Matrix Project

EE1390: Intro to AI and ML

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Problem Solving Strategy

1. Graphical Verification
   - Using Python
2. Theoretical Computation
   - Using Matrix

- If β is one of the angles between the normals of the ellipse $\mathbf{X}^T V \mathbf{X} = 9$ at the points $\begin{pmatrix} 3\cos\theta \\ \sqrt{3}\sin\theta \end{pmatrix}, \begin{pmatrix} -3\sin\theta \\ \sqrt{3}\cos\theta \end{pmatrix}$; $\theta \in (0, \frac{\pi}{2})$, $V = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$; then $\frac{2\cot}{\sin 2\theta}$ is equal to..

Analysis

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Graphical Analysis

Using python libraries, the following graphs are plotted

1.Normal to ellipse at point **A** and **B** intersecting at **N**

2.Polar graph of β for given value of θ

3.Graph of 2cot β Vs $\sin 2\theta$

ref: `https://github.com/tejasmeshram99/EE1390`

Figure 1

At $\theta = \frac{\pi}{4}$

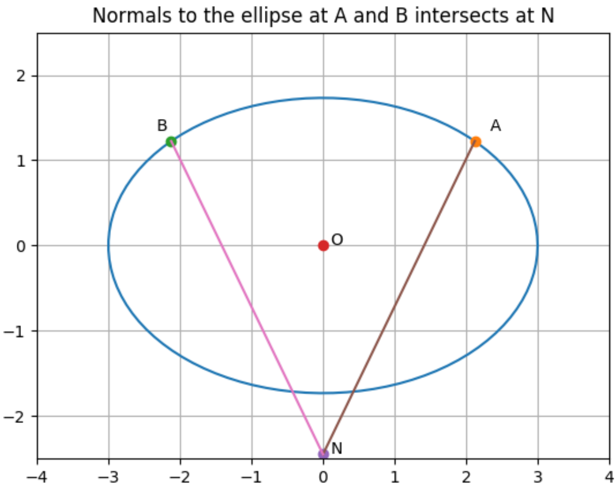


Figure 2

At $\theta = \frac{\pi}{4}$

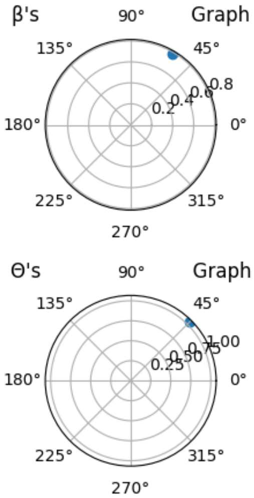
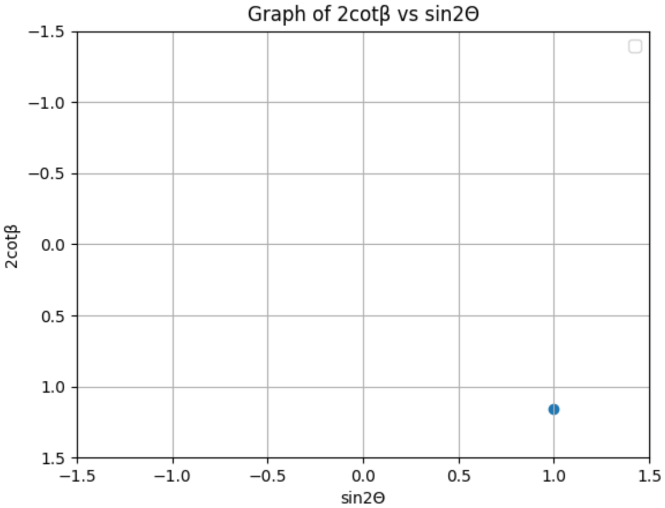


Figure 3

At $\theta = \frac{\pi}{4}$ point is $\begin{pmatrix} 1 & 2/\sqrt{3} \end{pmatrix}$.



Graphical Analysis

Results

For different values of $\theta \in (0, \pi/2)$, the slope of $2\cot\beta$ vs $\sin 2\theta$ turns out to be $\frac{2}{\sqrt{3}}$ or 1.155, which is independent of θ and β.

Analysis

Solution

- We've equation of the ellipse $\mathbf{X}^T V \mathbf{X} = 9$ and two points $\mathbf{A}$ and $\mathbf{B}$. Where, $V = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$, $\mathbf{A} = \begin{pmatrix} 3\cos\theta \\ \sqrt{3}\sin\theta \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -3\sin\theta \\ \sqrt{3}\cos\theta \end{pmatrix}$.

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- Equation of tangents at points $\mathbf{A}$ and $\mathbf{B}$ can be written as $\mathbf{A}^T V \mathbf{X} = 9 \implies \mathbf{n}_1^T \mathbf{X} = 9$
  where, $\mathbf{n}_1^T = \mathbf{A}^T V = \begin{bmatrix} 3\cos\theta & 3\sqrt{3}\sin\theta \end{bmatrix}$

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- Equation of tangents at points $\mathbf{A}$ and $\mathbf{B}$ can be written as $\mathbf{A}^T V \mathbf{X} = 9 \implies \mathbf{n}_1^T \mathbf{X} = 9$
  where, $\mathbf{n}_1^T = \mathbf{A}^T V = \begin{bmatrix} 3\cos\theta & 3\sqrt{3}\sin\theta \end{bmatrix}$
- $\mathbf{B}^T V \mathbf{X} = 9 \implies \mathbf{n}_2^T \mathbf{X} = 9$
  where, $\mathbf{n}_2^T = \mathbf{B}^T V = \begin{bmatrix} -3\sin\theta & 3\sqrt{3}\cos\theta \end{bmatrix}$

Solution(Cont'd)

- The angle between normal vectors n_1, n_2 is β, $0 \leq \beta \leq \pi$
  $\cos\beta = \frac{n_1^T n_2}{\|n_1\|\|n_2\|}$;

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Solution(Cont'd)

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  $\cos\beta = \frac{n_1^T n_2}{\|n_1\|\|n_2\|}$;
- $\cot\beta = \frac{n_1^T n_2}{\sqrt{(\|n_1\|\|n_2\|)^2 - \left(n_1^T n_2\right)^2}} = \frac{\sin 2\theta}{\sqrt{3}}$
- Therefore, $\frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$.

References I

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Introduction to AI and ML, Spring, 2019.
github.com/gadepall/school/tree/master/linalg

Latex Beamer
https://www.overleaf.com/learn/latex/Beamer
http://detexify.kirelabs.org/classify.html

Thanks!!

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