Sub Code: BAST-105 **ROLL NO……………..……………..**

EVEN SEMESTER EXAMINATION, 2023 – 24

First yr B.Tech.

MATHEMATICS-II

Duration: 3:00 hrs **Max Marks: 100**

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	Answer any four parts of the following. a) Solve $\frac{dy}{dx} = (x+y)^2$. b) Solve $p - \frac{1}{p} - \frac{x}{y} + \frac{y}{x} = 0$. c) Solve $ylogydx + (x - logy)dy = 0$. d) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = xlogx$. e) Solve $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = Cos2x$ f) Solve $\frac{dx}{dt} = 7x - y\,,\ \frac{dy}{dt} = 2x + 5y$	5x4=20
Q 2.	Answer any four parts of the following. a) Prove that the function $u = e^{-x}(xsiny - ycosy)$is harmonic. b) Solve by the method of variation of parameter $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$. c) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \quad for\ m \neq n$ d) Prove that $x\,J'_n(x) = n\,J_n(x) - x\,J_{n+1}(x)$ e) Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. f) Eliminate the arbitrary constant f from the relation $z = y^2 + 2f(\frac{1}{x} + logy)$.	5x4=20
Q 3.	Answer any two parts of the following. a) Solve $(D^3 - 3D^2D' - 4DD'^2 + 12D'^3)z = \sin(y+2x)$ b) Find the Fourier series expansion for the function $f(x) = x.Sinx\ , -\pi < x < \pi$. Hence deduce that $\frac{\pi-2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots$ … … … … … … …. c) By using the changing of independent variable method, Solve $x\frac{d^2y}{dx^2} + (4x^2 - 1)\frac{dy}{dx} + 4x^3\,y = 2x^3$	10x2= 20
Q 4.	Answer any two parts of the following. a) Test the series $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \cdots \ldots\ldots\infty$ b) Examine the convergence of the series $\sum(\sqrt[3]{n^3+1} - n)$ c) If $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ \pi(2-x) & 1 < x < 2 \end{cases}$, using half range cosine, show that $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots \ldots\ldots\ldots\ldots = \frac{\pi^4}{96}$	10x2= 20
Q 5.	Answer any two parts of the following. a) Solve $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}sin2x$ b) Apply Cauchy-Residue theorem to evaluate $\int_C \frac{3z^2+z+1}{(z^2-1)(z+3)}dz\ where\ \|z\| = 2$. c) Evaluate using Cauchy integral formula $\int_C \frac{z-1}{(z+1)^2(z-2)}dz$ where $\|z - i\| = 2$.	10x2= 20

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