Faculty of Engineering & Technology

First Semester B.Tech. (Applied Science Humanities)/AI/AI & DS/AI & ML/Robotics & AI (NEP) 2024-25 Examination

BASIC CALCULUS & DIFFERENTIAL EQUATIONS

Time : Three Hours]

[Maximum Marks : 70

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve Question No. 1 **OR** Question No. 2.

(3) Solve Question No. 3 **OR** Question No. 4.

(4) Solve Question No. 5 **OR** Question No. 6.

(5) Solve Question No. 7 **OR** Question No. 8.

(6) Solve Question No. 9 **OR** Question No. 10.

(7) Use of non-programmable calculator is permitted.

1. (a) If $y = (x^2 - 1)^n$ then prove that :

$$(x^2 - 1)y_{m+2} + 2(m + 1 - n)xy_{m+1} + (m - 2n)(m + 1)y_m = 0.$$ 7

(b) Evaluate :

(i) $\lim\limits_{x \to 0} x \tan\left(\frac{\pi}{2} - x\right)$ 3

(ii) $\lim\limits_{x \to 0} \left(\cot x - \frac{1}{x}\right)$. 4

OR

2. (a) Given $f(x) = x^3 + 8x^2 + 15x - 24$. Find the value of $f\left(\frac{11}{10}\right)$ by using Taylor's theorem. 7

(b) Evaluate :

(i) $\lim\limits_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ 4

(ii) $\lim\limits_{x \to \infty} \left(\frac{1}{x}\right)^{\frac{1}{x}}$. 3

3 (a) Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

where $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ 7

(b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u(1 - 4\sin^2 u).$$ 7

OR

4. (a) If $u = \frac{x + y}{1 - xy}$, $v = \tan^{-1} x + \tan^{-1} y$ then find $\frac{\partial(u, v)}{\partial(x, y)}$.

State whether u & v are functionally related. If so find the relation between them 7

(b) Prove that the rectangular solid of maximum volume which can be inscribed in a given sphere is a cube. 7

5. (a) Investigate the value of λ and μ so that the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution, (ii) unique solution, (iii) an infinite solution. 7

(b) Find the eigen values and corresponding eigen vectors for the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & 2 & 0 \end{bmatrix}$ 7

OR

6. (a) Investigate the linear dependence of vectors $X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$ and $X_4 = (-3, 7, 2)$. Find the relation if possible. 7

(b) By using Cayley Hamilton's theorem find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. 7

7. (a) Solve : $x\left(\frac{dy}{dx} + y\right) = 1 - y$. 7

(b) Solve : $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y(\log y)^2}{x^2}$. 7

OR

8. (a) Solve $ye^{xy}\,dx + (xe^{xy} + 2y)\,dy = 0$. 7

(b) A constant emf E volts is applied to a circuit containing a constant resistance R ohms in series and a constant inductance L henries. If the initial current is zero, show that the current builds up to half its theoretical max in $\dfrac{L\log 2}{R}$ seconds. 7

9. (a) Solve : $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 10y = -37\sin 3x$. 7

(b) Solve $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = \sin(e^x)$ by method of variation of parameters. 7

OR

10. (a) Solve : $x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + 5y = 10 - \dfrac{4}{x}$. 7

(b) An emf E sin pt is applied at t = 0 to a circuit containing a capacitance C and inductance L. The current i satisfies the equation $L\dfrac{di}{dt} + \dfrac{1}{C}\int i\,dt = E\sin pt$. If $p^2 = \dfrac{1}{LC}$ and initially the current i and the charge q are zero. Show that the current at time t is $\dfrac{Et}{2L}\sin pt$, where $i = \dfrac{dq}{dt}$. 7