

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS / B.TECH(NEW)APM / TT / AUE / CHE / ME /  
PE / CE / SEM-4 / M-402 / 2012**

**2012**

**MATHEMATICS-III**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

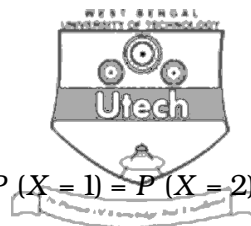
*Candidates are required to give their answers in their own words  
as far as practicable.*

**GROUP – A**

**( Short Answer Type Questions )**

Answer any *ten* questions :  $10 \times 2 = 20$

1. a) If  $f(x) = x + x^2$ ,  $-\pi \leq x \leq \pi$  be represented in a Fourier series as  $a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ , then find the value of  $a_0$ .
- b) State Fourier Integral theorem.
- c) If  $F(s)$  is the Fourier transform of  $f(x)$ , then find the Fourier transform of  $f(ax)$ .
- d) Find the residue of  $f(z) = \frac{2 + 3 \sin \pi z}{z(z-1)^2}$  at  $z = 1$ .
- e) Find the value of  $\int_C \frac{z}{z-1} dz$  where  $C$  is the curve defined by  $|z| = \frac{1}{2}$ .
- f) Find the poles of the function  $f(z) = z^2 / \left\{ (z-1)^2 (z+2) \right\}$ .



- g) If a Poisson variate  $X$  is such that  $P(X=1) = P(X=2)$ , Find  $P(X=4)$ .
- h) For any two events  $A$  and  $B$  (may not be mutually exclusive), prove that  
 $P(A+B) = P(A) + P(B) - P(AB)$ .
- i) Find the value of  $p$  so that the function  $(2x - x^2 + py^2)$  is harmonic.
- j) Find the value of  $J_1(x)$ .
- k) Prove that  $(2n+1)x P_n = (n+1)P_{n+1} + nP_{n-1}$
- l) Find the ordinary and singular points of the differential equation  $x^2(1+x)^2 \frac{d^2y}{dx^2} + (x^2-1) \frac{dy}{dx} + 2xy = 0$ .
- m) Find the bilinear transformation which maps  $z = 0, 1, \infty$  onto  $\omega = -1, -i, 1$  respectively.
- n) Find the value of  $\int_{-1}^1 P_0(x) dx$  where  $P_n(x)$  is a Legendre's polynomial of degree  $n$ .
- o) Write down the Bessel's equation of order 2.

### GROUP - B

Answer any *five* questions taking at least *one* question from each Modules. :  $5 \times 10 = 50$

### Module I : Fourier Series and Fourier Transform

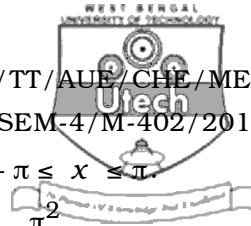
2. a) Expand  $f(x) = x$ ,  $-\pi \leq x \leq \pi$  in Fourier series. Hence deduce that  $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \dots$  4 + 1

- b) Find the Fourier sine transform of  $f(x)$ , where

$$f(x) = \begin{cases} 1, & 0 < x \leq \pi \\ 0, & x > \pi \end{cases}$$

and hence evaluate the integral  $\int_0^\infty \left( \frac{1 - \cos p\pi}{p} \right) \sin px \, dp$ .

3 + 2



3. a) Find the Fourier series of  $f(x) = x^2$ ,  $-\pi \leq x \leq \pi$ .  
Hence prove that  $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$ . 4 + 1
- b) Find the function whose Fourier cosine transform is  $\frac{\sin as}{s}$ . 5

**Module II : Calculus of Complex variable**

4. a) Using Cauchy's Residue theorem, prove that  $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta} = \frac{2\pi}{\sqrt{3}}$ . 5
- b) Evaluate  $\oint_C \frac{e^{2z}}{(z+1)^4} dz$   
where  $C$  is the circle  $|z| = 3$ . 5
5. a) If  $u = x^2 - y^2$  and  $v = -\frac{y}{x^2 + y^2}$ , then prove that both  $u$  and  $v$  are harmonic. 5
- b) Find the zeros and their orders of the function  $f(z) = \frac{z^5 - 1}{z^2 + 5}$ . 5
6. a) Expand  $f(z) = \frac{z}{(z-1)(z-2)}$  in Laurent's series for  $1 < |z| < 2$ . 5
- b) Show that the function  $f(z) = \sqrt{|xy|}$  is not analytic at the origin, although the Cauchy-Riemann conditions are satisfied at that point. 5

**Module III : Probability**

7. a) Two urns contain respectively 5 white, 7 black balls and 4 white, 2 black balls. One of the urns is selected by the toss of a fair coin and then 2 balls are drawn without replacement from the selected urn. If both balls are white, what is the probability that the first urn is selected? 5



- b)  $X$  is a continuous random variable having probability

$$\text{density function } f(x) = \begin{cases} \frac{4x}{5}, & 0 < x \leq 1 \\ \frac{2(3-x)}{5}, & 1 < x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the mean value of  $x$  and the distribution function.

3 + 2

8. a) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or by 8 ?  
b) A random variable  $X$  has the following probability function :

$X$	0	1	2	3	4	5	6	7
$P(x)$	0	$K$	$2K$	$2K$	$3K$	$K^2$	$2K^2$	$7K^2 + K$

Obtain the value of  $K$  and estimate  $P(X < 6)$  and  $P(0 < X < 5)$ .

5

#### Module IV : PDE and Series solution of ODE.

9. Use Laplace transform to solve the one dimensional wave

$$\text{equation } \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (x > 0, t > 0)$$

$$\text{where } u(x, 0) = 0, \quad \frac{\partial u}{\partial t}(x, 0) = 0, \quad x > 0$$

$$\text{and } u(0, t) = F(t), \quad u(\infty, t) = 0, \quad t \geq 0.$$

10

10. Solve the one dimensional heat equation  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$  by the method of separation of variables.

10

11. a) Find the series solution of ODE

$$\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + x^2 y = 0 \quad \text{about } x = 0.$$

5

$$\text{b) Show that } \int_{-1}^1 x^2 p_{n-1}(x) p_{n+1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$

5