Final Assessment Test– Jan/Feb 2023

VIT
Vellore Institute of Technology

Course: BMAT101L - Calculus

Class NBR(s): 5026 / 5039 / 5044 / 5047 / 5055 / 5058 / 5061 / 5420 / 5487 / 5519 / 5521 / 5524 / 5526 / 5528 / 5664 / 5692 / 5699 / 6207 / 6430 / 6502 / 6510 / 6547

Slot: B2+TB2

Time: Three Hours

Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Consider the function $f(x) = x(6-2x)^2$
   (i) Identify where the extrema of f occur.
   (ii) Find the intervals on which f is increasing and decreasing.
   (iii) Find the intervals on which f is concave up and concave down.

2. (i) Find the area of the region enclosed by $y = x^4$ and $y = 8x$. [5]
   (ii) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the y- axis. [5]

3. If z is a function of x and y, where $x = e^u \cos v$, $y = e^u \sin v$ prove that $y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$.

4. Using Taylor's series expansion expand $e^x \sin y$ in powers of x and y up to third degree terms.

5. The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

6. By changing the order of integration, evaluate
   $\int_0^1 \int_y^{2-y} xy\,dx\,dy.$

7. By transforming into spherical polar coordinates, evaluate
   $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} x\,dx\,dy\,dz.$

8. Using Gamma function, evaluate
   $\left(\int_0^\infty x e^{-x^8} dx\right)\left(\int_0^\infty x^2 e^{-x^4} dx\right).$

9. Show that $\vec{F} = (2x + yz)\hat{i} + (4y + zx)\hat{j} - (6z - xy)\hat{k}$ is irrotational. Hence find its scalar potential φ.

10. If $r = |\vec{r}|$, where $\vec{r}$ is the position vector of the point (x, y, z), then prove that $\nabla^2 r^n = n(n+1)r^{n-2}$.

11. Verify Green's theorem in the plane for
    $\int_C \{(2x - y)dx + (x + y)dy\}$ where C is the boundary of the circle $x^2 + y^2 = a^2$.

12. Verify Stokes theorem for the function $\vec{F} = x^2\vec{i} + xy\vec{j}$, integrated round the square in the plane $z = 0$ whose sides are along the lines $x = 0, y = 0, x = a, y = a$.

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