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Paper Code : PCC- CS401/PCC-CS401/PCCCS 401/PCCCS401 Discrete Mathematics

UPID : 004407

Time Allotted : 3 Hours *Full Marks :70*

The Figures in the margin indicate full marks.
Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer *any ten* of the following : [1 x 10 = 10]

(I) Explain the difference between a permutation and a combination, and provide an example of each.

(II) What is the truth value of $\neg(P \wedge Q)$-$(P \wedge Q)$ if P is true and Q is false?

(III) What is an abelian group?

(IV) What is the chromatic number of a graph?

(V) What is composition of mapping?

(VI) If you have 6 different colors of socks, how many ways can you choose 2 pairs (4 socks total)?

(VII) What is the truth value of $(P \wedge \neg Q) \vee (Q \wedge \neg P)(P \wedge \neg Q) \vee (Q \wedge \neg P)$ if P and Q are both true?

(VIII) Define a group in abstract algebra and explain the group axioms.

(IX) What is the minimum number of edges required for a connected graph with n vertices to be a tree?

(X) Define the union of two sets.

(XI) In a city, there are 30 traffic lights. If each traffic light can be either red, yellow, or green, what can you guarantee about the colors of some traffic lights?

(XII) If $p \rightarrow (q \wedge r)$ and $r \rightarrow s$ are both true, what can you conclude about $p \rightarrow s$?

Group-B (Short Answer Type Question)

Answer *any three* of the following : [5 x 3 = 15]

2. Prove by mathematical induction $3n < n!$ for all positive integers $n \geq 6$ [5]
3. A drawer contains ten black and ten white socks. You reach in and pull some out without looking at them. What is the least number of socks you must pull out to be sure to get a matched pair? Explain how the answer follows from the pigeonhole principle. [5]
4. What is De Morgan's Law in propositional logic? Provide both forms of the law. [5]
5. In a ring R if $x^3 = x$ for all $x \in R$, then show that R is commutative. [5]
6. Consider a connected undirected graph G with n vertices and m edges. Prove that if G has no cycles, then m must be less than n. [5]

Group-C (Long Answer Type Question)

Answer *any three* of the following : [15 x 3 = 45]

7. (a) Let G be a group in which for some integer $n > 1$, $(ab)^n = a^n b^n$ for all $a, b \in G$. Show that [5]
   a) $G^n = \{ x^n \mid x \in G \}$ is a normal subgroup of G.
   b) $G^{n-1} = \{ x^{n-1} \mid x \in G \}$ is a normal subgroup of G.

   (b) If $\phi : G \rightarrow H$ is a homomorphism and G is abelian, then $\mathrm{Im}\phi = \{ \phi(g) \mid g \in G \}$ is abelian. [5]

   (c) Prove that any group of order 15 is cyclic. [5]

8. (a) prove that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology. [5]

   (b) Without truth table find the DNF and CNF of the expression $\sim(p \vee q) \leftrightarrow (p \wedge q)$ [10]

9. (a) State and prove Division Algorithm [8]

   (b) Determine the Recursive formula for the sequance 3,9,21..........Also find the 11th term [7]

10. (a) By principle of inclusion and exclusion find the number of 10 combinations of the elements of the set S containing 5 a's, 4 b's 5 c's and 7 d's [8]

    (b) State the first form of Pigeonhole Principle and the prove that in a party where guests are handshaking among themselves.there will always be at least two guests who have shaken hands the same number of times. [7]

11. (a) Consider the following directed weighted graph G with vertices V={A,B,C,D,E} and edges with their corresponding weights: [8]

(A,B,5)
(A,C,3)
(B,C,2)
(B,D,6)
(C,D,7)
(C,E,4)
(D,E,5)

Apply Dijkstra's shortest path algorithm to find the shortest paths from vertex AA to all other vertices in the graph.

(b) Prove that a simple graph with n vertices and k components can have atmost 1/2(n-k)(n-k+1) edges [7]

*** END OF PAPER ***