



Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech(New)/CSE/IT/SEM-4/M-401/2013

2013

MATHEMATICS-III

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :
 $10 \times 1 = 10$

i) The number of generations of an infinite cyclic group is

- a) 1 b) 2
c) infinite d) none of these.

ii) The standard deviation of a sample mean for SRSWR is

- a) σ^2/n b) σ/\sqrt{n}
c) σ/n d) n .

- 2



vii) The distribution for which the mean and variance are equal is

- a) Poisson
- b) normal
- c) binomial
- d) exponential.

viii) In a Binomial (n, p) distribution, if its mean and variance are 2 and $4/3$ respectively, then the values of n and p are

- a) $8, \frac{1}{4}$
- b) $6, \frac{1}{3}$
- c) $4, \frac{1}{2}$
- d) none of these.

ix) If G is a connected planar graph with n vertices, e edges and f faces, then $n - e + f = 2$. This statement is

- a) True
- b) False.

x) The mean of Binomial variate is

- a) np
- b) $np (1 - p)$
- c) \sqrt{np}
- d) none of these.



xi) Kuratowski's graph is a

- a) planar graph
- b) regular graph
- c) tree
- d) none of these.

xii) The order of the dihedral group D_4 is

- a) 4
- b) 6
- c) 8
- d) 64.

xiii) Every finite integral domain is a field. This statement is

- a) True
- b) False.

xiv) If A and B are two subgroups of a group G , then which of the following is always a subgroup of G ?

- a) $A \cup B$
- b) $G - A$
- c) $G - B$
- d) $A \cap B$.

xv) The symmetric group S_3 has

- a) 6 elements
- b) 8 elements
- c) 9 elements
- d) none of these.



GROUP - B
(Short Answer Type Questions)

Answer any *three* of the following.

3 × 5 = 15

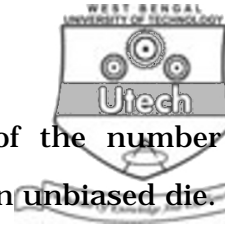
2. Let $(Q, +)$ be the additive group of rational numbers and (Q^+, \cdot) be the multiplicative group of positive rational numbers. Are these two groups isomorphic? Justify your answer.
3. Prove Baye's theorem for repeated trials.
4. Examine whether function $|x|$ in $(-1, 1)$ and zero elsewhere is a density function.
5. Show that a connected graph is Eulerian if and only if each of its vertices is of even degree.
6. Show that a field does not contain any zero divisor.

GROUP - C
(Long Answer Type Questions)

Answer any *three* of the following.

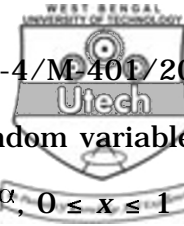
3 × 15 = 45

7. a) Prove that a planar graph with n vertices, e number of edges and k number of components determines f number of regions, where $f = e - n + k + 1$. 8
- b) Let \bar{X} be the sample mean of samples of size n drawn at random from a population which is normally distributed with mean μ and variance σ^2 . Find the standard error of the statistic \bar{X} . 7



8. a) Find the mathematical expectation of the number of points obtained in a single throw of an unbiased die. 5
- b) Define Poisson distribution and find its mean and variance. 5
- c) Let f be a ring homomorphism from the ring Z of integers into itself such that $f(1) = 1$. Determine the homomorphism f . 5
9. a) Show that any simple connected planar graph with n vertices ($n \geq 3$) has at most $(3n - 6)$ edges. 7
- b) Prove that every nontrivial subgroup of the additive group Z of integers is cyclic. 4
- c) Let R and S be two rings and $f : R \rightarrow S$ be a ring homomorphism. Show that kernel of f is a subring of R . 4
10. a) Determine the mean and variance of exponential distribution. 6
- b) Show that every cyclic group is commutative. 4
- c) Let H be a normal subgroup of a group G and G/H be the set of all cosets of H in G . Show that G/H forms a group under the composition

$$(aH) \cdot (bH) = (ab)H \text{ for all } a, b \in G. \quad 5$$



11. a) The probability density function of a random variable X is assumed to be of the form $f(x) = cx^\alpha$, $0 \leq x \leq 1$ for some number and constant c . If $\{X_1, X_2, \dots, X_n\}$ is a random sample of size n , find the maximum likelihood estimate of α . 5
- b) Let S' be the set defined by $S' = \{z \in C : |z| = 1\}$, where C is the set of all complex numbers. Show that S' forms a commutative group under usual multiplication of complex numbers. 5
- c) Let R be the additive group of real numbers and C^* be the multiplicative group of nonzero complex numbers. If $f: R \rightarrow C^*$ is a group homomorphism defined by $f(x) = e^{2\pi ix}$ for all $x \in R$, find the kernel of f . 5
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