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**CSE303**

Enrol. No. ....

[ET]

END SEMESTER EXAMINATION : APRIL-MAY, 2019

**ANALYSIS AND DESIGN OF ALGORITHMS**

*Time : 3 Hrs.*

*Maximum Marks : 70*

**Note:** *Attempt questions from all sections as directed.  
Use of Non Programmable Scientific Calculator  
is allowed.*

**SECTION – A (30 Marks)**

*Attempt any five questions out of six.*

*Each question carries 06 marks.*

1. Show that if  $f(n)$  and  $g(n)$  are monotonically increasing functions, then so are the functions  $f(n) + g(n)$  and  $f(g(n))$ , and if  $f(n)$  and  $g(n)$  are in addition nonnegative, then  $f(n) * g(n)$  is monotonically increasing.
2. Discuss recurrence relation for Tower of Hanoi (TOH) problem. Solve this recurrence to find the complexity using recursion tree.
3. (a) Use a recursion tree to give an asymptotically tight solution to the recurrence

P.T.O.

$T(n) = T(\alpha \cdot n) + T(1 - \alpha)n + cn$ ,  $0 < \alpha < 1$  and  $\alpha, c$  are constants. (3)

- (b) The recurrence  $T(n) = 7T(n/2) + n^2$  describes the running time of an algorithm A. A competing algorithm A' has running time of  $T'(n) = aT'(n/4) + n^2$ . What is the largest integer value for 'a' such that A' is asymptotically faster than A? (3)

4. Illustrate solution for 4-queen problem using backtracking. Also analyse its complexity.
5. Pan has discovered a way of multiplying  $68 \times 68$  matrices using 132,464 multiplications, a way of multiplying  $70 \times 70$  matrices using 143,640 multiplications, and a way of multiplying  $72 \times 72$  matrices using 155,424 multiplications. Which method yields the best asymptotic running time when used in a divide-and-conquer matrix-multiplication algorithm? How does it compare to Strassen's algorithm?
6. What do you mean by NP Hard and NP Complete? Give steps for NP Completeness.

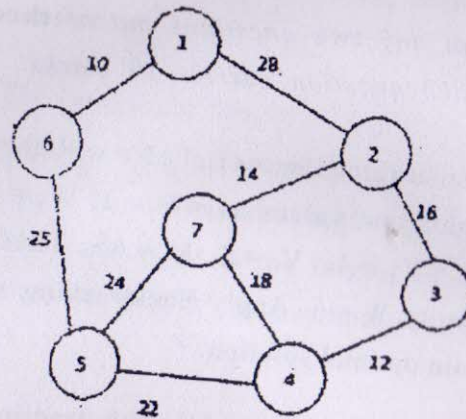
### SECTION - B (20 Marks)

Attempt any two questions out of three.

Each question carries 10 marks.

7. (a) Consider three items  $\langle I_1, I_2, I_3 \rangle$  with the respective weights and values as  $\langle W_1 = 1, W_2 = 4, W_3 = 3 \rangle$  and  $\langle V_1 = 4, V_2 = 5, V_3 = 6 \rangle$ . The knapsack capacity  $W = 6$ . Apply Backtracking approach to obtain optimal solution. (6)
- (b) State the Job-Sequencing with deadlines problem. Find an optimal sequence to the  $n=5$  Jobs where profits  $(P_1, P_2, P_3, P_4, P_5) = (20, 15, 10, 5, 1)$  and deadlines  $(d_1, d_2, d_3, d_4, d_5) = (2, 2, 1, 3, 3)$ . (4)
8. (a) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is  $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$ . (7)
- (b) Show that the running time of quick sort when array A contains distinct elements and is sorted in decreasing order. Also discuss best case running time for quick sort. (3)
9. Apply Kruskal Algorithm to the following Graph. Write their time complexity and find the minimum cost.





**SECTION - C**  
(Compulsory)

(20 Marks)

10. (a) Given two arrays A and B each with  $n$  numbers  $a_1, \dots, a_n$  and  $b_1, \dots, b_n$  respectively, the algorithm needs to output the order  $a_1, b_1, a_2, b_2, \dots, a_n, b_n$ . E.g. if  $n = 4$  and  $A = (1, 3, 5, 7)$  and  $B = (2, 4, 6, 8)$ , then the output is  $(1, 2, 3, 4) (5, 6, 7, 8)$ . The output must be in the same array as the input. You can think of the input as an array of size  $2n$  which contains A and B and your algorithm must rearrange these values as specified. The problem is allowed  $O(\log n)$  amount of temporary space. Give an  $O(n \log n)$ -time algorithm that uses  $O(\log n)$  temporary space. (8)

- (b) Construct a problem for the following pseudo code and derive the time complexity. Also show the real life application of this problems.

```
function check_all_permutations(adj[], n) for i = 0 to n
    p[i]=i
    while next permutation is possible
        valid=true for i = 0 to n-1 if adj[p[i]][p[i+1]]==false
            valid = false break
        if valid == true return true
    p = get_next_permutation(p) return false
```

(4)

- (c) Imagine you have a collection of  $N$  wines placed next to each other on a shelf. For simplicity, let's number the wines front left to right as they are standing on the shelf with integers front 1 to  $N$ , respectively. The price of the  $i^{\text{th}}$  wine is  $p_i$ . (prices of different wines can be different).

Because the wines get better every year, supposing today is the year 1, on year  $y$  the price of the  $i^{\text{th}}$  wine will be  $y \cdot p_i$ , i.e.  $y$ -times the value that current year.

You want to sell all the wines you have, but you want to sell exactly one wine per year, starting on this year.

One more constraint - on each year you are allowed to sell only either the leftmost or the rightmost wine on the shelf and you are not allowed to reorder the wines on the shelf (i.e. they must stay in the same order as they are in the beginning).

- (i) You want to find out, what is the maximum profit you can get, if you sell the wines in optimal order?" Write the name of approach/ technique also.
- (ii) If the prices of the wines are (in the order as they are placed on the shelf, from left to right) :  $p_1=1, p_2=4, p_3=2, p_4=3$ . Provide the optimal solution. (8)

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Please read Q10 (b) as under :-

10 (b) Construct a problem for the following pseudo code and derive the time complexity. Also show the real life application of this problems.

```
function check_all_permutations(adj[[]], n)
  for i = 0 to n
    p[i] = i
    while next permutation is possible
      valid = true
      for i = 0 to n-1
        if adj[p[i]][p[i+1]] == false
          valid = false
          break
      if valid == true
        return true
      p = get_next_permutation(p)
  return false
```