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VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES

FALL Semester 2023-2024
Continuous Assessment Test – I

Programme Name & Branch : B.Tech.,
Slot: C1+TC1+TCC1
Course Name & code: Complex Variables and Linear Algebra & BMAT201L
Exam Duration: 90 Min. **Maximum Marks: 50**

Answer all questions (5X10=50)

1. Show, by considering the function $f(z)$ defined by $f(z)=\dfrac{2xy(x+iy)}{x^2+y^2}$ for $z\neq 0$ and $f(0)=0$, that the C-R equations are not the sufficient conditions for a function to be analytic.

2. Show that $v(x,y)=x^3-3x^2y+2x+1+y^3-3xy^2$ is a harmonic function and obtain an analytic function for which $v(x,y)$ is the imaginary part. Also find the conjugate harmonic of $v(x,y)$.

3. Find the Bilinear transformation which maps $z_1=-2i,\ z_2=i,\ z_3=\infty$ onto $w_1=0,\ w_2=-3,\ w_3=\dfrac{1}{3}$ respectively. Also find

   i) the invariant points of the transformation.

   ii) the image of $|z|<1$ in the w-plane.

4. Find the Laurent's series for the function $f(z)=\dfrac{1}{(z+1)(z+2)^2}$ in the following regions:

   i) $|z-1|<2$ ii) $2<|z-1|<3$

5. a) Find the location and nature of singularity of $f(z)=\dfrac{1-e^z}{z^3}$.

   b) Using Cauchy integral formula, evaluate $\displaystyle\int_C \frac{z+1}{z^3-2z^2}dz$, where C is a circle $|z-2-i|=2$.

(4M+6M)

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