	<u>Uflech</u>
Name:	
Roll No. :	In the property of the State of
Invigilator's Signature :	

CS / B.TECH (ME/PE/AUE) / SEM-3 / M-303 / 2010-11 2010-11

MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

 ${\it The figures in the margin indicate full marks.}$

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

$$10 \times 1 = 10$$

i) If f(z) = u(x, y) - iv(x, y) is analytic, then f'(z) equals to

a)
$$\frac{\partial u}{\partial x} - i \frac{\partial u}{\partial y}$$

b)
$$\frac{\partial u}{\partial x} + i \frac{\partial v}{\partial y}$$

c)
$$\frac{\partial v}{\partial y} - i \frac{\partial v}{\partial x}$$

d) none of these

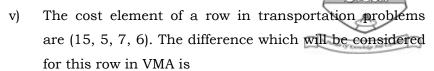
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- ii) Consider the differential equation xy'' + 2y' + xy = 0Then x = 0 is
 - a) an ordinary point
 - b) singular point but not a regular singular point
 - c) a regular singular point
 - d) none of these.
- iii) Give an LPP to maximize $Z=-5x_2$ subject to $x_1+x_2\leq 0,\, 0\cdot 5,\quad x_1\times 5x_2\geq 0 \ \ {\rm and} \ \ x_1\geq 0\,,\quad x_2\geq 0\,, \ {\rm using}$ graphical method we have
 - a) no feasible solution
 - b) unbounded solution
 - c) unique optimum solution
 - d) multiple optimum solution.
- iv) Which of the following is not correct?
 - a) It is not necessary for the aggregate demand to equal to be the aggregate supply in a transportation problem
 - b) An unbalanced transportation problem must be converted into a balanced problem before solving it
 - c) The cost element in a dummy row / column shall always be taken equal to zero
 - d) It is possible that in some cases both, dummy source and dummy destination may be required to convert an unbalanced transportation problem into a balanced one.

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o)	15
a)	15

b) 2

c) 1

d) 9.

vi) In an assignment problem involving 5 workers and 5 jobs, total number of assignment possible are

a) 5!

b) 10

c) 5

d) 25.

vii) The minimum number of lines covering all zeros in a reduced cost matrix of order n be

a) at most n

b) at least n

c) n-1

d) n + 1.

viii) The value of the integral $\oint C \frac{2z}{z-4}$ where C: |z-1|=2 is

a) $2\pi i$

b) πi

c) $4\pi i$

d) 0.

ix) The residue of $\frac{z^2}{z^2+3^2}$ at z=3i is

a) $-i\frac{3}{2}$

b) $i\frac{3}{2}$

c) 3

d) none of these.

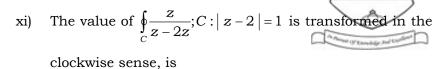
x) The function $f(z) = \overline{z}$ is

a) continuous at z = 0

b) differentiable at z = 0

c) analytic at z = 0

d) continuous at $z \neq 0$.



 πi a)

 $2\pi i$

 $-\pi i$ c)

d) $2\pi i$.

xii) The function
$$f(z) = \frac{e^{z^2}}{z^4}$$
 at $z = 0$ has

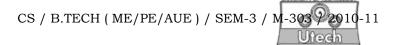
- a) an essential singularity
- b) a pole of order 4
- a simple pole c)
- d) non-singularity.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

- If f(z) is a analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2.$
- 3. Evaluate $\int_{C} \frac{e^{2z}}{(z+1)^4} dz$, C: |z| = 3.



- 4. Find the Tayler's series expansion of $\frac{1}{(z-1)(z-3)}$ when 0 < |z-1| < 2.
- 5. Prove that $\frac{d}{dx} \{x^{-n} J_n(x)\} = -x^{-n} J_{n+1}(x)$
- 6. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial t^2}$ given that u(0,t) = 0 $u(x,0) = f(x) \text{ and } \frac{\partial u(x,0)}{\partial t} = 0 \text{ where } 0 < x < 1.$

GROUP - C

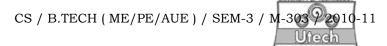
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Obtain series solution of the equation

$$(1+x^2)y'' + xy' - y = 0$$
.

- b) Express $f(x) = x^4 + 3x^3 x^2 + 5x 2$ in terms of Legendre's polynomials. 8 + 7
- 8. a) Solve: (mz ny) p + (nx ly) q = (ly mx)
 - b) Solve: $(D^2 DD' + D' 1)z = \cos(x + 2y)$



- c) Use the method of separation of variables to solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u \text{ where } u(x,0) = 6e^{-3x}.$ 5 + 5 + 5
- 9. a) If $u-v=(x-y)(x^2+4xy+y^2)$ and f(z)=u+iv is an analytic function of z=x+iy, find f(z) in terms of z.

b) Prove that
$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$$
 8 + 7

- 10. a) Find the optimal solution of the Transportation problem.
 - b) Find the alternative optimal solution (if any):

	D_1	D_2	D_3	D_4	Capacities
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O ₃	4	2	5	9	20
Demands	20	40	30	10	

8 + 7

11. a) Solve by Big M – method

$$Max Z = x_1 + 2x_2 + 3x_3 - x_4$$

s.t.

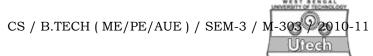
$$x_1 + 2x_2 + 3x_3 = 15$$

$$2x_1 + x_2 + 5x_3 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 = 10$$

$$x_1, x_2, x_3, x_4 \ge 0$$

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b) The owner of a small machine shop has four machines available to assign jobs for the day. Five jobs are offered with expected profit for each machine on each machine on each job which are as follows:

		Jobs					
		A	В	С	D	E	
	1	62	72	50	111	82	
Machines	2	71	84	61	73	59	
	3	87	92	111	71	81	
	4	48	64	87	77	80	

Find by using the assignment method, the assignment of machines to the job that will result in a maximum profit which job should be declined.

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