# Problem Formulation

$$
\begin{aligned}
\min_{x(\cdot),\, u(\cdot),\, z(\cdot),\, s(\cdot),\, s^{\mathrm{e}}} \quad & \int_0^T l(x(\tau), u(\tau), z(\tau), p) + \frac{1}{2} s(\tau)^\top Z s(\tau) + z_s^\top \left| s(\tau) \right| \mathrm{d}\tau + \\
& m(x(T), z(T), s(T), p) + \frac{1}{2} s(T)^\top Z^{\mathrm{e}} s(T) + z_s^{\mathrm{e}\top} \left| s(T) \right| \\
\text{s.t.} \quad & x(0) - \bar{x}_0 = 0, \\
& f_{\mathrm{impl}}(x(t), \dot{x}(t), u(t), z(t), p) = 0, && t \in [0,\, T), \\
& \underline{h} \leq h(x(t), u(t), p) + J_{\mathrm{sh}} s_{\mathrm{h}} \leq \bar{h}, && t \in [0,\, T), \\
& \underline{x} \leq J_{\mathrm{bx}} x(t) + J_{\mathrm{sbx}} s_{\mathrm{bx}}(t) \leq \bar{x}, && t \in [0,\, T), \\
& \underline{u} \leq J_{\mathrm{bu}} u(t) + J_{\mathrm{sbu}} s_{\mathrm{bu}}(t) \leq \bar{u}, && t \in [0,\, T), \\
& \underline{g} \leq C x(t) + D u(t) + J_{\mathrm{sg}} s_{\mathrm{g}} \leq \bar{g}, && t \in [0,\, T), \\
& s(t) \geq 0, && t \in [0,\, T) \\
& \underline{h}^{\mathrm{e}} \leq h^{\mathrm{e}}(x(T), p) + J_{\mathrm{sh}}^{\mathrm{e}} s_{\mathrm{h}}^{\mathrm{e}} \leq \bar{h}^{\mathrm{e}}, \\
& \underline{x}^{\mathrm{e}} \leq J_{\mathrm{bx}}^{\mathrm{e}} x(T) + J_{\mathrm{sbx}} s_{\mathrm{bx}}^{\mathrm{e}} \leq \bar{x}^{e}, \\
& \underline{g}^{\mathrm{e}} \leq C^{\mathrm{e}} x(T) \leq \bar{g}^{\mathrm{e}} + J_{\mathrm{sg}}^{\mathrm{e}} s_{\mathrm{g}}^{\mathrm{e}}, \\
& s^{\mathrm{e}} \geq 0
\end{aligned}
$$

Hereby:

- $x \in \mathbb{R}^{n_x}$ state vector
- $u \in \mathbb{R}^{n_u}$ control vector
- $z \in \mathbb{R}^{n_z}$ algebraic state vector
- $s \in \mathbb{R}^{n_s}$ slack variables, which are concatenated as $s = (s_{\mathrm{bu}}, s_{\mathrm{bx}}, s_{\mathrm{g}}, s_{\mathrm{h}})$
- $s^{\mathrm{e}} \in \mathbb{R}^{n_s}$ terminal slack variables, which are concatenated as $s^{\mathrm{e}} = (s_{\mathrm{bx}}^{\mathrm{e}}, s_{\mathrm{g}}^{\mathrm{e}}, s_{\mathrm{h}}^{\mathrm{e}})$
- $p \in \mathbb{R}^{n_p}$ parameters

# 1 Dynamics

The function $f_{\mathrm{impl}} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_x + n_z}$ describes the dynamics as a fully implicit DAE.
We offer to discretize $F$ with a classic implicit Runge-Kutta (`irk`) or a structure exploiting implicit Runge-Kutta method (`irk_gnsf`).
Additionally, we offer an explicit Runge-Kutta integrator (`erk`), which can be used with explicit ODE models, i.e. models of the form

$$f_{\mathrm{expl}}(x, u, p) = \dot{x}$$

| Mathematical Expression | string identifier | data type |
|---|---|---|
| $f_{\mathrm{impl}}$ respectively $f_{\mathrm{expl}}$ | `dyn_expr_f` | `CasADi` expression |
| - | `dyn_type` | string (`explicit` or `implicit`) |

# 2 Cost

There are different `acados` modules to model the cost functions.

- $l : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_z} \to \mathbb{R}$ is the Lagrange objective term.

- $m : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}$ is the Mayer objective term.

to decide which one is used set `cost_type` for $l$, `cost_type_e` for $m$.

### Cost module: `auto`

Set `cost_type` to `auto` (default). Hereby we detect if the cost function specified is a linear least squares term and transcribe it in the corresponding form. Otherwise, it is formulated using the external cost module. Note: slack penalties are optional and will be detected form the expressions in future versions.

| Mathematical Expression | string identifier | data type |
|---|---|---|
| $l$ | `cost_expr_ext_cost` | `CasADi` expression |
| $m$ | `cost_expr_ext_cost_e` | `CasADi` expression |
| $Z$ | `cost_Z` | double |
| $z_s$ | `cost_z` | double |
| $Z^{\mathrm{e}}$ | `cost_Z_e` | double |
| $z_s^{\mathrm{e}}$ | `cost_z_e` | double |

### Cost module: `external`

Set `cost_type` to `ext_cost`.

| Mathematical Expression | string identifier | data type |
|---|---|---|
| $l$ | `cost_expr_ext_cost` | `CasADi` expression |
| $m$ | `cost_expr_ext_cost_e` | `CasADi` expression |
| $Z$ | `cost_Z` | double |
| $z_s$ | `cost_z` | double |
| $Z^{\mathrm{e}}$ | `cost_Z_e` | double |
| $z_s^{\mathrm{e}}$ | `cost_z_e` | double |

### Cost module: `linear least squares`

Set `cost_type` to `linear_ls`.
The Lagrange cost term has the form

$$l(x, u, z) = \left\| \underbrace{V_x x + V_u u + V_z z}_{y} - y_{\mathrm{ref}} \right\|_W$$

with matrices $V_x, V_u, V_z, W$ of appropriate dimensions.
Similarly, the Mayer cost term has the form

$$m(x, u, z) = \left\| \underbrace{V_x^{\mathrm{e}} x}_{y^{\mathrm{e}}} - y_{\mathrm{ref}}^{\mathrm{e}} \right\|_{W^{\mathrm{e}}}$$

with matrices $V_x^{\mathrm{e}}, W^{\mathrm{e}}$ of appropriate dimensions.

| Mathematical Expression | string identifier | data type |
|---|---|---|
| $V_x$ | `cost_V_x` | double |
| $V_u$ | `cost_V_u` | double |
| $V_z$ | `cost_V_z` | double |
| $W$ | `cost_W` | double |
| $y_{\mathrm{ref}}$ | `cost_y_ref` | double |
| $V_x^{\mathrm{e}}$ | `cost_V_x_e` | double |
| $W^{\mathrm{e}}$ | `cost_W_e` | double |
| $y_{\mathrm{ref}}^{\mathrm{e}}$ | `cost_y_ref_e` | double |
| $Z$ | `cost_Z` | double |
| $z_s$ | `cost_z` | double |
| $Z^{\mathrm{e}}$ | `cost_Z_e` | double |
| $z_s^{\mathrm{e}}$ | `cost_z_e` | double |

## Cost module: `nonlinear least squares`

Set `cost_type` to `nonlinear_ls`.
The cost function has the same form as in the linear least squares module.
The only difference is that $y$, respectively $y^{\mathrm{e}}$ are defined as `CasADi` expressions, instead of the matrices $V_x, V_u, V_z$, respectively $V_x^{\mathrm{e}}$

| Mathematical Expression | string identifier | data type |
|---|---|---|
| $y$ | `cost_expr_y` | `CasADi` expression |
| $W$ | `cost_W` | double |
| $y_{\mathrm{ref}}$ | `cost_y_ref` | double |
| $y^{\mathrm{e}}$ | `cost_expr_y_e` | `CasADi` expression |
| $y_{\mathrm{ref}}^{\mathrm{e}}$ | `cost_y_ref_e` | double |
| $Z$ | `cost_Z` | double |
| $z_s$ | `cost_z` | double |
| $Z^{\mathrm{e}}$ | `cost_Z_e` | double |
| $z_s^{\mathrm{e}}$ | `cost_z_e` | double |

# 3 Path Constraints

| Mathematical Expression | string identifier | data type |
|---|---|---|
| $\bar{x}_0$ | `constr_x0` | double |
| $J_{\mathrm{bx}}$ | `constr_Jbx` | double |
| $\underline{x}$ | `constr_lbx` | double |
| $\bar{x}$ | `constr_ubx` | double |
| $J_{\mathrm{bu}}$ | `constr_Jbu` | double |
| $\underline{u}$ | `constr_lbu` | double |
| $\bar{u}$ | `constr_ubu` | double |
| $C$ | `constr_C` | double |
| $D$ | `constr_D` | double |
| $\underline{g}$ | `constr_lg` | double |
| $\bar{g}$ | `constr_ug` | double |
| $h$ | `constr_expr_h` | `CasADi` expression |
| $\underline{h}$ | `constr_lh` | double |
| $\bar{h}$ | `constr_uh` | double |
| $J_{\mathrm{sbx}}$ | `constr_Jsbx` | double |
| $J_{\mathrm{sbu}}$ | `constr_Jsbu` | double |
| $J_{\mathrm{sg}}$ | `constr_Jsg` | double |
| $J_{\mathrm{sbx}}$ | `constr_Jsh` | double |

# 4 Terminal Constraints

| Mathematical Expression | string identifier | data type |
|---|---|---|
| $J_{\mathrm{bx}}$ | `constr_Jbx_e` | double |
| $\underline{x}^{\mathrm{e}}$ | `constr_lbx_e` | double |
| $\bar{x}^{\mathrm{e}}$ | `constr_ubx_e` | double |
| $C^{\mathrm{e}}$ | `constr_C_e` | double |
| $\underline{g}^{\mathrm{e}}$ | `constr_lg` | double |
| $\bar{g}^{\mathrm{e}}$ | `constr_ug` | double |
| $h^{\mathrm{e}}$ | `constr_expr_h_e` | `CasADi` expression |
| $\underline{h}^{\mathrm{e}}$ | `constr_lh_e` | double |
| $\bar{h}^{\mathrm{e}}$ | `constr_uh_e` | double |
| $J_{\mathrm{sbx}}$ | `constr_Jsbx` | double |
| $J_{\mathrm{sg}}^{\mathrm{e}}$ | `constr_Jsg_e` | double |
| $J_{\mathrm{sbx}}^{\mathrm{e}}$ | `constr_Jsh_e` | double |