PL 09 - HYPOTHESIS TESTING

Video 1 : Introduction to Hypothesis Formulation

" Am I testing an ASSUMPTION or the STATUS QUO that already exists ? (water bottle)
Or am I testing a CLAIM or an ASSERTION beyond what I already know or can know ? "
(hybrid engine)

- NULL and ALTERNATIVE hypothesis

-- By definition, the null & alternative hypothesis are opposites ; mutually exclusive.
  -- The null is either rejected or it is not.
  -- Only if the null is rejected can we proceed to the alternative.

-- Researchers can start with either the null or alternative and then form the other as complement to the first.
  -- So, it mainly depends on POV.

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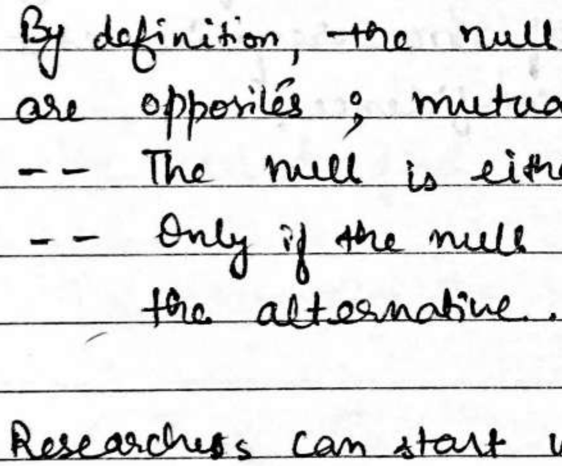
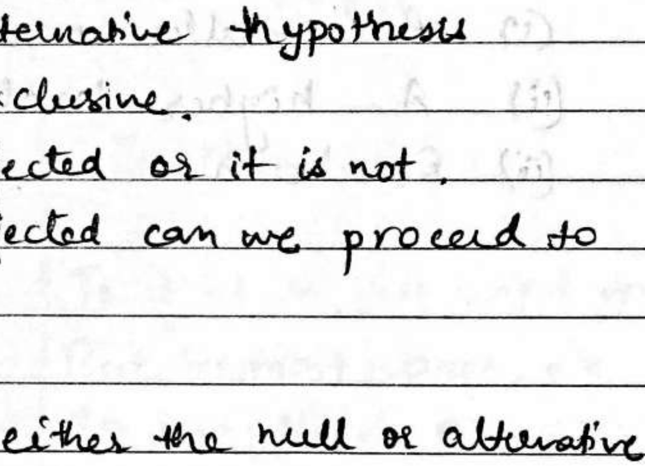


"This is Accepted as True, let's TEST it ".



" This might be True, lets test it. If not the truth is something else"

Video 2: Null and Alternative Hypothesis

- Null hypothesis – H_0 (symbol)

  -- "Null" means nothing new or different;
  -- assumption or status quo maintained

- Alternative hypothesis – H_a (symbol)

  -- The "alternative" is simply "the other option" when the null is rejected; nothing more.

- Properties

H_0	H_a
• Assumption, status quo, nothing new	• Rejection of an assumption
• Assumed to be "true"; a given	• Rejection of an assumption or the given.
• Negation of the research question	• Research question to be "proven"
• Always contains an equality ($=, \leq, \geq$)	• Does not contain equality. ($\neq, <, >$)

- NULL and ALTERNATIVE Statements

-- All Statistical conclusions are made in <u>reference</u> to the <u>null hypothesis</u>

-- As researchers we either REJECT the null hypothesis or fail to reject the null hypothesis. " We do not accept the null ".

-- If we <u>reject</u> the null hypothesis, then we conclude the <u>data supports</u> the <u>alternative hypothesis</u>.

-- However, if we <u>fail to reject</u> the <u>null hypothesis</u>, it <u>does not mean</u> we <u>have proven</u> the null hypothesis is " true ".
b/coz afterall it was an ASSUMPTION.

- <u>Words to Prefer</u>

$\boxed{\text{SUPPORT}}$ over Truth

$\boxed{\text{INDICATE}}$ over Certain

$\boxed{\text{INFER}}$ over Prove

Video 3: Null and Hypothesis Alternative Problems

Q) According to US Dept of Agr, in 2006 the average farm size was 2.3 km².
This decade long trend has to increas due to large agribusiness.

A business analyst wishes to test if now (2013) farm size is larger than it was in 2006.

Establish a null and Alternative hypothesis

-- What is our Assumption?
We assume that there has been no change in farm size since 2006.
This is our NULL hypothesis.

-- Are we testing a preliminary claim?
Yes, we wish to see the farm size has INCREASED since 2006.

So,

$H_0 \leq$ $H_0 \leq 2.3\ km^2$

$H_a >$ $H_a > 2.3\ km^2$

If the data indicates that farm size has increased then, we REJECT the NULL; Reject our Assumption.

Video 4 : Type 1 and Type 2 Errors

- Type 1 error :-

  -- Rejection of the assumption (null hypothesis) when it should not have been rejected.

  -- Incorrectly rejecting the null hypothesis.

  -- e.g. In this case, a false alarm.

- Type 2 error :

  -- Failure to reject the assumption (null) when it should have been rejected.

  -- Incorrectly NOT REJECTING the null hypothesis.

- Example : H_0 = problem is annoying but not serious
  H_a = problem is serious, everything NOT OK

		Actual Condition	
		No serious defect	Serious Defect
Conclusion	Accept H_0 (no serious defect)	Correct conclusion	Type II error
	Reject H_a (Serious defect)	Type I error	Correct conclusion

In general, Type 2 errors are more destructive.

Video 5: Type 1 and Type 2 Errors Examples

Q) Revisiting the same problem of Farm size.

Our Assumption :
-- There has been no change in farm size since 200[illegible]

$H_0 : \mu \leq 2.3\ km^2$ $H_a : \mu > 2.3\ km^2$

		Actual Condition	
		$\mu \leq 2.3\ km^2$	$\mu > 2.3\ km^2$
Conclusion	Do not Reject - H_0	Correct	Type 2 error
	Reject - H_a	Type 1 error	Correct

- Causes of Type 1 and Type 2 errors

-- When selecting samples we are always subject to the laws of the chance
-- We may by random chance alone, select a sample that is not Representative of the population.
  -- Selection of under-filled or over-filled bottles
  -- Select a sample of very small or very large far[illegible]
-- Our sampling methods/techniques may be FLAWED.

Video 6 : Hypothesis Testing : Visualizing Type 1 & 2 Errors

- The Actual vs Hypothesized Mean

$H_0 : \mu = \mu_0$ $H_a : \mu \neq \mu_0$

μ = actual mean of the population under analysis
μ_0 = the hypothesized mean of the population under "

" Does the Actual Mean Align with the hypothesized mean? ".

We will test that question using
SAMPLE MEANS & CONFIDENCE INTERVALS.

- The Two-tailed Test Rejection Region

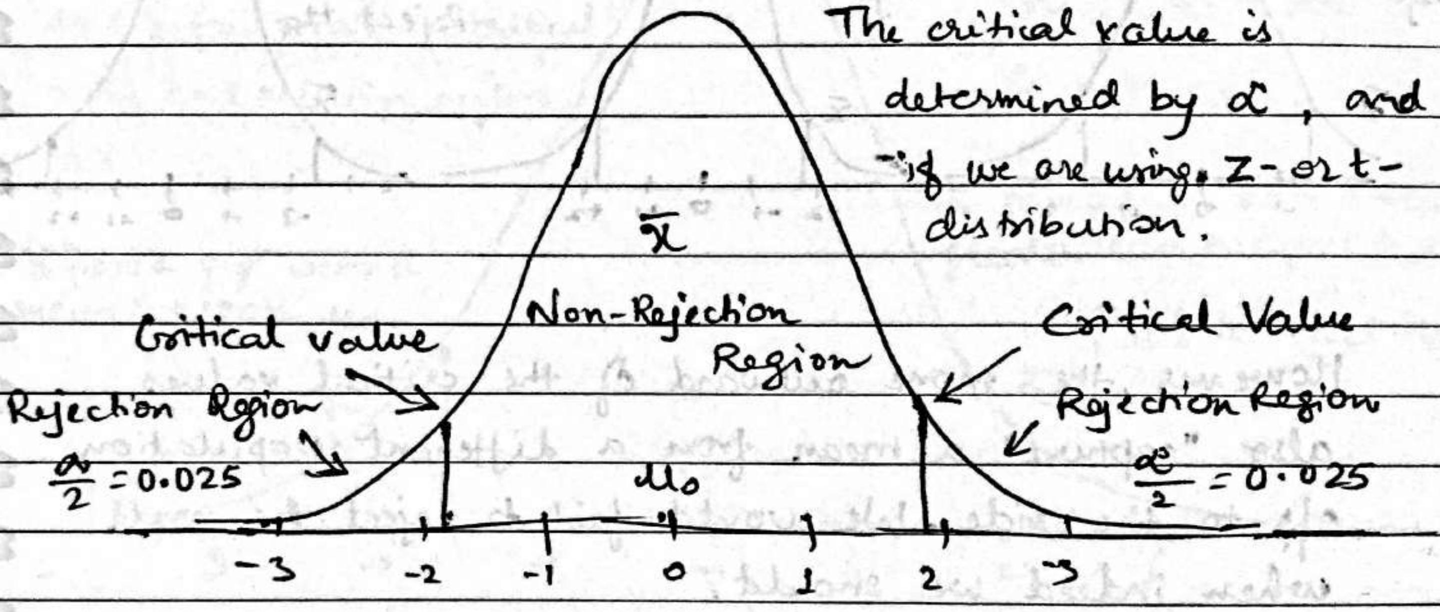
$H_0 : \mu = \mu_0$

$H_a : \mu \neq \mu_0$

$\alpha = 0.05$

If the null hypothesis is correct, then $(\alpha \times 100)\%$ of the sample means should be in the non-rejection region.

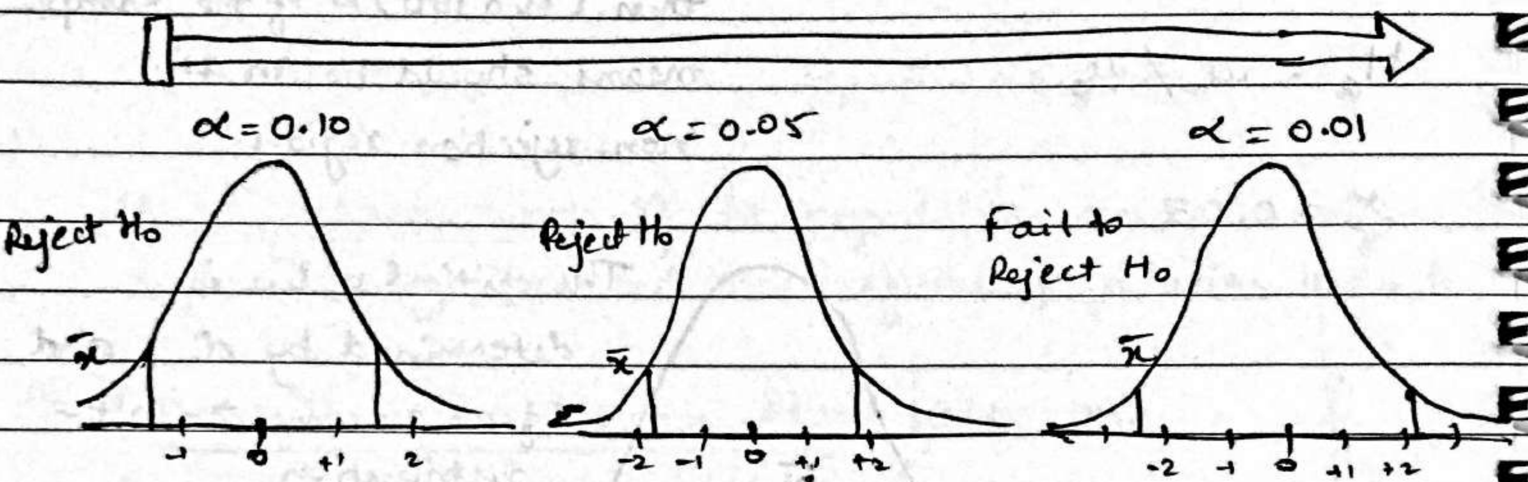
The critical value is determined by α, and if we are using Z- or t-distribution.



"Did our sample come from the same population we assume is underlying the null hypothesis?

If so, then we expect our sample mean to be inside the critical region 90%/95%/99% of the time depending on what we choose for α.

- As α decreases so does the chance of Type 1 error. The critical value to reject the null hypothesis moves outward, thus "capturing" more sample means.



However, the move outward of the critical values also "capture" a mean from a different population off to the side. We would fail to reject the null when indeed we should.
Thus the chance of Type II error increases as α decreases.

- ## The One-tailed (lower) Test Rejection Region

-- In a one-tailed hypothesis test, all of the α is in one tail or the other depending on the alternative hypothesis.

The trick is that H_a "points" to the tail where the critical value and rejection region are.

$H_0 : \mu \geq \mu_0$

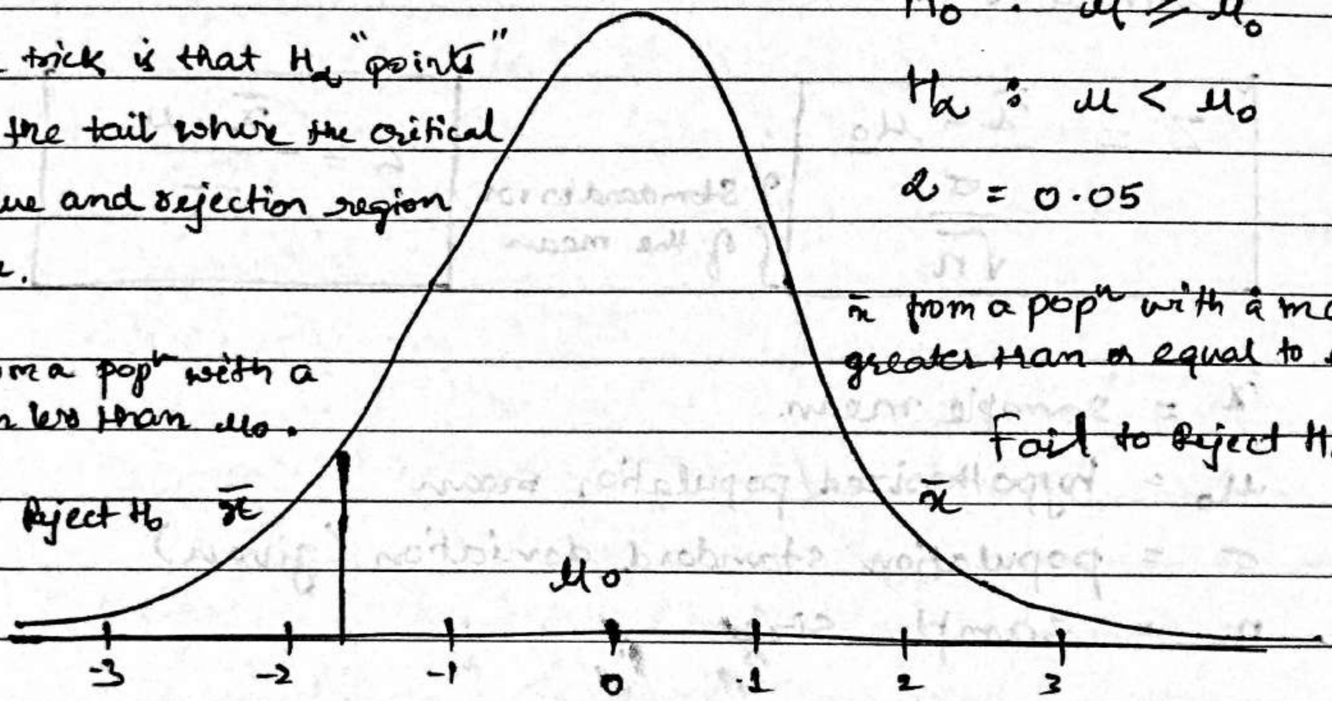
$H_a : \mu < \mu_0$

$\alpha = 0.05$

$\bar{x}$ from a pop^n with a mean less than μ_0.

$\bar{x}$ from a pop^n with a mean greater than or equal to μ_0

Fail to Reject H_0



- ## Type 1 and Type 2 error Review

		Actual Condition	
		H_0 "true"	H_a "true"
CONCLUSION	Do not Reject H_0	$\bar{x}$ correctly inside non-rejection region	Type 2 error: $\bar{x}$ inside non-rejection region by Chance
	Reject H_0	Type 1 error: $\bar{x}$ outside non-rejection region due to chance	$\bar{x}$ correctly outside the non-rejection region.

$\bar{x}$ = sample mean

Video 7 : Hypothesis Testing : Single Sample Hypothesis Z-test concepts : with known σ

- Z - test for a single mean.

Formula :

$$\boxed{Z = \frac{\bar{x} - \mu_0}{\underbrace{\frac{\sigma}{\sqrt{n}}}_{\text{Standard error of the mean}}}} \qquad \boxed{Z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}}$$

$\bar{x}$ = sample mean

μ_0 = hypothesized population mean

σ = population standard deviation (given)

n = sample size

** Once we find Z ~~value~~ test value, we answer. Is this Z-test value in the nonrejection or the rejection region?

Video 8 : Single Sample Hypothesis Z-test Examples

• **Hypothesis Testing Procedure : Follow the ORDER**

1. Start with a well-developed, clear research problem
2. Establish hypotheses, both null & alternative.
3. Determine appropriate statistical test and sampling distribution. (like Z- or t-test & others)
4. Choose the Type 1 error rate. (Type 1 $\propto \frac{1}{\text{Type 2 error}}$)

5. State the decision rule
6. Gather Sample data. (form your question first,)

~~7. State statistical conclusion~~

7. Calculate test statistics.
8. State statistical conclusion.
9. Make decision or inference based on conclusion

• σ known or unknown

-- As with confidence intervals, there are 2 types of single-sample hypothesis tests.
-- σ known - use normal standard / Z-distribution
-- σ unknown - use t-distribution
-- Always good to check the sample data for NORMALITY

Q.) A report from 6 years ago indicated that the average gross salary for a business analyst was \$69,872. We wish to test this figure against current salaries to see if the current salaries are statistically different from the old ones.

Based on other studies, we assume $\sigma = \$13,985$
Sample of 112 current salaries are taken.

Remember the Hypothesis Testing Procedure

$\boxed{\text{Step 1 : Establish Hypothesis}}$

$H_0 : \mu = 69,873$ $H_a : \mu \neq 69,873$

$\boxed{\text{Step 2 : Determine Appropriate Statistical Test and Sampling Distribution.}}$

This will be a two-tailed test.
Salaries could be higher or lower.

Since σ is known, we will use <u>z-distribution</u>.

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$ We'll use this formula.

Step 3 : Specify the Type 1 error rate
(significance level)

$\alpha = 0.05$ - (It means, i am ohk with the possibility of making Type I error 5% of the time)

Step 4 : State the Decision Rule

If $Z > 1.96$, reject H_0
If $Z < -1.96$, reject H_0

Step 5 : Gather Data

After gathering data of 112 salaries, we find

$n = 112$, $\bar{x} = \$72,180$

Step 6 : Calculate test Statistic

$\bar{x} = \$72,180$
$\mu = \$69,873$
$\sigma = \$13,985$
$n = 112$

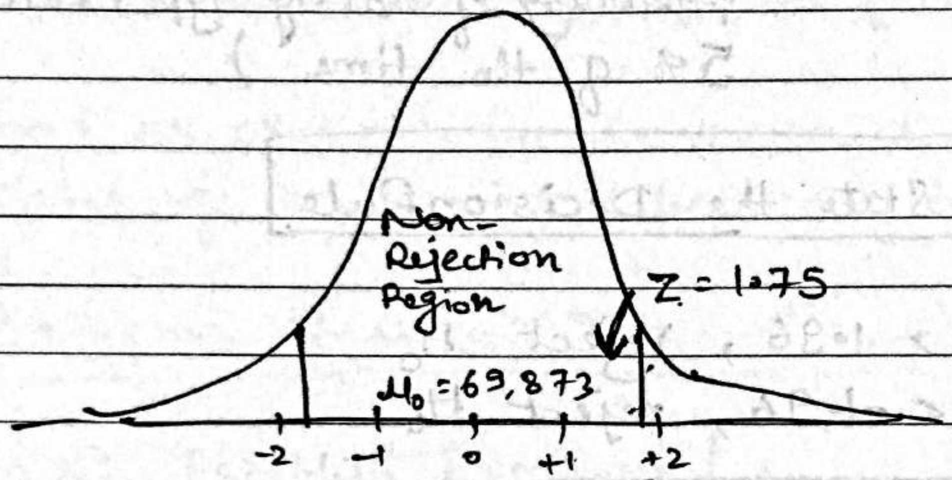
$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$ - (Replace the values in this formula)

$Z = 1.75$

Step 7 and 8 : State Statistical Conclusion

$H_0: \mu = \$69,873$

$H_a: \mu \neq \$69,873$



Since the test statistic is inside the nonrejection region and not beyond the critical value, we fail to reject the null hypothesis

- Note : For other example, go through the video again.

Video 9: Single Sample Hypothesis Z-test Alpha and p-Values.

How does the critical $\bar{x}$ and error risk change as α changes?

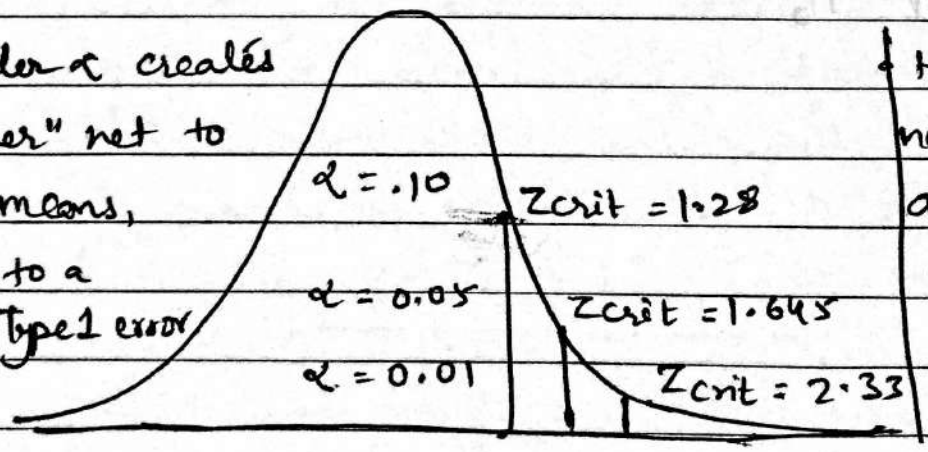
$\alpha = .01$, $Z_{crit} = 2.33$ $\quad 2.33 = \dfrac{\bar{x} - 3}{\frac{1.5}{\sqrt{225}}} \Rightarrow \bar{x} = 3.233$

$\alpha = 0.05$, $Z_{crit} = 1.645$ $\quad - - - - \Rightarrow \bar{x} = 3.1645$

$\alpha = 0.10$, $Z_{crit} = 1.28$ $\quad - - - - \Rightarrow \bar{x} = 3.128$

The ALPHA (α) effect

- A smaller α creates a "wider" net to "catch" means, leading to a smaller Type 1 error



However a wider net may capture a mean that belongs to a different distrib

- <u>The p-value method</u>

-- Based on our $\alpha = 0.1$ we know that 1% of our area (probability) is in the upper tail past our $z_{crit} = 2.33$.

In the p-value method, we ask how much area (probability) is above our test statistic of $z = 2.5$

Using a Z-table or Excel we can find this AREA/probability = 0.0062

Since this is less than $\alpha = 0.01$, we would reject H_0

Video10 : Hypothesis Testing : Single Sample T-test with unknown σ

- Revisit : Hypothesis Testing Procedure.

- There are 2 types of single-sample hypothesis tests.
  (i) σ known, Z-test
  (ii) σ unknown, t-test

- In t-distribution,
  → Every sample size has its own t-distribution with n-1 degrees of freedom.

- Always good to check the sample data for NORmality.

- General T-distribution Patterns

-- A smaller sample size means more sampling error.

-- This sampling error due to small n means a higher probability of extreme sample means.

-- More probability in the tails means the center hump of the t-distribution must come downward.

-- This process "squishes" the distribution slightly downward.

-- Given the same α and s, a smaller n will push the critical values further outward in the tails due to the uncertainty associated with the small n.

FORMULA

$$t = \frac{\bar{x} - \mu_0}{\underbrace{\frac{s}{\sqrt{n}}}_{\text{standard error of the mean}}}$$

$$t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}}$$

s = sample standard dev

n = sample size

$\bar{x}$ - sample mean

μ_0 - hypothesised population mean

Video 11: Single Sample Hypothesis t-test Examples

- Screenshots from the video attached.

Video 12: Screenshots attached.

Video 13: Screenshots attached.

Video 14: Screenshots attached.

F

$\bar{x}$ – sample mean

$$\mu_0 + Z_0 \frac{\sigma}{\sqrt{n}} = \mu_a - Z_\beta \frac{\sigma}{\sqrt{n}}$$

$$\Rightarrow n = \frac{(Z_a + Z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2}$$

Z_a = z value giving area of α
Z_β = z value giving area of β
σ = the population standard deviation
μ_0 = the value of the population mean in the null hypothesis
μ_a = the value of the population mean used chosen for Type 2 error.