Lab Exercise - Practice | Week 7

1. Implement the divide and conquer algorithm **for finding the element with a given rank** in an array. (Code the select (A,k) program discussed in class).

Input: Array A of integers; an integer k.
Output: Element in A whose rank is k.

Do not use helper functions for sorting or partitioning. Implement the Quicksort partition function in-place, i.e. without using any additional memory.

2. Given two sorted arrays A and B, each of size n, and a number k, write a O(logn) algorithm to find the **element with rank k** in the union of arrays A and B. You should not use additional memory other than a few temporary pointers. A solution that merges the two arrays won't work. Why?

3. Given a set of n point (x1,y1), (x2,y2),...(xn,yn) implement a O(nlogn) divide and conquer algorithm to find the **closest pair** among them. The closest pair are the distinct points (xi,yi) and (xj,yj) such that the Euclidean distance between them is the least.

4. Implement an efficient algorithm to find **articulation point** in a connected directed graph.

5. For this problem, we will explore the issue of truthfulness in the Stable Matching Problem and specifically in the **Gale-Shapley algorithm**. The basic question is: Can a man or a woman end up better off by lying about his or her preferences? More concretely, we suppose each participant has a true preference order. Now consider a woman w. suppose w prefers man m to m', but both m and m' are low on her list of preferences. Can it be the case that by switching the order of m and m' on her list of preferences (i.e., by falsely claiming that she prefers m' to m) and running the algorithm with this false preference list, w will end up with a man m'' that she truly prefers to both m and m'? (We can ask the same question for men, but will focus on the case of women for purposes of this question.)

Resolve this question by doing one of the following two things:
(a) Give a proof that, for any set of preference lists, switching the order of a pair on the list cannot improve a woman's partner in the Gale- Shapley algorithm; or
(b) Give an example of a set of preference lists for which there is a switch that would improve the partner of a woman who switched preferences.