MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BSC 301/BSC301 Mathematics-III (Differential Calculus)

UPID : 003445

Time Allotted : 3 Hours *Full Marks :70*

The Figures in the margin indicate full marks.
Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer *any ten* of the following : [1 x 10 = 10]

(I) Find the number of vertices in a graph with 15 edges if each vertex has degree 2.

(II) Determine whether the sequence $\{x_n\}$, where

$$x_n = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots\cdots + \frac{1}{n\cdot(n+1)}$$ converges or not.

(III) Is the function $f(x,y) = \begin{cases} \frac{xy}{xy+x-y}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ continuous at (1,2)?

(IV) Evaluate: $\int_0^{\pi/2} \int_0^2 r\, dr d\theta$

(V) Write the general solution of the differential equation $p = \cos(y - xp)$,where $p = \frac{dy}{dx}$.

(VI) Is the function $f(x,y) = \begin{cases} x^2 + y^2 + xy, & (x,y) \neq (2,3) \\ 10, & (x,y) = (2,3) \end{cases}$ continuous at (0,0) ?

(VII) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ converges or not.

(VIII) Find the degree of the homogeneous function

$$f(x,y) = \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$

(IX) Find the value of $\iint dxdy$ over the first quadrant of the circle having centre at the origin and radius 4 c.m.

(X) Evaluate: $\frac{1}{(D-2)(D-3)} e^{2x}$

(XI) If a graph has 5 vertices and 7 edges then write the size of its adjacency matrix.

(XII) Give an example of a bounded sequence which is not convergent.

Group-B (Short Answer Type Question)

Answer *any three* of the following : [5 x 3 = 15]

2. [5]

Let $f(x,y) = \begin{cases} \frac{x^3+y^3}{x-y}, & x \neq y \\ 0, & x = y \end{cases}$. Prove that $f(x,y)$ is not continuous at $(0,0)$ but $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at $(0,0)$.

3. [5]

Using the transformation $x + y = u,\ y = uv.$ Show that $\int_0^1 dx \int_0^{1-x} e^{\frac{y}{x+y}} dy = \frac{1}{2}(e-1)$.

4. [5]

Solve: $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$

5. [5]

Prove that the maximum degree of any vertex in a simple graph with n vertices is $n-1$.

Prove that the maximum number of edges in a connected simple graph with n vertices is $\frac{1}{2}n(n-1)$.

6. [5]

Using the transformation $x - y = u,\ x + y = v$ prove that $\iint_R \cos\frac{x-y}{x+y} dx\, dy = \frac{\sin 1}{2}$ where R is the region bounded by $x + y = 1$, $x = 0,\ y = 0$.

Group-C (Long Answer Type Question)

Answer *any three* of the following : [15 x 3 = 45]

7. (a) Prove that the minimum number of edges in a connected graph with n vertices is $n-1$. [5]

(b) Prove that a complete graph with n vertices consists of $\frac{1}{2}n(n-1)$ number of edges. [5]

Show that a bipartite graph cannot contain a cycle of odd length.

(c) Prove that there exist no simple graph with five vertices having degrees 4,4,4,2,2 . [5]

Draw, if possible, a simple graph with five vertices having degrees 2,3,3,3,3 .

8. (a) [5]

Show that the sequence $\left\{\sqrt{5}, \sqrt{5+\sqrt{5}}, \sqrt{5+\sqrt{5+\sqrt{5}}}, \cdots\right\}$ tends to a definite finite limit. Also find the limit.

(b) [5]

Discuss the convergence of the power series

$$1 + \frac{3}{7}x + \frac{3\cdot 6}{7\cdot 10}x^2 + \frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3 + \cdots \quad (x > 0)$$

(c) [5]

Discuss the convergence of the series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$, x be any number.

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9. (a) [5]

Let $f(x,y) = \begin{cases} 1, & xy \neq 0 \\ 0, & xy = 0 \end{cases}$. Show that the two repeated limits exist at (0,0) and are equal but the simultaneous limit $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

(b) [5]

Let $f(x,y) = \begin{cases} x + y\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Verify that the simultaneous limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exists and the repeated limit $\lim_{x\to 0}\lim_{y\to 0} f(x,y)$ exists but the repeated limit $\lim_{y\to 0}\lim_{x\to 0} f(x,y)$ does not exist.

(c) [5]

Let $f(x,y) = \begin{cases} y\sin\frac{1}{x} + x\sin\frac{1}{y}, & xy \neq 0 \\ 0, & xy = 0 \end{cases}$. Show that at (0,0) the double limit exists but the repeated limits do not exist.

10. (a) [5]

Verify Stoke's theorem for $\vec{F} = (2x-y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

(b) [5]

Verify Green's theorem in the plane for $\oint_C \left[(xy+y^2)\,dx + x^2\,dy\right]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(c) [5]

Verify the Divergence Theorem for the vector function

$\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$, taken over the rectangular parallelopiped $0 \le x \le a, \quad 0 \le y \le b, \quad 0 \le z \le c$.

11. (a) [5]

Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \text{grad}\,(x^3+y^3+z^3-3xyz)$.

(b) [5]

Show that $\vec{F} = (6xy+z^3)\hat{\imath} + (3x^2-z)\hat{\jmath} + (3xz^2-y)\hat{k}$ is irrotational. Find a scalar function φ such that $\vec{F} = \vec{\nabla}\varphi$.

(c) [5]

If $r = |\vec{r}|$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then prove that $\vec{\nabla}\cdot\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$.

*** END OF PAPER ***