

Name :

Roll No. :

Invigilator's Signature :

CS/B.TECH(OLD)/ME/PE/AUE/SEM-3/M-303/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Graph sheet(s) will be supplied by the institution on demand.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) $J_{-\frac{1}{2}}$ is given by

a) $\sqrt{\frac{2\pi}{x}} \sin x$

b) $\sqrt{\frac{2\pi}{x}} \cos x$

c) $\sqrt{\frac{2}{\pi x}} \cos x$

d) $\sqrt{\frac{2}{\pi x}} \sin x$

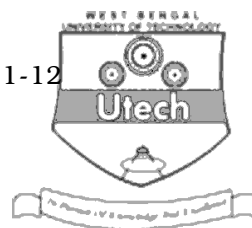
ii) The equation $u_{xx} + u_{yy} = 0$ is

a) parabolic

b) hyperbolic

c) elliptic

d) none of these.



iii) The function $f(z) = \frac{1}{(z+1)^2}$

- a) is analytic
- b) has a pole of order 2 at $z = -1$
- c) has removable singularity at $z = -1$
- d) has essential singularity at $z = 0$.

iv) The order and degree of the the P.D.E.

$$\frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial x} \right)^2 = 0$$

are

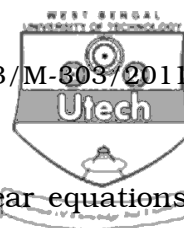
- a) 2,2
- b) 2,1
- c) 1,2
- d) none of these.

v) The residue of $\frac{z^2}{z^2 + a^2}$ at $z = ia$ is

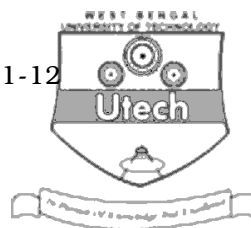
- a) $-\frac{1}{2}(ia)$
- b) $\frac{1}{2}(ia)$
- c) a
- d) ia .

vi) The number of initial basic feasible solution in a transportation problem is

- a) at most $(m + n - 1)$
- b) at least $(m + n - 1)$
- c) equal to $(m + n - 1)$
- d) none of these.



- vii) Given a system of m simultaneous linear equations in n unknowns ($m < n$). Then there will be
- a) n basic variables b) m basic variables
- c) $(n - m)$ basic variables d) $(n + m)$ basic variables.
- viii) If P denotes Legendre polynomial, then $P_0(x) =$
- a) x b) x^2
- c) 1 d) 2 .
- ix) The minimum number of lines covering all Zeros in a reduced cost matrix of order n can be
- a) at most n b) at least n
- c) $n - 1$ d) $(n + 1)$
- x) The value of $\lim_{x \rightarrow 0} \frac{xy}{x^2 + 2y^2}$ is
- a) 0 b) $1/2$
- c) 1 d) none of these.
- xi) In an Assignment problem involving 5 workers and 5 Jobs, total number of assignment possible are
- a) $5!$ b) 10
- c) 5 d) 25 .
- xii) Consider the differential equation $xy'' + 2y' + xy = 0$. Then $x = 0$ is
- a) an ordinary point
- b) singular point but not a regular singular point
- c) a regular singular point
- d) none of these.

**GROUP – B****(Short Answer Type Questions)**Answer any *three* of the following.

3 × 5 = 15

2. If $f(z)$ is a analytic function of z , prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$$

3. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given that $u(0, t) = 0$

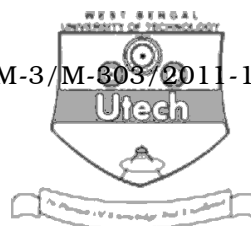
$$u(x, 0) = f(x) \text{ and } \frac{\partial u(x, 0)}{\partial t} = 0, \text{ where } 0 < x < 1 .$$

4. Show that transportation problem can be expressed as an LPP.
5. Obtain the general solution of the partial differential equation

$$p \tan x + q \tan y = \tan z ,$$

$$\text{where } p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}$$

6. Evaluate $\int_0^{1+i} (x^2 + iy) dz$ along $y = x^2$.

**GROUP – C****(Long Answer Type Questions)**Answer any *three* of the following. $3 \times 15 = 45$

7. a) Solve the following problem by simplex method :

$$\text{Max } Z = 20x_1 + 12x_2 + 8x_3$$

$$\text{subject to } 4x_1 + 4x_2 + 4x_3 \leq 1200$$

$$3x_1 + 4x_2 + 3x_3 \leq 900$$

$$2x_1 + x_2 + x_3 \leq 400$$

$$\forall x_i \geq 0 \quad i=1,2,3$$

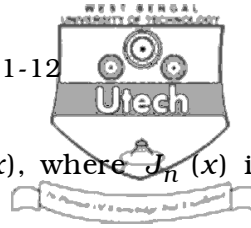
8

- b) The manager of an oil refinery must decide on the optimum mix of 2 possible blending processes of which the inputs and production runs are follows :

	Input		Output	
Process	Crude A	Crude B	Gasoline X	Gasoline Y
1	6	4	6	9
2	5	6	5	5

The maximum amounts available of crudes A and B are 250 units and 200 units respectively. Market demand shows that at least 150 units of Gasoline X and 130 units of Gasoline Y must be produced. The profits per production run from processes 1 and 2 are Rs. 4 and Rs. 5 respectively. Write the mathematical form of this problem for maximizing the profit.

7



8. a) Prove that $J_2'(u) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$, where $J_n(x)$ is the Bessel function of first kind. 7

b) Prove that $\int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{2n+1}$

where $P_n(x)$ denotes the Legendre's function. 8

9. a) Find the bi-linear transformation which maps the points $z = 2, i, -2$ into $w = 1, i, -1$ respectively. 5

- b) Expand $l_n(1+z)$ in power of $1/z$ and indicate the region of convergence. 5

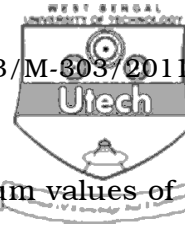
- c) Evaluate $\oint_C \bar{z} dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by the straight line joining $z = 0$ and $z = 4 + 2i$. 5

10. a) Determine all basic feasible solutions of the set of equations :

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

Find the order of degeneracy. 8



- b) Show graphically the maximum, minimum values of the objective function for the following are same :

$$Z = 5x_1 + 3x_2$$

$$\text{subject to } x_1 + x_2 \leq 6$$

$$2x_1 + 3x_2 \geq 3$$

$$0 \leq x_1 \leq 3, 0 \leq x_2 \leq 3.$$

7

11. a) Use residue theorem to evaluate

$$\int_0^{2\pi} \frac{1}{5-4\sin\theta} d\theta$$

8

- b) Find the complete integral of the partial differential equation $p^2q(x^2+y^2)=p^2+q$, where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$

$$z = z(x, y)$$

7

=====