Sub Code: BAST 401 **ROLL NO……………..……………..**

EVEN SEMESTER EXAMINATION, 2022 – 23

IInd yr B.Tech. – Computer Science & Eng/ Chemical Eng

Mathematics -III

Duration: 3:00 hrs **Max Marks: 100**

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	Answer any four parts of the following. a) Find Fourier sine transform of $\frac{e^{-ax}}{x}, a>0$. Hence find Fourier sine transform of $\frac{1}{x}$. b) Using Laplace transform find the value of $\int_0^\infty e^{-3t} t \sin t \, dt$. c) Find the real root of the equation $\cos x - xe^x = 0$, correct to three decimal places using Newton-Raphson method. d) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's one-third rule. e) Solve the initial value problem $\frac{du}{dt} = -2tu^2,\ u(0)=1$, using Runge-Kutta fourth order method with $h=0.2$ on the interval $[0, 0.4]$. f) Define Skewness and Kurtosis of a distribution. In a certain distribution the first four moments about the point $x=4$ are $-1.5, 17, -30$ and 108. Find the moments about mean also calculate β_1 and β_2.	5x4=20
Q 2.	Answer any four parts of the following. a) Using Fourier transform solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, x>0, t>0$ subject to the conditions (i) $u=0$ when $x=0, t>0$ (ii) $u = \begin{cases} 1, & 0<x<1 \\ 0, & x \ge 1 \end{cases}$ when $t=0$ and (iii) $u(x,t)$ is bounded. b) Solve by Laplace transform: $\frac{d^2y}{dt^2} + y = t\cos 2t, t>0$ given that $y = \frac{dy}{dt} = 0$, for $t=0$. c) Prove that: (i) $1+\left(\frac{\delta^2}{2}\right) = \sqrt{1+\delta^2\mu^2}$ (ii) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$ d) Using Lagrange's interpolation formula, find $y(10)$ from the following table: <table><tr><td>X</td><td>5</td><td>6</td><td>9</td><td>11</td></tr><tr><td>Y</td><td>12</td><td>13</td><td>14</td><td>16</td></tr></table>e) Find the co-efficient of correlation for the following table:	5x4=20

X	10	14	18	22	26	30
Y	18	12	24	6	30	36

f) Find the moment generating function of the discrete Poisson distribution given by $P(x) = e^{-\lambda} \frac{\lambda^x}{\lfloor x}$. Also find the first and second moments about the mean.

Q 3. Answer any two parts of the following. (10x2= 20)

a) Find the Fourier transform of e^{-x^2}. Hence find the Fourier transform of $F(x) = e^{-ax^2}, (a > 0)$

b) Define Unit step function. Also evaluate inverse laplace transform of $\frac{p^2}{(p^2+a^2)(p^2+b^2)}$.

c) Given $\frac{dy}{dx} = x - y^2$; $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$ and $y(0.6) = 0.1762$. Compute $y(0.8)$ using Milne's method.

Q 4. Answer any two parts of the following. (10x2= 20)

a) Estimate the production of cotton in the year 1935 from the following data:

Year(x)	1931	1932	1933	1934	1935	1936	1937
Production	17.1	13	14	9.6	-	12.4	18.2

b) Find $\frac{dy}{dx}$ at $x = 0.1$ from the following table

x	0.1	0.2	0.3	0.4
y	0.9975	0.9900	0.9776	0.9604

c) By the method of least squares, find the curve $y = ax + bx^2$ that best fits the following data:

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

Q 5. Answer any two parts of the following. (10x2= 20)

a) State and prove convolution theorem of Laplace transform.

b) The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface:

x = height	100	150	200	250	300	350	400
y = distance	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Using Newton's forward interpolation formula find the value of y when $x = 160\, fit$.

c) In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are legible: Variance of $x = 9$; Regression equations: $8x - 10y + 66 = 0$, $40x - 18y = 214$. What were (i) the mean values of x and y (ii) The standered deviation of y and the co-efficient of correlation between x and y?

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