Answer ALL the Questions

S.No.	Questions	Marks
1.	(i) Prove that an analytic function with its derivative zero is constant. (ii) Find the analytic function $f(z) = u + iv$ given $v = \left(r - \dfrac{1}{r}\right)\sin\theta, \; r \neq 0$.	(5M+5M)
2.	Determine the function $u = 2xy + 3xy^2 - 2y^3$ is harmonic. Find the conjugate harmonic function v and express $u + iv$ an analytic function $f(z)$ of z.	(10M)
3.	Use the transformation $\omega = z^2$ to find the image of (i). The straight lines through the origin of the z-plane are mapped onto the straight lines through the origin of the ω-plane and (ii). Find the image of the circle $|z| = a$.	(10M)
4.	Find the bilinear transform which maps the points $z = 2, i, -2$ into the points $\omega = 1, i, -1$ and also find the invariant points of this transform.	(10M)
5.	Determine the Laurent series expression $f(z) = \dfrac{1}{(z+2)(1+z^2)}$ valid for the regions $(i)\,|z| < 1$, $(ii)\,1 < |z| < 2$, $(iii)\,|z| > 2$	(10M)