# Problem 1: Sets in Complex plane

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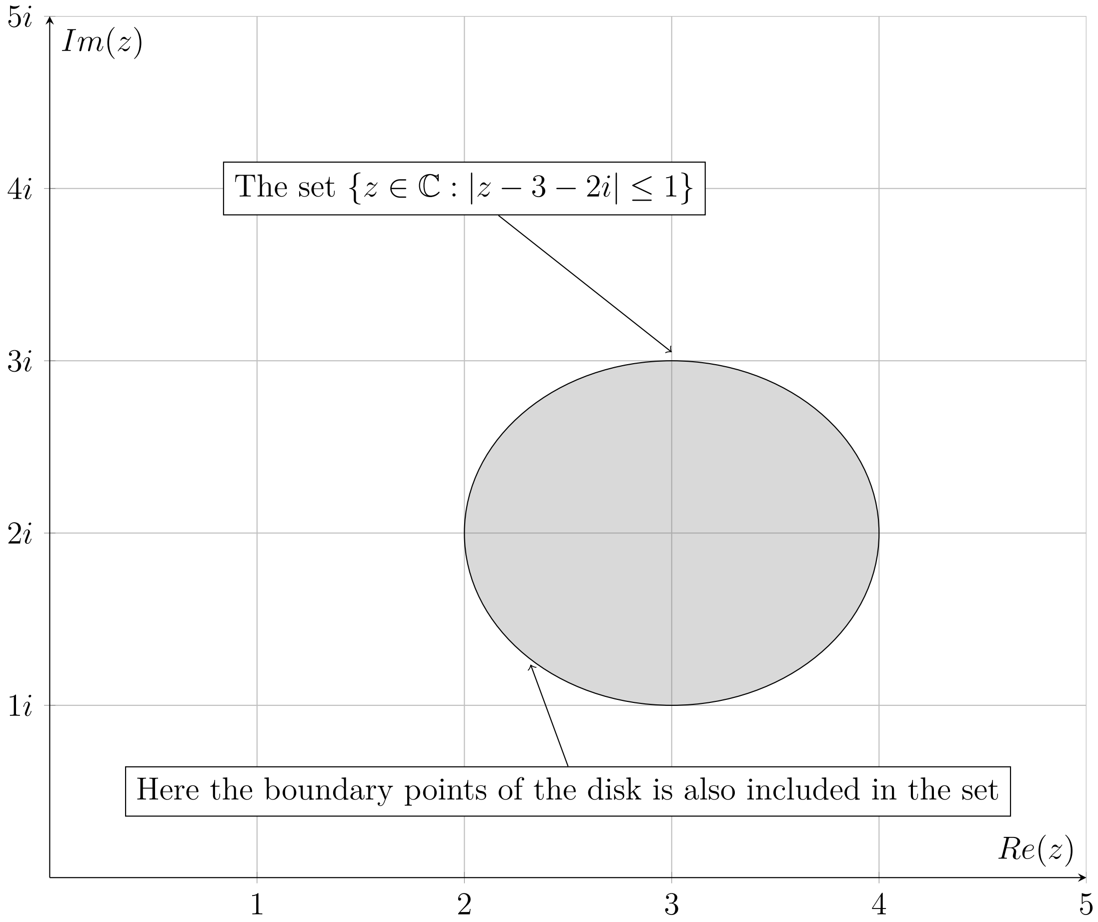
## Graph the following sets:

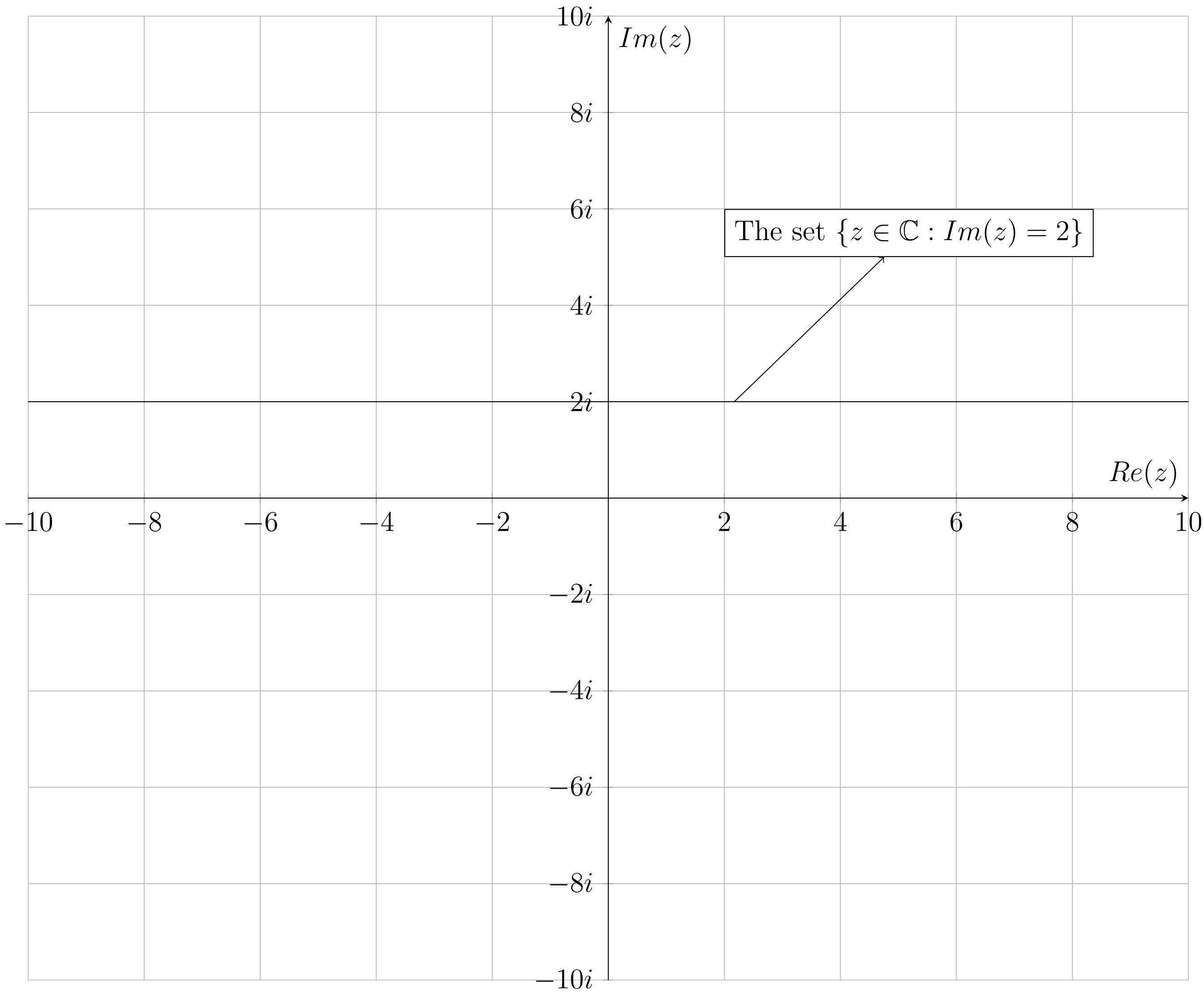
(a) $\{z \in \mathbb{C} : |z - 3 - 2i| \leq 1\}$




$$\begin{aligned}|z - 3 - 2i| &= |x + iy - 3 - 21| \\ &= |(x-3) + i(y-2)| \\ &= \sqrt{(x-3)^2 + (y-2)^2}\end{aligned}$$

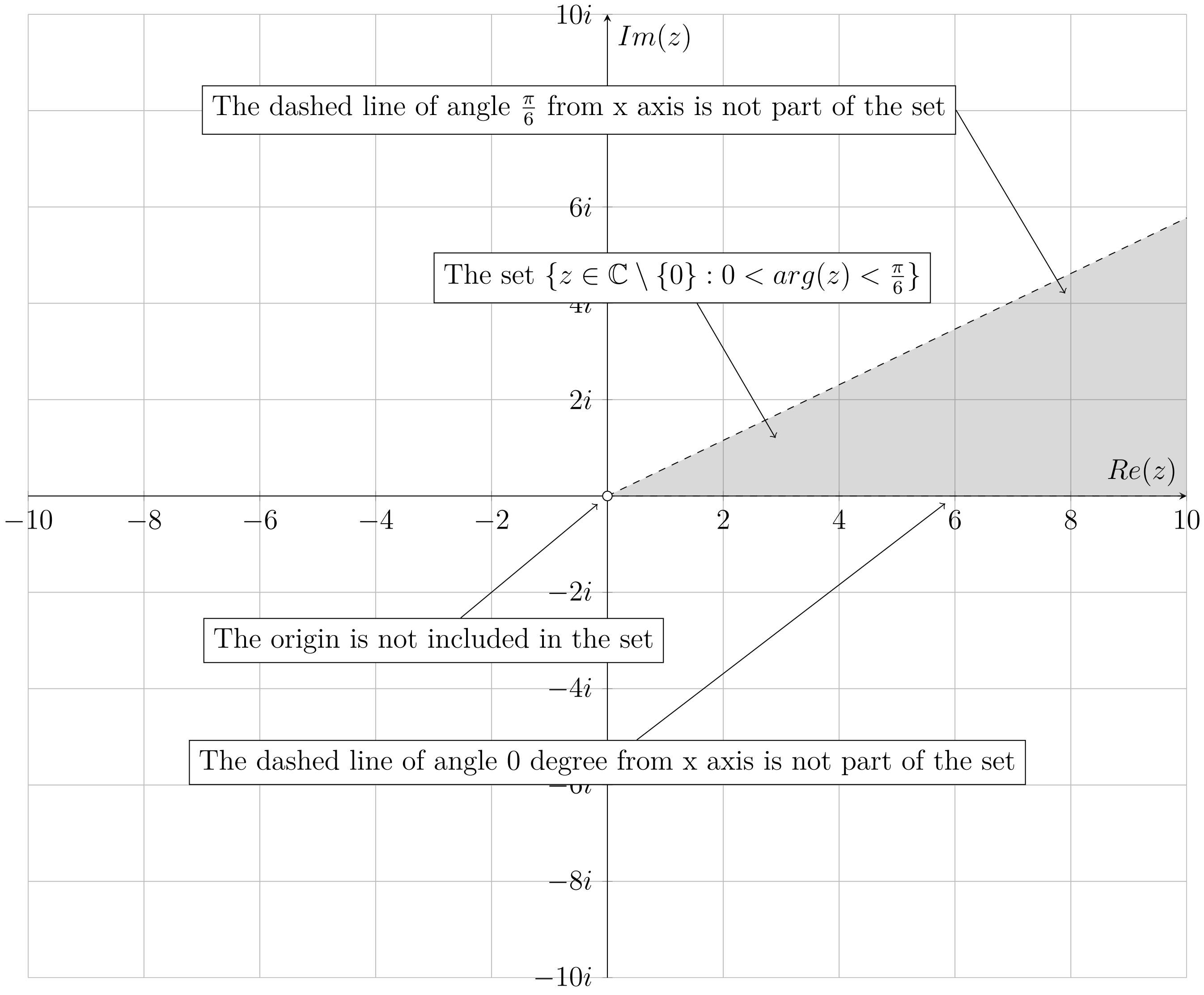
Thus we have a disc of radius 1 and centre $(3, 2)$. The set $\{z \in \mathbb{C} : |z - 3 - 2i| \leq 1\}$ contains all the interior points of the disc and also the boundary points.

(b) $\{z \in \mathbb{C} : Im(z) = 2\}$




The equation $Im(z) = 2$ gives a straight line passing through the point $2i$ and parallel to the real axis. The set $\{z \in \mathbb{C} : Im(z) = 2\}$ contains every points on the given straight line. Note that this set does not contain any interior points because it is just a straight line.

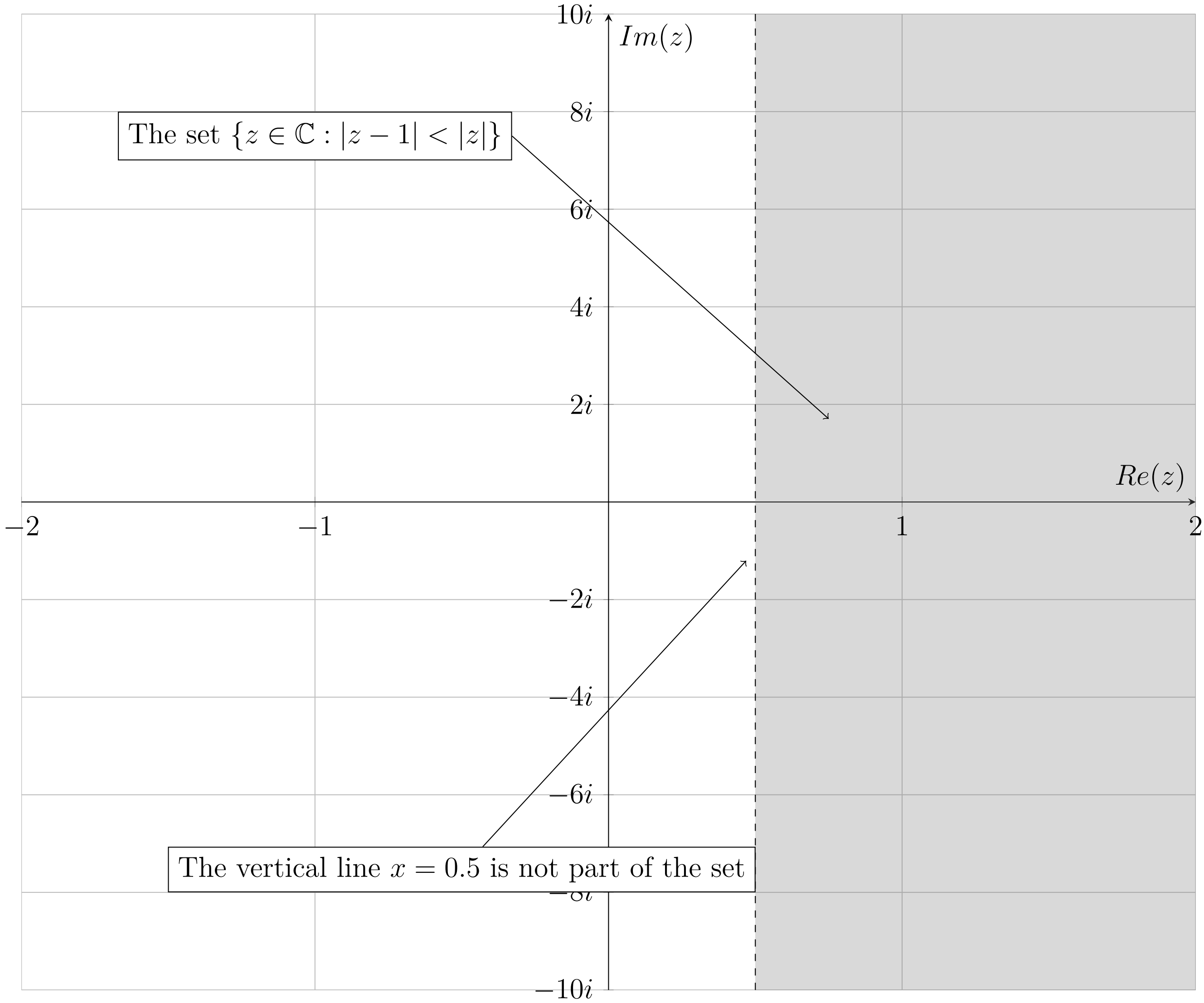
(c) $\{z \in \mathbb{C} \setminus \{0\} : 0 < arg(z) < \frac{\pi}{6}\}$




$$Let, z = x + iy$$
$$arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\implies \tan(arg(z)) = \frac{y}{x}$$
$$\implies y = \tan(arg(z))x$$

This set has excluded the origin. The straight lines $y = 0$ of domain: $(0, \infty)$ and $y = \frac{1}{\sqrt{3}}x$ i.e. line drawn $\frac{\pi}{6}$ rad from the x axis of domain: $(0, \infty)$ are not included in the set. The line of $y = 0$ is dashed but became invisible because of the axis lines.

(d) $\{z \in \mathbb{C} : |z-1| < |z|\}$




Let, $z = x + iy$

$$\begin{aligned} |z-1| &< |z| \\ |(x-1)+iy| &< |z| \\ \sqrt{(x-1)^2+y^2} &< \sqrt{x^2+y^2} \\ x^2-2x+1+y^2 &< x^2+y^2 \\ x &> \frac{1}{2} \end{aligned}$$

$\therefore x \in \left(\frac{1}{2}, \infty\right)$ and $y \in \mathbb{R}$.

The vertical line $x = 0.5$ is not part of the set.