

## ODD SEMESTER EXAMINATION, 2024 – 25

2<sup>nd</sup> Year (3<sup>rd</sup> Sem) B.Tech.

## MATHEMATICS- III

Duration: 3:00 hrs

Max Marks: 100

*Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.*

Q 1.	<p>Answer any two parts of the following. (10x2= 20)</p> <p>a) (i) Find the Fourier transform of</p> $f(x) = \begin{cases} 1 - x^2, & \text{for }  x  \leq 1 \\ 0 & \text{for }  x  > 1. \end{cases}$ <p>(5 marks)</p> <p>(ii) Find the Fourier sine transform of <math>f(x) = \frac{e^{-ax}}{x}</math>. (5 marks)</p> <p>b) Express the function</p> $f(x) = \begin{cases} 1, & \text{When }  x  \leq 1 \\ 0, & \text{When }  x  > 1. \end{cases}$ <p>As a Fourier integral. Hence evaluate <math>\int_0^\infty \frac{\sin \lambda \cos \lambda}{\lambda} d\lambda</math>. (10 marks)</p> <p>c) Solve <math>\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}</math> for <math>0 \leq x &lt; \infty</math>, <math>t &gt; 0</math> given the condition</p> <p>(i) <math>u(x, 0) = 0</math> for all <math>x \geq 0</math></p> <p>(ii) <math>\frac{\partial u}{\partial x}(0, t) = -a</math> (constant)</p> <p>(iii) <math>u(x, t)</math> is bounded. (10 marks)</p>
Q 2.	<p>Answer any two parts of the following. (10x2= 20)</p> <p>a) (i) Obtain the Laplace transform of <math>t^2 e^t \sin 4t</math>. (5 marks)</p> <p>(ii) Find the inverse Laplace transform of <math>\frac{s+4}{s(s-1)(s^2+4)}</math>. (5 marks)</p> <p>b) Find the Laplace transform of periodic function and find the Laplace transform of the following periodic function with period <math>\frac{2\pi}{\omega}</math> defined as</p> $f(t) = \begin{cases} \sin \omega t, & 0 \leq t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} \leq t \leq \frac{2\pi}{\omega} \end{cases}$ <p>(10 marks)</p> <p>c) Using Laplace transforms, find the solution of initial value problem</p> $y'' - 4y' + 4y = 64 \sin 2t$ $y(0) = 0, y'(0) = 1. \quad (10 \text{ marks})$
Q 3.	<p>Answer any two parts of the following. (10x2= 20)</p> <p>a) (i) By using Newton – Raphson's method find the root of <math>x^4 - x - 10 = 0</math>, which is near to <math>x = 2</math> correct to three places of decimal. (5 marks)</p> <p>(ii). Given <math>f(0) = 3</math>, <math>f(1) = 12</math>, <math>f(2) = 81</math>, <math>f(3) = 200</math>, <math>f(4) = 100</math> and <math>f(5) = 8</math>. Form a difference table</p>

	and find $\Delta^5 f(0)$ . b) Given values	(5 marks)																								
	<table><tr><td>x</td><td>5</td><td>7</td><td>11</td><td>13</td><td>17</td></tr><tr><td>F(x)</td><td>150</td><td>392</td><td>1452</td><td>2366</td><td>5202</td></tr></table> Find the value $f(9)$ by using Lagrange's formula.	x	5	7	11	13	17	F(x)	150	392	1452	2366	5202	(10 marks)												
x	5	7	11	13	17																					
F(x)	150	392	1452	2366	5202																					
	c) Estimate the sale for 1966 using the following table by Newton-Gregory formula																									
	<table><tr><td>Year</td><td>1931</td><td>1941</td><td>1951</td><td>1961</td><td>1971</td><td>1981</td></tr><tr><td>Sale in Thousand</td><td>12</td><td>15</td><td>20</td><td>27</td><td>39</td><td>52</td></tr></table>	Year	1931	1941	1951	1961	1971	1981	Sale in Thousand	12	15	20	27	39	52	(10 marks)										
Year	1931	1941	1951	1961	1971	1981																				
Sale in Thousand	12	15	20	27	39	52																				
Q 4.	Answer any two parts of the following. a) (i) Calculate the value of the integral $\int_4^{5.2} \log x \, dx$ by Trapezoidal rule. (ii) Using Milne predictor formula find $y$ when $x = 0.8$ given $\frac{dy}{dx} = x - y^2$ , $y(0) = 0$ , $y(0.2) = 0.02$ , $y(0.4) = 0.0795$ , $y(0.6) = 0.1762$ . d) b) Find $\int_0^1 \frac{dx}{1+x^2}$ by using (i). Simpson's $\frac{1}{3}$ rule (ii). Simpson's $\frac{3}{8}$ rule c) Given $\frac{dy}{dx} = y - x$ , $y(0) = 2$ . Find, using Runge-Kutta fourth order formula $y(0.1)$ and $y(0.2)$ correct to four decimal place.	(10x2= 20) (5 marks) (5 marks) (10 marks) (10 marks)																								
Q 5.	Answer any two parts of the following. a) (i) Define Skewness and its type. Calculate Karl Pearson's coefficient of skewness for distribution having mean=3.41, median = 3.4 and S. D. =0.70. (ii) Define Kurtosis. For a distribution the second and fourth central moments are 2 and 17 respectively. Find the moment coefficient of Kurtosis. b) Obtain the regression equation $Y$ on $X$ by least square method for the following data, also estimate the value of $Y$ when $X = 10$ . <table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>9</td><td>9</td><td>10</td><td>12</td><td>11</td></tr></table> c) Fit a least square geometric curve $y = ax^b$ for the following data <table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>0.5</td><td>2</td><td>4.5</td><td>8</td><td>12.5</td></tr></table>	x	1	2	3	4	5	y	9	9	10	12	11	x	1	2	3	4	5	y	0.5	2	4.5	8	12.5	(10x2= 20) (5 marks) (5 marks) (10 marks) (10 marks)
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