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# 2012 MATHEMATICS - III

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

# GROUP - A ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following:

The period of  $\sin 2x$  is

 $10 \times 1 = 10$ 

	c)	2	d)	2 π.	
ii)	If $F\{f(x)\} = F(s)$ represents the Fourier transform of $f(x)$ , then $F\{f(x-a)\}$ (a being a constant) equals				
	a)	$e^{isa}F(s)$	b)	F(s/a)	
	c)	$e^{-isa} F(s)$	d)	$\frac{1}{a^2} F (as).$	
	/T/1	1 6 1 1 0			

iii) The value of  $\alpha$  such that  $3y - 5 x^2 + \alpha y^2$  is a harmonic function is

a) 5

i)

a)

1

b) 0

b) π

c) - 5

d) 3.

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is

a) π

b) – π

c)  $-2 \pi i$ 

- d)  $2 \pi i$ .
- v) The residue of the function  $f(z) = z^2/(z-1)^2(z+2)$  at the pole z = -2 is
  - a)  $\frac{9}{4}$

b)  $\frac{2}{9}$ 

c)  $\frac{4}{9}$ 

- d)  $\frac{9}{2}$ .
- vi) Four coins are tossed simultaneously. The probability of getting 2 heads is
  - a)  $\frac{3}{8}$

b)  $\frac{1}{8}$ 

c)  $\frac{3}{4}$ 

- d)  $\frac{1}{4}$ .
- vii) The random variable X has the following p.d.f.:

$$f(x) = \begin{cases} k, -2 < x < 2 \\ 0, \text{ otherwise} \end{cases}$$

The value of the constant k is

a)  $\frac{1}{8}$ 

b)  $\frac{1}{12}$ 

c)  $\frac{1}{2}$ 

d)  $\frac{1}{4}$ 



viii) The variance of a Poisson distribution with parameter  $\lambda$ is



b)  $\frac{1}{\lambda}$ 

c) λ d)  $\frac{1}{\lambda^2}$ .

A solution u(x, y) of the PDE  $u_{xx} - u = 0$  is ix)

a) 
$$A(y) e^{x} + B(y) e^{-x}$$

b) 
$$A(x) e^{y} + B(x) e^{-y}$$

c) 
$$A(x) e^{y} + B(y) e^{x}$$

- d) none of these.
- The value of  $J_{-\frac{1}{2}}(x)$  is x)

a) 
$$\sqrt{\frac{2}{\pi x}} \sin x$$
 b)  $\sqrt{\frac{2}{\pi x}} \cos x$ 

b) 
$$\sqrt{\frac{2}{\pi x}} \cos x$$

c) 
$$-\sqrt{\frac{2}{\pi x}} \left( \frac{\sin x}{x} \right)$$

c) 
$$-\sqrt{\frac{2}{\pi x}} \left( \frac{\sin x}{x} \right)$$
 d)  $-\sqrt{\frac{2}{\pi x}} \left( \frac{\cos x}{x} \right)$ .

- The value of Legendre's polynomial  $P_1$  (x) is xi)
  - a) 0

c) x

- d)  $x^2$ .
- xii) If  $\alpha \neq \beta$ , then the value of  $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx$  is
  - a)  $\frac{1}{2} \{J_{n+1}(\alpha)\}^2$
- c)  $\frac{1}{2} \{J_n(\alpha)\}^2$
- d) none of these.



- xiii) If  $P_n(x)$  is the Legendre's polynomial of degree n,  $\int_{-1}^{1} P_n(x) dx$  in
  - 1, when n = 0
- b) 0, when n = 0
  - c) 2, when n = 0 d) none of these.
- xiv) If f(z) = u(x, y) i v(x, y) is analytic, then f'(z)equals
  - a)  $\frac{\partial u}{\partial x} i \frac{\partial u}{\partial y}$  b)  $\frac{\partial u}{\partial x} + i \frac{\partial u}{\partial y}$
  - c)  $\frac{\partial v}{\partial x} i \frac{\partial v}{\partial u}$
- d) none of these.

#### **GROUP - B**

#### (Short Answer Type Questions)

Answer any *three* of the following.  $3 \times 5 = 15$ 

- Find the Fourier series for  $f(x) = e^{-x}$  in the interval  $0 < x < 2\pi$ .
- 3. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1, |x| \le 1 \\ 0, |x| > 1 \end{cases}$$

Hence evaluate  $\int_0^\infty \frac{\sin x}{x} dx$ .

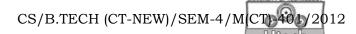
3 + 2

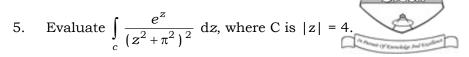
Show that the polar form of Cauchy - Riemann equations are 4.

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}.$$

Deduce that  $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$ . 3 + 2

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6. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution.

3 + 2

#### GROUP - C

#### (Long Answer Type Questions)

Answer any three of the following questions.

 $3 \times 15 = 45$ 

- 7. a) If  $f(x) = |\cos x|$ , expand f(x) as a Fourier series in the interval  $(-\pi, \pi)$ .
  - b) Using Parseval's identities, prove that

$$\int_0^\infty \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)}$$

- c) Find Fourier sine transform of  $e^{-|x|}$ . Hence show that  $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, m > 0.$
- 8. a) Find the analytic function, whose real part is  $\sin(2x)/(\cos h(2y) \cos(2x))$ . 3+2
  - b) Show that under the transformation  $w = \frac{z-i}{z+i}$ , real axis in the z plane is mapped into the circle |w| = 1. Which portion of the z plane corresponds to the interior of the circle.
  - c) Evaluate  $\int_0^{2+i} (\bar{z})^2 dz$  along (i) the line y = x/2, (ii) the real axis to 2 and then vertically to 2 + i. 2 + 3

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- 9. a) A has one share in a lottery in which there is 1 prize and 2 blanks; B has three shares in a lottery in which there are 3 prizes and 6 blanks; compare the probability of A's success to that of B's success.
  - b) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts?
- 10. a) Solve the following equation by the method of separation of variables  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

subject to the conditions u(0, y) = u(l, y) = u(x, 0) = 0and  $u(x, 0) = \sin \frac{n \pi x}{l}$ .

b) Show that the solution of the heat equation

$$K \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$
, -  $\infty < x < \infty, t > 0$  subject to the condition

$$u(x, t) = 0$$
 at  $x = \pm \infty$ ,  $\frac{\partial u}{\partial x} = 0$  at  $x = \pm \infty$  and

 $u(x, 0) = f(x), -\infty < x < \infty$  can be written in the form

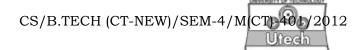
$$u(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s)e^{-Ks^2t - isx} ds$$

where F(s) is the Fourier transform of f(x). 8 + 7

11. a) Obtain the series rotation of the equation

$$x(1-x)\frac{d^2y}{dx^2}-(1+3x)\frac{dy}{dx}-y=0.$$

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- b) Show that  $J_n(x) = \frac{1}{\pi} \int_0^{\pi} \cos(n \theta x \sin \theta) d\theta$ , *n* being an integer. 8+7
- 12. a) Express  $f(x) = x^4 + 3x^3 x^2 + 5x 2$  in terms of Legendre polynomial.
  - b) Show that  $\int_{-1}^{1} x^{2} R_{n}(x) R_{n}(x) dx = \frac{2n(n+1)}{n}$

$$\int_{-1}^{1} x^{2} P_{n-1}(x) P_{n+1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

8 + 7