Practice Questions on Mathematics

1) Solve the equation $x^3 - 7x^2 + 36 = 0$, given that one root is double of another.
2) If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$, where *a*, *b* and *c* are different, calculate the value of *abc*.
3) (a) Find the values of λ for which the following equations become consistent.

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$
$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$
$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

   (b) Find the ratios of *x*: *y*: *z* for smallest value of λ. (c) What happens to the equations for greatest value of λ.
4) For a matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$, find the characteristic equation and thereafter A^{-1}. (**Hint:** Use Cayley Hamilton theorem)
5) Two cards are drawn in succession from a pack of 52 cards. Find the probability that the first is a king and the second a queen if the first card is (a) replaced, (b) not replaced.
6) Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ using cofactor method and Gauss Jordan Method

Answers:

1) Roots are = 3, 6, -2
2) *abc* = 1
3) (a) λ = 0, λ = 3; (b) *x* = *y* = z; (c) for λ = 3, all equations become same
4) Characteristic equation: $\lambda^3 - 20\lambda + 8 = 0$; $A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$ **(this is same as solved example in slides)**
5) (a) 1/169; (b) 4/663
6) $A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$