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MATH114 Enrol. No. ....62....

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END SEMESTER EXAMINATION : DECEMBER, 2023

APPLIED MATHEMATICS - I

Time : 3 Hrs. *Maximum Marks : 60*

Note: ***Attempt questions from all sections as directed.***

SECTION – A (24 Marks)

Attempt any **four** *questions out of* **five.**
Each question carries **06** *marks.*

✓1. Using Gauss-Elimination method: solve the following system of equations:

$$x+2y+z=2$$
$$3x+y-2z=2$$
$$4x-3y-z=3$$

2. If $y = \sin^{-1} x$, then prove that

$$(1 - x^2)y^{(n+2)} - (2n+1)xy^{(n+1)} - n^2 y^{(n)} = 0.$$

3. Using Green's theorem. Find the value of $\int_c (x^2 + xy)dx + (x^2 + y^2)dy$ where c is the square formed by the lines $y = \pm 1$, $x = \pm 1$.

4. Change the order of integration in

$$I = \int_0^{2a} \int_{\sqrt{2ax - x^2}}^{\sqrt{2ax}} f(x, y)\ dxdy.$$

5. Apply Euler's theorem on homogenous function to prove the following:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \text{Sin}2u, \text{ given that}$$

$$u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right).$$

SECTION - B

(20 Marks)

Attempt any **two** *questions out of* **three**.

Each question carries **10** *marks.*

6. (a) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$. (5)

   (b) Expand $e^x \cos y$ in integral powers of x and y up to second degree. (5)

7. (a) Find all the extreme and saddle points of the function: $f(x: y) = xy - x^2 - y^2 - 2x - 2y + 4$ (5)

   (b) Determine the rank of the following matrix:

$$\begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$ (5)

8. Apply Stoke's theorem to evaluate

$$\oint_C [(x+y)dx + (2x-z)dy + (y+z)dz]$$

Where C is the boundary of the triangle with vertices (2,0,0), (0,3,0), (0,0,6).

SECTION – C (16 Marks)

(Compulsory)

9. (a) **Evaluate:** $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz dx dy$ (8)

(b) Determine for what values of λ and μ the following equations:

$x+y+z=6, \; x+2y+3z=10, \; x+2y+\lambda z=\mu$

have

(i) no solution

(ii) a unique solution

(iii) infinite no. of solutions. (8)