Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.TECH(OLD) / ME / PE / AUE / SEM-3 / M-303 / 2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be supplied by the institution on demand.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) $J_{-\frac{1}{2}}$ is given by

   a) $\sqrt{\frac{2\pi}{x}} \sin x$ b) $\sqrt{\frac{2\pi}{x}} \cos x$

   c) $\sqrt{\frac{2}{\pi x}} \cos x$ d) $\sqrt{\frac{2}{\pi x}} \sin x$

   ii) The equation $u_{xx} + u_{yy} = 0$ is

   a) parabolic b) hyperbolic

   c) elliptic d) none of these.

iii) The function $f(z)=\dfrac{1}{(z+1)^2}$

a) is analytic

b) has a pole of order 2 at $z = -1$

c) has removable singularity at $z = -1$

d) has essential singularity at $z = 0$.

iv) The order and degree of the the P.D.E.

$$\frac{\partial^2 z}{\partial x \partial y}+\left(\frac{\partial z}{\partial x}\right)^2=0$$

are

a) 2,2 b) 2,1

c) 1,2 d) none of these.

v) The residue of $\dfrac{z^2}{z^2+a^2}$ at $z = ia$ is

a) $-\frac{1}{2}(ia)$ b) $\frac{1}{2}(ia)$

c) a d) ia.

vi) The number of initial basic feasible solution in a transportation problem is

a) at most $(m + n - 1)$ b) at least $(m + n - 1)$

c) equal to $(m + n - 1)$ d) none of these.

vii) Given a system of m simultaneous linear equations in n unknowns ($m < n$). Then there will be

a) n basic variables b) m basic variables

c) $(n - m)$ basic variables d) $(n + m)$ basic variables.

viii) If P denotes Legendre polynomial, then $P_0(x) =$

a) x b) x^2

c) 1 d) 2.

ix) The minimum number of lines covering all Zeros in a reduced cost matrix of order n can be

a) at most n b) at least n

c) $n - 1$ d) $(n + 1)$

x) The value of $\lim\limits_{x \to 0} \dfrac{xy}{x^2 + 2y^2}$ is

a) 0 b) 1/2

c) 1 d) none of these.

xi) In an Assignment problem involving 5 workers and 5 Jobs, total number of assignment possible are

a) 5! b) 10

c) 5 d) 25.

xii) Consider the differential equation

$xy'' + 2y' + xy = 0$. Then $x = 0$ is

a) an ordinary point

b) singular point but not a regular singular point

c) a regular singular point

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If $f(z)$ is a analytic function of z , prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2=4\left|f'(z)\right|^2$$

3. Solve the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial t^2}$ given that $u(0, t) = 0$

$u(x, 0) = f(x)$ and $\frac{\partial u(x,0)}{\partial t}=0$, where $0 < x < 1$.

4. Show that transportation problem can be expressed as an LPP.

5. Obtain the general solution of the partial differential equation

$$p \tan x + q \tan y = \tan z ,$$

where $p=\frac{\partial z}{\partial x}, \quad q=\frac{\partial z}{\partial y}$

6. Evaluate $\int_0^{1+i}(x^2+iy)\, dz$ along $y = x^2$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Solve the following problem by simplex method :

Max $Z = 20x_1 + 12x_2 + 8x_3$

subject to $4x_1 + 4x_2 + 4x_3 \leq 1200$

$3x_1 + 4x_2 + 3x_3 \leq 900$

$2x_1 + x_2 + x_3 \leq 400$

$\forall x_2 \geq 0 \quad i = 1,2,3$ 8

b) The manager of an oil refinery must decide on the optimum mix of 2 possible blending processes of which the inputs and production runs are follows :

	Input		Output	
Process	Crude A	Crude B	Gasoline X	Gasoline Y
1	6	4	6	9
2	5	6	5	5

The maximum amounts available of crudes *A* and *B* are 250 units and 200 units respectively. Market demand shows that at least 150 units of Gasoline *X* and 130 units of Gasoline *Y* must be produced. The profits per production run from processes 1 and 2 are Rs. 4 and Rs. 5 respectively. Write the mathematical form of this problem for maximizing the profit. 7

8. a) Prove that $J_2'(u) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$, where $J_n(x)$ is the Bessel function of first kind. 7

b) Prove that $\int_{-1}^{+1} [P_n(x)]^2 \, dx = \frac{2}{2n+1}$

where $P_n(x)$ denotes the Legendre's function. 8

9. a) Find the bi-linear transformation which maps the points $z = 2, i, -2$ into $w = 1, i, -1$ respectively. 5

b) Expand $l_n(1+z)$ in power of $1/z$ and indicate the region of convergence. 5

c) Evaluate $\oint_C \bar{z} \, dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by the straight line joining $z = 0$ and $z = 4 + 2i$. 5

10. a) Determine all basic feasible solutions of the set of equations :

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

Find the order of degeneracy. 8

b) Show graphically the maximum, minimum values of the objective function for the following are same :

$Z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \leq 6$

$2x_1 + 3x_2 \geq 3$

$0 \leq x_1 \leq 3, \; 0 \leq x_2 \leq 3.$ 7

11. a) Use residue theorem to evaluate

$$\int_0^{2\pi} \frac{1}{5-4\sin\theta}\, d\theta$$ 8

b) Find the complete integral of the partial differential equation $p^2 q(x^2+y^2) = p^2 + q$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

$z = z\,(x, y)$ 7