VIT
Vellore Institute of Technology

Final Assessment Test – November 2019

Course: MAT2002 - Applications of Differential and Difference Equations

Class NBR(s): 0391 / 0392 / 0393 / 0394 / 0395 / 7204 Slot: B1+TB1

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

Answer any FIVE Questions
(5 X 20 = 100 Marks)

SEARCH VIT QUESTION PAPERS ON TELEGRAM TO JOIN

1. a) Find the Fourier series expansion of the function $f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$, $f(x+2)=f(x)$ and hence find the value of the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots$ **[10]**

   b) The turning moment M units of a crank shaft of a steam engine are given for a series of values of the crank angle θ. Obtain the first three terms of sine series to represent M. Also verify the value M from the obtained function at $\theta=60^\circ$. **[10]**

θ:	0°	30°	60°	90°	120°	150°
M :	0	5224	8097	7850	5499	2656

2. a) i) If one of the eigen values of $A=\begin{bmatrix}7&4&-4\\4&-8&-1\\4&-1&-8\end{bmatrix}$ is -9. Find the other two eigen values. **[3]**

   ii) Verify the Cayley-Hamilton theorem for $A=\begin{bmatrix}1&1\\0&1\end{bmatrix}$. Hence find the value of the matrix given by A^5+A^3. **[7]**

   b) Reduce the quadratic form $4x_1^2+4x_2^2+x_3^2-2x_1x_2$ into canonical form by orthogonal transformation. Also find the nature, rank, index and signature of the quadratic form. **[10]**

3. a) Obtain the general solution of the differential equation given by $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=e^{-2x}\cos 3x.$ **[10]**

   b) Solve: $(x^2D^2-2xD-4)y=x^2+2\log x$. **[10]**

4. a) Solve the differential equation $y'''-3y''+3y'-y=t^2e^t$ given that $y(0)=1, y'(0)=0$ and $y''(0)=-2$ using the Laplace transform. **[10]**

   b) Find the general solution to $\begin{aligned}x_1' &= -x_1-2x_2+3\\ x_2' &= 3x_1+4x_2+3\end{aligned}$ subject to the conditions $x_1(0)=-4,\ x_2(0)=5$. **[10]**

5. a) Examine whether the two-point BVP given by $u''+\lambda u=0,\ u'(0)=0,\ u'(L)=0$ represents a Sturm-Liouville problem. If so, find the eigen values and the eigen functions of that problem. **[10]**

   b) Find the power series solution of $x^2y''+(x^2-x)y'+y=0$ about $x=0$. **[10]**

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$\sum$ $\sum_{n=0}$ $\sum_{n=0}$ $\sum$

6. a) Determine f_0, f_1, f_2 in the sequence $\{f_n\}$, when $Z\{f_n\} = F(z)$ is given by $\dfrac{z^3 + 5z^2 + 3z - 1}{(z-1)^3(z+2)}$. **[10]**

b) Find the inverse Z-transform of **[5+5]**

i) $\dfrac{10z}{(z-1)(z-2)}$ using the method of partial fraction.

ii) $\dfrac{z^2}{(z+a)^2}$ using convolution theorem.

7. a) Solve the difference equation $x_{t+2} - 5x_{t+1} + 6x_t = 4^t + t^2 + 3$ by the method of undetermined co-efficients. **[10]**

b) Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y(0) = y(1) = 0$ using the Z-transform. **[10]**

⇔⇔⇔

$\sum_{n=0}$

$a^k + ak^2 + b$

$\dfrac{z}{z-1}$

a^{n-1}

$\dfrac{1}{z-a}$

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