VIT
Vellore Institute of Technology

Winter semester –(2019-2020)

Continuous Assessment Test – I

Programme Name & Branch: B.Tech

Course Name:Complex Variables and Partial Differential Equations

Course Code: MAT3003 Slot: C1+TC1+TCC1+V2

Exam Duration: 90 minutes Maximum Marks: 50

Answer All the Questions (5 × 10 = 50)

1. Find the constant a so that $u(x,y) = ax^2 - y^2 + xy$ is harmonic. Find an analytic function $f(z)$ for which u is the real part. Also find its harmonic conjugate. [10M]

2. In a two dimentional fluid flow, if $xy(x^2 - y^2)$ represent the stream function, find the corresponding velocity potential and also the complex potential. [10M]

3. Find the bilinear transformation that maps the points $1 + i, -i, 2 - i$ of the z- plane into the points $0, 1, i$ of the ω-plane. Hence find the invariant points of the transformation. [10M]

4 a). Find the image of the rectangular region bounded by the lines $x = 1$, $x = 3, y = 1$ and $y = 2$ under the transformation $\omega = z^2$. [5M]

b). Find the image of the triangular region in the z-plane bounded by the lines $x = 0, y = 0, x + y = 1$ Under the transformation $\omega = 2z$. [5M]

5. Expand $\frac{1}{z(z-1)}$ as Laurent's series

(i) about $z = 0$ in powers of z

(ii) about $z = 1$ in powers of $z - 1$.

Also state the region of validity. [10M]