Simulation and Risk Analysis

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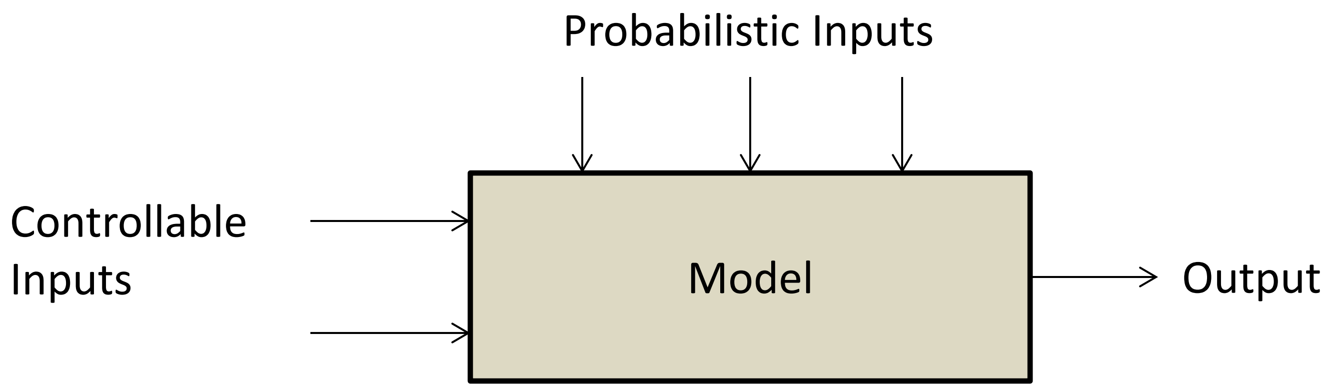
Simulation

- It is the process of generating probabilistic inputs and computing the value of the output.
- A method for learning about a real system by experimenting with a model that represents the system.
- A quantitative approach to decision making.
- It is not an optimization technique.

Simulation Model



The mathematical expressions and logical relationships that describe how to compute the value of the outputs given the values of the inputs.

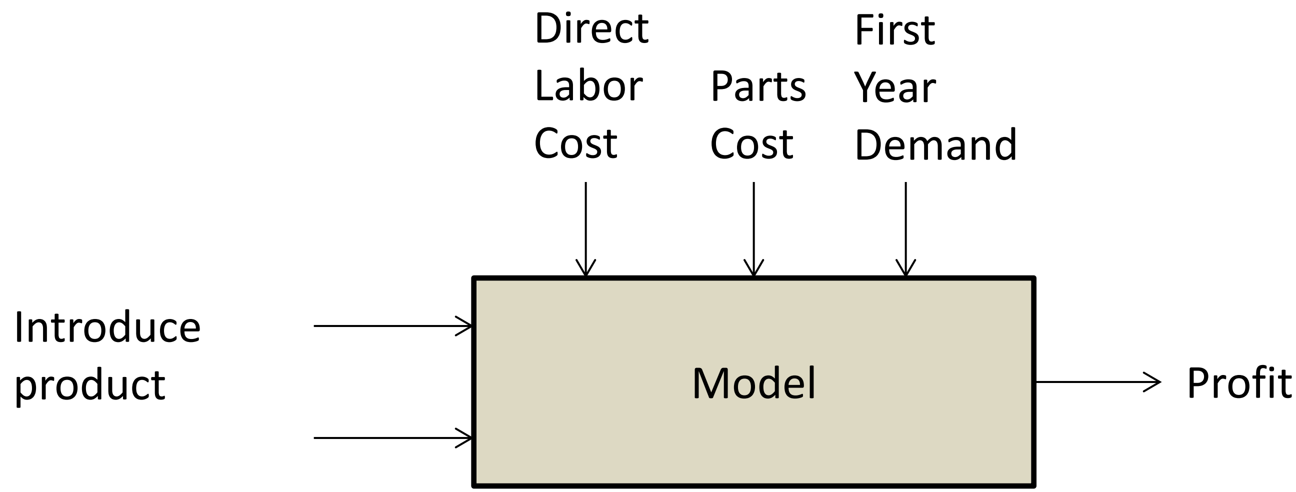
Any simulation model has two inputs:

- controllable inputs
- probabilistic inputs

Simulation Experiment

- Select the value or values for controllable inputs.
- The values for the probabilistic inputs are randomly generated.
- Study how using the variety of input values in simulation model affect or change the output.
- By viewing the simulation results, decision recommendations for the controllable inputs that will provide the desired output for the real system.

Example: New Product Development



To determine the probability that a new product will be profitable.

- Controllable input: whether to introduce product.
- Probabilistic input: demand, parts cost, labor cost
- Output: profit

Risk Analysis

- It is the process of predicting the outcome of a decision in the face of uncertainty.
- A problem that involves considerable uncertainty: the development of a new product.
- It is about both the probability of loss and the magnitude of loss.

PortaCom Project

PortaCom manufactures personal computers and related equipment. PortaCom's product design group developed a prototype for a new high-quality portable printer. The new printer features an innovative design and has the potential to capture a significant share of the portable printer market.

Preliminary marketing and financial analyses provide the following

Selling price = $249 per unit

Administrative cost = $400,000

Advertising cost = $600,000

The cost of direct labor, the cost of parts, and the first-year demand for the printer are not known with certainty and are considered probabilistic inputs. PortaCom's best estimates of these inputs are $45 per unit for the direct labor cost, $90 per unit for the parts cost, and 15,000 units for the first-year demand.

PortaCom would like an analysis of the first-year profit potential for the printer. Because of the PortaCom's tight cash flow situation, management is particularly concerned about the potential for a loss.

PortaCom What-If Analysis

- A what – if analysis involves generating values for the probabilistic inputs (direct labor cost, parts cost, and first-year demand) and computing the resulting value for the output (profit).
- With a selling price of \$249 per unit and administrative plus advertising costs equal to \$400,000 + \$600,000=\$1,000,000. The PortCom profit model is

Profit=(\$249 – Direct labor cost per unit- parts cost per unit)(demand) - \$1,000,000

PortaCom Profit Model

Letting,

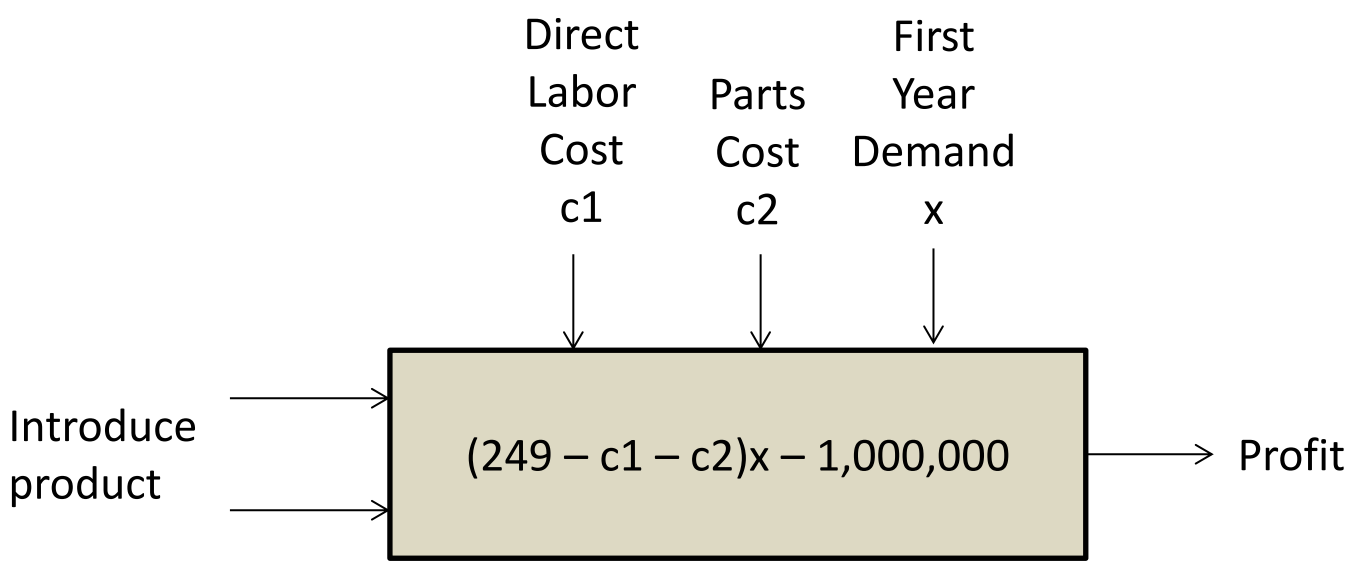
$c1$ = direct labor cost per unit

$c2$ = parts cost per unit

x = first-year demand

The profit model for the first year can be written as follows:

$$\text{Profit} = (249 - c1 - c2)\,x - 1{,}000{,}000$$



PortaCom What-If Analysis

Base case scenario:

$$\text{Profit} = (249 - 45 - 90)(15000) - 1{,}000{,}000 = 710{,}000$$

Suppose that PortaCom believes that direct labor costs could range from \$43 to \$ 47 per unit, parts cost could range from \$80 to \$100 per unit, and first year demand could range from 1500 to 28,500 units.

Worst Case Scenario:

$$\text{Profit} = (249 - 47 - 100)(1500) - 1{,}000{,}000 = -847{,}000$$

Best Case Scenario:

$$\text{Profit} = (249 - 43 - 80)(28{,}500) - 1{,}000{,}000 = 2{,}591{,}000$$

Conclusion

- The profit value can range from a loss of \$847,000 to a profit of \$2,591,000 with a base case scenario value of \$710,000.
- Although the base case profit of \$710,000 is possible, the what-if analysis indicates that either a substantial loss or a substantial profit is possible.
- What-is analysis doesn’t indicate the likelihood of the various profit or loss values. We do not know the probability of loss.

Using Simulation to perform Risk Analysis

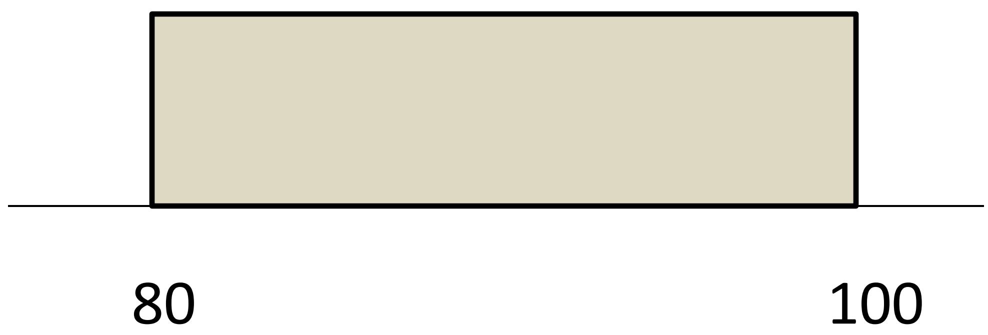
- It is like playing out many what-if scenarios by randomly generating values for the probabilistic inputs.
- It allows us to assess the probability of a profit and the probability of a loss.
- It requires generating values for the probabilistic inputs that are representative of what we may observe in practice.
- To generate such values we must know the probability distribution for each probabilistic input.

Probabilistic Inputs: Direct Labor Cost

Direct Labor Cost per Unit	Probability
Rs 43	0.1
Rs 44	0.2
Rs 45	0.4
Rs 46	0.2
Rs 47	0.1

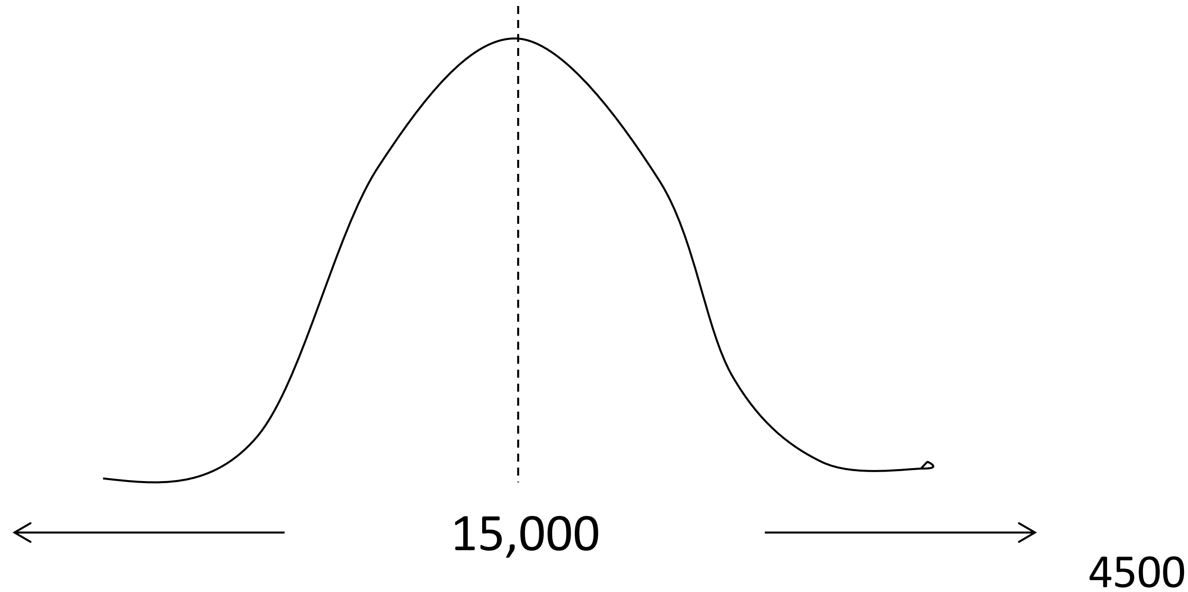
It will range from Rs 43 to Rs 47 per unit and is described by the discrete probability distribution.

Probabilistic Inputs: Parts Cost



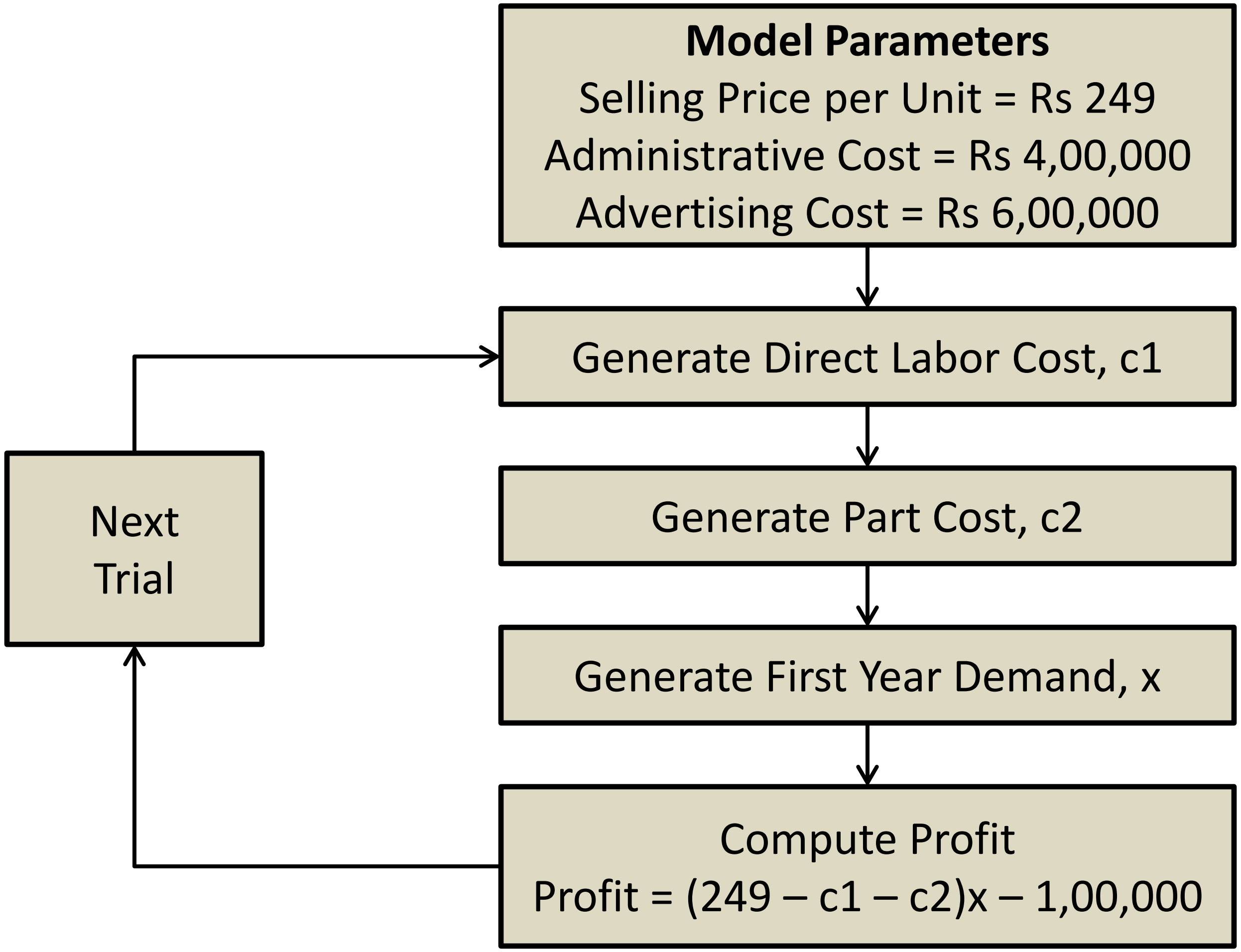
It will range from Rs 80 to Rs 100 per unit and is described by the uniform probability distribution.

Probabilistic Inputs: First Year Demand



The first year demand is described by the normal distribution. The mean or expected value of first-year demand is 15,000 units. The standard deviation of 4500 units describes the variability in the first-year demand.

Flowchart for PortaCom Simulation



Generating Probabilistic Input Values

- Random numbers and the probability distributions associated with each probabilistic input are used to generate representative values.
- Using computer generated random numbers to generate a value of
  - Direct labor cost per unit //discrete probability distribution
  - Parts cost per unit //uniform probability distribution
  - First-year demand per unit //normal probability distribution

Computer Generated Random Number

- They are randomly selected decimal numbers from 0 up to, but not including, 1.
- They are equally likely and are uniformly distributed over the interval from 0 to 1.
- It can be obtained using built-in functions available in computer simulation packages and spreadsheets.
  – RAND() function in Excel.

Random Number Intervals for Generating Values of Direct Labor Cost Per Unit

Direct Labor Cost per Unit	Probability	Interval of Random Numbers
Rs 43	0.1	0.0 but less than 0.1
Rs 44	0.2	0.1 but less than 0.3
Rs 45	0.4	0.3 but less than 0.7
Rs 46	0.2	0.7 but less than 0.9
Rs 47	0.1	0.9 but less than 1.0

Random Generation of 10 values for the Direct Labor Cost Per Unit

Trial	Random Number	Direct Labor Cost
1	0.9109	47
2	0.2841	44
3	0.6531	45
4	0.0367	43
5	0.3451	45
6	0.2757	44
7	0.6859	45
8	0.6246	45
9	0.4936	45
10	0.8077	46

Generating Values for Parts Cost Per Unit

With a uniform probability distribution, the following relationship between the random number and the associated value of the parts cost is used.

$$\text{Parts Cost} = a + r(b - a)$$

where

r – random number between 0 and 1

a – smallest value for parts cost

b – largest value for parts cost

For PortaCom example , a = 80 and b = 100. This leads to the following formula for generating the parts cost

$$\text{Parts Cost} = 80 + r\,(100 - 80) = 80 + r20$$

Random Generation of 10 values for the Parts Cost Per Unit

Trial	Random Number	Direct Labor Cost
1	0.2680	85.36
2	0.5842	91.68
3	0.6675	93.35
4	0.9280	98.56
5	0.4180	88.36
6	0.7342	94.68
7	0.4325	88.65
8	0.1186	82.37
9	0.6944	93.89
10	0.7869	95.74

Generating Values for First-Year Demand

The first-year demand is normally distributed with a mean of 15,000 units and a standard deviation of 4500 unit.

Using Excel the following formula can be placed into a cell to obtain a value for a probabilistic input that is normally distributed:

=NORMINV(RAND(), Mean, Standard Deviation)

For PortaCom problem

=NORMINV(RAND(),15000,4500)

Random Generation of 10 values for First-Year Demand

Trial	Random Number	Direct Labor Cost
1	0.7005	17,366
2	0.3204	12,900
3	0.8968	20,686
4	0.1804	10,888
5	0.4346	14,259
6	0.9605	22,904
7	0.5646	15,732
8	0.7334	17,804
9	0.0216	5,902
10	0.3218	12,918

PortaCom Simulation Results for 10 trials

Trial	Labor Cost per Unit	Parts Cost per Unit	Units Sold	Profit
1	47	85.36	17,366	1,025,570
2	44	91.68	12,900	461,828
3	45	93.35	20,686	1,288,906
4	43	98.56	10,888	169,807
5	45	88.36	14,259	648,911
6	44	94.68	22,904	1,526,769
7	45	88.65	15,732	814,686
8	45	82.37	17,804	1,165,501
9	45	93.89	5,902	-350,131
10	46	95.74	12,918	385,585
Total	449	912.64	151,359	7,137,432
Average	44.90	91.26	15,136	713,743

Excel Worksheet for PortaCom Problem

Inventory Simulation

Butler Electrical Supply Company is the distributor of home ventilation fan. Each fan costs Butler \$75 and sells for \$125. Thus the butler realizes a gross profit of \$125-\$75=\$50 for each fan sold.

Monthly demand for the fan is described by a normal distribution with a mean of 100 units and a standard deviation of 20 units.

Butler receives monthly deliveries from its suppliers and replenishes its inventory to a level of Q at the beginning of each month. This beginning inventory level is referred to as the replenishment level.

Inventory Simulation

If monthly demand is less than the replenishment level, an inventory holding cost of $15 is charged for each unit that is not sold.

However, if monthly demand is greater than the replenishment level, a stock out occurs and a shortage cost is incurred.

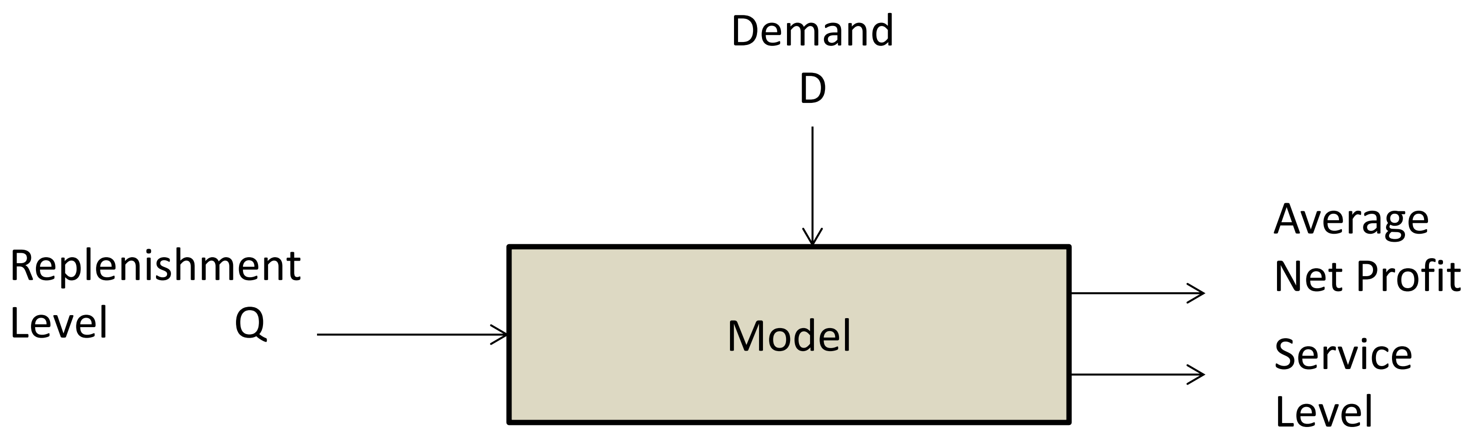
Because Butler assigns a goodwill cost of $30 for each customer turned away, a shortage cost of $30 is charged for each unit of demand that cannot be satisfied.

Inventory Simulation

Management would like to use a simulation model to determine the average monthly net profit resulting from using a particular replenishment level.

Management would also like information on the percentage of total demand that will be satisfied. This percentage is referred to as the service level.

Butler Inventory Simulation Model



where

Controllable Input – replenishment level (Q)
Probabilistic Input - monthly demand (D)
Output – Average Net Profit and Service Level

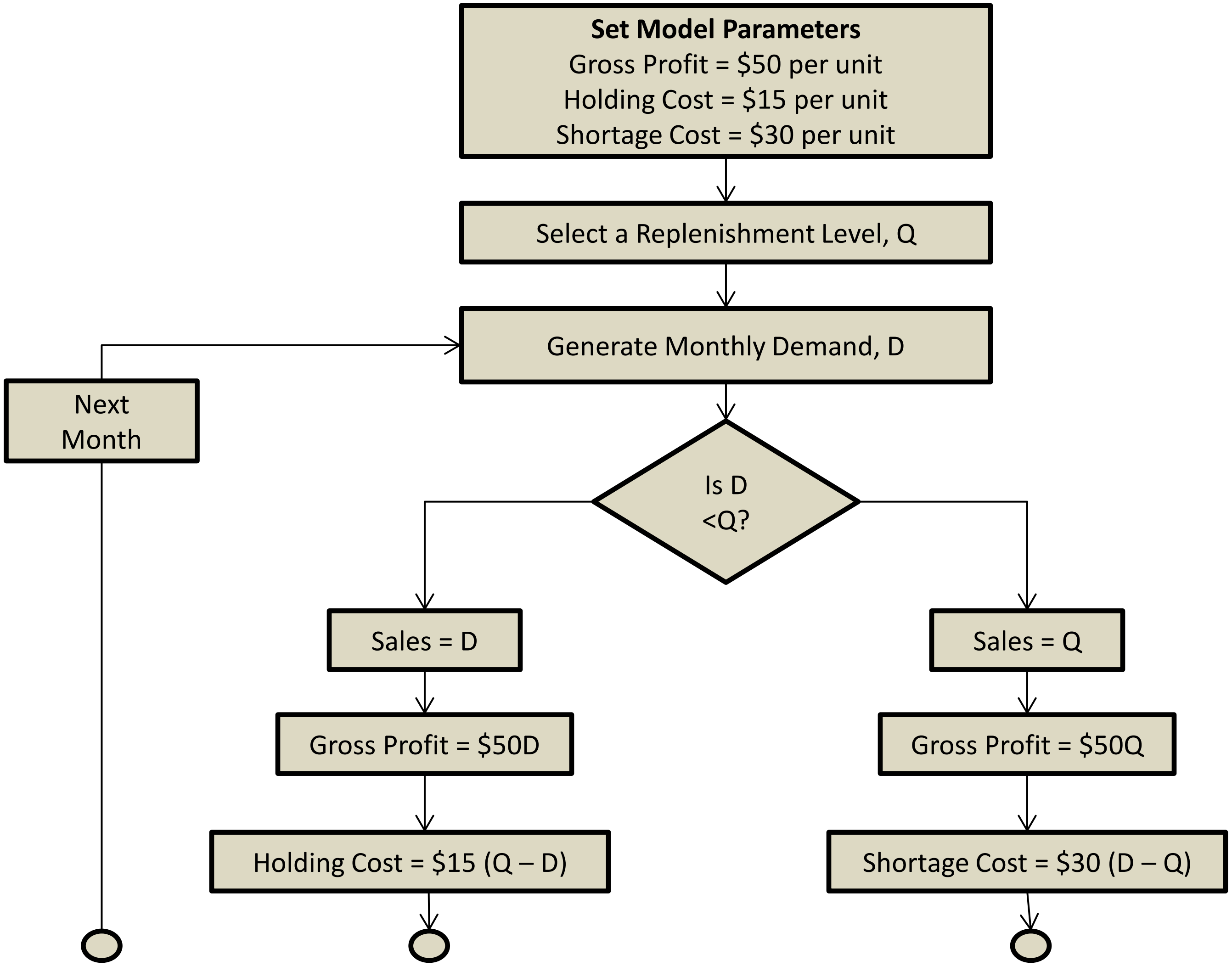
Case 1: D < Q

Gross profit = $50D
Holding cost = $15 (Q – D)
Net profit = Gross profit – Holding cost = $50D - $15(Q - D)

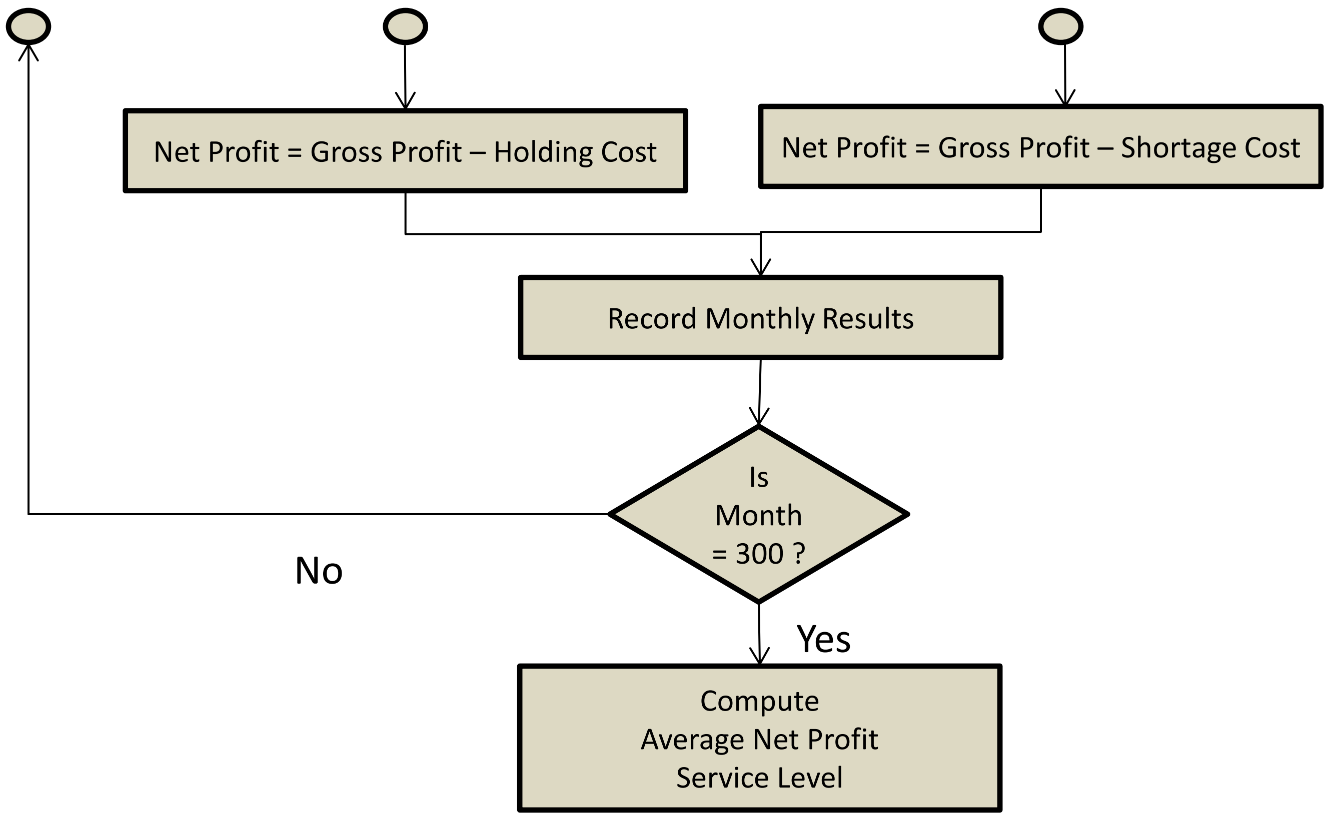
Case 2: D >= Q

Gross profit = $50Q
Shortage cost = $30 (D – Q)
Net profit = Gross profit – Shortage cost = $50Q - $30(D - Q)

Flowchart for Butler Inventory Simulation



Flowchart for Butler Inventory Simulation



Butler Inventory Simulation Results For Five Trials With Q=100

Month	Demand	Sales	Gross Profit ($)	Holding Cost ($)	Shortage Cost ($)	Net Profit ($)
1	79	79	3,950	315	0	3,635
2	111	100	5,000	0	330	4,670
3	93	93	4,650	105	0	4,545
4	100	100	5,000	0	0	5,000
5	118	100	5,000	0	540	4,460
Totals	501	472	23,600	420	870	22,310
Average	100	94	$4,720	$84	$174	$4,462

Excel Worksheet for the Butler Inventory Problem

Applications

- Airline Booking: the number of reservations an airline should accept for a particular flight.
  - Output: profit for the flight
  - Probabilistic input: number of passengers with a reservation who show up and use their reservation.
  - Controllable input: number of reservations accepted for a flight.
- Inventory Policy: to choose an inventory policy that will provide good customer service at a reasonable cost.
  - Output: total inventory and service level
  - Probabilistic input: product demand and delivery lead time
  - Controllable input: order quantity and reorder point

Applications

- Traffic Flow: the effect of installing a left turn signal on the flow of traffic through a basic intersection.
  - Output: waiting time for vehicles to get through the intersection.
  - Probabilistic input: number of vehicle arrivals, fraction that want to make left turn.
  - Controllable inputs: length of time the left turn signal is on.
- Waiting Lines: the waiting times for customers at a bank's automated teller machine (ATM).
  - Output: customer waiting time
  - Probabilistic input: customer arrival and service time
  - Controllable input: number of ATM machines installed.

Summary

- Simulation is a method that can be used to describe or predict how a system will operate given certain choices for the controllable inputs and randomly generated values for the probabilistic inputs.
- Management scientist often use simulation to determine values for the controllable inputs that are likely to lead to desirable system outputs.
- In this sense, simulation can be effective tool in designing a system to provide good performance.

PortaCom Risk Analysis in Python

```
from scipy.stats import rv_discrete
from scipy.stats import uniform
from scipy.stats import norm
import pandas as pd

sp_per_unit=249
admin_cost=400000
advt_cost=600000

#parts cost with uniform distribution
low=80
high=100

#demad with normal distribution
mean=15000
std_dev=4500

trials=500

sample = rv_discrete(values=([43, 44, 45, 46, 47],[0.1, 0.2, 0.4, 0.2, 0.1]))
df=pd.DataFrame({"Labor Cost" : sample.rvs(size=trials)})
df["Part Cost"]=uniform .rvs(low, high, size = trials).round(2)
df["Demand"] = norm.rvs(loc=mean, scale=std_dev, size = trials).round()
df["Profit"]= (sp_per_unit-df["Labor Cost"]-df["Part Cost"])*df["Demand"]-admin_cost-
advt_cost
print(df)
print("Mean Profit", df["Profit"].mean())
print("Standard Deviation", df["Profit"].std())
print("Minimum Profit", df["Profit"].min())
print("Maximum Profit", df["Profit"].max())
num_of_loss=df[df["Profit"]<0]["Profit"].count()
print("Number of losses", num_of_loss)
print("Probability of loss", num_of_loss/trials)
```

References

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- James Evans, Business Analytics: Methods, Models and Decisions, Second Edition, Pearson Publication, 2017.
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- Statistical functions (scipy.stats) https://docs.scipy.org/doc/scipy/reference/stats.html