VIT
Vellore Institute of Technology

Final Assessment Test – April 20[illegible]9

Course: MAT1014 - Discrete Mathematics and Graph The[illegible]y

Class NBR(s): 0693 / 0695 / 0696 / 0698 / 0699/ 0702 / 0705 / 0706 / 0708 / 0717 / 0902 / 0903 / 1755

Slot: A2+[illegible]2+TAA2+V3

Time: Three Hours

Max. Mark[illegible]00

Answer any FIVE Questions

(5 X 20 = 100 Marks)

a) Show the following equivalent formula without constructing the truth table. [5]

$(p \vee q) \vee (q \vee \neg r) \wedge (p \vee r) \Leftrightarrow p \vee q$

b) Prove Demorgan laws in two variables and Compute equivalent of the following using Demorgan law [5]

" It is not the case that roses are red and violets are blue "

c) Explain PCNF and PDNF. Obtain PCNF and PDNF of the statement formula $(p \wedge q) \vee (\neg p \wedge q \wedge r)$. [10]

2. a) (i) Define tautology in statements. Show that SVR is tautologically implied from PVQ, P→R and Q→S [10]

(ii) Prove the following by a counter example $(\forall x)\{p(x) \vee q(x)\} \rightarrow (\forall x)\{p(x)\} \vee (\forall x)\{q(x)\}$ is not logically valid.

b) Verify validity of the following arguments. [10]

Every living thing is a plant or animal.

John's gold fish alive and is not a plant.

All animals have hearts.

Therefore, John's gold fish has heart

3. a) Define a subgroup with an example. Prove that "A non-empty subset H of a group G is a subgroup of G if and only if [10]

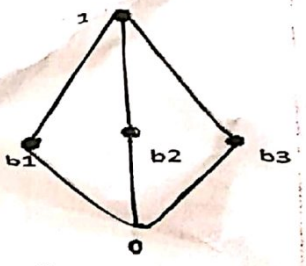
$a, b \in H \Rightarrow ab^{-1} \in H$

b) Prove that "If G is a group in which $(ab)^k = a^k.b^k$ for any three consecutive integers k, then G is abelian". [10]

4. a) Define Poset with an example. Suppose that (S, $\leq_1$) and (T, $\leq_2$) are two posets. Show that $(S \times T, \leq)$ is a poset, where $(s,t) \leq (u,v)$ if and only if $s \leq_1 u$ and $t \leq_2 v$. [10]

b) (i) Define a Lattice. Let S = { a, b, c }. Show that the power set of S is a lattice under set inclusion and draw the corresponding Hasse Diagram. [10]

(ii) Show that, in a distributive lattice the complement of any element is unique if it exists.



Show that the lattice in above Hasse diagram is not distributive using complements

5 a) Express $x_1 \oplus x_2$ as sum of products canonical and $x_1 * x_2$ as product sums canonical form. [10]

b) Use k-maps to simplify the sum of the products canonical form [10]

$w\bar{x}\bar{y}z + w\bar{x}yz + w\bar{x}\bar{y}\bar{z} + wx\bar{y}z + wx\bar{y}\bar{z} + \bar{w}x\bar{y}\bar{z} + \bar{w}xyz$

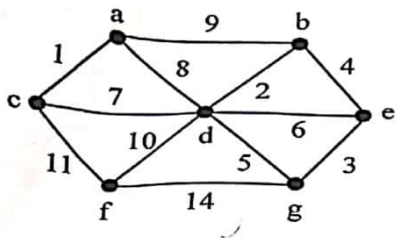
Verify your solution using the operations of Boolean algebra.

Possession of Mobile Phone in the exam hall even in switched off condition is a malpractice.

6. a) Show that in any graph $\sum_{v \in V} d(v) = 2 \in$, where $\in$ is the total number of edges. Hence prove that $\delta \leq \frac{2 \in}{\nu} \leq \Delta$, Where δ and Δ are minimum and maximum degrees vertices, ν is total no. of vertices in G. [10]

b) (i) Show that there are eleven non-isomorphic simple graphs on four vertices.
(ii) Explain Eulerian and Hamiltonian Graphs with suitable examples. [10]

7. a) Explain Prim's Algorithm and compute the minimum spanning tree of the following by using above algorithm. [10]



b) Explain Chromatic number and Chromatic Polynomial with an example. [10]
Prove that the chromatic polynomial of a tree with n vertices is $\lambda(\lambda - 1)^{n-1}$.