① Solve the equation $x^3-7x^2+36=0$, given that one root is double of another.

$\Rightarrow$ Let the roots be $= \alpha_1, \alpha_2, \alpha_3$ such that $\alpha_2=2\alpha_1$

From the properties

$$\alpha_1+\alpha_2+\alpha_3 = -\frac{a_1}{a_0} = \frac{7}{1} = 7 \quad \cdots ①$$

$$\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1 = \frac{a_2}{a_0} = \frac{0}{1} = 0 \quad \cdots ②$$

$$\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0} = -36 \quad \cdots ③$$

From ① - ③ put $\alpha_2 = 2\alpha_1$

$$\alpha_1+2\alpha_1+\alpha_3 = 3\alpha_1+\alpha_3 = 7 \quad \cdots ④$$

$$\alpha_1(2\alpha_1)+(2\alpha_1)\alpha_3+\alpha_3\alpha_1 = 0 \Rightarrow 2\alpha_1^2+3\alpha_1\alpha_3=0 \quad ⑤$$

$$\alpha_1(2\alpha_1)\alpha_3 = -36 \Rightarrow 2\alpha_1^2\alpha_3 = -36 \quad \cdots ⑥$$

~~From ④ $\alpha_3 = 7-3\alpha_1$ put in ⑥~~

$$\cancel{2\alpha_1^2(7-3\alpha_1) = -36}$$

$$\cancel{14\alpha_1^2 - 6\alpha_1^3 = -36}$$

From ④ and ⑤ put $\alpha_3 = 7-3\alpha_1$

$$2\alpha_1^2+3\alpha_1(7-3\alpha_1)=0$$

$$2\alpha_1^2+21\alpha_1-9\alpha_1^2=0$$

$$-7\alpha_1^2+21\alpha_1=0 \Rightarrow 7\alpha_1^2 = 21\alpha_1$$

$$\boxed{\alpha_1=3} \Rightarrow \alpha_3 = 7-3\alpha_1 = 7-3\times 3$$

$$\boxed{\alpha_3=-2}$$

$$\alpha_2 = 2\alpha_1 = 2\times 3 = 6 \quad \boxed{\alpha_2=6}$$

② If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$ where a, b, c are differ[ent] calculate the value of abc

From property ⑤

$$\Rightarrow \begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & -1 \\ b & b^2 & -1 \\ c & c^2 & -1 \end{vmatrix}$$

From property ④

$$= abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

From property ②

$$= abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (abc-1) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

since $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$ as a, b, c are all different

$$(abc-1) = 0$$

$$\Rightarrow \boxed{abc = 1}$$

⑥ Find the inverse of matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ using

(i) cofactor method (ii) Gauss Jordan Method

$\Rightarrow$ (i) cofactor method :-

cofactors of 1, 2, 3, and 4 are

$= 4, -3, -2, 1$

$$\text{Determinat} = \Delta = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$= 1\times 4 - 2\times 3 = 4 - 6 = -2$$

Adj. A is a transpose of cofactor matrix A

$$A^{-1} = \frac{\text{Adj. } A}{|A|} = \frac{1}{|A|}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

(ii) Gauss Jordan Method

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$R_1 + R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix}$$

$-R_2/2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

③ Find the values of λ for which the equations

$(\lambda-1)x + (3\lambda+1)y + 2\lambda z = 0$ ...①

$(\lambda-1)x + (4\lambda-2)y + (\lambda+3)z = 0$ ..②

$2x + (3\lambda+1)y + 3(\lambda-1)z = 0$ ..③

are consistent, and find the ~~values~~ ratio of $x:y:z$ when λ has the smallest of these values. What happens when λ has the greater of these values.

$\Rightarrow$ The given equations will be consistent, if

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0 \Rightarrow \overset{\text{operate } R_2-R_1}{\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ 0 & \lambda-3 & 3-\lambda \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix}}$$

operate C_3+C_2

$$\Rightarrow \begin{vmatrix} \lambda-1 & 3\lambda+1 & 5\lambda+1 \\ 0 & \lambda-3 & 0 \\ 2 & 3\lambda+1 & 6\lambda-2 \end{vmatrix} = 0$$

$$(\lambda-3)\begin{vmatrix} \lambda-1 & 5\lambda+1 \\ 2 & 2(3\lambda-1) \end{vmatrix} = 0 \Rightarrow 2(\lambda-3)\left[(\lambda-1)(3\lambda-1)-(5\lambda+\right. = 0$$

$$2(\lambda-3)\left[3\lambda^2-\lambda\overbrace{-3\lambda+1-5\lambda}^{3\lambda+1-5\lambda}-1\right] = 0 \Rightarrow 2(\lambda-3)\left[3\lambda^2-4\lambda\right] =$$

$6\lambda(\lambda-3)^2 = 0 \Rightarrow \lambda=0 \quad \lambda=3$

① For $\lambda=0$ ①, ② and ③ becomes

$-x+y=0$
$-x-2y+3z=0$
$2x+y-3z=0$

solving
$x=y=z$

⑪ when $\lambda=3 \Rightarrow$ All equation become identical.

④ For a matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ Find the characteristic equation and thereafter A^{-1}

$\Rightarrow$ The characteristic equation is

$$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 3-\lambda & -3 \\ -2 & -4 & -4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(3-\lambda)(-4-\lambda) - 12\right] + (-1)\left[-4-\lambda-6\right]$$

$$+ 3\left[-4 + 2(3-\lambda)\right] = 0$$

$$(1-\lambda)\left[-12+4\lambda-3\lambda+\lambda^2-12\right] + 10+\lambda - 12 + 18 - 6\lambda = 0$$

$$(1-\lambda)\left[\lambda^2+\lambda-24\right] + 16 - 5\lambda = 0$$

$$\cancel{\lambda^2} + \lambda - 24 - \lambda^3 - \cancel{\lambda^2} + 24\lambda + 16 - 5\lambda = 0$$

$$-\lambda^3 + 20\lambda - 8 = 0 \quad \Rightarrow \quad \lambda^3 - 20\lambda + 8 = 0$$

From Cayley Hamilton theorem

$$A^3 - 20A + 8 = 0$$

multiply the equation by A^{-1}

$$A^2 - 20I + 8A^{-1} = 0$$

$$A^{-1} = -\frac{1}{8}\left[A^2 - 20I\right] = \frac{5}{2}I - \frac{1}{8}A^2$$

$$A^{-1} = \frac{5}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{8}\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

$$A^{-1} = \frac{5}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{8}\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

$$= \frac{5}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{8}\begin{bmatrix} 1+1-6 & 1+3-12 & 3-3-12 \\ 1+3+6 & 1+9+12 & 3-9+12 \\ -2-4+8 & -2-12+16 & -6+12+16 \end{bmatrix}$$

$$= \frac{5}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{8}\begin{bmatrix} -4 & -8 & -12 \\ 10 & 22 & 6 \\ 2 & 2 & 22 \end{bmatrix}$$

$$= \begin{bmatrix} 5/2 + \frac{1}{2} & 1 & 3/2 \\ -5/4 & 5/2 - \frac{11}{4} & -3/4 \\ -1/4 & -1/4 & 5/2 - 11/4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

⑤ Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and the second a queen if the first card is ① replaced, ⑪ not replaced

$\Rightarrow$ The probability of drawing a king $= \frac{4}{52}$

$= 1/13$

If the card is replaced, the pack will have 52 cards.

The probability of drawing queen $= \frac{1}{13}$

since both events independent and occuring in succession

probability $= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

if the card is not replaced after drawing a king

probability of drawing queen $= \frac{4}{51}$

probability of drawing both cards in succession $= \frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$