

Mathematics-III / Applied Mathematics-III

P. Pages : 3

Time : Three Hours



SPM/KW/22/2535/2541/2547

Max. Marks : 70

- Notes :
1. All questions carry marks as indicated.
 2. Solve Question 1 OR Questions No. 2.
 3. Solve Question 3 OR Questions No. 4.
 4. Solve Question 5 OR Questions No. 6.
 5. Solve Question 7 OR Questions No. 8.
 6. Solve Question 9 OR Questions No. 10.
 7. Use of non programmable calculator is permitted.

1. a) Find L.T. of $\frac{e^{-at} - e^{-bt}}{t}$, hence evaluate $\int_0^{\infty} \frac{e^{-at} - e^{-bt}}{t} dt$ 5

b) Find $L^{-1} \left\{ \frac{1}{(s+1)(s^2+1)} \right\}$ by using Convolution theorem. 4

c) Find Fourier transform of $f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$, hence find $\int_0^{\infty} \frac{\sin x}{x} dx$ 5

OR

2. a) Find $L \left\{ \int_0^t \sin u du \right\}$ 4

b) Solve $\frac{d^2 y}{dt^2} + 9y = \cos 2t$, given $y(0) = 1$, $y(\pi/2) = -1$ 6

c) Solve the integral equation 4
 $\int_0^{\infty} f(x) \cos \lambda x dx = e^{-\lambda}, \lambda > 0$

3. a) Find Z-transform $\sin n\theta \cos n\theta$. 7

b) By using convolution theorem, 7
 $z^{-1} \left\{ \frac{z^2}{(z-1)(z-3)} \right\}$

OR

4. a) Using Power series method, find $z^{-1} \left\{ \frac{1}{z^2 - 3z + 2} \right\}$, for the region $|z| < 1$. 7

b) Solve the difference equation by Z-transform. 7

$$y_{n+2} + 5y_{n+1} + 6y_n = 6^n, \text{ given } y(0) = 0, y(1) = 1.$$

5. a) Reduce the given matrix in diagonal form, $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}$ 7

b) Using Sylvester's Theorem, solve if $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$, show that $\log_e e^A = A$. 7

OR

6. a) Find the singular values of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and find the singular value decomposition of A. <https://www.rtmnuonline.com> 7

b) Find the largest eigen value and the corresponding eigen vector for the matrix. 7

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

7. a) A density function of random variable X is. 7

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Find

i) $E(X)$

ii) $\text{Var}(X)$

iii) σ_X

iv) $E[(X-1)^2]$

b) If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs- 7

i) Exactly 2

ii) More than 5

iii) Between 1 and 3

iv) At the most 2,

v) At least 2 bulbs will be defective

OR

8. a) Find the moment generating function of the random variable 7

$$X = \begin{cases} 1/2, & \text{Prob. } 1/2 \\ -1/2 & \text{Prob. } 1/2 \end{cases}$$

Also find first four moments about the origin.

- b) A machine produces bolts which are 10% defective. Find the probability that in a random sample of 400 bolts produced by this machine. 7
- Between 30 and 50
 - at the most 30,
 - 55 or more of the bolts will be defective.

9. a) Find the mode from the following data. 7

Age	0-6	6-12	12-18	18-24	24-30	30-36	36-42
Frequency	6	11	25	35	18	12	6

- b) Calculate the mean and standard deviation for the following data. 7

Size of item	6	7	8	9	10	11	12
Frequency	3	6	9	13	8	5	4

OR

10. a) Let x_1, x_2, x_3 are three variates measured from their mean with 7
- $n = 10, \Sigma x_1^2 = 90, \Sigma x_2^2 = 160, \Sigma x_3^2 = 40, \Sigma x_1 x_2 = 60, \Sigma x_1 x_3 = 40$ and $\Sigma x_2 x_3 = 60$ calculate the multiple correlation coefficient $R_{1.23}$.

- b) Calculate coefficient of skewness of the following distribution. 7

x	0	1	2	3	4	5	6	7	8
f	7	12	32	56	70	56	28	8	1
