Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH (CT-OLD)/SEM-3/M (CT)-301/2011-12

2011

APPLIED MATHEMATICS

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) If $f(z) = \dfrac{\sin z}{z^3}$ then $z = 0$ is a pole of order 2.

a) True   b) False.

ii) If $f(z) = \dfrac{z+1}{z^4 - 2z^3}$, then $z = 0$ is a pole of order

a) 3   b) 2

c) 1   d) 4.

iii) Residue of $f(z) = \dfrac{2 + 3\sin \pi z}{z(z-1)^2}$ at $z = 0$ is

a) 1   b) 2

c) 3   d) *i*.

iv) cos nx is a periodic function where fundamental period is

a) 2π b) $n\pi$

c) $\dfrac{2\pi}{n}$ d) $2n\pi$.

v) Value of the integral $\int_{-\pi}^{\pi} \sin(mx) \cos nx \, dx$ is

(where m, n are unequal positive numbers)

a) π b) 2π

c) 0 d) none of these.

vi) The inverse Fourier Transform of

$F(s) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} \, dx$ is given by

a) $\int_{-\infty}^{\infty} F(s) e^{-isx} \, ds$

b) $\dfrac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-isx} \, ds$

c) $\dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} \, ds$

d) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{isx} \, ds.$

vii) The residue of $f(z) = \frac{z+1}{z^2 - 2z}$ at the pole $z = 0$ is

a) $-\frac{1}{2}$ b) 0

c) $\frac{1}{2}$ d) $\frac{3}{2}$.

viii) The number of poles of

$f(z) = \frac{z}{(z-1)(z-2)(z-3)}$ inside the circle

$| z - 2 | = 2$ is

a) 3 b) 1

c) 2 d) 0.

ix) The value of the integral $\oint_C \frac{dz}{z-4}$ where $C : | z | = 1$

is

a) $2\pi i$ b) $-2\pi i$

c) 0 d) none of these.

x) The complete solution of $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ is

a) $z = ax + (1 + a) y + c$

b) $z = ax + (1 - a) y + 2$

c) $z = x + (1 + a) y + c$

d) $z = ax + (1 - a) y + c$.

xi) If the events A and B are such that A and B are independent, then $P (B / A)$ equals to

a) $P (B)$ b) $P (A)$

c) 1 d) 0.

xii) Value of the expectation $E \left\{ X + E (X) \right\}$ is

a) 1 b) 0

c) mean of X d) none of these.

xiii) Probability of obtaining 3 heads with a toss of 4 coins is

a) $\frac{1}{4}$ b) $\frac{3}{8}$

c) $\frac{5}{16}$ d) $\frac{3}{16}$.

xiv) 3 dice and 3 coins are rolled out. The number of elementary events in the sample space is

a) $6^3 \infty 2^3$ b) $3 \infty 2$

c) 2^2 d) $6^3 \infty 2^2$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Evaluate the limit : $\lim_{z \varnothing i} \left(\frac{iz^4 - 1}{z - i} \right)$.

3. State Cauchy's integral formula and use it to evaluate $\int_C \frac{e^{z+1}}{z^2+4}\,dz$ where C is the circle $|z-i| = 2$.

4. Find the value of $10^{1/2}$ correct up to 4 significant figures using Newton-Raphson method.

5. Using Fourth order Runge-Kutta method, solve

$$\frac{dy}{dx} = x^2 + y^2 \,, \; y(0) = 1$$

at $x = 0{\cdot}2$ using a step length $h = 0{\cdot}1$.

6. Define Fourier cosine transform. Solve the integral equation

$$\int_0^\infty f(x)\cos\lambda x\,dx = e^{-\lambda} \,.$$

7. Find the complete solution of the following *p.d.e.* :

$p + q = x$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions.

3 × 15 = 45

8. a) Obtain the Fourier series to represent x^2 in $-\pi \le x \le \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{6} \,.$$

b) Using Cauchy's Residue theorem, prove that

$$\oint_C \frac{z \cos z}{\left(z - \frac{\pi}{2} \right)^3} \, dz = -2\pi i. \qquad 8 + 7$$

9. a) Use Laurent's series to find the residue of $\frac{e^{2z}}{(z-1)^2}$ at $z = 1$.

b) Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's series about $z = 1$ and determine the region of convergence. 8 + 7

10. a) Prove that $\int_0^{+\infty} \frac{dx}{x^2+1} = \frac{\pi}{2}$, by using Cauchy's residue theorem.

b) Find the Fourier series of the function

$$f(x) = \begin{cases} \pi + 2x & \text{if } -\pi < x < 0 \\ \pi - 2x & \text{if } 0 \le x < \pi \end{cases}$$

Hence, deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots \infty = \frac{\pi^2}{8}. \qquad 7 + 8$$

11. a) If $\mathcal{F}\{f(x)\} = F(s)$ be the Fourier transform of $f(x)$, show that

$$\mathcal{F}\{f(x+a)\} = e^{-ias} F(s).$$

b) Find out the Fourier transform of $K.e^{-x^2/2}$, K is a constant.

c) Show that $P\left(\overline{A}\ /\ B\right) = 1 - P(A\ /\ B)$, where A and B are any two events. 5 + 6 + 4

12. a) The probability that a teacher will give a surprise test during any class of a particular day is $\frac{1}{5}$. If a student is absent, on two days, what is the probability that he will miss at least one test ?

b) If $f(x) = \frac{1}{4} - Kx, \quad 0 \le x \le 4$

$= 0, \qquad$ otherwise

is the *p.d.f.* of a random variable X,

determine —

i) the value of K and

ii) $P(\,|X - 2| < 0{\cdot}5\,)$.

Also find out the mean of X. 7 + 8