Module 1

Q1.

If $f(x, y) = x^3y - xy^3$, find $\left[\dfrac{1}{\dfrac{\partial f}{\partial x}} + \dfrac{1}{\dfrac{\partial f}{\partial y}} \right]_{\substack{x=1 \\ y=2}}$ **Ans.** $-\dfrac{13}{22}$

Q 2.

If $x = r \cos \theta$, $y = r \sin \theta$, prove that

(i) $\dfrac{\partial r}{\partial x} = \dfrac{\partial x}{\partial r}$, $r \cdot \dfrac{\partial \theta}{\partial x} = \dfrac{1}{r} \cdot \dfrac{\partial x}{\partial \theta}$ (ii) $\dfrac{\partial^2 \theta}{\partial x^2} + \dfrac{\partial^2 \theta}{\partial y^2} = 0$

Q 3

Find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$.

4. $f(x, y) = \begin{cases} \dfrac{x\,y}{x^2 + y^2}, & \text{when } x \neq 0,\ y \neq 0 \\ 0, & \text{when } x = 0,\ y = 0 \end{cases}$

at origin.   **Ans.** Not continuous at origin.

5. $f(x, y) = \begin{cases} x^3 + y^3, & \text{when } x \neq 0,\ y \neq 0 \\ 0, & \text{when } x = 0,\ y = 0 \end{cases}$

at origin.   **Ans.** Continuous at origin.

Q6.

Show that the function

$$f(x,y) = x^3 + y^3 - 63(x + y) + 12xy$$

is maximum at $(-7, -7)$ and minimum at $(3, 3)$.