Problem 2: nth roots of unity

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To find the 6^{th} roots of unity:

we know that

$$(1)^{\frac{1}{n}} = e^{i\frac{2k\pi}{n}}, \qquad k = 0, 1, ..., n-1$$

Thus for 6^{th} roots we will get,

$$\begin{aligned}
(1)^{\frac{1}{6}} &= e^{i\frac{\pi k}{3}} \\
\text{when } k = 0 &\implies e^0 = 1 \\
\text{when } k = 1 &\implies e^{i\frac{\pi}{3}} = \frac{1}{2}(1 + i\sqrt{3}) \\
\text{when } k = 2 &\implies e^{i\frac{2\pi}{3}} = -\frac{1}{2}(1 - i\sqrt{3}) \\
\text{when } k = 3 &\implies e^{i\pi} = -1 \\
\text{when } k = 4 &\implies e^{i\frac{4\pi}{3}} = -\frac{1}{2}(1 + i\sqrt{3}) \\
\text{when } k = 5 &\implies e^{i\frac{5\pi}{3}} = \frac{1}{2}(1 - i\sqrt{3})
\end{aligned}$$

Thus the 6^{th} roots of unity are ± 1, $\pm\frac{1}{2}(1+i\sqrt{3})$ and $\pm\frac{1}{2}(1-i\sqrt{3})$. The below graph shows the 6^{th} roots of unity and the roots are represented by blue dots and the red lines shows that every roots are $60°$ apart from each other.