VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act 1956)

Fall Semester 2019-2020

Continuous Assessment Test – I

Programme Name & Branch: B. Tech.

Course Name & Code: Applied Linear Algebra – MAT3004 Slot: C2+TC2+TCC2

Exam Duration: 90 minutes Maximum Marks: 50

Answer All the Questions (5 × 10 = 50)

1. Solve by Gauss Elimination Method

$$x + y + z = 3$$
$$x + 2y + 2z = 5$$
$$3x + 4y + 4z = 11$$

2. Solve by LU decomposition Method

$$x + y - z = 4$$
$$x - 2y + 3z = -6$$
$$2x + 3y + z = 7$$

3. Prove that the set V of all polynomials of degree $\leq n$ including the zero polynomial is vector space over the field R under usual polynomial addition and scalar multiplication

4. (a) Show that each of the following subsets of R^3 is not a subspace

   a) $S = \{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}$

   b) $S = \{(x, y, z) | x + y + z = 1\}$

   c) $S = \{(x, y, z) | x \geq y \geq z\}$ **(5 marks)**

   (b) Let V be a vector space and let $u, v, w \in V$. Show that the vectors $u - v, v - w$ and $w - u$ are linearly dependent. **(5 marks)**

5. (a) Let V be the vector space of all 2×2 matrices whose entries are real numbers. Let $S = \left\{A \in V \middle| A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}, a, b \in R\right\}$. **(4 marks)**

   i. Find a basis of S

   ii. Find the dimension of S

   (b) Prove that the vector space V is the direct sum of subspaces U and W if and only if there exists a unique $u \in U$ and $w \in W$ such that $v = u + w$. **(6 marks)**