Faculty of Science & Technology

Fourth Semester B.Tech. (Computer Science and Engineering) CE/IT/CT (C.B.C.S.) Examination

DISCRETE MATHEMATICS AND GRAPH THEORY

Time : Three Hours] [Maximum Marks : 70

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve Question **1 OR** Question No. **2.**

(3) Solve Question **3 OR** Question No. **4.**

(4) Solve Question **5 OR** Question No. **6.**

(5) Solve Question **7 OR** Question No. **8.**

(6) Solve Question **9 OR** Question No. **10.**

(7) Illustrate your answers wherever necessry with the help of neat sketches.

(8) Use of non programmable calculator is permitted.

1. (a) By the principle of mathematical induction, show that

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 1$$ 7

(b) Let F be the set of all one-one and onto mappings from X to X, where X = {1, 2, 3}. Find all elements of F and also find inverse of each element. 7

OR

2. (a) If the relation matrices of two relations R and S are given by

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

then find R,S and show that $M_{ROS} \neq M_{SOR}$ 7

(b) Define characteristic function. Using property of characteristic function prove that :

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $(A')' = A$

3. (a) If u = {a, b, c, d}, A and B are two fuzzy sets defined on U as given below : 6

	a	b	c	d
A	0.5	0.8	0.0	0.3
B	0.2	1.0	0.1	0.7

Find :

(i) $A \cup B$

(ii) $A \cap B$ and show that $(A \cup B)' = A' \cap B'$ 7

(b) Define the following terms :

(i) α–cut

(ii) Normal fuzzy set

(iii) Height of fuzzy set.

Also find 0.4 – cut and height of fuzzy set A, where

A = {(1, 0.2), (2, 0.6), (3, 0.5), (4, 0.0), (5, 0.3), (6, 0.7), (7, 0.0), (8, 0.0), (9, 0.1), (10, 0.4)} 7

OR

4. (a) If $A = \frac{0.2}{x_1} + \frac{0.5}{x_2} + \frac{0.6}{x_3}$ and

$B = \frac{0.1}{x_1} + \frac{0.4}{x_2} + \frac{0.5}{x_3}$ then find

(i) A–B and

(ii) $A \oplus B$. 7

(b) If R and S are fuzzy relations given by

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.9 \\ 0.8 & 0.6 \end{bmatrix} \end{array}, \quad S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.8 \\ 0.7 & 0.9 \end{bmatrix} \end{array}$$

then find $R \cup S$, $R \cap S$, $\bar{R}, \bar{S}$. 7

5. (a) Show that the set A = {1, 2, 3} under multiplication modulo 4 is not a group, but set B = {1, 2, 3, 4} is a group under multiplication modulo 5. 7

(b) Prove that every field is an integral domain. 7

OR

(Contd.)

6. (a) If R is a ring then prove that for all a, b ∈ R

(i) a. 0 = 0. a = 0

(ii) a(−b) = −(ab) = (−a) b

(iii) a. (b−c) = ab−ac

(iv) (−a) (−b) = (ab) 7

(b) Prove that the set Q^+ of all positive rational numbers form an abelian group for operation $*$ defined as $a * b = \frac{ab}{2}, \forall a, b \in Q^+$. 7

7. (a) Define :

(i) Null graph

(ii) Node base

(iii) Path

(iv) Isomorphic graphs

(v) Trail

(vi) Reachable Node

(vii) Root of the tree. 6

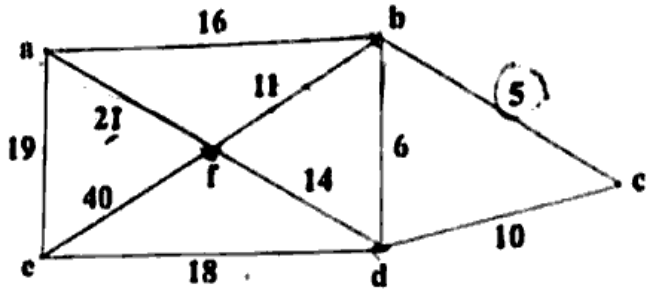
(b) Draw the diagraphs corresponding to adjacency matrices $A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$ and

$B = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$ also show that these graphs are isomorphic to each other. 6

(c) Construct a tree diagram for the following algebraic expression. Also find corresponding binary tree $[3(1-x) \div \{4+\{7-(y+2)\}\}]. \{7+(x \div y)\}$ 6

OR

8. (a) Apply Prims algorithm to construct a minimal spanning tree for the weighted graph given below :



6

(b) Draw the diagraphs corresponding to matrix :

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Find AA^T, $A^T A$, A^2 and interprete the result. 6

(c) Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10},

T = {(2,3), (2,1), (4,5), (4,6), (5,8), (6,7), (4,2), (7,9), (7,10)}

Identify the root and show that T is a rooted tree. Also find corresponding binary tree. 6

9. (a) Solve the recurrence relation

$a_n - 8\,a_{n-1} + 21\,a_{n-2} - 18\,a_{n-3} = 0$,

$a_0 = 1$, $a_1 = 1$, $a_2 = 2$. 5

(b) Find the minimum number of student in a class to be sure that four out of them are born in the same month. 5

OR

10. (a) Find the closed form of generating function for the following :

(i) 3, −3, 3, −3, 3, −3, ...

(ii) 1, 0, −1, 0, 1, 0, −1, 0, ... 5

(b) 5 men and 4 women are required to seat in a row such that the women occupy the even places. How many arrangements are possible ? 5