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Vellore Institute of Technology

DEPARTMENT OF MATHEMATICS

SCHOOL OF ADVANCED SCIENCES

Re-Continuous Assessment Test - I

Course Code: MAT1011 | **Slot: G1+TG1**

Class Nbr: VL2018191000558 | **Course Name: Calculus for Engineers**

Max. Marks: 50 | **Date: 17.10.2018**

Course Teacher: Dr Hemant Kumar Nashine | **Duration: 90 Minutes**

Answer ALL the Questions
Each question carries 10 marks

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(1) Determine the critical points, [redacted], local maxima and local minima of $f(x)=\frac{3}{4}(x^2-1)^{2/3}$. Then identify the intervals on which f is concave up and concave down, and find the points of inflection.

(2) (a) Find the volume of the solid generated by revolving the plane region bounded by the lines $x+y=1, x=0$ and $y=0$ about the y-axis.

(b) Determine the area of the region enclosed by $y = x^2$ and $y=\sqrt{x}$.

(3) Verify Rolle's theorem for the function $f(x) = log\left(\frac{x^2+ab}{x(a+b)}\right)$ on $[a,b]$, where $a > 0$. Find the values of $c \in (a,b)$ if exist.

(4) (a) Find the Laplace transforms of $\frac{(1-\cos t)}{t^2}$.

(b) Find the Laplace transform of following triangular wave of period $2a$

$$f(t) = \begin{cases} t, 0 < t < a \\ 0, a < t < 2a \end{cases}.$$

(5) Use convolution theorem to find the inverse Laplace transform of the function

$$F(s) = \frac{s}{(s^2+16)^2}.$$

$\frac{1}{s(s^2+4)}$ $\frac{1-\cos 2t}{4}$

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Department of Mathematics

School of Advanced Sciences

Continuous Assessment Test – II, Fall Semester-2018

Course Code : MAT1011 | **Duration: 90 Minutes**

Course: Calculus for Engineers | **Slot : G1+TG1**

Max. Marks : 50

Answer All the questions

1. (a) If $u=\frac{x}{y}$ and $v=\frac{x+y}{xy}$, then obtain $\frac{\partial(u,v)}{\partial(x,y)}$. [7 M]

(b) What rate is the area of a rectangle changing if its length is 15 cm and increasing at 3 m/sec while its width is 6 cm and increasing at 2 cm/ sec.

[8 M]

2. Let the profit function be $P(x,y)=(\sin x)(\sin y)\sin(x+y)$, where $0<x<\frac{\pi}{2}$ and $0<y<\frac{\pi}{2}$. Then obtain the point at that maximum profit occurs.

[15 M]

3. Change the order of integration in the integral $\int_0^1\int_x^{\sqrt{2-x^2}}\frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$ and hence evaluate it. [10 M]

4. Using cylindrical polar co-ordinates, find the volume of the cylinder with base radius a and height h. [10 M]

Final Assessment Test – November 2018

Course: MAT1011 - Calculus for Engineers

Class NBR(s): 0268 / 0272 / 0277 / 0281 / 0286 / 0288 / 0294 / 0297 / 0304 / 0316/ 0321 / 0326 / 0494 / 0513 / 0533 / 0538 / 0546 / 0550 / 0558 / 0564 / 0570 / 0579 / 7307

Slot: G1+TG1

Time: Three Hours

Max. Marks: 100

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Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) Consider the function $f(x) = x^4 - 4x^3 + 10$ [10]

   (i) Identify where the extrema of f occur.

   (ii) Find the intervals on which f is increasing and decreasing

   (iii) Find the intervals on which f is concave up and concave down

   b) (i) Find the area of the region enclosed by the line $y = 2$ and the curve $y = x^2 - 2$. [5]

   (ii) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1,\ x = 4$ about the line $y = 1$. [5]

2. a) Find the Laplace transform of the "half-sine wave rectifier" function [10]

$$f(t) = \begin{cases} a\sin\omega t & in\ 0 \le t \le \frac{\pi}{\omega} \\ 0 & in\ \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases} \quad \text{and } f\left(t + \frac{2\pi}{\omega}\right) = f(t).$$

   b) Using convolution theorem, find the inverse Laplace transform of $\frac{1}{(s-2)(s+2)^2}$. [10]

3. a) If z is a function of x and y, where $x = e^u \cos v$ and $y = e^u \sin v$, prove that $y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$ [5]

   b) Find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ if u $= \frac{2yz}{x}$, $v = \frac{3zx}{y}$, $w = \frac{4xy}{z}$. [5]

   c) Using the gamma function, prove that $\left(\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}}\right)\left(\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta\right) = \pi$. [10]

4. a) Find the Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$ up to the third degree terms. [10]

   b) Show that the greatest rectangle that can be inscribed in the ellipse $4x^2 + 9y^2 = 36$, having its sides parallel to the coordinate axes, is 12. [10]

5. a) By changing the order of integration, evaluate $\int_0^1 \int_y^{2-y} xy\,dx\,dy$. [10]

   b) Evaluate $\iiint \sqrt{1 - x^2 - y^2 - z^2}\, dxdydz$, taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$, by transforming into spherical polar coordinates. [10]

6. a) Obtain the directional derivative of $\varphi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. [6]

b) Prove that $\vec{F} = (2x + yz)\hat{\imath} + (4y + zx)\hat{\jmath} - (6z - xy)\hat{k}$ is irrotational vector and hence find its scalar potential. [6]

c) If $r = |\vec{r}|$, where $\vec{r}$ is the position vector of the point (x, y, z), with respect to the origin, prove that (i) $\nabla f(r) = \frac{f'(r)}{r}\vec{r}$ [8]

(ii) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

7. a) Verify Green's theorem in the plane for $\oint_C (2x - y)dx + (x + y)dy$, where C is the boundary of the circle $x^2 + y^2 = a^2$. [10]

b) Verify Stoke's theorem when $\vec{F} = (2xy - x^2)\hat{\imath} - (x^2 - y^2)\hat{\jmath}$ and C is the boundary of the region enclosed by the parabolas $y^2 = x$ and $x^2 = y$. [10]

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