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Paper Code : BSCAIDS301/BSCAIML 301/BSCAIML301/BSCICB301 Linear Algebra

UPID : 003902

Time Allotted : 3 Hours *Full Marks :70*

The Figures in the margin indicate full marks.

Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer *any ten* of the following : [1 x 10 = 10]

(I) A complete inner product space is known as _______space.

(II) If A is an orthogonal matrix then the rows of A are linearly ________ .

(III) If $T:V\to W$ be a mapping then Nullity T + Rank T will be equal to _________.

(IV) If (a,b) and $(1/\sqrt{2}, -1/\sqrt{2})$ in R^2 are orthonormal vectors then the value of a+b is ____

(V) In Schur's decomposition of a matrix A if $A=PUP^{-1}$ then the matrix U is _____________.

(VI) If $T:R^2\to R^2$ be a mapping defined by $T(x,y)=(x+y, x)$ then the nullity of T is __________ .

(VII) If $T:V\to V$ be a singular linear mapping with eigenvalues 1,2 and 3 and dim(V)=4 then find the trace of matrix of T.

(VIII) Let α, β be two vectors in an inner product space. If $||\alpha+\beta||=||\alpha-\beta||$ then the value of $<\alpha,\beta>$ is ______

(IX) If P be a subspace of dimension k of a n - dimensional Euclidean space V. Then find the dimension of the orthogonal complement of P.

(X) If (1,2) and (2,-k) are orthogonal vectors in R^2 then what is the value of k?

(XI) If $T_1,T_2 : R^2\to R^2$ be two linear mappings such that $T_1 T_2 + T_1^2 = I$ then find Rank of T_1.

(XII) If T be a linear mapping on a finite dimensional vector space V such that $T^2 = I$ then the nullity of T is __________

Group-B (Short Answer Type Question)

Answer *any three* of the following : [5 x 3 = 15]

2. Find the matrix of T for the mapping $T: R^3\to R^3$ defined by $T(x,y,z)=(y+z , x+z , x+y)$ ∀ (x,y,z) R^3 with respect to the bases {(0,1,1),(1,0,1),(1,1,0)} and {(1,0,0),(0,1,0),(0,0,1)}. [5]

3. Prove that the set of vectors {(1,2,2), (2,-2,1), (2,1,-2)} is an orthogonal basis of the Euclidean space R_3 with the standard inner product. Express (4,3,2) as a linear combination of these basis vectors. [5]

4. If S is a subspace of R^4 generated by the vectors (1,2,-1,0) and (1,-1,0,1). Determine the orthogonal complement of S. [5]

5. If A be a symmetric matrix of order m and P be an mXn matrix, prove that P^tAP is a symmetric matrix. [5]

6. Find orthogonal complement of the row space of the matrix [5]

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 1 \end{pmatrix}$$

Group-C (Long Answer Type Question)

Answer *any three* of the following : [15 x 3 = 45]

7. (a) Prove that for all α, β in a Euclidean space V $< \alpha+\beta , \alpha-\beta >=0$ if and only if $||\alpha|| = ||\beta||$. [5]

(b) Prove that the set of vectors {(1,0,0,1), (-1;0,2,1) ,(1,3,1,-1) , (1,-1,1,-1)} is an orthogonal basis of the Euclidean space R^4 with standard inner product. [5]

(c) Find the projections of the vector (1,2,3,1) along the basis vectors given above. [5]

8. (a) Determine the linear mapping $T: R^3\to R^2$ which maps the basis vectors (1,0,0), (0,1,0), (0,0,1) to the vectors (1,1), (2,3), (3,2) respectively. Find T(6,0,-1) and T(1,1,0). [5]

(b) Find Ker T and Im T. [5]

(c) Prove that T is not one-one but onto. [5]

(a) Find all possible linearly independent eigenvectors of the given matrix. [6]

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

(b) Diagonalize the above matrix if possible. [4]

(c) Prove that eigenvalues of a symmetric matrix are all real. [5]

10. (a) $T:R^3 \to R^3$ is a linear transformation defined by [5]
$T(x,y,z)=(x-y,x+2y,y+3z)\ \forall\ x,y,z\ R$
Find nullity T and Rank T. Hence check whether T is bijective.

(b) Find T^{-1} if it exists. [5]

(c) Find matrix of T^{-1} with respect to the standard basis if T^{-1} exists. [5]

11. (a) Let $D:P_3 \to P_3$ be a linear mapping defined by $D(f(x))=\frac{df}{dx}\ \forall\ f(x)\ P_3$, where P_3 is the set of all real polynomials of degree less than or equal to n. Find matrix of T. [5]

(b) Determine nullity and rank of T. [5]

(c) Check whether T is diagonalizable. [5]

*** END OF PAPER ***