	Utech
Name :	(4)
Roll No.:	To Albana (V Executings 2nd Explicate)
Inviailator's Sianature :	

CS/B.TECH (CT-OLD)/SEM-3/M (CT)-301/2011-12 2011

APPLIED MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

 $10 \times 1 = 10$

- i) If $f(z) = \frac{\sin z}{z^3}$ then z = 0 is a pole of order 2.
 - a) True

- b) False.
- ii) If $f(z) = \frac{z+1}{z^4-2z^3}$, then z = 0 is a pole of order
 - a) 3

b) 2

c) 1

- d) 4.
- iii) Residue of $f(z) = \frac{2 + 3 \sin \pi z}{z(z-1)^2}$ at z = 0 is
 - a) 1

b) 2

c) 3

d) i.

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- cos nx is a periodic function where fundamental period
 - a)

b) $n\pi$

- d) $2n\pi$.
- Value of the integral $\int \sin(mx) \cos nx \, dx$ is

(where m, n are unequal positive numbers)

a)

b) 2π

c)

- d) none of these.
- The inverse Fourier Transform of vi)

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$
 is given by

a)
$$\int_{0}^{\infty} F(s) e^{-isx} ds$$

b)
$$\frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$$

c)
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$$

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d)
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{isx} ds$$
.

- vii) The residue of $f(z) = \frac{z+1}{z^2-2z}$ at the pole z=0 is
 - a) $-\frac{1}{2}$

c) $\frac{1}{2}$

- d) $\frac{3}{2}$.
- viii) The number of poles of

$$f(z) = \frac{z}{(z-1)(z-2)(z-3)}$$
 inside the circle

$$|z-2| = 2 \text{ is}$$

a)

b) 1

c)

- d) 0.
- The value of the integral $\oint_C \frac{dz}{z-4}$ where C: |z| = 1

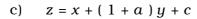
is

a) $2\pi i$ b) $-2\pi i$

- d) none of these.
- The complete solution of $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ is

a)
$$z = ax + (1 + a)y + c$$

b)
$$z = ax + (1 - a)y + 2$$



d)
$$z = ax + (1 - a)y + c$$
.



If the events A and B are such that A and B are independent, then P (B / A) equals to

P(B)a)

P(A)b)

c) 1 d) 0.

xii) Value of the expectation $E\{X + E(X)\}$ is

1 a)

- b) 0
- c) mean of X
- d) none of these.

xiii) Probability of obtaining 3 heads with a toss of 4 coins is

a)

d) $\frac{3}{16}$.

xiv) 3 dice and 3 coins are rolled out. The number of elementary events in the sample space is

- $6^3 \propto 2^3$ a)

 2^{2} c)

d) $6^3 \propto 2^2$.

GROUP - B

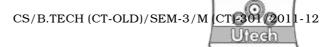
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

Evaluate the limit : $\lim_{z \varnothing i} \left(\frac{iz^4 - 1}{z - i} \right)$.

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- 3. State Cauchy's integral formula and use it to evaluate $\int \frac{e^{z+1}}{z^2+4} dz \text{ where } C \text{ is the circle } |z-i| = 2.$
- 4. Find the value of $10^{1/2}$ correct up to 4 significant figures using Newton-Raphson method.
- 5. Using Fourth order Runge-Kutta method, solve

$$\frac{dy}{dx} = x^2 + y^2$$
, $y(0) = 1$

at x = 0.2 using a step length h = 0.1.

6. Define Fourier cosine transform. Solve the integral equation

$$\int_{0}^{\infty} f(x) \cos \lambda x \, dx = e^{-\lambda} .$$

7. Find the complete solution of the following p.d.e.:

$$p + q = x$$
.

GROUP - C

(Long Answer Type Questions)

Answer any three of the following questions.

$$3 \times 15 = 45$$

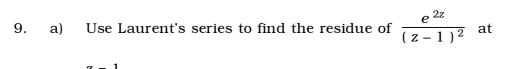
8. a) Obtain the Fourier series to represent x^2 in $-\pi \le x \le \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6} .$$



b) Using Cauchy's Residue theorem, prove that

$$\oint_C \frac{z \cos z}{\left(z - \frac{\pi}{2}\right)^3} dz = -2\pi i.$$



- b) Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's series about z=1 and determine the region of convergence. 8+7
- 10. a) Prove that $\int_{0}^{+\infty} \frac{dx}{x^2 + 1} = \frac{\pi}{2}$, by using Cauchy's residue theorem.
 - b) Find the Fourier series of the function

$$f(x) = \begin{cases} \pi + 2x & \text{if } -\pi < x < 0 \\ \pi - 2x & \text{if } 0 \le x < \pi \end{cases}$$

Hence, deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty = \frac{\pi^2}{8} . 7 + 8$$

11. a) If $F\{f(x)\} = F(s)$ be the Fourier transform of f(x), show that

$$F\left\{ f\left(\,x+a\,\right)\,\right\} = e^{\,-\,i\alpha s}\;\;F\left(\,s\,\right)\,.$$



- b) Find out the Fourier transform of $K.e^{-\chi^2/2}$, K is a constant.
- c) Show that $P(\overline{A}/B) = 1 P(A/B)$, where A and B are any two events. 5 + 6 + 4
- 12. a) The probability that a teacher will give a surprise test during any class of a particular day is $\frac{1}{5}$. If a student is absent, on two days, what is the probability that he will miss at least one test?

b) If
$$f(x) = \frac{1}{4} - Kx$$
, $0 \le x \le 4$

$$= 0$$
, otherwise

is the p.d.f. of a random variable X,

determine —

- i) the value of K and
- ii) P(|X-2| < 0.5).

Also find out the mean of X.

7 + 8