equations of equilibrium for system of concurrent forces in a plane:

We can find the resultant R of several forces F_1, F_2, F_3, F_4 --- using method of projection as explained earlier. If their resultant is zero the particle is said to be in equilibrium.

i.e, $R = \sqrt{R_x^2 + R_y^2}$; $R_x = 0$, $R_y = 0$

$\left.\begin{array}{l} \Sigma f_x = 0 \\ \Sigma f_y = 0 \end{array}\right\}$ Are called equations of equilibrium. If the two eqns of equilibrium can be solved to find max of two unknowns.

Two forces are acting at a point as shown in fig. Determine the magnitude & direction of resultant



$R_x = \Sigma f_x$

$= P\cos 15° + Q\cos 45°$

$= 50\cos 15 + 100\cos 45$

$= 50 \times 0.96 + 100 \times 0.707$

$= 70.71 + 48.$

$R_x = 118.71\ N.$

$R_y = \Sigma f_y$

$= P\sin 15° + Q\sin 45°$

$= 50\sin 15 + 100\sin 45°$

$= 50 \times 0.25 + 70.71$

$= \cancel{103.2} + 83.65 \Rightarrow R_y = 83.65\ N$

$R_y = 103.21\ N$

$R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{24744.36}$

$\boxed{R = 157.30\ N}$

$\cancel{\tan\phi = \frac{R_y}{R_x}}$

$= \cancel{\frac{103.21}{118.71}}$

$\phi = 0.715$

$R = \sqrt{R_x^2 + R_y^2}$

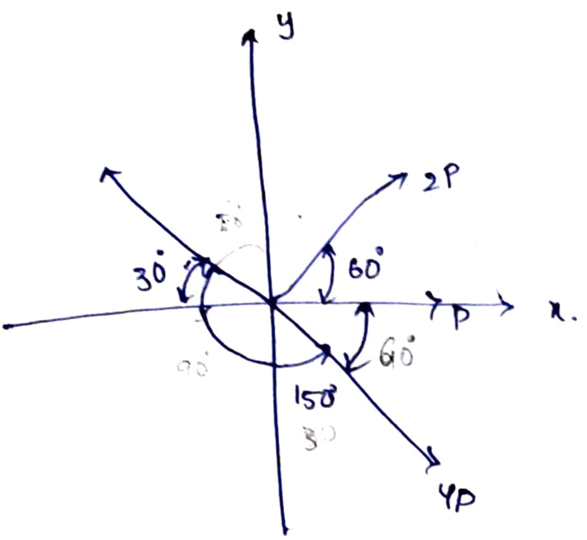
$= \sqrt{21158.32}$

$= 145.46\ N$

$\tan\phi = \frac{R_y}{R_x}$

$\phi = 35.104$

Find the magnitude & direction of resultant R of 4 concurrent forces acting as shown in fig



$$\Rightarrow \quad R_x = \Sigma f_x$$

$$= P + 2P\cos 60^\circ \cancel{+P}\, 3\sqrt{3}\cos 30^\circ + 4P\cos 60^\circ$$

$$= P + 2P \times \frac{1}{2} - 3\sqrt{3} \cdot \frac{\sqrt{3}}{2} P + 4P \cdot \frac{1}{2}$$

$$= P + P - \frac{9P}{2} + 2P$$

$$R_x = -P/2$$

$$R_y = \Sigma f_y .$$

$$= 2P\sin 60^\circ + 3\sqrt{3}P\sin 30^\circ - 4P\sin 60^\circ$$

$$= \cancel{2}P \times \frac{\sqrt{3}}{\cancel{2}} + 3\sqrt{3} \times \frac{1}{2} P - \overset{2}{\cancel{4}}P \cdot \frac{\sqrt{3}}{\cancel{2}}$$

$$= \sqrt{3}P + \frac{3\sqrt{3}}{2}P - 2P\sqrt{3}$$

$$= -\sqrt{3}P + \frac{3\sqrt{3}}{2}P$$

$$= \frac{\sqrt{3}}{2}P$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{P^2/4 + \frac{3}{4}P^2} \Rightarrow R = P.$$

R_x

$$= \frac{\frac{\sqrt{3}}{2} P}{-P/2}$$

$$= \frac{\sqrt{3}\cancel{P}}{\cancel{2}} \times \frac{-\cancel{2}}{\cancel{P}}$$

$$\tan\phi = -\sqrt{3}.$$

$$\phi = -60^\circ$$

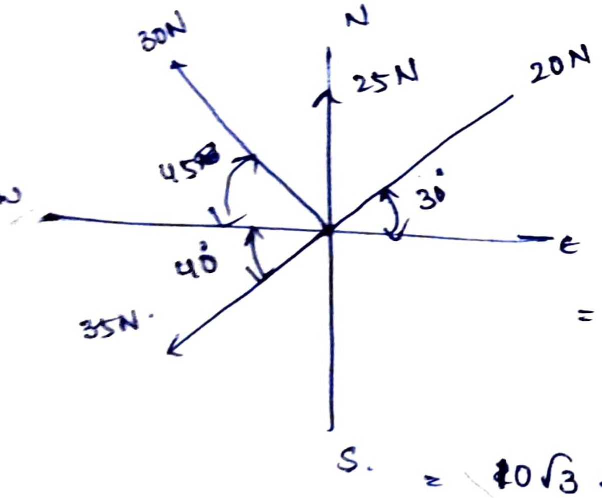
Following forces act a point

(i) 20N inclined at 30° towards north of east

(ii) ~~2g~~N towards north

(iii) 30N inclined at 45° towards north of west

(iv) 35N inclined at 40° towards south of west



$\Rightarrow$

$$R_x = \Sigma f_x$$

$$= 20\cos 30 \cancel{+25} + 30\cos 45 - 35\cos 40^\circ$$

$$= 10\sqrt{3} \cancel{+25} + 30/\sqrt{2} - 35$$

$$= 10\sqrt{3} \cancel{+25} + 30/\sqrt{2} - 26.81$$

$$= \cancel{17.32}\ 63.533 - 26.81$$

$$= \cancel{36.7}\ 11.72\,N$$

$$\tan\phi = \frac{R_y}{R_x}$$

$$= \frac{\frac{\sqrt{3}}{2}P}{-P/2}$$

$$= \frac{\sqrt{3}\cancel{P}}{\cancel{2}} \times \frac{-\cancel{2}}{\cancel{P}}$$

$$\tan\phi = -\sqrt{3}.$$

$$\phi = -60^\circ$$

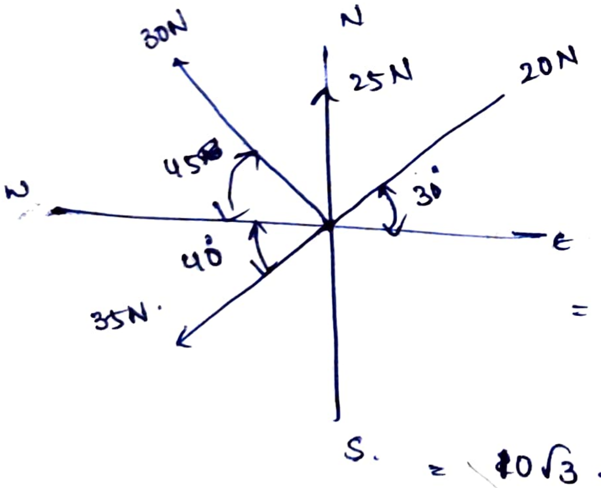
Following forces act a point

(i) 20N inclined at 30° towards north of east

(ii) 25N towards north

(iii) 30N inclined at 45° towards north of west

(iv) 35N inclined at 40° towards south of west



$\Rightarrow$

$$R_x = \Sigma f_x$$

$$= 20\cos 30 \cancel{+25} + 30\cos 45 - 35\cos 40^\circ$$

$$= 10\sqrt{3} \cancel{+25} + 30/\sqrt{2} - 35$$

$$= 10\sqrt{3} \cancel{+25} + 30/\sqrt{2} - 26.81$$

$$= \cancel{17.32}\ 63.533 - 26.81$$

$$= \cancel{36.7}\ 11.72\,N$$

$$R_x = 10\sqrt{3} - 30/\sqrt{2} - 35\cos 40^\circ$$

$$= 17.32 - 21.21 - 26.81$$

$$= -30.07 \text{ N}$$

$$R_y = \Sigma f_y$$

$$= 20\sin 30 + 25 + 30\sin 45 - 35\sin 40^\circ$$

$$= 35 + \frac{30}{\sqrt{2}} - 35\sin 40$$

$$= 35 - 22.49 + 21.21$$

$$= 33.723 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(30.07)^2 + (33.723)^2}$$

$$= 45.6 \text{ N}$$

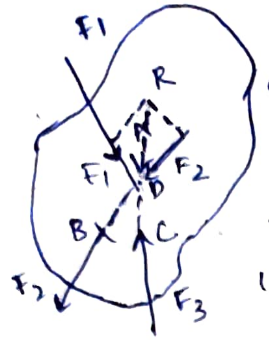
$$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{33.72}{-30.07}\right)$$

$$= -47.7^\circ \text{ or } 132.3$$

12-10-22

EQUILIBRIUM OF A BODY SUBJECTED TO THREE FORCES:-

(Theorm of 3 forces)

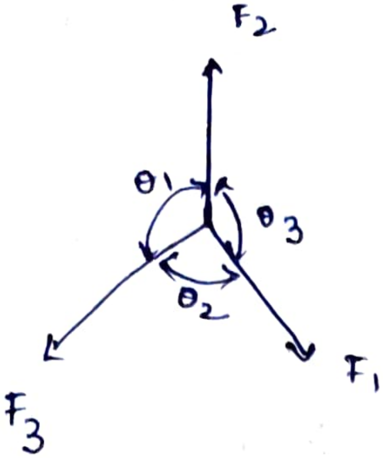


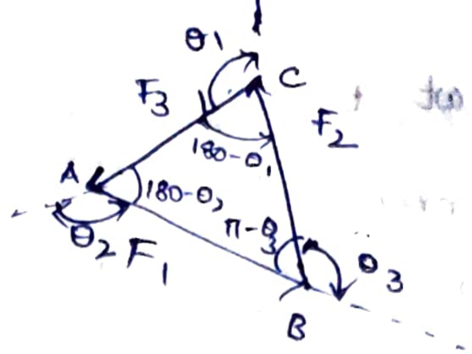
⇒ When the body is acted upon by 3 coplanar forces it can be in equillibrium if (their) either the lines of action the 3 forces intersect at one point (concurrent) (or) they are parallel

⇒ let us consider a rigid body with 3 non parallel ~~planar~~ forces f_1, f_2, f_3 acting at points A, B & C respectively. Let the lines of action of forces f_1 & f_2 intersect at D. Transmit the forces f_1 & f_2 to act point at D. Replace the forces f_1 & f_2 by their resultant R acting at D.

The 3rd force F_3 and resultant R can keep the body in equllibrium if they have the same action of lines (or) are collinear. So, the 3rd force F_3 must allow pass through the point D. So we can conclude that three forces are concurrent

Three concurrent forces in equillibrium must form a closed triangle of force when drawn in head to tail fashion as shown in fig.





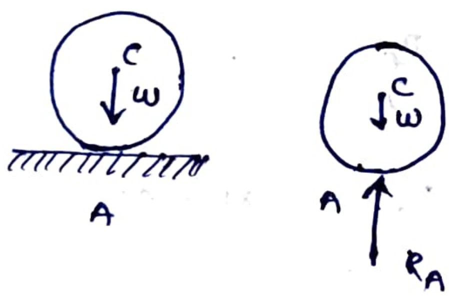
LAMI'S THEORM :- (Law of Sines)

If a body is in equillibrium under the action of 3 forces, each force is proportional to the sine of the angle b/w the other two forces

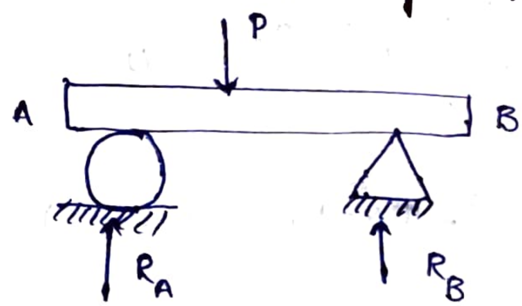
$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

Types of supports and its reactions:-

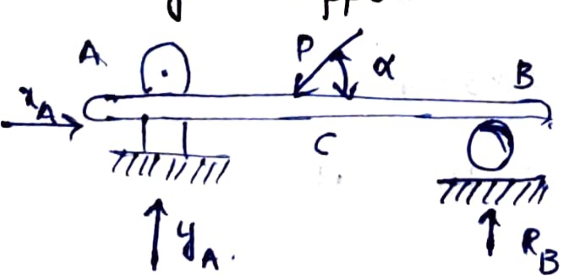
1) Friction less support



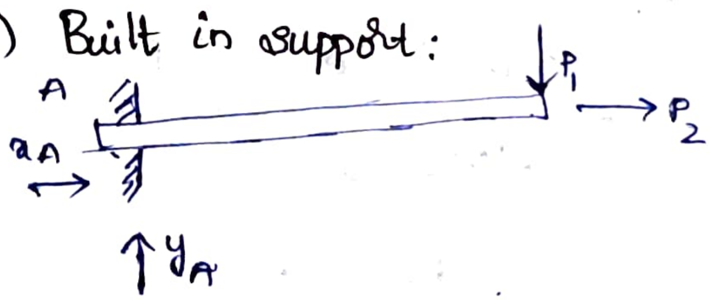
2) Roller and knife edge support:-



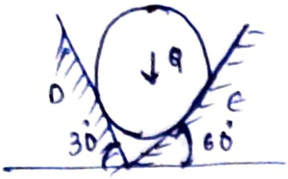
3) Hinged support



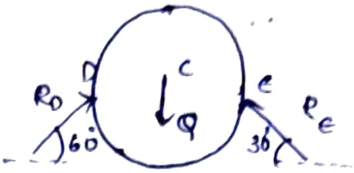
4) Built in support:

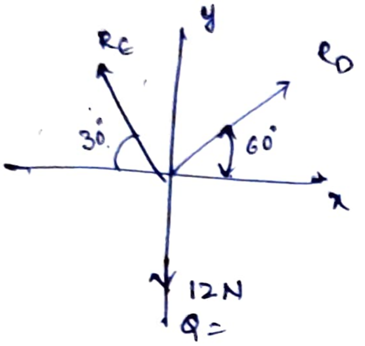


A ball of wt $t = 12$ kg trough as shown in fig. Determine the forces exerted on its sides of trough at D & E if all surfaces are perfectly smooth



⇒





$[~~R_x = R_D \cos 60° +~~]$

$R_x = \Sigma f_x$

$= R_D \cos 60° + R_E \cos 30°$

$R_y = \Sigma f_y$

$= R_D \sin 60 + R_E ~~\cos~~ \sin 30 - 12N$

$\frac{R_D}{2} + \frac{\sqrt{3} R_E}{2} = 0$

$\frac{\sqrt{3} R_D}{2} + \frac{R_E}{2} = 12\,N$

$R_D + \sqrt{3} R_E = 0$ — (1)

$\sqrt{3} R_D + R_E = 24N$ — (2)

$\cancel{\sqrt{3}} R_D - 3 R_E = 0$

(−) $\cancel{\sqrt{3}} R_D + R_E = 24$

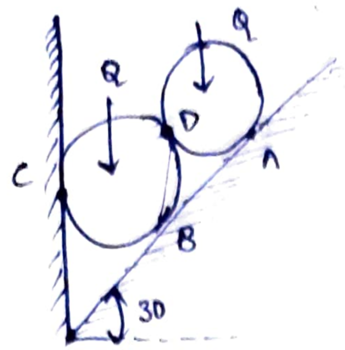
$\cancel{-}4R_E = \cancel{-}24$

$R_E = 6N$

$R_D = \sqrt{3} R_E$

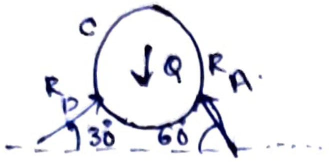
$R_D = \sqrt{3} \times 6$

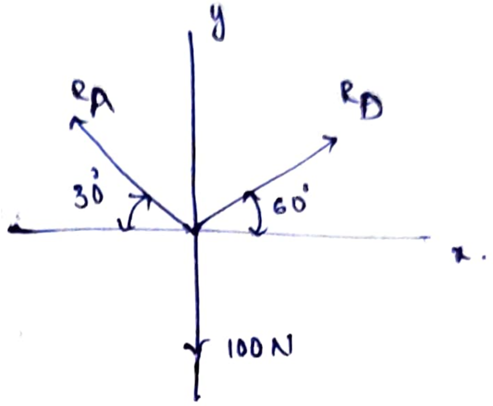
$R_D = 10.39\,N.$



Two identical rollers each of wt Q = 100 N are supported by an inclined plane & vertical wall as shown in fig. Assuming smooth surfaces, find the rxns induced at the points of support A, B, C

→ 1st roller :-





$R_D \cos 60° - R_A \cos 30 = 0$

$\left[\cancel{R_D = \sqrt{3} R_A}\right]$ ∴ $\sqrt{3} R_D = R_A$

$R_D \sin 60 + R_A \sin 30 = 100$

$\sqrt{3} R_D + R_A = 200$

$\sqrt{3}(\sqrt{3} R_D) + R_D = 200$

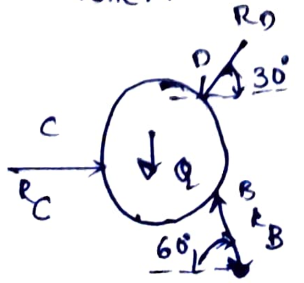
$4 R_D = 200$

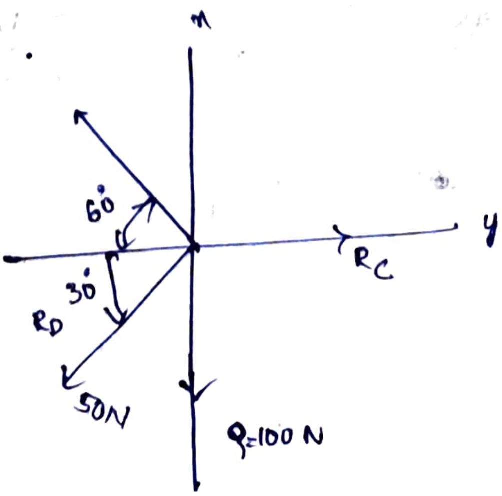
$R_D = 50\,N$

$R_A = \sqrt{3} R_D$

$R_A = 86.6\,N$

2nd roller :-





$\Sigma f_x = 0$ $\quad\quad\quad$ $\Sigma f_y = 0$

$R_C - R_B \cos 60 - R_D \cos 30 = 0$ $\quad\quad$ $R_B \sin 60 - R_D \sin 30 - 100 = 0$

$R_C - R_B/2 - \cancel{50}^{25} \times \frac{\sqrt{3}}{\cancel{2}} = 0$ $\quad\quad$ $\frac{\sqrt{3} R_B}{2} - 25 - 100 = 0$

$R_C - 72.168 - 43.30 = 0$ $\quad\quad$ $\sqrt{3} R_B / 2 = 125$

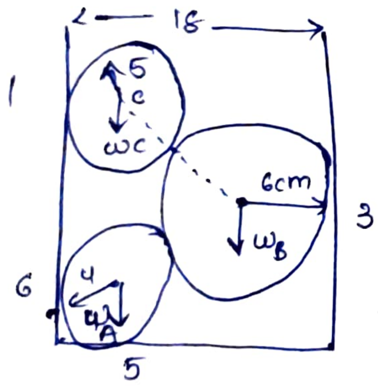
$R_C = 115.47 N$ $\quad\quad$ $\sqrt{3} R_B = 250$

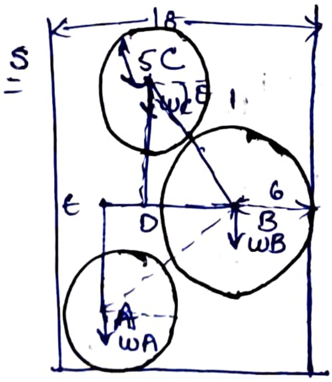
$R_B = \frac{250}{\sqrt{3}}$

$R_B = 144.33 N$

⇒ 3 cylinders are(~~placed~~) piled with a rectangular channel as shown in fig. Determine the rxn R_6 b/w cyclinder A & vertical wall of channel given

$w_A = 15 N$, $w_B = 40 N$, $w_C = 20 N$





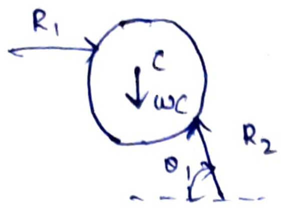
From ΔBCD, $\cos\theta_1 = \frac{BD}{BC} = \frac{18-5-6}{5+6} = 7/11$

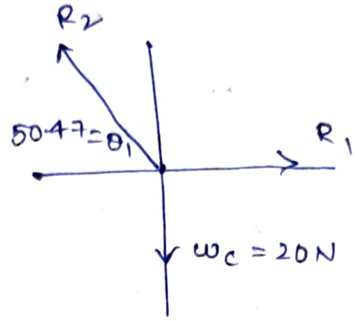
$\theta_1 = 50.47$

From ΔBeA, $\cos\theta_2 = \dfrac{Be}{AB} = \dfrac{18-4-6}{6+4} = 8/10$

$\theta_2 = 36.86$

cylinder-C :-





$\Sigma f_x = 0$

$R_1 \cos 0^\circ - R_2 \cos 50.47 = 0$

$R_1 = R_2 \cos 50.47$

$R_1 = R_2 (0.6364)$

$R_1 = 16.49 N$

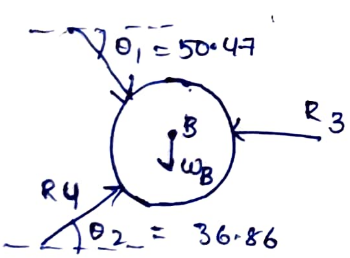
$\Sigma f_y = 0$

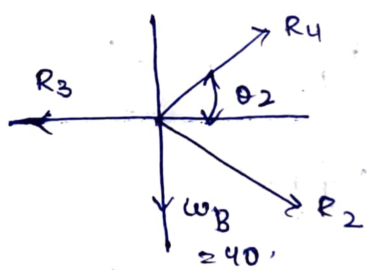
$R_2 \sin\theta_1 - 20 = 0$

$R_2 \sin 50.47 = 20$

$R_2 = 25.9 N.$

cylinder B :-





$\Sigma f_x = 0$

$R_4 \cos\theta_2 - R_3 + R_2 \cos\theta_1 = 0$

$(R_4 (0.8) - R_3) + (25.9) \cos(\theta_1) = 0$

$0.8 R_4 - R_3 + 16.48 = 0$

$0.8 R_4 - R_3 = -16.48$

$\Sigma f_y = 0$

$R_4 \sin\theta_2 - R_2 \sin\theta_1 - 40 = 0$

$R_4 (0.599) - R_2 (0.771) = 40$

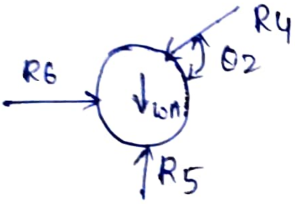
$R_4 (0.599) = 19.97 + 40$

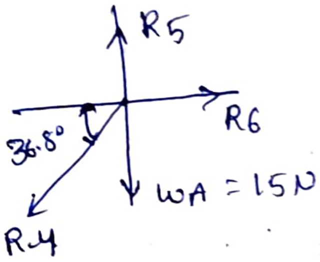
$R_4 = 100.11\ N$ $\qquad$ $R_2 = 25.9\ N$

$0.8 R_4 - R_3 = -16.48$

$R_3 = 96.57\ N$

cylinder - A:-





$\Sigma f_x = 0$

$R6 - R4 \cos 36.86 = 0$

$R6 = 100.11 (\cos 36.86)$

$\boxed{R6 = 80.091N}$

$\Sigma f_y = 0$

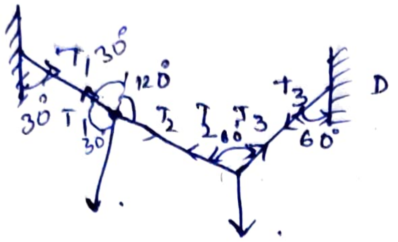
$R5 - R4 \sin 36.86 - 15 = 0$

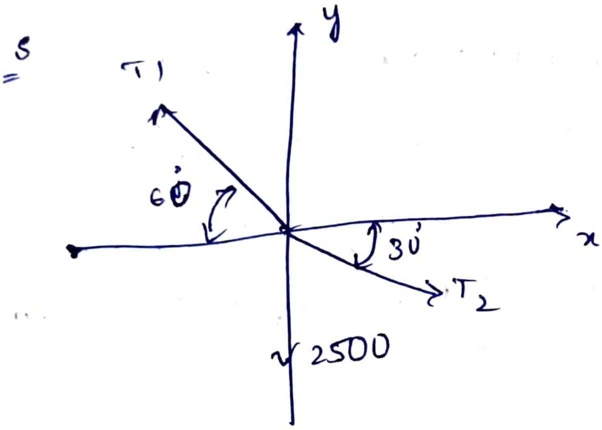
$R5 - (100.11) - 15 = 0$

$\boxed{R5 = 75.052N}$

18-10-22

Two equal loads of 2500 N are supported by a flexible string ABCD at points B, C. Find the tensions in the portion AB, BC, CD of the string.





$T_2 \cos 30° - T_1 \cos 60 = 0.$

$\frac{\sqrt{3} T_2}{2} - \frac{1}{2} = 0$

$T_1 = \sqrt{3} T_2$

$-2500 - T_2 \sin 30 + T_1 \sin 60 = 0$

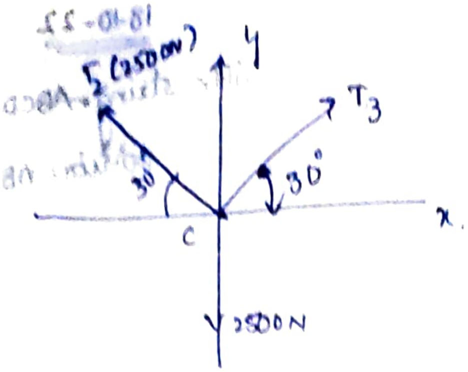
$-2500 - T_2/2 + (\sqrt{3} T_2) \times \frac{\sqrt{3}}{2} = 0$

$-2500 - \frac{T_2}{2} + \frac{3T_2}{2} = 0$

$2500 = \frac{2T_2}{2}$

$\boxed{T_2 = 2500 N}$

$T_1 = 4330.1 N$



$T_3 \cos 30 + T_2 \cos 30 = 0$

$T_3 \cancel{\cos} \sqrt{3} \times \frac{1}{2} + 4330N = 0$

$T_3 = \cancel{4330}\ 2500\,N$

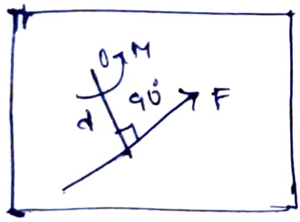
$T_3 = 2500\,N$

$\Rightarrow$ i.e, $T_1 = 4330\,N$; $T_2 = 2500N$; $T_3 = 2500N$

Moment of force :-

A force can rotate a nut when applied by a wrench (or) can open a door while the door rotates on its hinges. A force does come up can produce rotary motion besides producing a translatory motion. The ~~weather~~ measure of this turning effect produced by force on a body can be called as moment of force

Moment of force about an axis :-

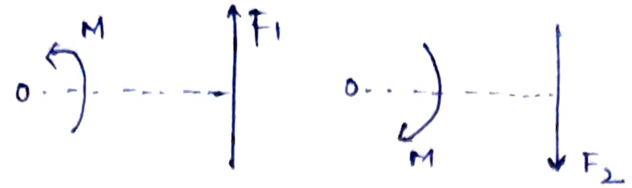
-> 

⇒ · Moment of force about an axis through a point is equal to product of force and ⊥r distance of point from line of action of ~~point~~ force.

$M_O = F \times d$ [N.m] → Joule

= Pascal (N/m^2)

→ The point O is called moment centre and the distance d is called arm of force



(+ve)

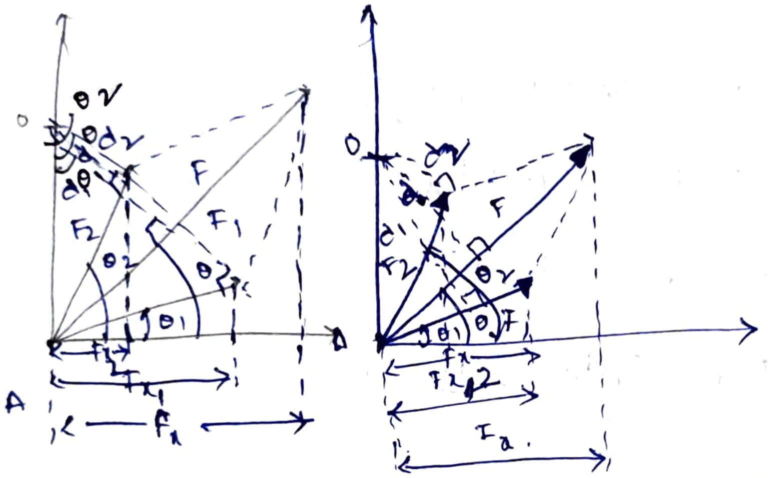
(anti clockwise)

(-ve)

(clockwise)

[upwards = clock wise
downwards - anti clock wise

VARIGNON'S THEORM:



→ The moment of a force about an axis is equal to the sum of the moments of its components about the same axis.

→ Consider a force F acting at a point A and having components F_1 & F_2 in any two directions. let us choose any point O lying in plane of forces as moment centre. Attach a 2 rectangular axis the y axis is along the line AO and x axis is along the ⊥r to it as shown in fig.

Moment of force about O = F x d.

$$F \times d = F(OA\cos\theta).$$

$$= OA \times F\cos\theta$$

$$F \times d = OA \times F_x \quad \rightarrow (1)$$

$\Rightarrow$ Moment of force F_1 about 'O'

$$F_1 \times d_1 = F_1(OA\cos\theta_1)$$

$$F_1 \times d_1 = OA \times Fx_1 \qquad -(2)$$

Moment of force F_2 about 'O'

$$F_2 \times d_2 = F_2(OA\cos\theta_2)$$

$$F_2 \times d_2 = OA \times Fx_2 \qquad -(3)$$

Adding 2, 3,

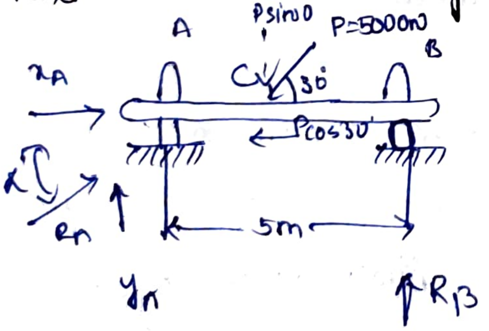
$$F_2 d_1 + F_2 d_2 = OA(Fx_1 + Fx_2)$$

$$F_2 d_1 + F_2 d_2 = OA \times Fx.$$

$$F_1 d_1 + F_2 d_2 = F \times d$$

$$\boxed{Fd = F_1 d_1 + F_2 d_2}$$

A force of 2500N is applied at centre c of a beam AB of length 5m. Find the rxns at the hinge and roller supports



$$\Sigma M_A = 0$$

$$R_B \times 5 - 5000\sin 30 \times 2.5 = 0.$$

$$5R_B - 2500 \times 2.5 = 0.$$

$$\boxed{R_B = 1250\ N}$$

$\Rightarrow \quad \Sigma f_x = 0.$

$-P\cos 30 + X_A = 0.$

$X_A = \dfrac{5000 \times \sqrt{3}}{2}$

$x_A = 4330\,N$

$\Sigma f_y = 0.$

$y_A + R_B - 5000\sin 30 = 0$

$y_A + 1250N = 2500.$

$\boxed{y_A = 1250N}$

$R_A = \sqrt{(4330N)^2 + (1250)^2}$

$R_A = 4506.81\,N$

$\alpha = \tan^{-1}\left(\dfrac{y_A}{x_A}\right) = \tan^{-1}\left(\dfrac{1250}{4330}\right)$

$\boxed{\alpha = 16.1^\circ}$