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Total No. of Pages : 03

Total No. of Questions : 9

B.Tech. (AI&ML)/ AI and Data Science / AI / (Cyber Security)/ Computer Engineering / (Data Science)/ (CSE) / IT/ (IOT)/(Internet of Things and Cyber Security including Block Chain Technology) (Sem-4)

DISCRETE MATHEMATICS

Subject Code : BTCS-401-18

M.Code : 77626

Date of Examination : 15-06-2023

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A** is **COMPULSORY** consisting of **TEN** questions carrying **TWO** marks each.
2. **SECTION-B** contains **FIVE** questions carrying **FIVE** marks each and students have to attempt any **FOUR** questions.
3. **SECTION-C** contains **THREE** questions carrying **TEN** marks each and students have to attempt any **TWO** questions.

SECTION-A

1. Answer briefly :

- a) Find the Cartesian product $A \times A$ if $A = \{0, 1, 3\}$.
- b) Determine the domain and range of the relation

$$R = \{x, y : x \in N, y \in N \text{ and } x + y = 10\}$$

- c) How many 4-digit numbers can be formed by using the digit 2,4,6,8.
- d) From any 26 points within a rectangle 20 cm by 15 cm, show that at least two are within 5cm of each other.
- e) Write down the truth table of $p \leftrightarrow q \leftrightarrow r$.

- f) Draw a multigraph G whose adjacency matrix $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 2 \\ 0 & 2 & 0 \end{bmatrix}$.

- g) Define a complete binary tree.

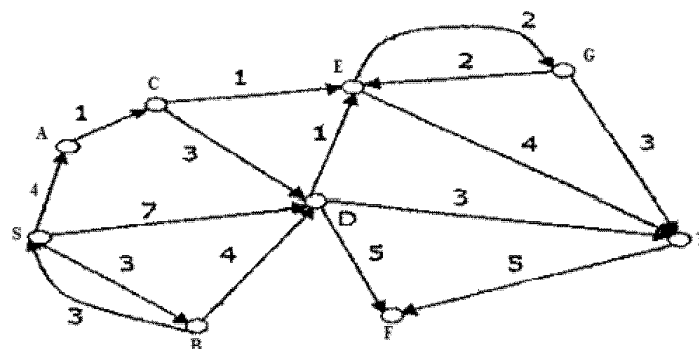
- h) State fundamental theorem on Relations,
- i) What will be the chromatic number of complete graph with n - vertices.
- j) The number of diagonals of a polygon is 20. Find the number of its sides.

SECTION-B

2. Show that intersection of two partial order relations is a partial order relation. But union of two partial order relations need not be a partial order relation. Give suitable example.
3. Give an example of a non abelian group G and a normal subgroup H of G such that quotient group G/H is abelian.
4. a) How many numbers greater than 1000000 can be formed by using digits 1, 2, 0, 2, 4, 2, 4.
b) Find the number integers between 1 and 60 which are divisible by 2 nor by 3 and nor by 5.
5. a) Prove that $p \rightarrow q \wedge r = p \rightarrow q \wedge p \rightarrow r$.
b) **Check the validity of the following argument :**
If I work, I cannot study. Either I work or pass mathematics.
I passed mathematics, Therefore I study.
6. Prove that in a graph the number of vertices of odd degree is even.

SECTION-C

7. Find the shortest path between A to T using Dijkstra's algorithm for the following graph :



8. a) If H is a subgroup of G of index 2 in G . The H is normal subgroup of G .
- b) Simplify the Boolean expression $f(x, y, z) = x \wedge y' \wedge z \vee x \wedge y \wedge z$. And find its conjunctive normal forms.
9. a) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be a real valued function defined by $f(x) = 2x^3 - 1, x \in \mathbb{R}$ and $g(x) = \left[\frac{x+1}{2} \right]^{\frac{1}{3}}, x \in \mathbb{R}$. Show that each f and g is inverse of other.
- b) If $f: \mathbb{N} \rightarrow \mathbb{N}$ and $f(j) = j(\text{mod } 4)$. Determine whether f is one to one or onto or both or neither.