MAT 3004 - ALA - CAT II

A2 Slot - key.

(R. Padma)

1. Let $A = \begin{bmatrix} 1 & 3 & 2 & 0 & 0 \\ 2 & 1 & -5 & 1 & 2 \\ 3 & 2 & 5 & 1 & -2 \\ 5 & 8 & 9 & 1 & -2 \\ 9 & 9 & 4 & 2 & 0 \end{bmatrix}$

$R_2 \to -2R_1 + R_2$; $R_3 \to -3R_1 + R_3$; $R_4 \to -5R_1 + R_4$; $R_5 \to -9R_1 + R_5$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 \\ 0 & -5 & -9 & 1 & 2 \\ 0 & -7 & -1 & 1 & -2 \\ 0 & -7 & -1 & 1 & -2 \\ 0 & -18 & -14 & 2 & 0 \end{bmatrix}$$

$R_4 \to -R_3 + R_4$; $R_4 \leftrightarrow R_5$; $R_4 \to R_4/2$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 \\ 0 & -5 & -9 & 1 & 2 \\ 0 & -7 & -1 & 1 & -2 \\ 0 & -9 & -7 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$R_2 \to -R_3 + R_2$; $R_3 \to -R_4 + R_3$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 \\ 0 & 2 & -8 & 0 & 4 \\ 0 & +2 & +6 & 0 & -2 \\ 0 & -9 & -7 & 1 & 0 \end{bmatrix}$$

$R_3 \to -R_2 + R_3$; $R_2 \to R_2/2$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 \\ 0 & 1 & -4 & 0 & 2 \\ 0 & 0 & 14 & 0 & -6 \\ 0 & -9 & -7 & 1 & 0 \end{bmatrix}$$

$R_4 \to R_4 + 9R_2$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 \\ 0 & 1 & -4 & 0 & 2 \\ 0 & 0 & 14 & 0 & -6 \\ 0 & 0 & -43 & 1 & 18 \end{bmatrix}$$

$R_3 \to 3R_3 + R_4$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 \\ 0 & 1 & -4 & 0 & 2 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -43 & 1 & 18 \end{bmatrix}$$

$R_4 \to -43R_3 + R_4$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 \\ 0 & 1 & -4 & 0 & 2 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -42 & 18 \end{bmatrix}$$

$R_4 \to R_4/42$, $R_3 \to -R_3$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 \\ 0 & 1 & -4 & 0 & 2 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -\frac{3}{7} \end{bmatrix}$$

$R_3 \to R_4 + R_3$; $R_2 \to 4R_3 + R_2$; $R_1 \to -2R_3 + R_1$

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 6/7 \\ 0 & 1 & 0 & 0 & \frac{2}{7} \\ 0 & 0 & 1 & 0 & -\frac{3}{7} \\ 0 & 0 & 0 & 1 & -\frac{3}{7} \end{bmatrix}$$

$R_1 \to -3R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{7} \\ 0 & 0 & 1 & 0 & -\frac{3}{7} \\ 0 & 0 & 0 & 1 & -\frac{3}{7} \end{bmatrix}$$

The reduced row-echelon form of Q is

$$U_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{7} \\ 0 & 0 & 1 & 0 & -\frac{3}{7} \\ 0 & 0 & 0 & 1 & -\frac{3}{7} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

A basis of $V+W = \{ \bar{v}_1 = (1,2,3,5,9),\ \bar{v}_2 = (3,1,2,8,9),\ \bar{v}_3 = (2,-5,5,9,4),\ \bar{u}_1 = (0,1,1,1,2) \}$.

A basis of $N(Q) = N(U_1) = \{ (0, -\frac{2}{7}, \frac{3}{7}, \frac{3}{7}, 0) \}$.

or $\{(0,-2,3,3,0)\}$.

$\dim(V+U) = 4$, $\dim(V \cap W) = 1$.

2 a) $A = \begin{bmatrix} 2 & -3 & -7 & 11 \\ 3 & -1 & -7 & 13 \\ 1 & 2 & 0 & 2 \end{bmatrix}$

$R_1 \leftrightarrow R_3$; $R_2 \to -3R_1 + R_2$; $R_3 \to -2R_1 + R_3$

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & -7 & -7 & 7 \\ 0 & -7 & -7 & 7 \end{bmatrix}$$

$R_3 \to -R_2 + R_3$; $R_2 \to R_2/(-7)$

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank $(A) = 2 \neq 3$ = no. of rows of A.

$\therefore$ A does not have a right inverse.

2b) Let $f(x) = ax^2 + bx + c$. The equations are

$a + b + c = 2$; $a - b + c = -8$; $4a + 2b + c = 1$

The row echelon form of $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & -8 \\ 4 & 2 & 1 & 1 \end{array}\right]$ is $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right]$

The polynomial is $-2x^2 + 5x - 1$

3a) $\{2t+1, 2t-1\}$ is a basis of $P_1(\mathbb{R})$ and

$$at+b = \frac{a+2b}{4}(2t+1) + \frac{a-2b}{4}(2t-1)$$

$$\therefore T(at+b) = \frac{a+2b}{4}(t^2-1) + \frac{a-2b}{4}(t^2+t)$$

$$= \frac{a}{2}t^2 + \frac{a-2b}{4}t - \frac{a+2b}{4}$$

b) $L(x,y) = (x, x+y, y)$

$L(x,y) = (0,0,0) \Rightarrow x=0, y=0.$

$\therefore \ker(L) = \{\bar{0}\} \Rightarrow L$ is 1-1.

L is obviously not onto as $\dim(\mathbb{R}^2) \neq \dim(\mathbb{R}^3)$

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4. $$(x,y)_\alpha = \begin{bmatrix} \frac{2x-y}{5} \\ \frac{3y-x}{5} \end{bmatrix}$$

$S(3,1) = (9,2)$, $S(1,2) = (8,-1)$. $\therefore [S]_\alpha = \begin{bmatrix} \frac{16}{5} & \frac{17}{5} \\ -\frac{3}{5} & \frac{-11}{5} \end{bmatrix}$

$T(3,1) = (4,7)$, $T(1,2) = (3,-1)$ $\therefore [T]_\alpha = \begin{bmatrix} 1/5 & 7/5 \\ \frac{17}{5} & -\frac{6}{5} \end{bmatrix}$

$$[S+T]_\alpha = \begin{bmatrix} 17/5 & 24/5 \\ 14/5 & -17/5 \end{bmatrix}$$

$$[T \circ S]_\alpha = [T]_\alpha \cdot [S]_\alpha = \frac{1}{5}\begin{bmatrix} -1 & -12 \\ 58 & 71 \end{bmatrix}$$

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5 a) $\bar{w}_1 = 1\cdot\bar{v}_1 + 2\cdot\bar{v}_2 - \bar{v}_3$

$= (1,0,1) + 2(1,1,0) - (0,0,1)$

$= (3,2,0)$

$\bar{w}_2 = 1\cdot\bar{v}_1 + 1\cdot\bar{v}_2 - \bar{v}_3 = (2,1,0)$

$\bar{w}_3 = 2\cdot\bar{v}_1 + 1\cdot\bar{v}_2 + \bar{v}_3 = (3,1,3)$

b) Determinants are not equal.

Hence the matrices are not similar.

$$\det\left(\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 2 \\ 0 & 2 & 3 \end{bmatrix}\right) = 0, \quad \det\left(\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 1 \end{bmatrix}\right) = -11$$