## Indian Institute of Information Technology and

## Management, Gwalior

Mathematics-II (IMAS-1201)
Mid-Term Examination (Session 2022-23)

Maximum Time: 2 Hours

Max marks: 30

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## Attempt all the questions

1. (a) Find Laplace transform of the following functions

$$f(t) = \frac{\cos\sqrt{t}}{\sqrt{t}}$$
  
$$f(t) = \sin 2t \ \delta(t - \pi/4) + \frac{\pi}{2t} \ \delta(t - \pi/2).$$

(b) Find the solution of the following integral equation

$$y(t) = 1 + t + \int_0^t (t - u)y(u)du.$$

2. (a) Solve the following differential equation

$$y'' + y = t \cos 2t$$
,  $y(1) = 0$ ,  $y'(1) = 0$ .

(b) Find the Fourier series expansion of

$$f(x) = x^2, \qquad -2 \le x \le 2,$$

and prove that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}.$$

(a) Discuss Gibbs phenomena of the following function

$$f(x) = \begin{cases} 1, & -\pi < x < 0, \\ -1, & 0 \le x < \pi. \end{cases}$$

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Solve the following heat equation

$$u_t(x,t) = u_{xx}(x,t), \qquad 0 < x < \infty,$$
  $u(x,0) = 0, \qquad \text{when } x > 0,$ 

$$u(x,0)=0, \qquad \text{when } x>0,$$

$$\frac{\partial u}{\partial x} = -\mu$$
, when  $x = 0$ ,  $u, \frac{\partial u}{\partial x} \to 0$ , as  $x \to \infty$ ,  $t > 0$ .

(b) Find the inverse Fourier transform of  $(\sqrt{\pi} \omega e^{-\omega^2/8})(4\sqrt{2}i)$ .

Best wishes