VIT
Vellore Institute of Technology

Final Assessment Test – Jan / Feb 2023

Course: BMAT101L - Calculus

Class NBR(s): 5450 / 5470 / 5473 / 6119 Slot: D1+TD1

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Find the volume of the solid generated by revolving the region $R = \{(x,y)|0 \leq x \leq 2, (x-1)^2 \leq y \leq 1\}$ around the line $x = -1$.
2. Find the critical points and local maxima and minima of the function $y = \frac{3}{4}(x^2-1)^{2/3}$. Identify the intervals on which the function is concave up and concave down. Also identify the inflection points.
3. The pressure, volume and temperature of an ideal gas are related by the equation $PV = 8.31T$. Find the rate at which the pressure is changing when the temperature is 300 k and increasing at a rate of 0.1 K/sec and the volume is 100 l and increasing at a rate of 0.2 l/sec .
4. Determine the extreme values of $f(x,y) = x + 2y$ on the circle $x^2+y^2 = 1$ using Lagrange's multiplier method.
5. Find the second degree Taylor polynomial of the function $f(x,y) = \log(1 + x + 2y)$ at (2,1).
6. Evaluate $\int_0^1 \int_{-\sqrt{y}}^{y^2} (6x - y)\, dx\, dy$ by changing the order of integration.
7. Evaluate $\iiint \sqrt{1-x^2-y^2-z^2}\; dxdydz$ taken throughout the volume of the sphere $x^2+y^2+z^2 = 1$, by transforming in to spherical polar coordinates.
8. Find $\int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\, d\theta \;X\; \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{sin\theta}}\, d\theta$
9. Find the directional derivative of the function $\phi = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at (-1,2,1)
10. Prove that if $\bar{r}$ is the position vector of any point in space, then $r^n\bar{r}$ is irrotational.
11. Find the work done in moving a particle in the force field $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the straight line from (0,0,0) to (2,1,3)
12. Apply Greens theorem to evaluate $\oint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$, where c is the boundary of the area enclosed by the x-axis and upper half of the circle $x^2 + y^2 = a^2$

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