

Practice Question on Optimization 1

- 1) Suppose a company sell Q ($= 100000$) units of the product over a given year at a fixed price with demand spread evenly over the year. Production cost per run is a linear function of number of units produced per run. Inventory cost is proportional to number of units produced per run.
- Formulate the optimization problem for minimizing the cost.
 - Obtain the analytical expression for optimal number of units produced per run.
 - Also, check whether the obtained optimum is indeed minimum.
 - If fixed cost factor for production cost is 10000, variable cost factor for production cost is 4, and cost factor for inventory cost is 1, calculate the number of units produced per run.
 - Can you suggest any improvement in the problem definition?

Solution: Number of units produced per run is D and number runs per year are n .

$$\text{Inventory Cost} = K_1 D$$

$$\text{Production Cost per run} = K_2 + K_3 D$$

$$\text{Total Cost} = C = K_1 D + n(K_2 + K_3 D) = K_1 D + \frac{Q}{D}(K_2 + K_3 D) = K_1 D + \frac{K_2 Q}{D} + K_3 Q$$

$$\frac{dC}{dD} = 0 = K_1 - \frac{K_2 Q}{D^2} \Rightarrow D^{opt} = \sqrt{\frac{K_2 Q}{K_1}}$$

$$\frac{d^2 C}{dD^2} = \frac{2K_2 Q}{D^3} \Rightarrow +ve \Rightarrow \text{The obtained optimum is indeed minimum}$$

$$D^{opt} = \sqrt{\frac{K_2 Q}{K_1}} = \sqrt{\frac{10000 \times 100000}{1}} \approx 31622$$

The production cost varies non-linearly with number of units produced per run. In general, as the size increases, the operating cost varies with the lower exponent. Therefore, the above formulation can be improved further by assuming a nonlinear function for production cost. A sample representation is given below.

$$\text{Production Cost per run} = K_4 + K_5 D^{0.5}$$