ECO 764 - OPTIONS

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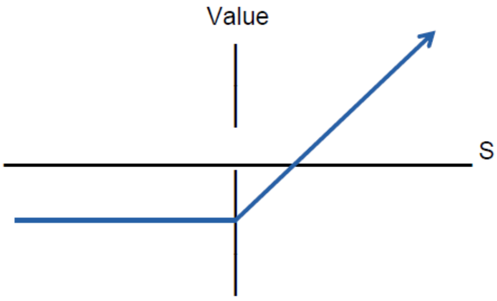
A. Strategy.

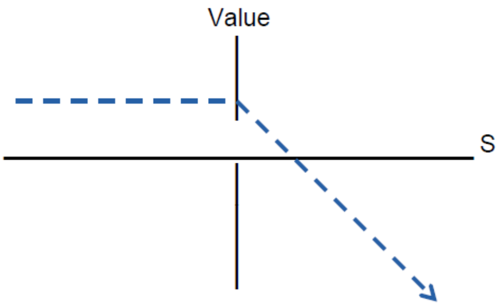
Option Buyer (Call)

1. If you think S will ↑, **buy a call**;
   a. If right (S↑), S > K, exercise.
      (buy @ K, sell @ S, worth (S-K)).
   b. If wrong (S↓), S < K,
      lose price of call.

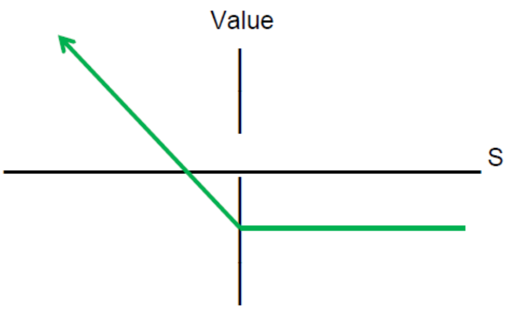


Option Writer (Call)

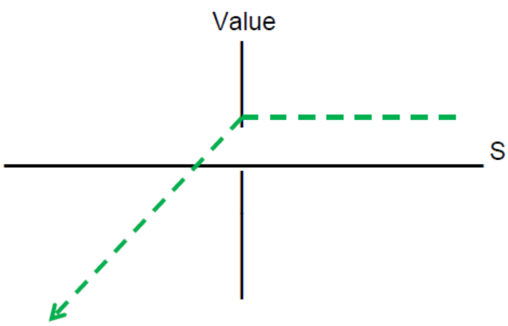
2. If you think S will ↓, **sell a call**;
   a. If right (S↓), S < K,
      keep price of call.
   b. If wrong (S↑), S > K,
      will be exercised.
      (must buy @ S, sell @ K, lose (S-K)).



3. If you think S will ↓, **buy a put**;
   a. If right (S↓), S < K, exercise.
      (buy @ S, sell @ K, worth (K-S)).
   b. If wrong (S↑), S > K,
      lose price of put.



4. If you think S will ↑, **sell a put**;
   a. If right (S↑), S > K,
      keep price of put.
   b. If wrong (S↓), S < K,
      will be exercised.
      (must buy @ K, sell @ S, lose (K-S)).



A *naked option* is an option that is not combined with an offsetting position in the underlying stock. The initial and maintenance margin required by the CBOE for a written naked call option is the greater of the following two calculations:

1. A total of 100% of the proceeds of the sale plus 20% of the underlying share price less the amount, if any, by which the option is out of the money
2. A total of 100% of the option proceeds plus 10% of the underlying share price.

For a written naked put option, it is the greater of

1. A total of 100% of the proceeds of the sale plus 20% of the underlying share price less the amount, if any, by which the option is out of the money
2. A total of 100% of the option proceeds plus 10% of the exercise price.

The 20% in the preceding calculations is replaced by 15% for options on a broadly based stock index because a stock index is usually less volatile than the price of an individual stock.

Table 11.1 Summary of the effect on the price of a stock option of increasing one variable while keeping all others fixed.

Variable	*European call*	*European put*	*American call*	*American put*
Current stock price	+	−	+	−
Strike price	−	+	−	+
Time to expiration	?	?	+	+
Volatility	+	+	+	+
Risk-free rate	+	−	+	−
Amount of future dividends	−	+	−	+

+ indicates that an increase in the variable causes the option price to increase or stay the same;
− indicates that an increase in the variable causes the option price to decrease or stay the same;
? indicates that the relationship is uncertain.

Europian Bounds

$c \leq S$ and $C \leq S$.

$c \geq S - Ke^{-rT}$

$P \leqslant K$

$p \leqslant Ke^{-rT}$

$p > \max\{ (Ke^{-rT} - S), 0 \}$

Put-call parity

		$S_T > K$	$S_T < K$
Portfolio A	**Call option**	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K
Portfolio C	Put option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

Summarize: If $S_T > K$, both portfolios are worth S_T at time T; if $S_T < K$, both portfolios are worth K at time T.

Both are worth: Max (S_T , K)
when the options expire at time T.

$$\text{Call (c)} + Ke^{-rT} + D = \text{Put (p)} + S_0$$

- Put-call parity is applicable only in case of European because the options cannot be exercised prior to time T.
- Since options are European, the portfolios have identical values at time T, they must have identical values today.

Arbitrage opportunity:

- If this were not the case, an arbitrageur could buy the less expensive portfolio and sell the more expensive one.

▼ Europian Options

$$\text{Profit} = \begin{cases} \text{Present value} = \left| P + S_0 - C - D - Ke^{-rT} \right| \\ \text{Future value} = \left| (P + S_0 - C - D)e^{rT} - K \right| \end{cases}$$

American Options

American Options

Put–call parity holds only for European options. However, it is possible to derive some results for American option prices. It can be shown (see Problem 11.18) that, when there are no dividends,

$$S_0 - K \leqslant C - P \leqslant S_0 - Ke^{-rT} \tag{11.7}$$

American Options: Calls on non dividend paying stock

- Evidence suggests that it is never optimal to exercise an American call option on a non-dividend-paying stock before the expiration date.
- A call option, when held instead of the stock itself, in effect insures the holder against the stock price falling below the strike price.
- Once the option has been exercised and the strike price has been exchanged for the stock price, this insurance vanishes.

$$\operatorname{Max}(S_0 - Ke^{-rT}, 0) \leq C \leq S_0$$

▼ American Options: Puts on non dividend paying stock

- It can be optimal to exercise an American put option on a non-dividend-paying stock early.

- **If the investor waits, the gain from exercise might be less than K, but it cannot be more than K, because negative stock prices are impossible.**

$$\max(K - S_0, 0) \le P \le K$$

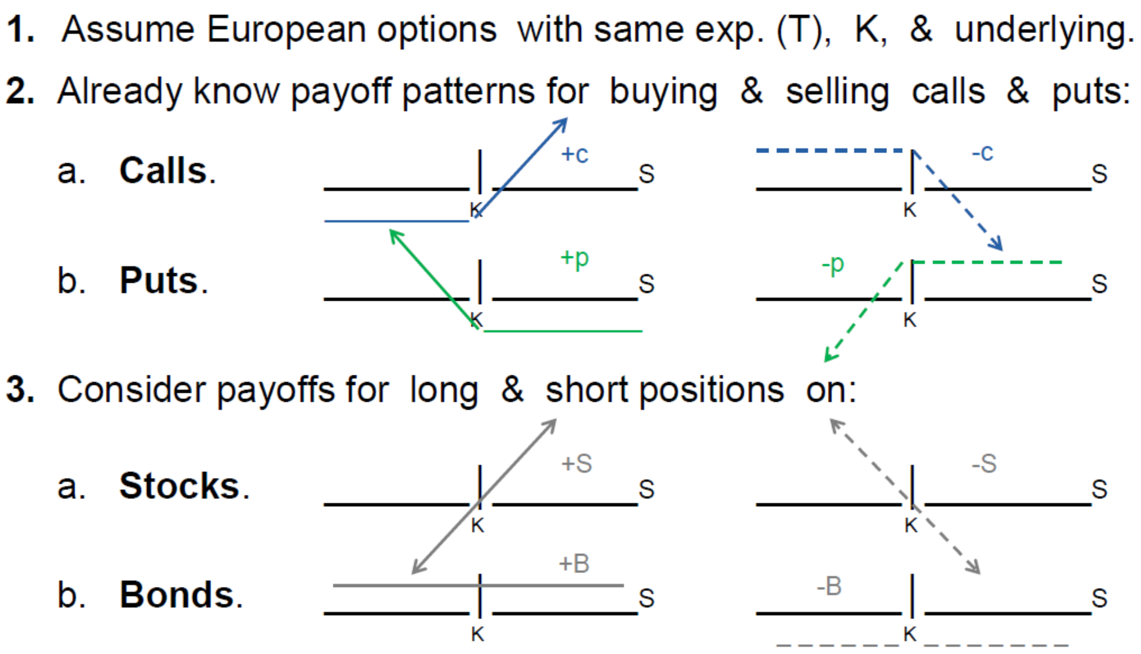
▼ Effect of Dividends

$$c \geq S - (Ke^{-rT} + D)$$

$$p \geq (Ke^{-rT} + D) - S$$

$$c = S + p - (Ke^{-rT} + D)$$

▼ Trading Strategies with Options



- **Writing a covered call:**
  The portfolio consists of a long position in a stock plus a short position in a European call option.
- **Reverse of writing a covered call:**
  A short position in a stock is combined with a long position in a call option.
- **Protective put strategy:**
  The investment strategy involves buying a European put option on a stock and the stock itself.
- **JUST TRANSPOSE THE TWO GRAPHS TO GET THE RESULT**

▼ Bull Spreads

Calls→ Buy at K1 and Sell at K2

Figure 12.2 Profit from bull spread created using call options.

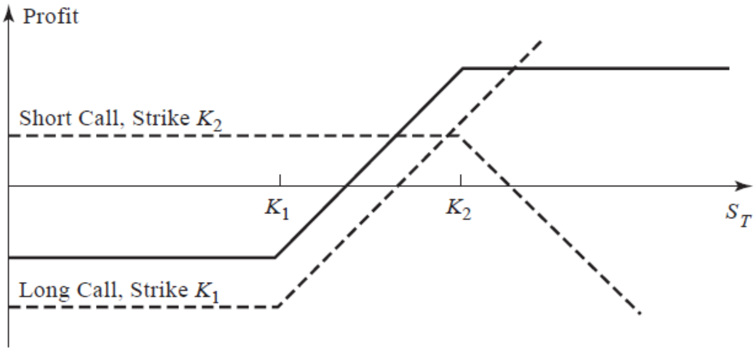


Table 12.1 Payoff from a bull spread created using calls.

Stock price range	*Payoff from long call option*	*Payoff from short call option*	*Total payoff*
$S_T \leqslant K_1$	0	0	0
$K_1 < S_T < K_2$	$S_T - K_1$	0	$S_T - K_1$
$S_T \geqslant K_2$	$S_T - K_1$	$-(S_T - K_2)$	$K_2 - K_1$

Profit = Total Payoff - cost of the strategy (c1-c2)

▼ Bear Spreads

Puts→ Sell puts at K1 and Buy at K2

Figure 12.4 Profit from bear spread created using put options.

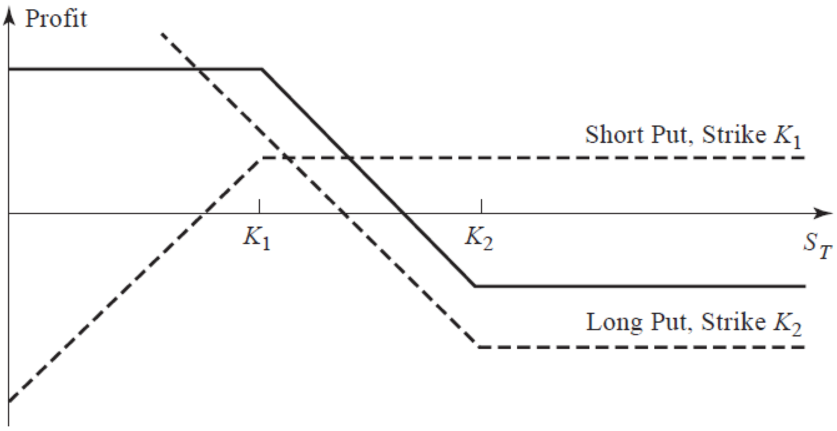
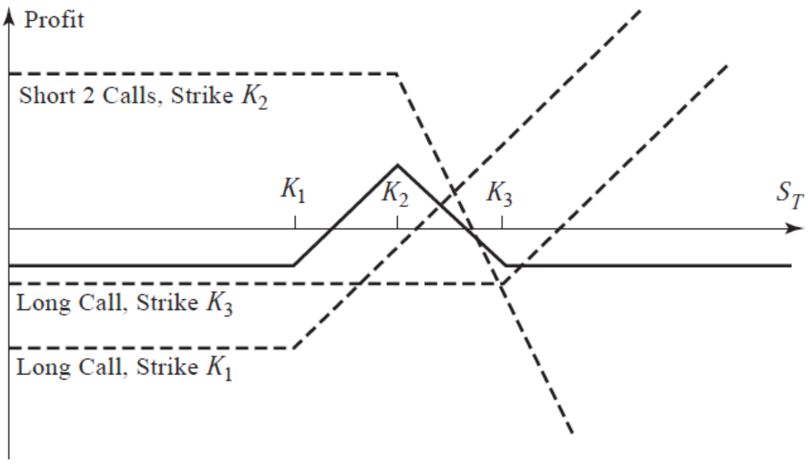


Table 12.2 Payoff from a bear spread created with put options.

Stock price range	*Payoff from long put option*	*Payoff from short put option*	*Total payoff*
$S_T \leqslant K_1$	$K_2 - S_T$	$-(K_1 - S_T)$	$K_2 - K_1$
$K_1 < S_T < K_2$	$K_2 - S_T$	0	$K_2 - S_T$
$S_T \geqslant K_2$	0	0	0

▼ Butterfly Spreads

Figure 12.6 Profit from butterfly spread using call options.



Calls → **Buy at K1 Sell 2 at K2 Buy at K3**

Table 12.4 Payoff from a butterfly spread.

Stock price range	*Payoff from first long call*	*Payoff from second long call*	*Payoff from short calls*	*Total payoff**
$S_T \leqslant K_1$	0	0	0	0
$K_1 < S_T \leqslant K_2$	$S_T - K_1$	0	0	$S_T - K_1$
$K_2 < S_T < K_3$	$S_T - K_1$	0	$-2(S_T - K_2)$	$K_3 - S_T$
$S_T \geqslant K_3$	$S_T - K_1$	$S_T - K_3$	$-2(S_T - K_2)$	0

* These payoffs are calculated using the relationship $K_2 = 0.5(K_1 + K_3)$.

⇒ Combinations:

▼ Straddle

Buy 1 Call and 1 Spot with same K

Figure 12.10 Profit from a straddle.

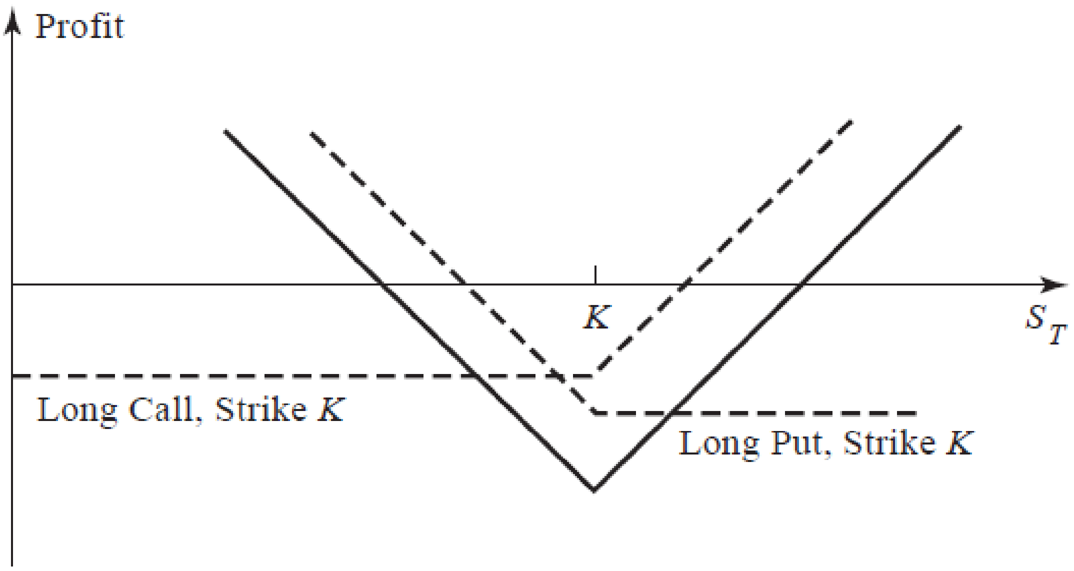


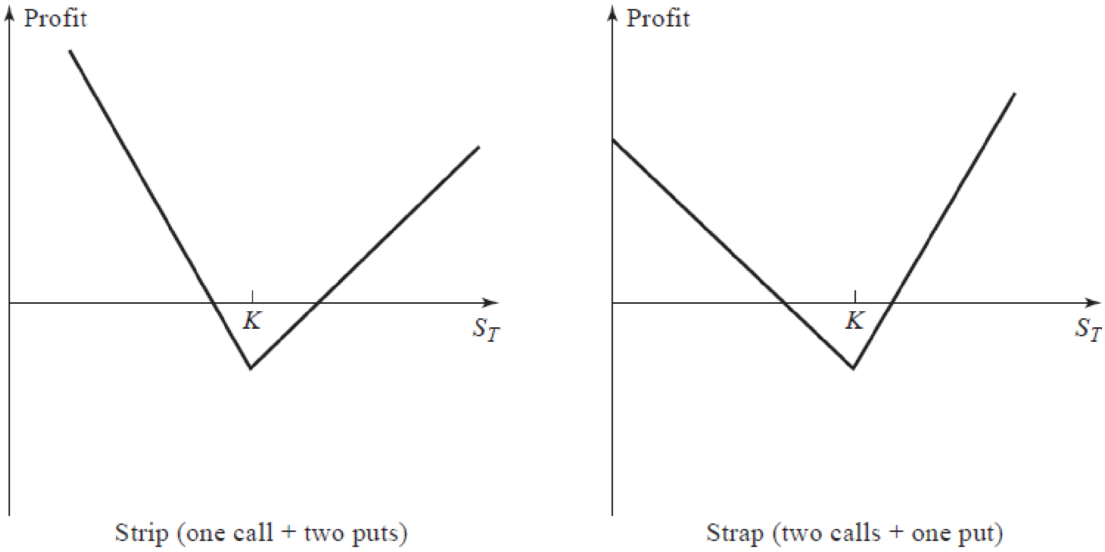
Table 12.5 Payoff from a straddle.

Range of stock price	*Payoff from call*	*Payoff from put*	*Total payoff*
$S_T \leqslant K$	0	$K - S_T$	$K - S_T$
$S_T > K$	$S_T - K$	0	$S_T - K$

▼ Strip and Strap

Buy 1(2) Call and 2(1) Spot with same K→ 2 based on stock price change beliefs

Figure 12.11 Profit from a strip and a strap.



▼ Strangle

Buy 1 Call and 1 Spot with different K

Figure 12.12 Profit from a strangle.

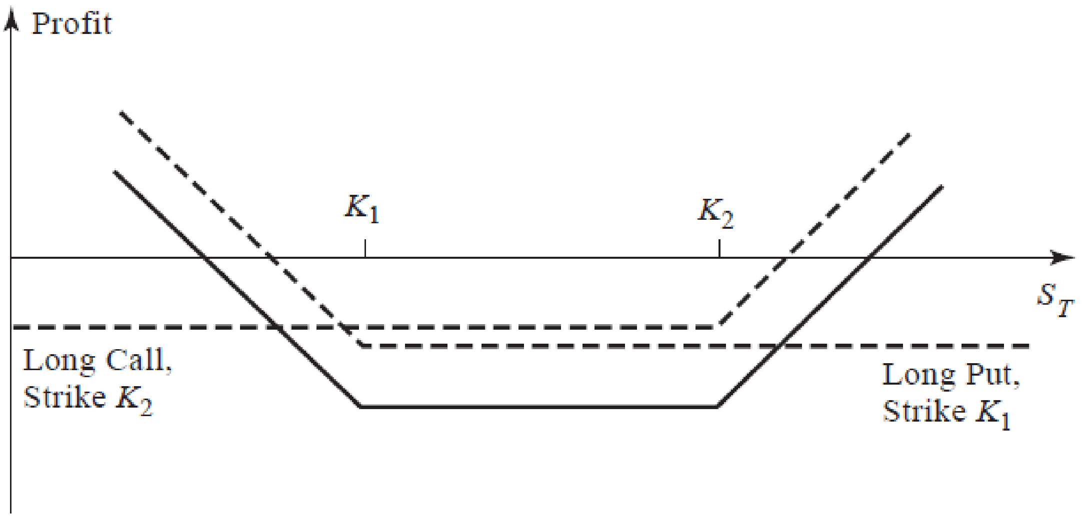


Table 12.6 Payoff from a strangle.

Range of stock price	*Payoff from call*	*Payoff from put*	*Total payoff*
$S_T \leqslant K_1$	0	$K_1 - S_T$	$K_1 - S_T$
$K_1 < S_T < K_2$	0	0	0
$S_T \geqslant K_2$	$S_T - K_2$	0	$S_T - K_2$

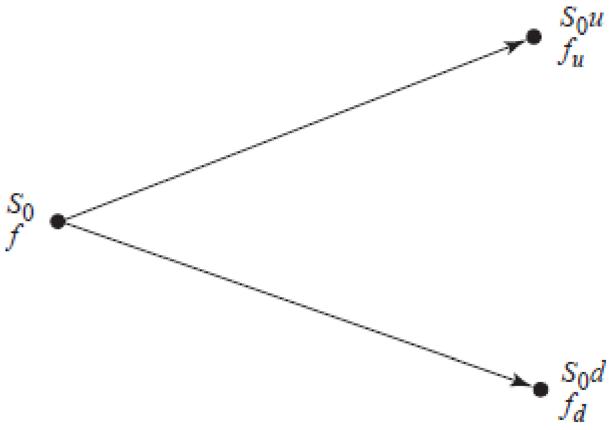
⇒ Binomial trees:

▼ A ONE-STEP BINOMIAL MODEL AND A NO ARBITRAGE ARGUMENT

- Long on Δ Shares and Short on 1 Europian Call
- Final value of the portfolio should be the same if Stock price increase/decrease
- Find value of Δ from (2)
- The value of this portfolio will then return with rate = r_f
- Find the current value of portfolio = $P\,e^{-r_f T}$
- Equate this with $S_0 - f$ where f is the initial price of Call Options
- Find the no arbitrageous value of f
- If f is more then P_0 is less and would earn with $r > r_f$
- If f is less then P_0 is more and would borrow with $r < r_f$

▼ Binomial Trees: Generalization

Figure 13.2 Stock and option prices in a general one-step tree.



The two are equal when

$$S_0u\Delta - f_u = S_0d\Delta - f_d$$

or

$$\Delta = \frac{f_u - f_d}{S_0u - S_0d} \tag{13.1}$$

In this case, the portfolio is riskless and, for there to be no arbitrage opportunities, it must earn the risk-free interest rate. Equation (13.1) shows that Δ is the ratio of the change in the option price to the change in the stock price as we move between the nodes at time T.

If we denote the risk-free interest rate by r, the present value of the portfolio is

$$(S_0 u \Delta - f_u)e^{-rT}$$

The cost of setting up the portfolio is

$$S_0 \Delta - f$$

It follows that

$$S_0 \Delta - f = (S_0 u \Delta - f_u)e^{-rT}$$

or

$$f = S_0 \Delta (1 - ue^{-rT}) + f_u e^{-rT}$$

Substituting from equation (13.1) for Δ, we obtain

$$f = S_0 \left(\frac{f_u - f_d}{S_0 u - S_0 d} \right) (1 - ue^{-rT}) + f_u e^{-rT}$$

or

$$f = \frac{f_u(1 - de^{-rT}) + f_d(ue^{-rT} - 1)}{u - d}$$

or

$$f = e^{-rT}[p f_u + (1 - p) f_d] \qquad \textbf{(13.2)}$$

where

$$p = \frac{e^{rT} - d}{u - d} \qquad \textbf{(13.3)}$$

▼ Binomial Trees: Risk Neutral Valuation

Returning to equation (13.2), the parameter p should be interpreted as the probability of an up movement in a risk-neutral world, so that $1 - p$ is the probability of a down movement in this world. (We assume $u > e^{rT}$, so that $0 < p < 1$.) The expression

$$p f_u + (1 - p) f_d$$

is the expected future payoff from the option in a risk-neutral world and equation (13.2) states that the value of the option today is its expected future payoff in a risk-neutral world discounted at the risk-free rate. This is an application of risk-neutral valuation.

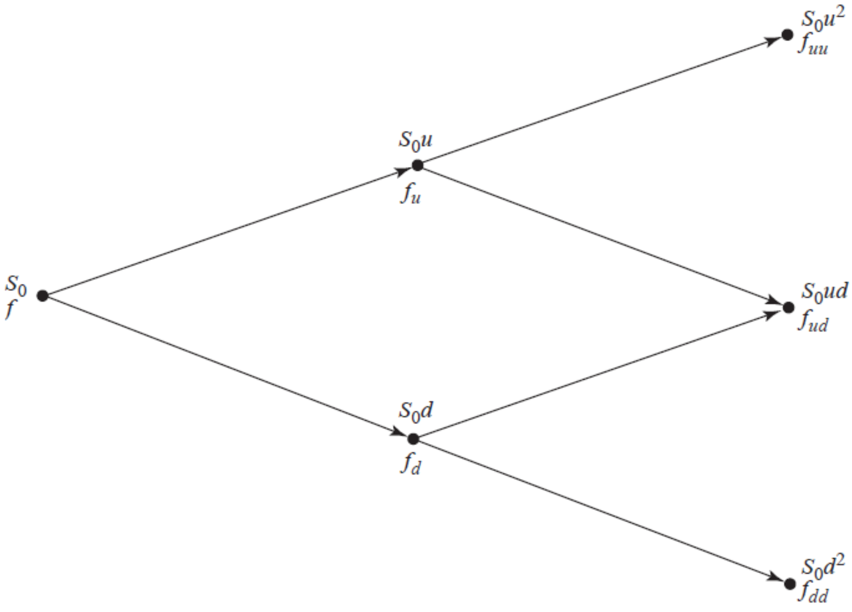
$$E(S_T) = pS_0 u + (1 - p)S_0 d$$

$$E(S_T) = pS_0(u - d) + S_0 d$$

$$E(S_T) = S_0 e^{rT}$$

▼ Two-Step Binomial Trees

Figure 13.6 Stock and option prices in general two-step tree.



Because the length of a time step is now Δt rather than T, equations (13.2) and (13.3) become

$$f = e^{-r\Delta t}[pf_u + (1-p)f_d] \tag{13.5}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{13.6}$$

Repeated application of equation (13.5) gives

$$f_u = e^{-r\Delta t}[pf_{uu} + (1-p)f_{ud}] \tag{13.7}$$

$$f_d = e^{-r\Delta t}[pf_{ud} + (1-p)f_{dd}] \tag{13.8}$$

$$f = e^{-r\Delta t}[pf_u + (1-p)f_d] \tag{13.9}$$

Substituting from equations (13.7) and (13.8) into (13.9), we get

$$f = e^{-2r\Delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}] \tag{13.10}$$

This is consistent with the principle of risk-neutral valuation mentioned earlier. The

Black-Scholes Model for Option Pricing

$$C_0 = S_0 N(d_1) - \frac{K}{e^{rT}} N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

- **Implied Volatility:** It is done by using the option price quoted in the market as an input and then solve for the volatil**ity.**

Real Options:

- The total cost of this firms would be $T = F + Vx$, where F is the fixed cost, V is the rate of variable cost and x is the amount of product produced.
- The profit of the plant in a month in which it operates at level x is profit (π) = $px - F - Vx$, where p is the market price of the product.
- If $p > V$, the firm will operate at x equal to the maximum capacity of the plant.
- If $p < V$, it will not operate.
- Hence, the firm has a continuing option to operate, with a strike price equal to the rate of variable cost.