

Sub: Design and Analysis of Algorithms

Course Code: CS203

Time: 2 Hours

Date: 27/Sept/2023

Max. Marks: 30

Note: Attempt all questions and answer them appropriately.

No doubts will be cleared during examination.

1. Let  $f(n)$  and  $g(n)$  be asymptotically nonnegative functions. Using the basic definition of  $\Theta$ -notation, prove that  $\max(f(n), g(n)) = \Theta(f(n) + g(n))$ . [2]
2. Is the array with values  $\{23, 17, 14, 6, 13, 10, 1, 5, 7, 12\}$  a max heap? [2]
3. Show that the worst-case running time of MAX-HEAPIFY on a heap of size  $n$  is  $\Omega(\log n)$ . (Hint: For a heap with  $n$  nodes, give node values that cause MAXHEAPIFY to be called recursively at every node on a simple path from the root down to a leaf.) [4]
4. Illustrate the operation of MAX-HEAP-INSERT ( $A, 10$ ) on the heap  
 $A = \{15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1\}$  [4]
5. Write pseudocode for the procedures HEAP-MINIMUM, HEAP-EXTRACT-MIN, HEAP-DECREASE-KEY, and MIN-HEAP-INSERT that implement a min-priority queue with a min-heap. [4]
6. Give an adjacency-list representation for a complete binary tree on 7 vertices. Give an equivalent adjacency-matrix representation. Assume that vertices are numbered from 1 to 7 as in a binary heap. [4]
7. Suppose that we represent the graph  $G = (V, E)$  as an adjacency matrix. Give a simple implementation of Prim's algorithm for this case that runs in  $O(V^2)$  time. [5]
8. Construct the minimum spanning tree (MST) for the given graph using Kruskal's Algorithm. And write the complexity. [5]

