Design and Analysis of Algorithms

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(a) **Aggregate Method:**

* In the aggregate method, we analyze total cost for a sequence of operations and then, divide it by no. of operations to get the average cost per operation.

(i) **Inserting an element without resizing:** for each insertion where there is enough space in table, the cost of constant. $O(1)$.

(ii) **Resizing:** Every time table doubles in size, the cost is proportional to no. of elements being copied to new table. If the table size is k before doubling, copying all k elements takes $O(k)$ time.

Total cost of n Insertions:

c> The cost of inserting n elements: $O(n)$ for insertions.

c> The cost of resizing: The first doubling involves copying 1 element, next involves copying 2, then 4 & so on.

c> The sum of these doubling operations is:

$O(1 + 2 + 4 + \dots + 2^k) \sim O(n)$

* Amortized cost per insertion: $\frac{O(n) + O(n)}{n} = O(1)$

So, using aggregate method, the amortized time complexity for inserting n elements is $O(1)$

(b.) **Accounting Method:**

The accounting method assigns "credits" to each operation to account for costs of future of expensive operations.

(i) **Assiging credits:**

Each insertion will be charged 3 credits.

c> 2 credits for insertion itself, which pays constant time $O(1)$ operation

↳ 1 credit to help pay for cost of future resizing operations.

② Cost of Insertion:

↳ when no resizing happens, the cost is exactly 1 credit for the insertion.

↳ when a resizing happens, it costs $O(i)$ for copying i elements, but since we have credit saved for each previous insertion, we have enough credits to cover resizing.

Resizing & Cost:

↳ when the table doubles, the cost of copying elements doubles as well.

↳ The total no. of credits that we collect is 3 credits per insertion for n insertions, resulting in $3n$ credits.

↳ Each resizing covered by the saved credits. The total no. of resizing operations is been proportional to no. of doublings about $\log n$ times.

Final Amortized Cost:

* Inserting n elements costs 1 credit each
* Total credits collected $= 3n$
* Cost of each resizing operating is already covered by saved ~~costs~~ credits.

Amortized Cost Per Insertion:

$$= \frac{3n}{n}$$
$$\approx O(1)$$

∴ $O(1)$ is amortized time complexity for inserting n elements using accounting method for a dynamic table that doubles in size.