Permutation-1

n-box, r-item (non-identical). print all possible arrangements of placing items in the box.

level and options → level → items
option → boxes.

print in Recursion
↳ level & option

$n=3, \quad r=2 \to \{1, 2\}$

_ _ _

→ 1 2 0
→ 1 0 2
→ 2 1 0
→ 2 0 1
→ 0 1 2
→ 0 2 1

Same type of arrangement but permuted

i i _

1 2 -
2 1 -

n-boxes

r-object

$$^{n}P_{r} = \frac{n!}{(n-r)!} = \underbrace{(n-0)(n-1)(n-2)(n-3)\ldots(n-(r-1))}_{r \text{ terms}}$$

$n=3$

$3 \times 2 = 6$

1 2 → r items

$n=4, \quad r=2$

$$^{4}P_{2} = \frac{4!}{2!} = 4 \times 3 = 12$$

12 ways.

level → items

options → boxes

n = 4, r = 2

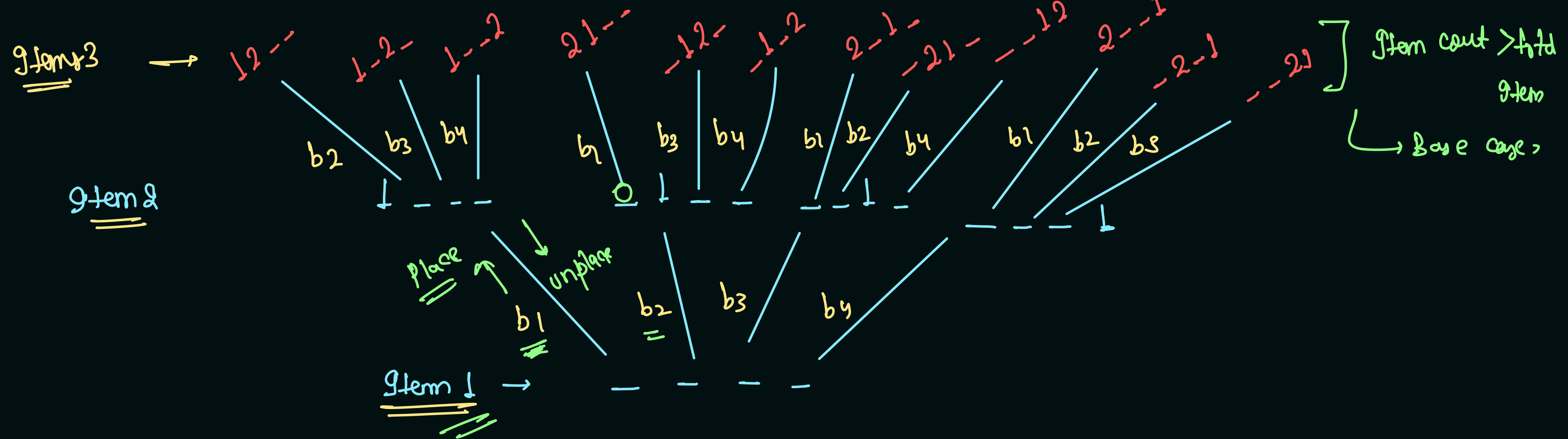
boxes, items

1 2 0 0	2 0 1 0
1 0 2 0	0 2 1 0
1 0 0 2	0 0 1 2
2 1 0 0	2 0 0 1
0 1 2 0	0 2 0 1
0 1 0 2	0 0 2 1

observation → we have permutation of same type of arrangement

→ why this happened

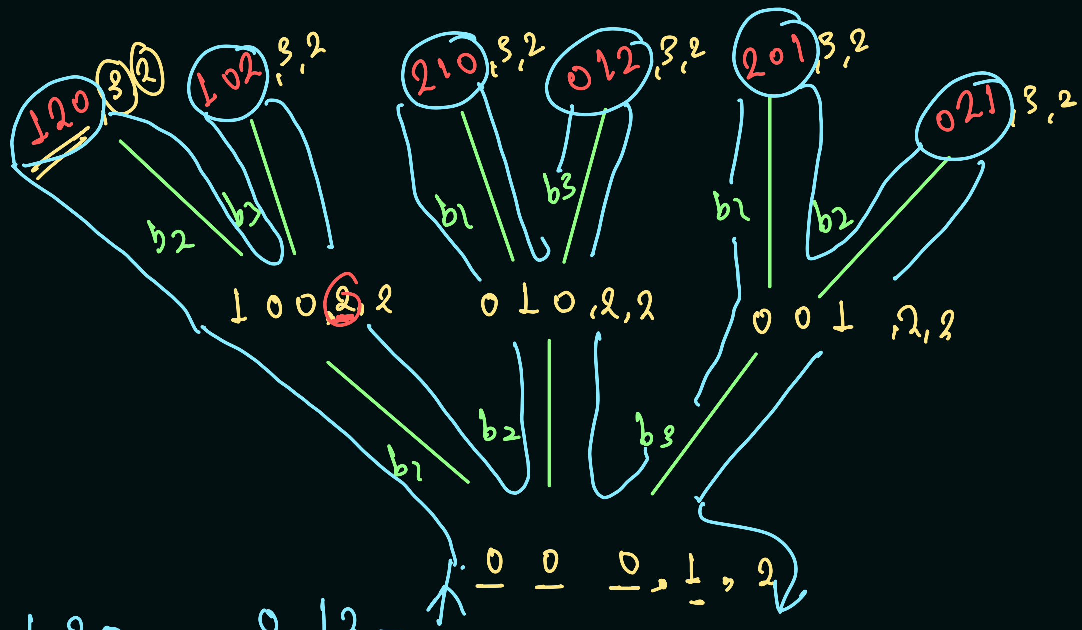
→ we have non identical items



ci →

n=3 → box   r=2 → item

```java
// ci-> current item, ti-> total items
public static void permutations(int[] boxes, int ci, int ti){
    if(ci > ti) {
        // print arrange of items in box and return
        for(int val : boxes) {
            System.out.print(val);
        }
        System.out.println();
        return;
    }
    // loop of options
    for(int b = 0; b < boxes.length; b++) {
        if(boxes[b] == 0) {
            // place current item at bth box
            boxes[b] = ci;
            // recursive call
            permutations(boxes, ci + 1, ti);
            // unplace current item at bth box
            boxes[b] = 0;
        }
    }
}
```



1 2 0	0 1 2
1 0 2 ⟷	2 0 1
2 1 0	0 2 1

Combination - 1

n-boxes, r- identical items, print all possibilities to arrange r items in n-boxes.

Hint

level - boxes

option → choice of items

$^{n}P_r = {}^{4}P_2 = \frac{4!}{2!} = \frac{4\times3\times\cancel{2!}}{\cancel{2!}} = 12$

Permutations - 12

n- boxes = 4   r- items = 2

$^{n}C_r = \frac{n!}{(n-r)!\,r!} \Rightarrow {}^{4}C_2 = \frac{4!}{2!\,2!} = \frac{\overset{2}{\cancel{4}}\times3\times\cancel{2!}}{\cancel{2!}\times\cancel{2!}} = 6$

Combinations → 6

Permutated

1 2 0 0	2 1 0 0
1 0 2 0	2 0 1 0
1 0 0 2	2 0 0 1
0 1 2 0	0 2 1 0
0 1 0 2	0 2 0 1
0 0 1 2	0 0 2 1

→ arrangements

→ non identical items

i i - -

i - i -

i - - i

- i i -

- i - i

- - i_1 i_2

combinations

- - i_2 i_1

→ identical items

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$$

${}^nC_r$ = possible no. of ways to place r-identical items in n-boxes

n=4

$$2^4 = {}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3$$

level - box

option → choices of item

↓

Yes or No

items are identicals

$$\frac{4!}{1!\,3!} = 4$$

$2^4 = 16$

${}^4C_4 = 1$

iiii

${}^4C_0 = 1$

${}^4C_1 = 4$

i---
-i--
--i-
---i

${}^4C_2 = 6$

ii--
i-i-
i--i
-ii-
-i-i
--ii

${}^4C_3 = 4$

iii-
ii-i
i-ii
-iii

Permutation 2

n- box , r- Item (non- Identical) print all possible arrangements of placing items in the box.

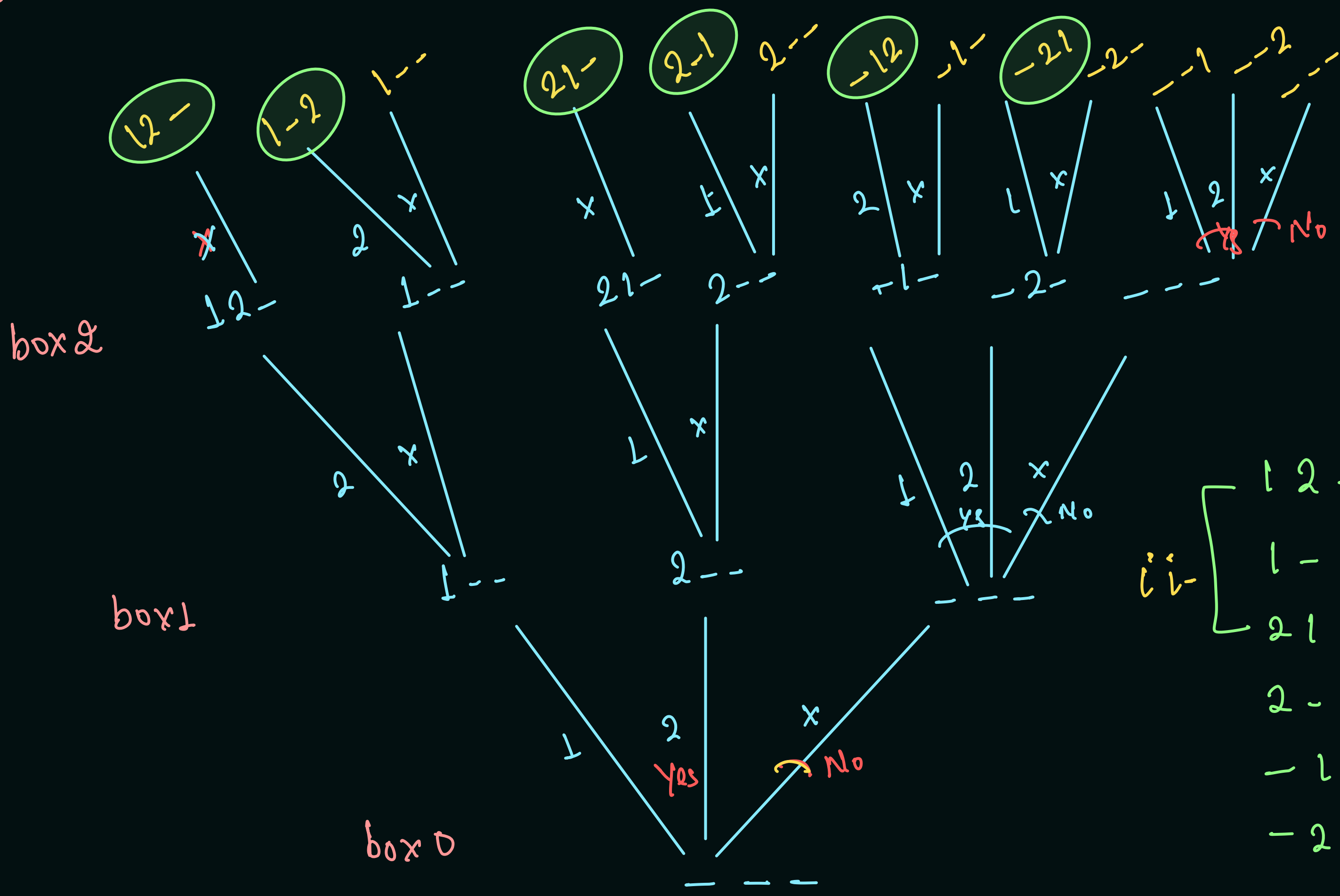
level → boxes

options → choices of items

$$^nC_r = \frac{n!}{(n-r)!\,r!} \quad , \quad ^nP_r = \frac{n!}{(n-r)!}$$

n=3 → box

r=2 → item



$$^nP_r = {^nC_r} \times r!$$

permute

Permute all the combination to get the permutation

$$\begin{array}{ll} \text{ii-} & 12- \\ & 1-2 \\ & 21- \\ & 2-1 \\ \end{array}$$

i-i

-ii: -12, -21

$$^3P_2 = \frac{3!}{1!} = 3 \times 2 \times 1 = 6$$

$$^3C_2 = \frac{3!}{1!\,2!} = 3$$

$$3 \times 2 = 6$$

Combination2:

n-boxes, r-identical items, print all possibilities to arrange r items in n-boxes.

Permutation

boxes = 3, item = 2

$$^nP_r = (n-0)(n-1)(n-2)\ldots(n-(r-1))$$

$$^3P_2 = \frac{3!}{1!} = 6$$

1	2	0	2	1	0
1	0	2	2	0	1
0	1	2	0	2	1

combination

boxes = 3, items = 2

$$^nC_r = \frac{n!}{(n-r)!\,r!} = \frac{^nP_r}{r!}$$

$$^3C_2 = \frac{3!}{2!\,1!} = \frac{3\times\cancel{2!}}{\cancel{2!}\times 1!} = 3$$

i	i	_
i	_	i
_	i	i

4-boxes 3 items → {1, 2, 3}

4-boxes 3 items → {i, i, i}

$$^4P_3 = \frac{4!}{1!} = 4\times 3\times 2 = 24$$

$$^4C_3 = \frac{4!}{3!\,1!} = \frac{4\times \cancel{3!}}{\cancel{3!}\times 1!} = 4$$

i i i _

1 2 3 0
1 3 2 0
2 1 3 0
2 3 1 0
3 1 2 0
3 2 1 0

i i _ i

1 2 0 3
1 3 0 2
2 1 0 3
2 3 0 1
3 1 0 2
3 2 0 1

i _ i i

1 0 2 3
1 0 3 2
2 0 1 3
2 0 3 1
3 0 1 2
3 0 2 1

_ i i i

0 1 2 3
0 1 3 2
0 2 1 3
0 2 3 1
0 3 1 2
0 3 2 1

figure out combination from permutation →

$^4P_3 = 4 \times 3 \times 2 = 24$

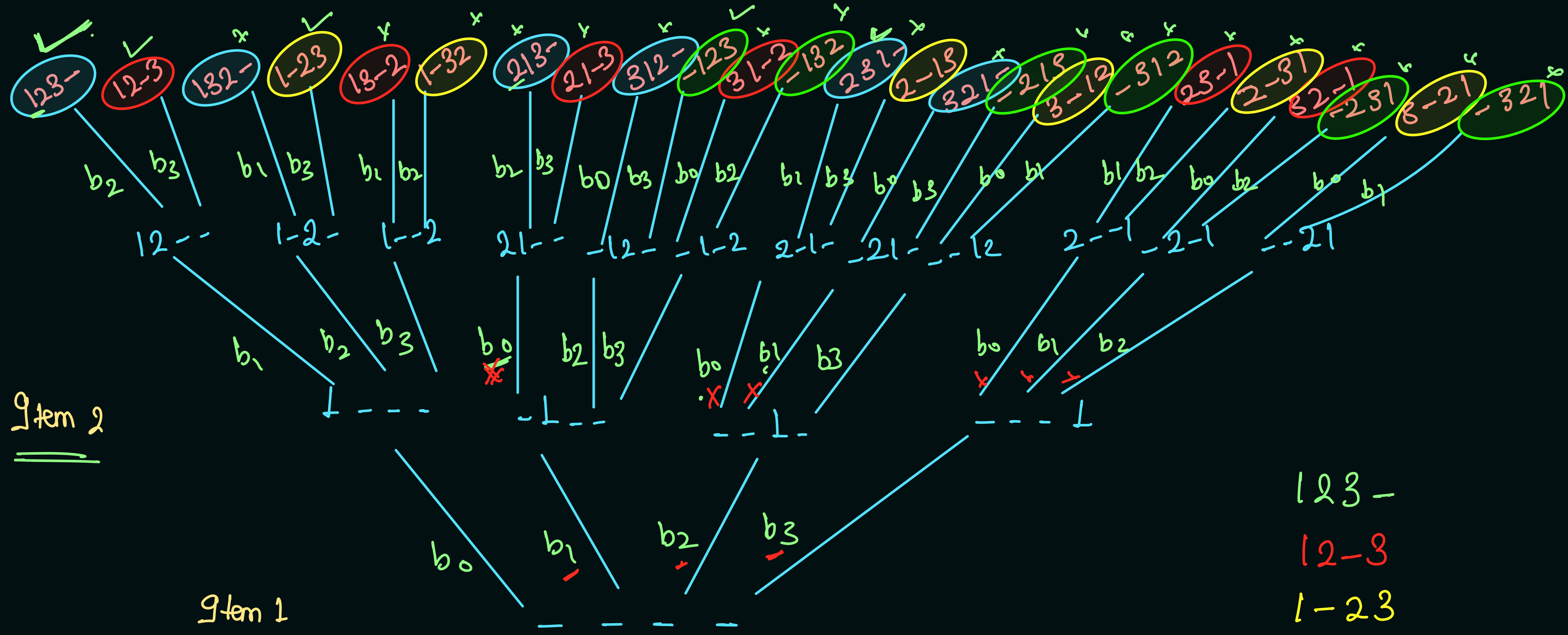
Permutation

level → Items

option → box

Observation → Make call is Sorted order of Index

Result



Item 3

Item 2

Item 1

123 –

12 – 3

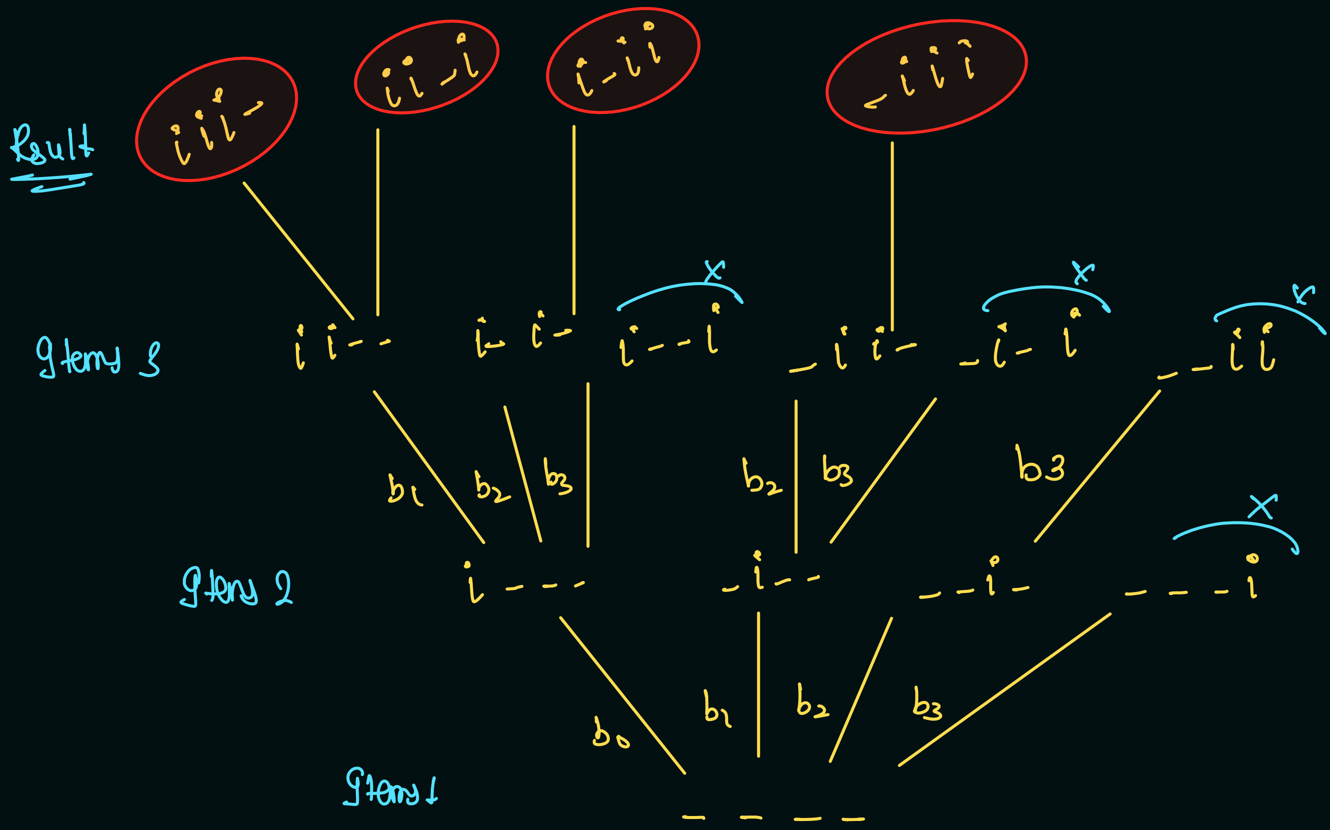
1 – 23

– 123

$${}^{4}C_3 = \frac{4!}{3!\,1!} = 4$$

level → items

option → boxes



i i i _

i i _ i

i _ i i

_ i i i