Fifth Week Report

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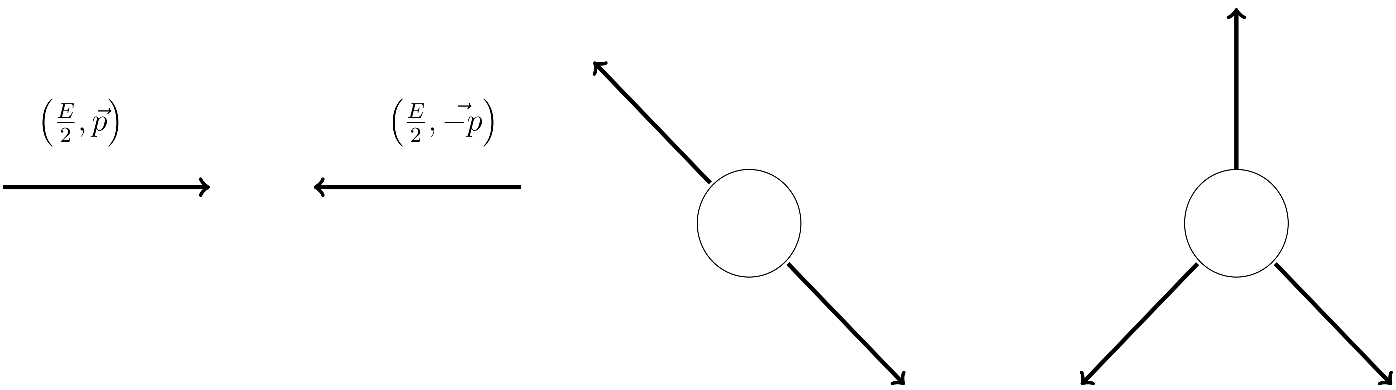
Feynman Diagram

On week 5, an introduction to Feynman diagram was given by Dr Raul Briceno. We all know that quantum mechanics is the mathematical framework which is used to describe the mechanics of the sub atomic particles moving in non relativistic manner, whereas in the Einstein's special relativity is a mathematical framework for describing particles which are moving in a relativistic manner.



(a) Two particles heading towards each other i.e. before collision

(b) After collision if the energy is $3m < E < 2m$

(c) After collision if the energy is $E > 3m$

Figure 1: Two identical particles undergoing collision or scattering process

In figure(1a), 2 particles of same magnitude of momentum is travel towards each other. Lets assume that the mass of each particle be m such that we expect that the energy of the particles during collision is $3m \leq E \leq 2m$ then the particles collide and moving in same other equal and opposite direction as shown in figure(1b). But if the energy of the particles is greater than $3m$ i.e. $E > 3m$, then the excessive energy is sufficient enough to create another particle as shown in figure(1c).

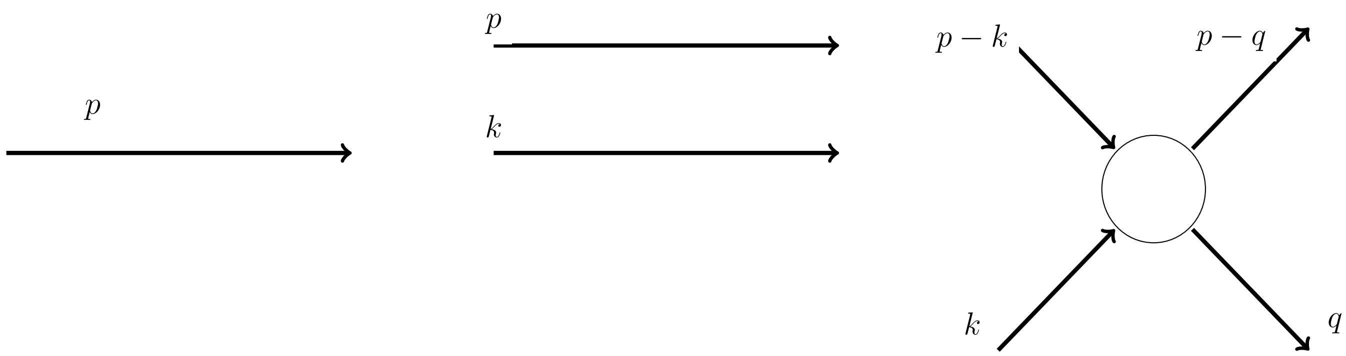
This phenomenon has failed explanation in quantum mechanics. This is where Quantum field theory(QFT) comes into play. From this theory, we consider particles are the excitation of some field which fills the entire universe. The interference of these fields gives the properties and interaction mechanisms.

The direct consequences of the QFT is we can write any observable as a Feynman diagram. To understand what are Feynman diagrams are, let us consider the simplest possible theory of QFT which is known as $\lambda\varphi^4$ field which is a 4 scalar field which is denoted as φ^4 and λ is tuning constant.

In this $\lambda\varphi^4$ either of the following can happen:

1. It propagates freely i.e. no interaction takes place as shown in figure(2a) & figure(2b). Note figure(2b) has a non zero probability so this cannot be discarded.

2. Some interactions takes place as shown in figure(2c). Note this is not the only interaction possible. There are infinitely many interactions can be taken place, like interactions can be happen twice or thrice and so on.



(a) A particle propagates freely with a momentum p

(b) Two particles of momenta p and k propagates without interacting

(c) Two particles undergoes into some interaction

Figure 2: Some Possible interactions in $\lambda\varphi^4$ field. Remember there are infinite possible interactions for a given system. Note that the time flows from left to right.

Now we know that for a particle at rest $E = mc^2$ so this tells us that the remaining energy is converted into mass i.e. a particle with mass $m = \Delta E$. But lets dive deep inside how mass energy equivalence principle and our theory of scalar field $\lambda\varphi^4$ works.

Figure 3: Possible decay channels available for some unstable QCD states are also shown.

The figure(3) shows energy of the system E along vertical axis, if $E < 2m$, then it is not even possible to have 2 particles after the process. Thus we consider only the case $E > 2m$. We can split the region into multiple bands between nm and $(n+1)m$ where $n \geq 2 \in \mathbb{Z}^+$ among them let's consider the regions between $2m$ and $3m$ (region 1), $3m$ and $4m$ (region 2), $4m$ and $5m$ (region 3).

- In region 1, the 2 incoming particles are undergoes some interaction and scattered. There is only 2 outgoing particles. Thus, no particle was created.
- In region 2, here it has enough energy to give 2 particles on the outgoing end. But even though it has enough energy to produce an extra particle, the dynamics of the theory of $\lambda\varphi^4$ field builds on the fact that only even number of external legs are allowed. Since the system having energy greater than $3m$ has 5 external legs (if we consider one particle is created), this is not allowed.
- In region 3, it can scattered as 2 particles. Also it can produce 2 other particles thus scatters into 4 particles. This is because the fact that it will have 6 external legs after a pair of particles are produced.

Now we will see how Feynman diagrams look and what are all the mathematical description can be obtained from those diagrams. To understand them we should now the Feynman rules for our theory. Thus, the Feynman rules for the $\lambda\varphi^4$ field are as follows:

1. **Rule 1:** Every vertex contributes to $-i\lambda$ where λ is some parameter which we don't know but we can tune in such a way to match with the experimental results.
2. **Rule 2:** For every external legs, a factor of $(+1)$ is multiplied.
3. **Rule 3:** For internal lines also known as propagators the term $\left(\frac{i}{p^2-m^2+i\epsilon}\right)$ where p is the 4 momentum of the propagators and for most of the case we ultimately set $\epsilon = 0$
4. **Rule 4:** For each intermediate undetermined momentum then we should integrate over it i.e. $\int \frac{d^4k_1}{(2\pi)^4}$, $\int \frac{d^4k_2}{(2\pi)^4}$ and so on.
5. **Rule 5:** Divide the expression by the symmetry factors. These symmetry factor are equal to number of permutations of internal lines which leave the diagram unchanged.

For example, lets look at the figure(4). Here the momentum flows are all marked

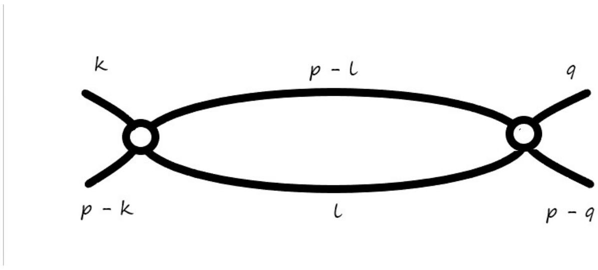


Figure 4: Example

The mathematical expression for the figure(4) is

$$
\begin{aligned}
&(-i\lambda).(-i\lambda)\frac{1}{2}\int \frac{d^4l}{(2\pi)^4}\left(\frac{i}{(p-l)^2-m^2+i\epsilon}\right)\left(\frac{i}{l^2-m^2+i\epsilon}\right) \\
&= \frac{\lambda^2}{2}\int \frac{d^4l}{(2\pi)^4}\left(\frac{1}{(p-l)^2-m^2+i\epsilon}\right)\left(\frac{1}{l^2-m^2+i\epsilon}\right)
\end{aligned}
$$