



SCAN ME



VIT<sup>®</sup>

Vellore Institute of Technology  
(Deemed to be University) under section 3 of UGC Act, 1956

### Continuous Assessment Test –II

Programme Name & Branch: B. Tech.

Exam Duration: 90 mins

Slot: C2+TC2+TCC2

Semester: Winter 2019-20

Maximum Marks: 50

Course Code & Title: MAT3004 Applied Linear Algebra

Exam Mode: Closed book

Answer any five questions  $5 \times 10 = 50$  Marks

1. Find a basis for the subspaces  $V + W$  and  $V \cap W$  of  $\mathbb{R}^4$  where the subspaces  $V$  and  $W$  are spanned by the set of vectors  $\{v_1 = (1, 1, 0, 0), v_2 = (1, 0, 1, 0)\}$  and  $\{w_1 = (0, 1, 0, 1), w_2 = (0, 0, 1, 1)\}$  respectively. Is  $V + W = V \oplus W$ ? (10)

2. a) Check the identity:  $\text{rank}(A) + \text{nullity of } A = \text{total number of columns of } A$  for the

matrix  $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & -1 & 3 \\ 7 & -8 & 3 \\ 5 & -7 & 0 \end{bmatrix}$ . (5)

- b) Can the matrix  $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 7 & 8 \end{bmatrix}$  have a left or right inverse? If so, find it. (5)

3. a) Find the equation of a circle that passes through the points  $(0, 0)$ ,  $(-1, -3)$  and  $(-3, -3)$ . (5)

- b) Is the mapping  $T: P_1(\mathbb{R}) \rightarrow P_2(\mathbb{R})$  defined by  $T(at + b) = at^2 + (a - b)t$  linear? (5)

4. a) Find the matrix of reflection about the line  $y = \sqrt{3}x$ . (5)

- b) Find the basis change matrix  $[id]_{\alpha}^{\beta}$  if  $\alpha = \{(1, 0), (0, 1)\}$  and  $\beta = \{(5, 1), (1, 2)\}$ . (5)

5. Let  $S$  and  $T$  be linear transformations from  $\mathbb{R}^3$  to  $\mathbb{R}^3$  defined as follows:

$$S(x, y, z) = (3x + 5y + z, -2x + y - z, x - z) \text{ and}$$

$$T(x, y, z) = (x - y, y - z, z - x). \text{ Find the associated matrices}$$

$$[S]_{\alpha}, [T]_{\alpha}, [S + T]_{\alpha}, [S \circ T]_{\alpha} \text{ and } [T^{-1}]_{\alpha} \text{ if } T^{-1} \text{ exists where } \alpha \text{ is the standard basis of } \mathbb{R}^3. (10)$$

6. Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be the linear transformation  $T(x, y, z) = (2x - y + z, x + y - 2z)$ . Let  $\alpha = \{v_1 = (1, 1, 1), v_2 = (1, 2, 1), v_3 = (0, 1, -1)\}$  and  $\beta = \{w_1 = (1, 2), w_2 = (2, 1)\}$  be ordered bases of  $\mathbb{R}^3$  and  $\mathbb{R}^2$  respectively. Find the associated matrix  $[T]_{\alpha}^{\beta}$ . (10)

$$\begin{matrix} 1 & 1 & 1 & -2 \\ 1 & 1 & 2 & -2 \\ 0 & 1 & 1 & -2 \end{matrix}$$