PL04 - INTRODUCTION to PROBABILITY

Video 1 : Permutations vs Combinations

- PERMUTATIONS - The no. of different ways that a certain no. of objects can be arranged in order from a larger no. of objects.

- If there are n objects, how many different ways can we make ordered lists of size r.

- $P(n, r)$

- COMBINATIONS - The no. of different ways that a certain no. of objects AS A GROUP can be selected from a large no. of objects.

- If there are n objects, how many different ways can we select groups or sets of size r.

e.g Top 3 in any order ; $C(10, 3) = 120$ possibilities

e.g Top 3 in exact order ; $P(10, 3) = 720$ possibilities

Video 2 : COMBINATIONS

COMBINATION FORMULA :

$$\boxed{C(n, r) = \frac{n!}{r!\,(n-r)!}}$$

n = the no. of things to choose from
r = no. of things chosen, group size
$!$ = factorial.

$$C(3, 1) = \frac{3!}{1!\,(3-1)!} = \frac{3 \times 2 \times 1}{1 \times 2 \times 1} = 3$$

Video 3: PERMUTATIONS

PERMUTATION FORMULA :

$$\boxed{P(n, r) = \frac{n!}{(n-r)!}}$$

n = the no. of things to make lists about
r = no. of things listed, ordered list size
$!$ = factorial.

$$P(3, 3) = \frac{3!}{0!} = 6$$

Video 4 ; Vido 5 ; WorkOut Examples - Skipped from Notes.

Video 6 : Combinations : Nearly Normal

- Combinations : $C(n, r)$ = n choose r

We know that n is total group size & r is selection size.

$\therefore$ r can be any natural number between 0 & n.

or $C(12, 0 \leq n \leq 12)$

or $\boxed{C(n, 0 \leq r \leq n)}$

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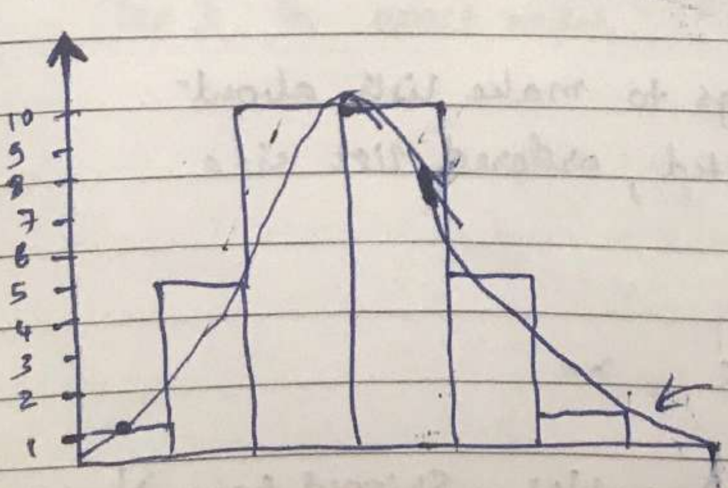
n	r	Expression	Combinations → frequency
5	0	C(5, 0)	1
5	1	C(5, 1)	5
5	2	C(5, 2)	10
5	3	C(5, 3)	10
5	4	C(5, 4)	5
5	5	C(5, 5)	1



Normal curve
Bell curve

- As we increase our group size and then calculate the frequency of each possible combination where $0 \leq r \leq n$, our histogram begins to look a lot like a NORMAL CURVE or "BELL CURVE".

- If the group size increases towards infinity, The BARS, would almost be squished together, almost invisible, creating a SMOOTH CURVE.

Video 7: Combinations - Under the Curve.

(Finite Maths & Statistics Overlap)

Question : The value of mortgage loans made by a bank were normally distributed (bell shaped) with a mean of \$150,000 and standard deviation of \$30,000. We need to Prepare of Report for

(i) What is the probability that a randomly selected mortgage was between \$90,000 & \$190,000 ?

(ii) In your report, include the dollar range of all mortgages in the middle 40% ($30\% \leq \$\$ \leq 70\%$) What are the endpoints, in terms of \$\$ amount, for that range ?

- All of the proportion bars in the histogram add up to 1.
  Their TOTAL AREA is 1.

- e.g $C(3,0) = 1$
  $C(3,1) = 3$
  $C(3,2) = 3$ Total combinations
  $C(3,3) = 1$ ↓

Sum of Frequency of each combination $= 1+3+3+1 = 8$

Proportion of a combination i.e $C(3,2) = \frac{3}{8}$

Similarly, each combination consitutes a part of the whole, so all proportions add up to 1

Hence, here, $\frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

- As 'n' becomes large "discrete example bars" approach a
- Therefore, All the "PROPORTION BARS" approach a

CONTINUOUS VARIABLE "CURVE", (but never actually gets there)

- The AREA under a continuous normal curve or bell curve is 1.

Video 8: Example

Video 9: Set Operations & Notations

Sets : Elements : Equivalent sets : Subsets : Proper Subsets

$\in$ $M=N$ or $N=M$ $E \subseteq N$ $E \subset N$

Empty sets - $Z = \{ \phi \}$ or $Z = \{ \}$

Also, $Z \subseteq N$, also $Z \subset N$

Infinite sets - $I = \{ 1, 2, 3, \dots \}$

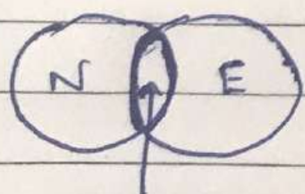
- Does not have a finite no. of elements.

Set Builder Notation - $e = \{ n \mid n$ is an odd integer less than 10$\}$

CONCLUSION :

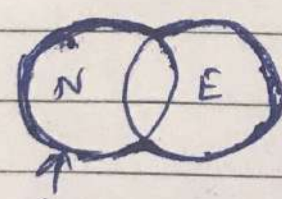
Set is a simple way of categorizing things based on common criterion.

Video 10 : Venn Diagram



Intersection

$N \cap E$

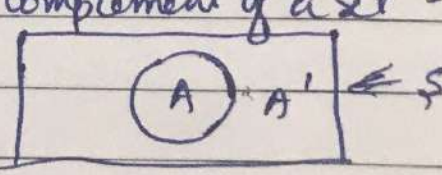


Union

$N \cup E$

Universal Set - ξ

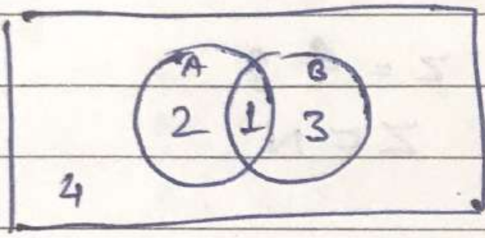
Complement of a set - A'



Video 11 : Venn Diagram Regio Method

- Assign Regions, a serial no. to the different parts of Venn-Diagram.

$U_r =$



$A = 2, 1$ $A \cap B = 1$

$B = 3, 1$ $A \cup B = 2, 1, 3$

Universal set $= 1, 2, 3, 4$ $A' = 3, 4$

$(A \cap B)' = 2, 3, 4$

$(A \cup B)' = 4$

$(A' \cap B') = 4$ $(A' \cup B') = 2, 3, 4$

Video 12 : Cardinality of a Union

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

~~Video 13:~~ Video 14 : Workout Example / Problems