MATH211

[ST]

END SEMESTER EXAMINATION : NOV.–DEC., 2019

APPLIED MATHEMATICS – III

Time : 3 Hrs.   *Maximum Marks : 70*

Note: *Attempt questions from all sections as directed.*
Use of scientific calculator is allowed.

SECTION – A   (30 Marks)
Attempt any **five** *questions out of* **six.**
Each question carries **06** *marks.*

1. Obtain the half range sine series for the function $f(x) = x^2$ in the interval $0 < x < 3$.

2. Obtain the first three coefficients in the Fourier cosine series for y, where y is given in the following table :

x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

3. Solve Lagrange's linear partial differential equation $y^2p - xyq = x(z - 2y)$.

P.T.O.

(664)

4. Form the partial differential equation by the elimination

   of f from $z = y^2 + 2 f\left(\dfrac{1}{x} + \log y\right)$.

5. Solve by method of separation of variables

   $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u$, given $u = 3e^{-y} - e^{-5y}$, when $x = 0$.

6. Solve the integral equation

   $$\int_0^\infty f(x) \cos px \, dx = \begin{cases} 1-p, & 0 \le p \le 1 \\ 0, & p \ge 1 \end{cases}$$

   Hence deduce that $\displaystyle\int_0^\infty \dfrac{\sin^2 t}{t^2} \, dt = \dfrac{\pi}{2}$.

SECTION – B (20 Marks)

*Attempt any **two** questions out of **three**.*
Each question carries 10 marks.

7. Find the Fourier cosine transform of $F(x) = \dfrac{1}{\left(1 + x^2\right)}$

   and hence find Fourier sine transform of $F(x) = \dfrac{x}{\left(1 + x^2\right)}$.

8. Find the temperaturein a bar of length 2 whose ends are kept at zero and lateral surface insulated if the initial temperature is $\sin\dfrac{\pi x}{2} + 3\sin\dfrac{5\pi x}{2}$.

9. (a) Obtain the Fourier series to represent the function $f(x) = |\sin x|$ for $-\pi < x < \pi$.   (7)

     (b) Classify the partial differential equation

$$x\frac{\partial^2 u}{\partial x^2} + t\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial t^2} = 0$$ (3)

SECTION – C   (20 Marks)

(Compulsory)

10. (a) Solve the non-homogeneous linear partial differential equation

$$(3D^2 - 2D'^2 + D - 1)z = 4e^{x+y}\cos(x+y).$$ (7)

     (b) Solve of Charpit's method $z = p^2 x + q^2 y$.   (7)

     (c) A string of length '1' is initially at rest in equilibrium position and at each of its points velocity is given

P.T.O.

$$\frac{\partial y}{\partial t} = b \, \sin^3 \frac{\pi x}{l}, \quad \text{at} \quad t = 0. \text{ Find the displacement}$$

y(x, t).   (6)