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DSGN & ANLY ALGORITHMS
5311 - Hands-On-6

(3.) Mathamatically derive average runtime Complexity of non-random Pivot Version of quickSort.

Ans: Usually the runtime Complexity depends on the choice of Pivot & how balanced Partitions are.

↳ If Pivot splits the array into two nearly equal halves, recurrence relation is

$$T(n) = T(n/2) + T(n/2) + O(n)$$

Also, which can been simplifies as

$$T(n) = 2T(n/2) + O(n)$$

↳ Expected Runtime Recurrence Relation which gives average Pivot selection, the recurrence becomes

$$T(n) \leq T(\alpha n) + T((1-\alpha)n) + O(n)$$

↳ The Solving Recurrence by Summing up to work done at each level of recursion.

In all total no. of levels is $O(\log n)$, & total work at each level remains $O(n)$, overall Complexity Sums to $O(n \log n)$

↳ The Average-case time Complexity is $O(n \log n)$

This is because:

↳ The Pivot on average results is nearly balanced Partitions.

↳ The total depth of recursion is $O(\log n)$ and Work Per level is $O(n)$

Even through worst-case complexity $O(n^2)$; in Practical scenarios, QuickSort performs close to $O(n \log n)$, making it one of fastest sorting algorithms.