Indian Institute of Technology
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Biomedical Imaging

CT IMAGING

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1 *Sinogram of a Shepp Logan Phantom*

- A sinogram is a graphical representation of the intensity values obtained from a set of projections of an object. It is the projected sum of the 1D Fourier transforms taken about the image.
- The sinogram is represented as a 2D plot where the x-axis corresponds to the projection angle θ and the y-axis corresponds to the radial distance ρ. Each point in the sinogram represents the integral of the the image .

```python
import numpy as np
import matplotlib.pyplot as plt
from skimage.data import shepp_logan_phantom
from skimage.transform import radon

def generate_sinogram_with_theta(phantom, theta, cmap='viridis'):
    sinogram = radon(phantom, theta=theta, circle=True)

    plt.subplot(121)
    plt.imshow(phantom, cmap='gray')
    plt.title('Shepp-Logan Phantom')

    plt.subplot(122)
    plt.imshow(sinogram, cmap=cmap, aspect='auto', extent=[min(theta), max(theta), 0, sinogram.shape[0]])
    plt.colorbar(label='Intensity')
    plt.title('Sinogram' )

    plt.show()

# Generate Shepp-Logan phantom
phantom = shepp_logan_phantom()

# Specify the sampling angle theta (in degrees) with a sampling rate of 0.05 degrees
theta_values = np.arange(0, 180,0.05)

# Generate sinogram with specified theta values
generate_sinogram_with_theta(phantom, theta_values)
```

Listing 1: Python code for generating sinogram

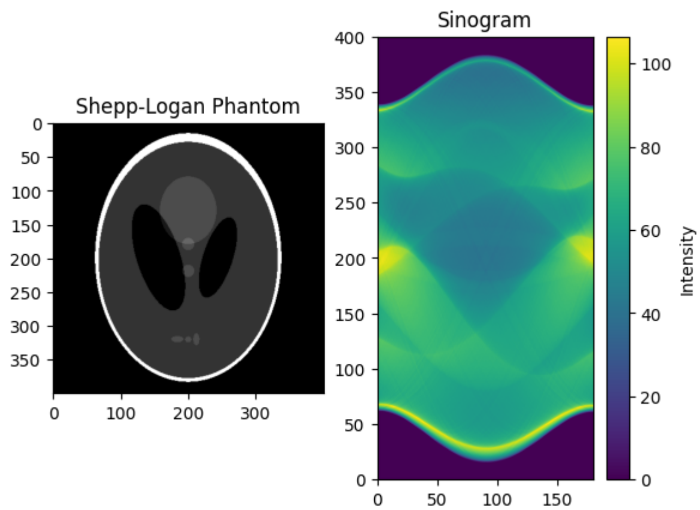

Figure 1: Sinogram at a sampling rate $\theta = 0.5$ degree

2 *Filtered Back Projection*

- Filtered back projection is a reconstruction technique to obtain the image after taking the radon transform
- It is used to convert the sinogram back into the 2-D image that the CT scan has taken.
- A filter is applied to the sinogram data to bring attention to other prominient features(frquency) in the image.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.ndimage import convolve1d
from skimage.data import shepp_logan_phantom
from skimage.transform import radon

def generate_sinogram_with_theta(phantom, theta, cmap='
    viridis'):
    sinogram = radon(phantom, theta=theta, circle=True)

    # Apply Mexican Hat filter
    mexican_hat_kernel = np.array([-1, 2, -1])
    filtered_sinogram = convolve1d(sinogram,
    mexican_hat_kernel, axis=0, mode='constant', cval=0)

    fig, axes = plt.subplots(1, 3, figsize=(15, 5))  # 1 row
    , 3 columns
```

```
    axes[0].imshow(phantom, cmap='gray')
    axes[0].set_title('Shepp-Logan Phantom')

    im1 = axes[1].imshow(sinogram, cmap=cmap, aspect='auto', extent=[min(theta), max(theta), 0, sinogram.shape[0]])
    axes[1].set_title('Original Sinogram')
    plt.colorbar(im1, ax=axes[1], label='Intensity')  # Add colorbar to the first subplot

    im2 = axes[2].imshow(filtered_sinogram, cmap=cmap, aspect='auto', extent=[min(theta), max(theta), 0, filtered_sinogram.shape[0]])
    axes[2].set_title('Sinogram with Mexican Hat Filter')
    plt.colorbar(im2, ax=axes[2], label='Intensity')  # Add colorbar to the second subplot

    plt.subplots_adjust(wspace=0.5)  # Adjust the width space between subplots

    plt.show()

# Generate Shepp-Logan phantom
phantom = shepp_logan_phantom()

# Specify the sampling angle theta (in degrees) with a sampling rate of 0.05 degrees
theta_values = np.arange(0, 180, 0.05)

# Generate sinogram with specified theta values and apply Mexican Hat filter
generate_sinogram_with_theta(phantom, theta_values)
```

Listing 2: Python code for generating sinogram with Mexican Hat filter

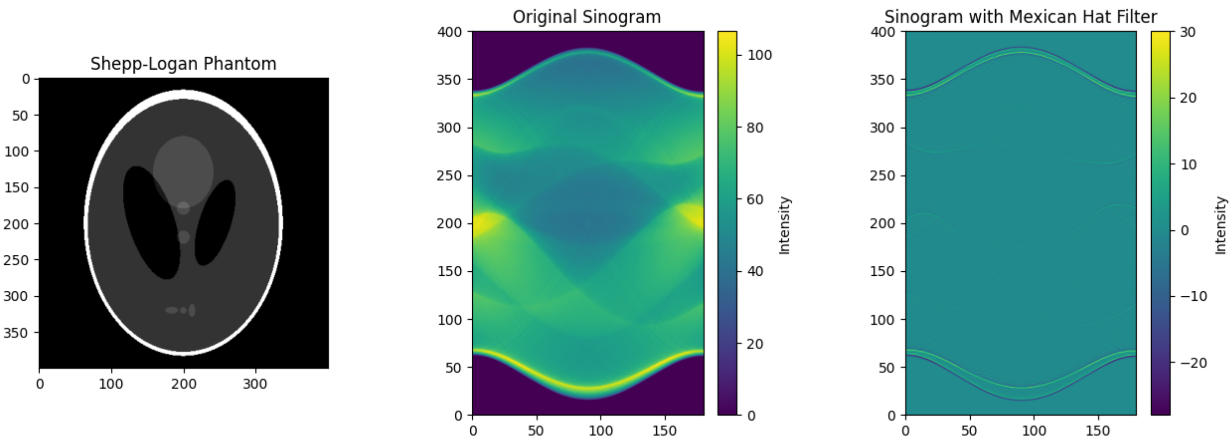

Figure 2: Singogram with Mexican Hat Filter

Comparing Properties

- As Observed the Mexican Hat filter enhances certain frequency components in the sinogram while suppressing others.
- The sinogram with the Mexican Hat filter is has sharper features
- The filtering process reduces noise in the sinogram, leading to a cleaner representation of the object's characteristics

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3 *Reconstruction of Image*

```python
import numpy as np
import matplotlib.pyplot as plt
from skimage.data import shepp_logan_phantom
from skimage.transform import radon, iradon

def generate_sinogram_with_theta(phantom, theta, cmap='
    viridis'):
    sinogram = radon(phantom, theta=theta, circle=True)
    plt.subplot(121)
    plt.imshow(phantom, cmap='gray')
    plt.title('Shepp-Logan Phantom')

    plt.subplot(122)
    plt.imshow(sinogram, cmap=cmap, aspect='auto', extent=[
    min(theta), max(theta), 0, sinogram.shape[0]])
    plt.colorbar(label='Intensity')
    plt.title('Sinogram' )

    plt.show()

    return sinogram

def reconstruct_image_from_sinogram(sinogram, theta):
    reconstructed_image = iradon(sinogram, theta=theta,
    circle=True)

    plt.imshow(reconstructed_image, cmap='gray')
    plt.title('Reconstructed Image')
    plt.show()

# Generate Shepp-Logan phantom
phantom = shepp_logan_phantom()

# Specify the sampling angle theta (in degrees) with a
    sampling rate of 0.05 degrees
theta_values = np.arange(0, 180, 5)

# Generate sinogram with specified theta values
sinogram = generate_sinogram_with_theta(phantom,
    theta_values)

# Reconstruct the image from the sinogram
reconstruct_image_from_sinogram(sinogram, theta_values)
```

Listing 3: Reconstructing Sinogram using Inverse Radon Transform

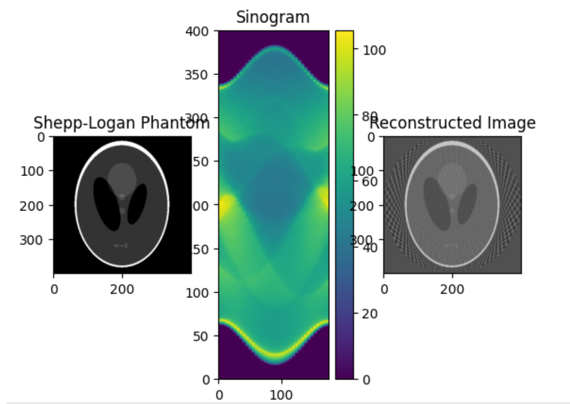

Figure 3: Reconstruction with Filter

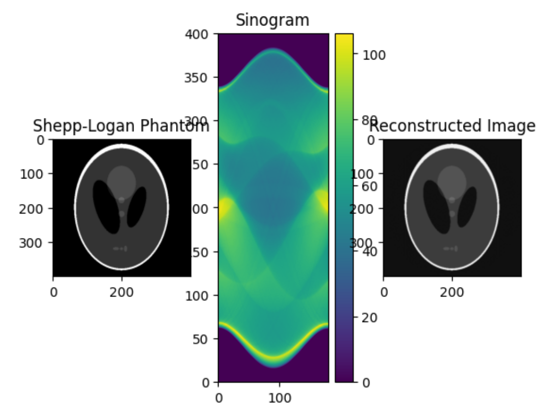

Figure 4: Reconstruction at sampling angle $\theta = 0.5$ degree i.e 360 projections

4 *Sinogram using 360-degree projection*

Effect of Sampling Rate on Image Reconstruction

- Insufficient sampling can lead to decreased image reconstruction quality
- When the angle of sampling is too sparse, multiple white streaks appear in the image due to lack of data in the unkown region
- Increasing the number of projections (higher sampling rate) will improves the resolution and characteristics of the reconstructed image.

5 *Reconstruction Using Central Slice Theorem*

```python
import numpy as np
import matplotlib.pyplot as plt
from skimage.data import shepp_logan_phantom
from skimage.transform import radon, iradon

# Generate Shepp-Logan phantom
phantom = shepp_logan_phantom()

# Perform Radon transform to obtain sinogram
sinogram = radon(phantom)

# Apply Central Slice Theorem by taking the inverse Radon
    transform
reconstructed_image = iradon(sinogram)

# Display the results
plt.figure(figsize=(10, 4))

plt.subplot(131)
plt.imshow(phantom, cmap='gray')
plt.title('Shepp-Logan Phantom')

plt.subplot(132)
plt.imshow(sinogram, cmap='viridis', aspect='auto', extent
    =[0, 180, 0, sinogram.shape[0]])
plt.title('Sinogram')

plt.subplot(133)
plt.imshow(reconstructed_image, cmap='gray')
plt.title('Reconstructed Image')

plt.show()
```

Listing 4: Reconstructing Sinogram using Central Slice Theorem

- the Central Slice Theorem states that the Fourier transform of a 1D projection of an object is equivalent to a central line in the 2D Fourier transform of the object itself.

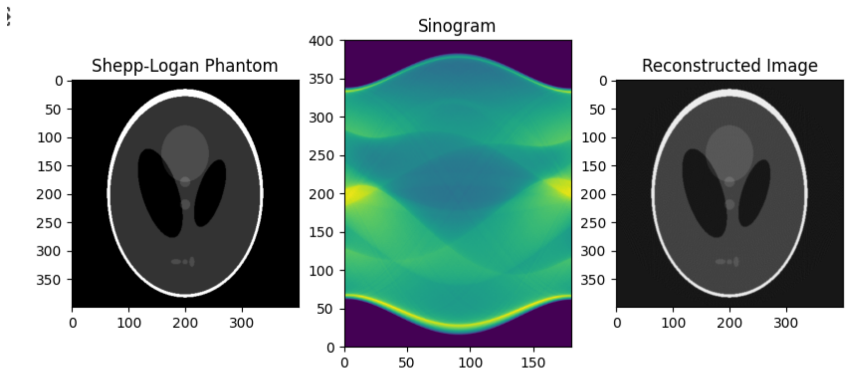

Figure 5: Reconstruction using central Slice Theorem

6 *Reconstruction After Filtering*

```python
import numpy as np
import matplotlib.pyplot as plt
from skimage import io, color, transform, img_as_float
from skimage.metrics import mean_squared_error

# Read the image

# Convert the image to grayscale if it is RGB
if image.ndim == 3:
    image = color.rgb2gray(image)

# Convert the image to double
image = img_as_float(image)

# Display the size of the matrix
print('Size of the image:')
print(image.shape)

# Define the filters
filters = ["ram-lak", "shepp-logan", "cosine", "hamming"]

# Initialize an array to store the mean squared errors
mse = np.zeros(len(filters))

# Loop over each filter
for i, filter_type in enumerate(filters):
    # Perform the radon transform
    sinogram = transform.radon(image, theta=np.arange(180))

    # Apply the filter
    filtered_sinogram = transform.iradon(sinogram, filter=
```

```
    filter_type)

    # Convert the reconstructed image to double
    filtered_sinogram = img_as_float(filtered_sinogram)

    # Compute the mean squared error
    mse[i] = mean_squared_error(image, filtered_sinogram)

    # Display the filter name and corresponding mean squared
     error
    print(f"{filters[i].capitalize()} Filter MSE: {mse[i]:.4
    f}")

    # Display the images
    plt.figure(figsize=(12, 4))
    plt.subplot(131)
    plt.imshow(image, cmap='gray')
    plt.title('Original Image')

    plt.subplot(132)
    plt.imshow(filtered_sinogram, cmap='gray')
    plt.title(f'Reconstructed Image ({filter_type.capitalize
    ()} Filter)\nMSE: {mse[i]:.4f}')

    plt.subplot(133)
    plt.plot(np.arange(180), sinogram.mean(axis=0), label='
    Sinogram')
    plt.legend()
    plt.title('Sinogram')

    plt.show()
```

Listing 5: Reconstructing Sinogram After Filtering

Mean Square Error of Filters:

- Filter: Ram-Lak, Mean Squared Error: 0.0006536
- Filter: Shepp-Logan, Mean Squared Error: 0.0006555
- Filter: Cosine, Mean Squared Error: 0.0007248
- Filter: Hamming, Mean Squared Error: 0.0008254

We can analyze the filtering accuracy of the filter based on the the mean square error

7 *Limitations of projection radiography*

- Projection radiography fails to provide details about the 3 dimensional structure as it provides a 2 dimensional image based on the central slice theroem.
- projection radiography provides static images and not suitable for real time imaging
- There are multiple possibilities of distortion of the image due to external factors like patient movement,equiment malfuction.

X-RAY IMAGING

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1 *Factor Affecting SNR*

Signal-to-Noise Ratio (SNR) in X-ray imaging is influenced by various factors

- Scatter Radiation: Scatter radiation contributes to noise in the image
- Detector Efficiency: The efficiency of the detector in converting X-ray photons
- Patient Thickness and Density: The composition and thickness of the patient's body influence the reduction in intensity of X-rays

If the SNR doubles then dose becomes four times

2 *Beam Hardening*

- Beam hardening is a phenomenon in X-ray imaging that occurs when a X-ray beam, which consists of a range of energy levels, passes through an object or a patient. As the X-ray beam pases through different materials of varying thickness and composition
- In this process lower-energy X-ray photons are preferentially absorbed, leading to an increase in the average energy of the remaining X-ray beam.

3 *Aliasing,bandwidth limiting,Nyquist condition*

- **Aliasing**:Aliasing occurs when a signal is under sampled, causing incomplete reconstruction of the signal waveform.
- **Bandwidth limiting**Bandwidth limiting restricts the range of frequencies in a signal,between the bandpass range fro avoiding interference of irrelaavnt frequencies.
- **Nyquist condition**The Nyquist condition sampling theorem states that the sampling rate must be at least twice the maximum frequency in a signal to prevent aliasing. This condition is $f_{sampling} > 2f_{max}$

4 *Percentage of X-rays are transmitted through the chest*

We know percentage of X-rays transmitted through the chest, the exponential attenuation formula:

$$I = I_o e^{-ux} \tag{1}$$

where:

- I is the intensity of the transmitted X-rays

- I_o is the initial intensity of the incident X-rays
- u is the linear attenuation coefficient
- x is the thickness of the material.

The linear attenuation coefficient u can be calculated using the half-value layer (HVL) (H) and the formula

$$u = \frac{ln2}{H} \tag{2}$$

Calculate u for muscle and bone

$$\mu_{\text{muscle}} = \frac{0.693}{3.5\,\text{cm}} \approx 0.198\,\text{cm}^{-1}$$

$$\mu_{\text{bone}} = \frac{0.693}{1.8\,\text{cm}} \approx 0.385\,\text{cm}^{-1}$$

$$I_{\text{muscle}} = I_0 \cdot e^{-\mu_{\text{muscle}} \cdot x_{\text{muscle}}} = I_0 \cdot e^{-0.198 \cdot 16}$$

$$I_{\text{bone}} = I_0 \cdot e^{-\mu_{\text{bone}} \cdot x_{\text{bone}}} = I_0 \cdot e^{-0.385 \cdot 4}$$

$$I_{\text{total}} = I_{\text{muscle}} \cdot I_{\text{bone}}$$

Percentage of X-Ray Transmitted

$$e^{-0.198} * e^{-0.385} = e^{-0.583} = 0.558$$

5 *Difference between X-Ray Images*

- Higher Effective Energy (140 keV): X-rays with higher energy penetrate tissues more effectively. This results in lower contrast in the image because the X-rays making it challenging to distinguish between different
- As observed the high energy X-Ray is completly
- Higher Effective Energy (140 keV): The X-rays with higher energy can penetrate the body more deeply, reaching the detector with less absorption of the X-Ray
- Lower Effective Energy (50 keV): Lower energy X-rays are more likely to be absorbed by tissues, leading to greater absorption by tissues.

6 *Opinion on getting Xray imaging for heart related diseases.*

Pros	**Cons**
• Diagnostic Tool: X-ray imaging can provide valuable information about the structure of the heart. • Quick and Non-Invasive: X-rays are relatively quick and non-invasive compared to some other imaging modalities. • Widely Available: X-ray facilities are widely available, making it a convenient option for initial screening.	• Radiation Exposure: X-rays involve exposure to ionizing radiation, which carries potential risks. • Limited Soft Tissue Detail: X-rays are better at visualizing bones and dense structures but have limitations in capturing detailed images of soft tissues. • May Require Contrast Agents: In some cases, contrast agents may be used, and individuals may have allergies or adverse reactions to these agents.

7 *Issue with X-Ray Machine*

The diagram incorrectly labeled the Multitap AC transformer as a "Multiple AC transformer" and the Timing circuit as a "Counting circuit."

The operation of an X-ray machine involves various components:

- Multitap AC transformer: This component adjusts for incoming line variations by selecting different taps to compensate. The number of outputs, or taps, can range from 2 to many, allowing for the selection of a higher or lower voltage tap based on the required X-ray exposure intensity.

- X-ray tube filament transformer: This transformer converts the AC line to supply power for heating the cathode filament. By selecting different taps, the filament heat can be adjusted, subsequently altering the X-ray tube current and total energy delivered to the patient.

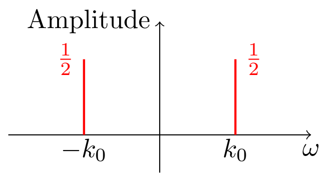
- X-ray tube high voltage transformer and bridge rectifier: This unit transforms the AC line into high DC voltage, essential for accelerating electrons from the cathode to the anode. The high DC voltage can be chosen using taps.

- Timing circuit: Contrary to the mislabeling as a "Counting circuit," the Timing circuit controls the timing aspects of X-ray exposure. It includes an electronic counter that regulates the turn-on, turn-off, and duration of X-ray exposure to the patient.

8 *Fourier Transforms and Spectra*

a. $\cos^2(k_0 z)$

$$\mathcal{F}\{\cos^2(k_0 z)\} = \frac{1}{2}\left[\delta(\omega - k_0) + \delta(\omega + k_0)\right]$$



b. $\sin^3(k_0 z)$

$$\mathcal{F}\{\sin^3(k_0 z)\} = \frac{3}{4i}\left[\delta(\omega - 3k_0) - \delta(\omega - k_0)\right]$$