*Name* : ..............................................................

*Roll No.* : ..............................................................

*Invigilator's Signature :* ..............................................

**CS/B.TECH(CSE/IT)(OLD)/SEM-4/M-401/2012**

**2012**

# MATHEMATICS

*Time Allotted* : 3 Hours | *Full Marks* : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP – A
### ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) Let $R$ be the relation on the set $A = \{a, b, c\}$ where $R = \{(a,a),(b,b),(a,b),(b,a)\}$. Then $R$ is

a) an equivalence relation

b) reflexive, symmetric but not transitive

c) reflexive but not symmetric and transitive

d) symmetric, transitive but not reflexive.

ii) A semigroup ( $G,*$ ) will be monoid if

a) associative law holds under ( * )

b) commutative law holds under ( * )

c) inverse element exists $\forall a \in G$

d) $G$ contains an identity element.

iii) The set of all residue class $Z_6$ contains

a) 6 elements  b) 5 elements

c) 7 elements  d) none of these.

iv) Solution of the recurrence relation $a_n = 3a_{n-1}$, $a_0 = 1$ is $a_n =$

a) $3^n$  b) $3^{n-1}$

c) $3^{n+1}$  d) none of these.

v) In a Boolean Algebra, $(a+b)' + (a+b')$ is

a) $b$  b) $a$

c) $a'$  d) $ab$.

vi) In a lattice { 1, 5, 25, 125 }, the complement of 25 is

a) 1  b) 5

c) 25  d) 125.

vii) If a graph has 4 vertices and 7 edges, then the order of Adjacency matrix is

a) 4 × 4  b) 4 × 7

c) 7 × 4  d) 7 × 7.

viii) A complete graph with $n$ vertices has

a) $(n-1)$ edges  b) $n(n-1)/2$ edges

c) $2n$ edges  d) $n(n+1)/2$ edges.

ix) The degree of any vertex of a null graph is

a) 0  b) 1

c) 2  d) none of these.

x) The hamming distance between 11010 and 10101 is

a) 2 b) 3

c) 4 d) 0.

xi) The order of Dihedral Group $D_4$ is

a) 4 b) 6

c) 8 d) 64.

xii) If the cyclic group $G$ contains 11 distinct elements, then it has

a) 2 generators b) 7 generators

c) 9 generators d) 10 generators.

xiii) Which of the following sets is closed under multiplication ?

a) $\{1, -1, 0, 2\}$ b) $\{1, i\}$

c) $\{1, \omega, \omega^2\}$ d) $\{1, \omega\}$.

xiv) A tree contains at least

a) one vertex b) two vertices

c) three vertices d) four vertices.

xv) A ring with zero divisors is called an integral domain.

a) True b) False.

**GROUP – B**

**( Short Answer Type Questions )**

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that centre of a group $G$, given by

$Z(G) = \{a \in G : ag = ga \forall g \in G\}$ is a normal subgroup of $G$.

3. Show that the ring of matrices of the form $\begin{bmatrix} 2\alpha & 0 \\ 0 & 2\beta \end{bmatrix}$, $\alpha, \beta \in Z$ contains divisors of zero. ( $Z$ = set of all integers and the operations are matrix addition and multiplication)

4. In a lattice $(L, \wedge, \vee)$ prove that $a \wedge b = a$ if and only if $a \vee b = b,\ a, b \in L$.

5. Express $E = y' + z(x' + y)$ as a full disjunctive normal form.

6. Draw the graph whose incidence matrix is

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

7. Define a planer graph. If $G$ be a connected planner graph with $n_v$ vertices, $n_e$ edges and $n_f$ faces, then show that $n_v - n_e + n_f = 2$.

## GROUP – C

### ( Long Answer Type Questions )

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Using generating function, solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $n > 1$ and $a_0 = 3$, $a_1 = 3$.

   b) Show that the set of all roots of the equation $x^4 = 1$ forms a group under multiplication.

   c) Show that the set of matrices $s = \left\{ \begin{pmatrix} \alpha & 0 \\ \beta & 0 \end{pmatrix} : \alpha, \beta \in \mathbb{R} \right\}$ is a left ideal but not a right ideal of 2 × 2 real matrices.

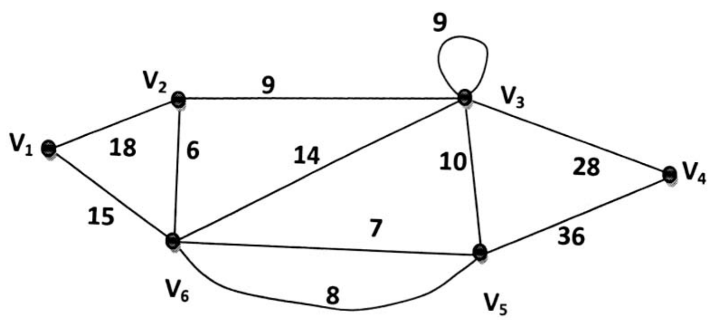
5 + 5 + 5

9. a) Show that for any two subgroups $H$ and $K$ of a group $G$, $H \cap K$ is also a subgroup of $G$.

   b) Show that the mapping $f : (\mathbb{Z}_6, +) \rightarrow (\mathbb{Z}_6, +)$ defined by $f(x) = 5x,\ x \in \mathbb{Z}_6$ is a group homomorphism. Find the ker $f$.

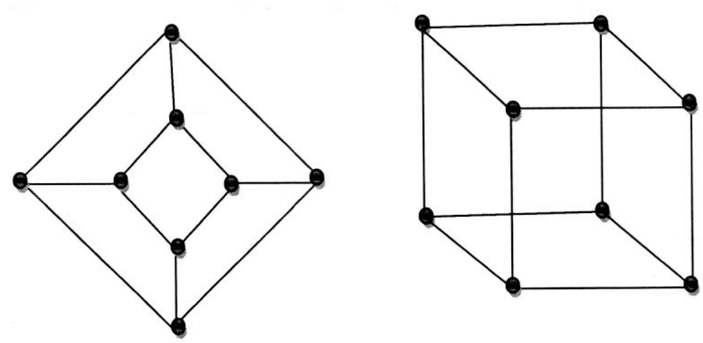
c) Prove that in a lattice $(L, \leq)$, for any $a, b, c \in L$, if $a \leq b \leq c \Rightarrow a \vee b = b \wedge c$ and $(a \wedge b) \vee (b \wedge c) = b = (a \vee b) \wedge (a \vee c)$. 5 + 5 + 5

10. a) Applying Dijkstra's algorithm, find the shortest path from $v_1$ to $v_4$ in the following graph :



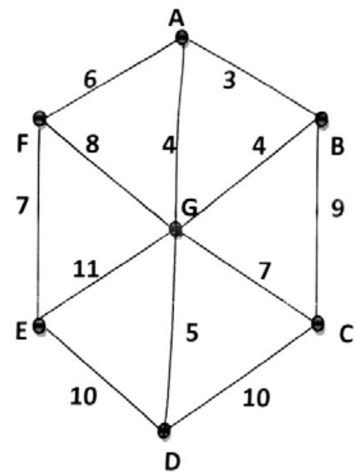

b) Prove that every cut set in a connected graph contains at least one branch of every spanning tree of graph.

c) Define Isomorphic graph. Examine whether the following two graphs are isomorphic or not.



6 + 4 + 5

11. a) Apply Prim's algorithm to find a minimal spanning tree for the following weighted graph :



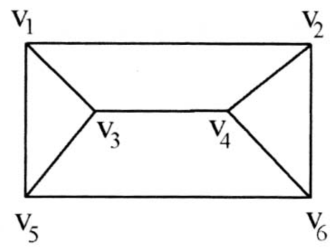

b) Show that if every element of a group is its own inverse, then it is an Abelian group.

c) Show that $f : R \rightarrow R, f(x) = 2x + 3$ is a bijective mapping. 6 + 4 + 5

12. a) Show that a ring $R$ satisfies cancellation law if and only if $R$ is without zero divisor. 5

b) A relation $\rho$ is defined on $Z$ by "$a \rho b$ if and only if $a^2 - b^2$ is divisible by 5" for $a, b \in Z$. Prove that $\rho$ is an equivalence relation on $Z$. Show that there are three distinct equivalence classes. 3 + 2

c) Draw the dual of the following graph :

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