VIT
Vellore Institute of Technology

Final Assessment Test – November 2019

Course: MAT2002 - Applications of Differential and Difference Equations

Class NBR(s): 0401 / 0402 / 0403 / 0540 / 0542 Slot: F1+TF1

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

Answer any FIVE Questions
(5 X 20 = 100 Marks)

1. a) If $f(x)$ is a periodic function defined over a period $(0, 2\pi)$ for $f(x) = \frac{1}{2}(3x^2 - 6x\pi + 2\pi^2)$. [10]

Prove that $f(x) = \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ and hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots$

b) Obtain the first two coefficients in the Fourier series for y, where y is given in the following table: [10]

x	0	1	2	3	4	5
y	9	18	24	28	26	20

2. a) Reduce the quadratic form $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ into a canonical form by an orthogonal transformation. Find its rank, index, signature and nature of the quadratic form. [10]

b) Use Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ to express [10]

$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

3. a) Solve the differential equation $(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = 4\cos[\log(x+1)]$ [10]

b) Solve $\frac{d^2y}{dx^2} + y = \sin x + \cos x$ by the method of undetermined coefficients. [10]

4. a) Reduce the third order equation $y'''+2y''-y'-2y=0$ to the system of first order linear equations and solve by matrix method. [10]

b) An e.m.f $E\sin pt$ is applied at $t = 0$ to a circuit containing a capacitance C and inductance L. The current 'i' satisfies the equation $L\frac{di}{dt} + \frac{1}{C}\int i\,dt = E\sin pt$. If $P^2 = \frac{1}{LC}$ and initially the current 'i' and the charge q are zero, show that the current at time t is $\left(\frac{Et}{2L}\right)\sin pt$, where $i = \frac{dq}{dt}$. [10]

5. a) Show that the BVP $\frac{d^2u}{dx^2} + \lambda u = 0, 0 < x < \pi$, $u'(0) = 0$, $u'(\pi) = 0$ is a SLP and hence find the eigen values and corresponding eigen functions. [10]

b) Obtain the series solution of the equation $2x^2\frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0$ [10]

6. a) *Using convolution theorem find the* inverse Z-transform of $\frac{z^2}{z^2+18z+81}$

b) Obtain the inverse Z-transform of the function $U(z) \Rightarrow \left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$. [10]

7. a) Solve the difference equation $x(k+2)+2(k+1)+x(k)=\sin k$, by the method of undetermined coefficients. [10]

b) Solve the difference equation $x(k+2)-\frac{3}{2}x(k+1)+\frac{1}{2}x(k)=1(k),\ x(0)=1, x(1)=\frac{5}{2}$ using Z-transform. [10]

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