MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BSC301   Mathematics-III (Differential Calculus)

UPID : 003445

Time Allotted : 3 Hours   *Full Marks :70*

The Figures in the margin indicate full marks.
Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer any ten of the following :   [1 x 10 = 10]

    (I)   What is the area of the region bounded by x-axis , $y=e^x$, x=0, x=1

    (II)   What is the general form of clairaut's equation?

    (III) If a graph has 5 vertices and 7 edges, then what is the size of its adjacency matrix?

    (IV)   On which region log(1+x) can be expanded in an infinite series ?

    (V)   If for any

    $$\vec{A}, \vec{\nabla} x \vec{A} = 0$$, then $\vec{A}$ will be called as?

    (VI)   Find the value of

    $$\int_{x=-1}^{1} \int_{y=-2}^{2} \int_{z=-3}^{3} xy^2 z^3 \, dx \, dy \, dz$$

    (VII) If $\int_c y \, dx + x \, dy = p$ where c is given by $x = \cos\theta, y = \sin\theta, 0 \le \theta \le \pi/2$, find value of p?

    (VIII) Find the value of

    $$\frac{1}{D^2 + 4}(\sin 2x),$$?

    (IX)   What is the eccentricity of the vertex of a graph having only one vertex?

    (X)   What is the natue of the series

    $$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots \ldots$$

    (XI)   If $f(x, y) = |x| + |y|$, find the value of $f_x(0, 0)$?

    (XII) If c is the circle $x^2 + y^2 = 4$, find the value of

    $$\int_c x^2 \, dx$$

Group-B (Short Answer Type Question)

Answer any three of the following   [5 x 3 = 15]

2. Test the series   [5]

    $$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$$

3. If $z = u^2 + v^3$, where u=Sin xy and $v = y^2$, Find   [5]

    $$\frac{\partial z}{\partial x} \text{ and } \frac{\partial z}{\partial y}$$

4. Varify that,   [5]

    $$e^{tan^{-1}x} = 1 + x + \frac{x^2}{2} - \frac{x^{-3}}{6} - \ldots$$

5. Find   [5]

    $$\frac{dy}{dx}$$ of the function $(\sin y)^x - (\cos x)^y = 0$

6. Find the general and singular solution of [5]

$$y = 4xp - 16y^3 p^2$$

Group-C (Long Answer Type Question)
Answer any three of the following

[15 x 3 = 45]

7. (a) Test the convergence of the series whose n_{th} term are [3]

$$(n^{\frac{1}{n}} - 1)^n$$

(b) Examine the convergence of the series [5]

$$\frac{1}{a} - \frac{1}{a+b} + \frac{1}{a+2b} - \frac{1}{a+3b} + \ldots \ldots (a > 0, b > 0)$$

(c) Assuming the validity of expansion, show that [7]

$$sinx = 1 - \frac{(x - \frac{\pi}{2})^2}{2!} + \frac{(x - \frac{\pi}{2})^4}{4!} - \ldots \ldots$$

8. (a) If u=log r and [5]

$$r^2 = x^2 + y^2 + z^2 \text{, Prove that } r^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 1$$

(b) Show that [5]

$$f(x, y) = 3x^3 + 4x^2 y - 3xy^2 - 4y \text{, neither a maximum nor a minimum at (0,0)}$$

(c) Determine the constant m so that the vector [5]

$$\vec{v} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + mz)\hat{k} \text{ is solenoidal}$$

9. (a) If [5]

$$u_n = \frac{3^n}{n+1} \text{, show that } [u_n] \text{ is monotonic increasing and bounded above, find its limit.}$$

(b) Expand e^x in power series of (x-1) [5]

(c) Examine the convergence of the series [5]

$$\sum u_n \text{, where } u_n = \frac{(n+1)(n+4)}{n(n+2)(n+5)}$$

10. (a) If u(x,y)=f(x²+2yz, y²+2zx), prove that [5]

$$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0$$

(b) If [5]

$$u = \tan^{-1}\left(\frac{x^{5/2} + y^{5/2}}{\sqrt{x} - \sqrt{y}}\right) \text{ show that } x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$$

(c) Show that the function f(x,y)=4x²y - y² - 8x⁴ has a maximum value at (0,0). [5]

11. (a) The given function [7]

$$f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}, (x, y) \neq (0, 0)$$

$$= 0, (x,y)=(0,0)$$

Find from definition fxy(0,0) and f_{yx}(0,0)

(b) If [3]

$$A = \pi h^2 \frac{sin\alpha}{1 - sin\alpha} \text{ find dA, where h and } \alpha \text{ are independent variables}$$

(c) If [5]

$$f(x, y) = \frac{x + y}{1 - xy} \text{ and } g(x, y) = \tan^{-1} x + \tan^{-1} y \text{ find } Jacobian \frac{\partial(f, g)}{\partial(x, y)}$$