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Paper Code : ME-605B

FINITE ELEMENT METHODS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

Choose the correct alternatives for any *ten* of the **following** : $10 \times 1 = 10$

i) The Galerkin approximation seeks to

a) Obtain an exact solution of the strong form

b) Obtain an exact solution of an initial value problem

c) Obtain an approximate solution of the weak form in a finite dimensional subspace of the solution space

d) None of these.

ii) Compatibility requirement of a finite element method formulation requires

a) The displacements at shared nodes of adjacent elements are equal

b) The elemental forces and external forces applied at the system nodes are equal

c) The system satisfies he boundary conditions

d) None of these.

iii) Boundary value problems are sometimes called

a) Field problem b) Universal problem

c) Critical problem d) none of these.

iv) $u = a + bx + cy$ is the deformation field in case of

a) constant strain field

b) linearly varying strain field

c) parabolic variation of strain field

d) cubic variation of strain field.

v) The field variables are the

a) independent variables

b) neither dependent nor independent variables

c) dependent variables

d) none of these.

vi) Finite element can be applied to the problems of

a) Solid mechanics b) Fluid mechanics

c) Thermal science d) All of these.

vii) Stiffness matrix is

a) symmetrical about top left to bottom right diagonal

b) non-symmetric

c) symmetrical about top right to bottom left diagonal

d) none of these.

viii) One of the essential qualities of shape functions is

a) smoothness

b) piecewise continuity

c) non-zero over the entire domain

d) none of these.

ix) Six nodded triangular element is called

a) Constant Strain Triangle

b) Linear Strain Triangle

c) Quadratic Strain Triangle

d) None of these.

x) For a beam element, if the displacement polynomial be $a + bx + cx^2 + dx^3$, the slope polynomial would be of degree

a) one b) two

c) three d) four.

xi) Elements that help in reducing a 3D problem to a 2D in a finite element formulation is called

a) CST element b) LST element

c) Line element d) Axisymmetric.

xii) A two-point Gaussian quadrature method exactly evaluates a

a) 1st degree polynomial

b) 2nd degree polynomial

c) 3rd degree polynomial

d) 4th degree polynomial.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Derive the shape function for a Linear Strain Triangle element.

3. Vertices of a CST element are (1, 1), (5, 1) and (2, 6). Its shape functions are N_1, N_2 and N_3 at a point $P(x, y)$, within the CST element. It has been found that $N_2 - N_1 = 0 \cdot 10$ and $N_2 - N_3 = 0 \cdot 25$. Find the x and y co-ordinates of the point P.

4. Explain the usefulness of the following :

a) Back substitution

b) Pascal's triangle.

5. If $[T]$ represents the transformation matrix between the local and global coordinate systems, then prove that the relation between the local stiffness matrix $[K^l]$ and the global stiffness matrix $[K]$ is given by

$$[K] = [T]^T [K^l] [T]$$

6. Discuss about plane stress and plane strain problems.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

a) A typical Gaussian quadrature formula is $\int_{-1}^{1} f(\xi)d\xi = W_1 f(\xi_1) + W_2 f(\xi_2)$. In the given formula, considering the following, determine ξ_1 and ξ_2. 6

(i) $f(\xi) = a_0 + a_1\xi + a_2\xi^2$

(ii) $W_1 = \frac{1}{2}$

(iii) $W_2 = \frac{3}{2}$.

b) An isoparametric element is shown in the figure-1

P is a point within the element $P(x, y) = P(2, 3)$. Find $P(\xi, \eta)$, where ξ and η are the generalized natural coordinates. 9

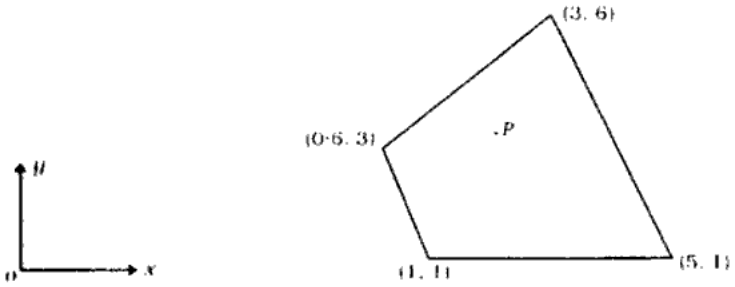

Fig.-1

8. Figure-2 depicts a tapered elastic bar subjected to an applied tensile load *P* at one end and attached to a fixed support at the other end. The cross-sectional area varies linearly from A_0 at the fixed support at $x = 0$ to $A_0/2$ at $x = L$. Calculate the displacement of the end of the bar (a) by modelling the bar as a single element having cross-sectional area equal to the area of the actual bar at its mid-point along the length, (b) using two bar elements of equal length and similarly evaluating the area at the midpoint of each. Also calculate the stresses in both the cases. Plot two graphs to show the variation in displacement and stress for both cases.

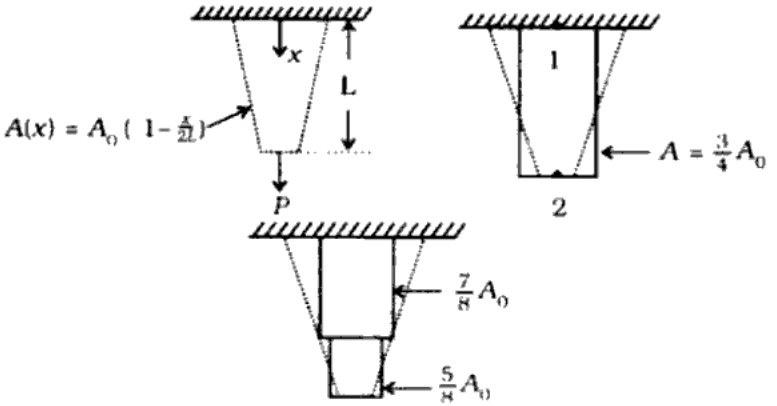

Fig.-2

9. a) Employing Euler's equation, find $y(x)$ for which $\left[\int_0^4 (1 + y'^2)dx\right]$ is an extremum.

Given that $y(0) = 2$ and $y(4) = 3$. 5

b) Using Rayleigh Ritz method, find $y(x)$ for which $\left[\int_0^2 (2y'^2 + y + 2x)dx\right]$ is an extremum. Boundary conditions are : $y(0)$ and $y(2) = 3$. 10

Q. Determine the displacement and rotation under the force and moment located at the centre of the beam shown in Figure-3. The beam has been discretized into the two elements shown in Figure-3. The beam is fixed at each end. A downward force of 10 kN and an applied moment of 20 kN-m act at the centre of the beam. Let E = 210 GPa and I = $4 \times 10^{-4} m^4$ throughout the beam length.

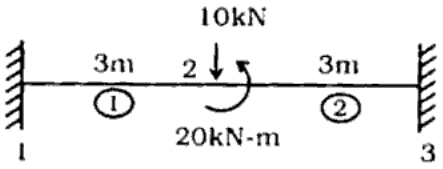

Fig.-3

a) Derive the expression for stiffness matrix for a constant strain element (CST). 7

b) Find the Jacobian transformation of a four noded quadrilateral element with (x, y) coordinates at the nodes i (0, 0), j (2, 0), k (2, 1) and l (0, 1). Also find the Jacobian of the point whose natural coordinates are (0, 0). 8

8. a) What is quadrature ? Derive the expression for two point Gaussian quadrature formula from fundamentals. 7

b) Using Gaussian quadrature (two points) evaluate the following integral and compare with the exact value : 8

$$I = \int_{4}^{6} \int_{-2}^{2} (1 - x)^2 (4 - y)^2 \, dx dy$$