Matrix Project

EE1390: Intro to AI and ML

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February 17, 2019

Problem Solving Strategy

1. Theoretical Computation
   - Using Matrix
2. Graphical Verification
   - Using Python

Matrix problem in coordinate geometry

From JEE Main 2018

- If β is one of the angles between the normals of the ellipse $\mathbf{X}^T V \mathbf{X} = 9$ at the points $\begin{bmatrix} 3\cos\theta \\ \sqrt{3}\sin\theta \end{bmatrix}$, $\begin{bmatrix} -3\sin\theta \\ \sqrt{3}\cos\theta \end{bmatrix}$; $\theta \epsilon (0, \frac{\pi}{2})$, V $= \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$; then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to

Analysis

Solution

- We've equation of the ellipse $\mathbf{X}^T V\mathbf{X} \;= 9$ and two points $\mathbf{A}$ and $\mathbf{B}$. Where, V $= \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$, $\mathbf{A}= \begin{bmatrix} 3\cos\theta \\ \sqrt{3}\sin\theta \end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix} -3\sin\theta \\ \sqrt{3}\cos\theta \end{bmatrix}$.

Solution

- We've equation of the ellipse $\mathbf{X}^T V\mathbf{X} \; = 9$ and two points $\mathbf{A}$ and $\mathbf{B}$. Where, V $= \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$, $\mathbf{A}= \begin{bmatrix} 3\cos\theta \\ \sqrt{3}\sin\theta \end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix} -3\sin\theta \\ \sqrt{3}\cos\theta \end{bmatrix}$.
- Equation of tangents at points $\mathbf{A}$ and $\mathbf{B}$ can be written as
$$\mathbf{A}^T V\mathbf{X} = 9 \implies \mathbf{n}_1^T\mathbf{X} \; = 9 \;, \; \mathbf{n}_1 = V\mathbf{A}$$
$$\mathbf{B}^T V\mathbf{X} = 9 \implies \mathbf{n}_2^T\mathbf{X} \; = 9 \;, \; \mathbf{n}_2 = V\mathbf{A}$$

Solution(Cont'd)

- The angle between normal vectors n_1, n_2 is β, $0 \leq \beta \leq \pi$
  $\cos\beta = \frac{n_1^T n_2}{\|n_1\|\|n_2\|}$.

Solution(Cont'd)

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  $\cos\beta = \frac{n_1^T n_2}{\|n_1\|\|n_2\|}$.
- So,
  $\cot\beta = \frac{n_1^T n_2}{\sqrt{(\|n_1\|\|n_2\|)^2 - \left(n_1^T n_2\right)^2}} = \frac{\sin 2\theta}{\sqrt{3}}$

Solution(Cont'd)

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  $\cos\beta = \frac{n_1^T n_2}{\|n_1\|\|n_2\|}$.
- So,
  $\cot\beta = \frac{n_1^T n_2}{\sqrt{(\|n_1\|\|n_2\|)^2 - \left(n_1^T n_2\right)^2}} = \frac{\sin 2\theta}{\sqrt{3}}$
- Therefore,
  $\frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$.

Analysis

Graphical Analysis

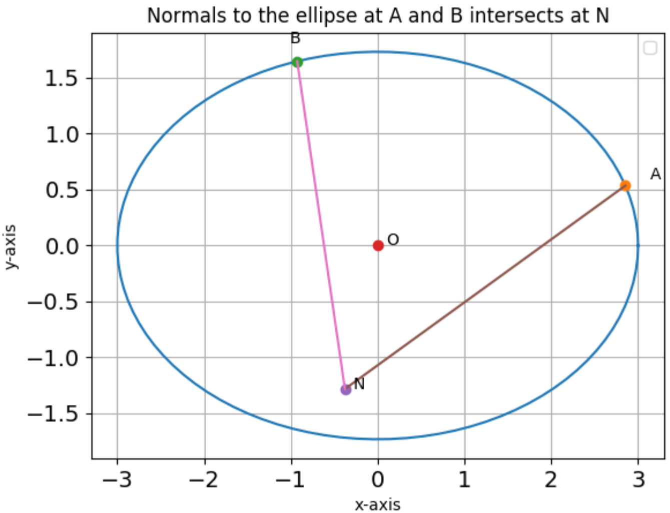
Using python libraries, graph of the following ellipse has been plotted.
https://github.com/AbhishekKrS/EE1390

Results

The value of $\frac{2\cot\beta}{\sin 2\theta}$ turns out to be $\frac{2}{\sqrt{3}}$ or 1.155, which is independent of $'\theta'$.

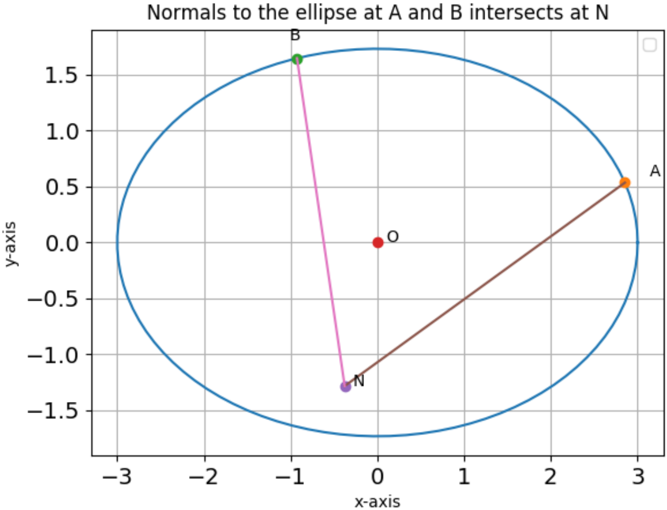
Figure

At $\theta = \frac{\pi}{10}$



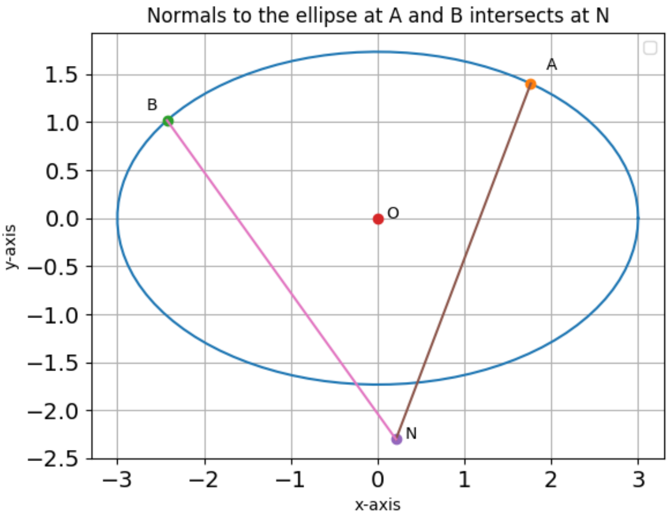
Figure

At $\theta = \frac{2\pi}{10}$



Figure

At $\theta = \frac{3\pi}{10}$



Figure

Matrix Project

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Theoretical Computation

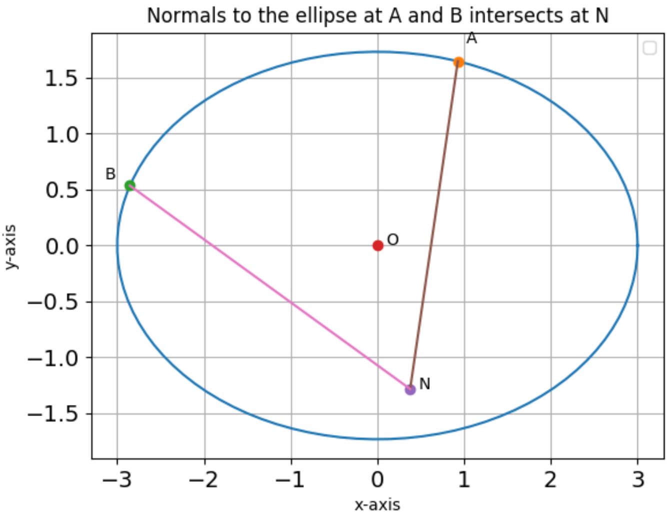
Using Matrix

Graphical Verification

Using Python

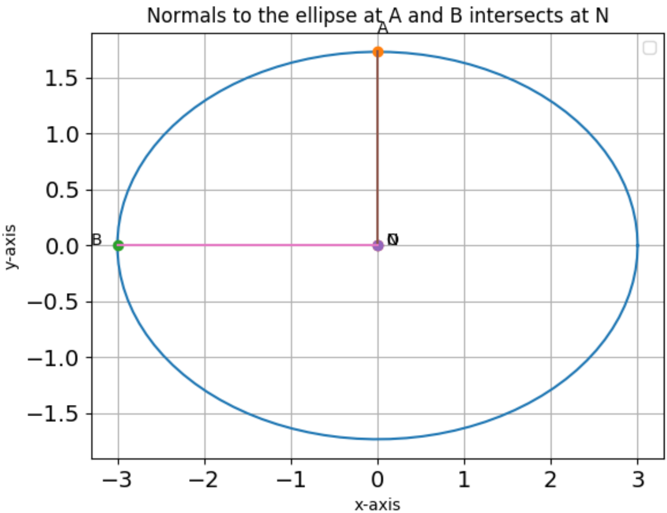
Summary

At $\theta = \frac{4\pi}{10}$



Figure

At $\theta = \frac{5\pi}{10}$



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Latex Beamer
https://www.overleaf.com/learn/latex/Beamer