VIT
Vellore Institute of Technology

Final Assessment Test – November 2019

Course: ECE2005 - Probability Theory and Random Processes

Class NBR(s): 0932 / 0939 Slot: C1+TC1

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. a) A random variable X has a probability density [5]

$$f_X(x) = \begin{cases} \left(\frac{3}{32}\right)(-x^2+8x-12), & 2 \le x \le 6 \\ 0, & elsewhere \end{cases}$$

SEARCH VIT QUESTION PAPERS ON TELEGRAM TO JOIN

Find the following moments:

(i) m_0 (ii) m_1 (iii) m_2 and (d) μ_2

b) Find the marginal densities of X and Y given the joint density function [5]

$$f_{X,Y}(x,y) = 2u(x)u(y)exp\left[-\left(4y+\frac{x}{2}\right)\right]$$

2. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn, Y denotes the number of red balls drawn.

a) Find the marginal distribution functions of X & Y

b) Find $P[Y \le 2 / X \le 1]$

c) $P[X+Y \le 3]$

3. Gaussian random variables X_1 and X_2 for which $\overline{X_1} = 2, \sigma_{X_1}^2 = 9, \overline{X_2} = -1, \sigma_{X_2}^2 = 4$ and $C_{X_1X_2} = -3$ are transformed to new random variables Y_1 and Y_2 according to

$Y_1 = -X_1 + X_2$

$Y_2 = -2X_1 - 3X_2$

Find $\overline{X_1^2}, \overline{X_2^2}, \rho_{X_1X_2}, \sigma_{Y_1}^2, \sigma_{Y_2}^2$ and $C_{Y_1Y_2}$.

4. Two Gaussian random variables X_1 and X_2 are defined by the mean and covariance matrices $[\bar{X}] = \begin{bmatrix}1\\2\end{bmatrix}, C_X = \begin{bmatrix}3 & -1\\-1 & 5\end{bmatrix}$ respectively. Two new random variables Y_1 and Y_2 are formed using the transformation

$$[T] = \begin{bmatrix}1 & \frac{3}{4}\\ \frac{3}{4} & 1\end{bmatrix}$$

Find the probability density function f(Y)

5. If $X(t)$ is a stationary random process having a mean value $E[X(t)] = 3$ and autocorrelation function $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$, find

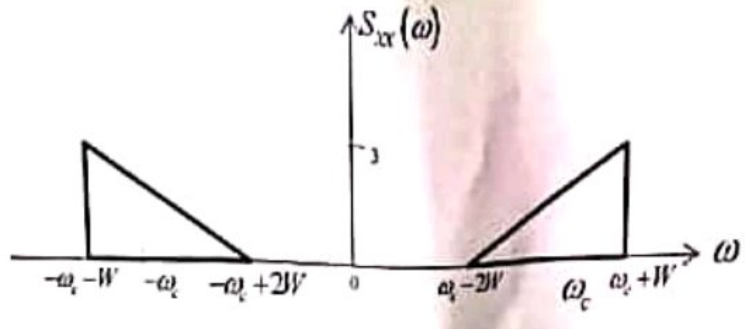
a) The mean value and

b) The variance of the random variable

$$Y = \int_0^2 X(t)\,dt$$

6. The sample function of a random process is defined by $Z(t) = X(t)\cos(\omega_c t) - Y(t)\sin(\omega_c t)$ in which $X(t)$ and $Y(t)$ are zero-mean, jointly wide-sense stationary. In terms of auto correlation functions $R_X(\tau)$ & $R_Y(\tau)$ and cross-correlation functions $R_{XY}(\tau)$ & $R_{YX}(\tau)$, determine sufficient conditions for $Z(t)$ to be wide-sense stationary.

7. A band limited random process $X(t)$ has the spectrum given in figure. Assume $\omega_c = 100$ rad/sec, $W = 10$ rad/sec



a) Compute the mean frequency of X(t).

b) Also compute its rms bandwidth.

8. A band limited random process $N(t)$ has the power density spectrum

$$S_{NN}(\omega) = \begin{cases} P\cos[\Pi(\omega - \omega_0)/W] & ; \frac{-W}{2} \le \omega - \omega_0 \le \frac{W}{2} \\ P\cos[\Pi(\omega + \omega_0)/W] & ; \frac{W}{2} \le \omega + \omega_0 \le \frac{W}{2} \\ 0 & ; \text{elsewhere} \end{cases}$$

Where P, W and $\omega_0 > W$ are real positive constants?

a) Find the power in $N(t)$.

b) Find the power spectrum $S_{XY}(\omega)$ of $X(t)$ when $N(t)$ is represented by

$$N(t) = X(t)\cos(\omega_0 t) - Y(t)\sin(\omega_0 t)$$

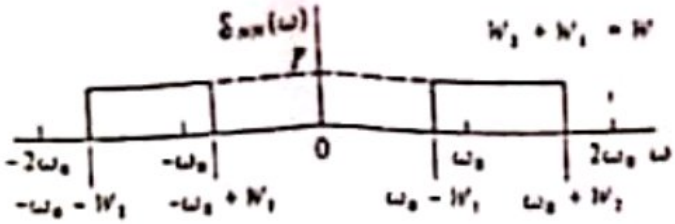
9. A stationary random signal X(t) has an auto correlation function $R_{XX}(\tau) = 10\delta(\tau)$. It is added to white noise [independent of X(t)] for which the power spectrum of noise $\frac{N_0}{2} = 10^{-3}$ and the sum is applied to a filter having a transfer function $H(\omega) = \frac{2}{(1+j\omega)}$

a) Find the signal component of the output power spectrum and the average power in the output signal

b) Find the power spectrum and average power in the output noise.

c) What is the ratio of the output signal's power to the output average noise power?

10. Let two random processes X(t) and Y(t) be both being zero mean WSS processes also have equal powers, same auto correlation function and same power spectrum. Consider the Narrow band process N(t) having the power density spectrum shown in Figure.



Let N(t) be represented as $N(t) = X(t)\cos(\omega_0 t) - Y(t)\sin(\omega_0 t)$

Using the given properties of the X(t) and Y(t), find and plot $S_{XX}(\omega)$, $S_{YY}(\omega)$ and $R_{XY}(\tau)$

11. The sum of a signal

$$x(t) = \begin{cases} 0 & , t < -3 \\ 6 + 2t & , -3 < t < 5 \\ 0 & , 5 < t \end{cases}$$

and white noise for which $\frac{N_0}{2} = 0.1 \ W/HZ$ is applied to a matched filter

a) What is the smallest value of t_0 required for the filter to be causal?

b) For the value of t_0 found in (a), sketch the impulse response of the matched filter

c) Find the maximum output signal-to-noise ratio it provides.

12. a) In an amplifier the first stage, in a cascade of 5 stages has $T_{e1} = 75K$ and $G_1 = 0.5$. Each succeeding stage has an effective input noise temperature and an available power gain that are each 1.75 times that of the stage preceding it. What is the cascade's effective input noise temperature?

b) Compare the analog and digital communications based on the noise performance.