

Duration: 03:00

Full Marks: 60

1 Answer All

- a Draw the graph of $f(x)=|x+5|$. 1
- b Find the curvature of $y=2x+5$. 1
- c Find the order and degree of $\sqrt{y''}+y=\sqrt[3]{y'}+4y$. 1
- d Solve $y'=-y/x$, $y(1)=1$. 1
- e Find the Wronskian of the bases x^4 , $x^4 \ln x$? 1
- f Solve $y''=y'$. 1
- g Define Linearly dependent and linearly independent with suitable examples. 1
- h Show that the vectors $(1, 1, 1)$, $(2, 3, 1)$ and $(3, 4, 2)$ are linearly dependent vectors. 1
- i Identify the type of matrix $\begin{bmatrix} 4i & 0 & i \\ 0 & i & 0 \\ i & 0 & 4i \end{bmatrix}$. 1
- j Define trace of a matrix. 1

2 Answer any Two

- a Find all the asymptotes of $x^3+y^3-3xy=0$. 2.5
- b Draw the graph of $f(x)=2+|x+1|$ 2.5
- c Find the radius of the curvature of $y^2=12x$ at $(3,6)$. 2.5
- d Find all the asymptotes of $r \log \theta = a$. 2.5

3 Answer any Two

- a Solve $y' + ky = e^{-kx}$. 2.5
- b Test for exactness and solve $(2x + e^x \sin y)dx + e^x \cos y dy = 0$ where $y(0) = \frac{\pi}{2}$ 2.5
- c Solve the linear equation $y'+y \cos x = \sin 2x$, $y(\pi)=0$. 2.5
- d Solve $y' + 1 = e^{x+y}$ 2.5

4 Answer any Two

- a Are the functions $f(x) = x|x|$ and $g(x) = x^2$ are linearly independent over the interval $[-1, 1]$? Show the work. 2.5
- b Solve $xy'' + y' = 0$. 2.5
- c Find a second-order homogeneous linear differential equation which has $\cos 2\pi x$, $\sin 2\pi x$ are solutions? 2.5

[P. T. O.]

d Solve $x^2 y'' + xy' + 9y = 0, y(1) = 2, y'(1) = 0$. 2.5

5 Answer any Two

a Verify whether the vectors $(-1, 5, 0)$, $(16, 8, -3)$ and $(-64, 56, 9)$ are LI or LD? 2.5

b Using Gauss elimination solve 2.5

$$4x - 8y + 3z = 16, \quad -x + 2y - 5z = -21, \quad 3x - 6y + z = 7$$

c Find the rank of the matrix $\begin{pmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{pmatrix}$. 2.5

d Find A^{-1} by Gauss Jordan elimination method where $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$ 2.5

6 Answer any Two

a Prove that the determinant of an orthogonal matrix is ± 1 . 2.5

b Find the eigenvalues for the unitary matrix $\begin{bmatrix} \frac{1}{2} & i\sqrt{\frac{3}{4}} \\ i\sqrt{\frac{3}{4}} & \frac{1}{2} \end{bmatrix}$. 2.5

c Show that $\begin{bmatrix} 0.96 & -0.28 \\ 0.28 & 0.96 \end{bmatrix}$ forms an orthonormal system. 2.5

d Find the spectrum of $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ 2.5

7 Answer any One

a Find all asymptotes of the curve $y^3 + 3xy^2 - x^2y - 3x^3 + y^2 - 2xy + 3x^2 + 4y + 5 = 0$ 5

b Show that $3\sqrt{3}/2$ is the least value of ρ for $y = \log x$. 5

8 Answer any One

a Solve $2 \sin y^2 dx + xy \cos y^2 dy = 0, y(2) = \sqrt{\frac{\pi}{2}}$. 5

b Solve the Bernoulli equation $xy' + 2y = 3x^3y^{4/3}$. 5

9 Answer any One

a Solve $(x^2 D^2 + 3xD + 1)y = 0, y(1) = 3, y'(1) = -4$ 5

b Solve $4x^2 y'' + 8xy' - 3y = 7x^2 - 15x^3$. 5

10 Answer any One

a Solve the given system of linear equations for all its solutions. 5

$$x_2 - 3x_3 + x_4 = 0$$

$$2x_1 - x_2 + x_3 = 0$$

$$2x_1 - 3x_2 + 4x_4 = 0$$

[P. T. O.]

- b Find the inverse of the given matrix using Gauss Jordan elimination $A = \begin{pmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{pmatrix}$ 5

11 Answer any One

- a Find all the eigenvalues and eigenvector corresponding to the largest eigenvalue of 5

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 5 & 0 & -1 \end{bmatrix}$$

- b Find a matrix P such that $P^{-1}AP$ will be a diagonal matrix with eigenvalues of A in the diagonal where 5

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$