MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BS-M101/BSM101 Mathematics –IA

UPID : 001004

Time Allotted : 3 Hours *Full Marks :70*

The Figures in the margin indicate full marks.
Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer *any ten* of the following : [1 x 10 = 10]

(i) f(x)=|x| satisfies Roll's theorem [-1,1]. State true or false with reasons.

(ii) Every square matrix A can be expressed as a sum of symmetric and skew symmetric matrix and it can be expressed as=

(iii) Collection of vectors containing null vectors is linearly _____.

(iv) The eigen value of the matrix A are a and b; then what are the eigen values of A^2 ?

(v) The value of $\Gamma(5/2) = ?$

(vi) $\lim_{x\to 0} \frac{1-Cos\,x}{Sin\,x} = ?$

(vii) Find the value of C for which the vector (1,2) and (3,C) linearly independent.

(viii) A set of vectors in a vector space V over a field F is orthogonal if $||\alpha||$ = _____.

(ix) What is eigen value of a square matrix?

Find the eigen value of the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(x) Define Beta and Gamma functions.

(xi) The product of two Orthogonal matrix is _____ matrix.

(xii) The product of any matrix and its transpose is_____ matrix.

Group-B (Short Answer Type Question)

Answer *any three* of the following : [5 x 3 = 15]

2. Check whether the vectors { (1, 1, 0), (1, 1, 1), (0, 1, -1) is a basis in R^3 or not. [5]

3. State Lagrange's mean value theorem. [5]

Prove that $\frac{x}{1+x} < \log(1+x) < x \;, x > 0$

4. Solve the following equations by matrix inversion method, if possible: [5]
2x+3y+z= 6; 3y+5z= 8; -x+y=0

5. Define subspace of a vector space. [5]

Let V={M_2 *set of all* 2×2 *matrices of the form* $\begin{pmatrix} a & b \\ c & d \end{pmatrix}, a,b,c,d \in R$} and S is the subset of V such that $S=\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a+b=0\}$ ~~is a subspace of V.~~ Prove that S is a subspace of V

6. Prove that B(m, n) = B(n, m). Also write the relation between Beta and Gamma functions and find B(3,4). [5]

Group-C (Long Answer Type Question)

Answer *any three* of the following : [15 x 3 = 45]

7. (a) Define kernel of a linear transformation between two vector spaces. [5]
Is $f(x) = Sin\,x$ a Linear transformation? Justify your answer.

(b) State Cayley Hamilton theorem for matrices. What is the Characteristic equation of a square matrix? Explain with an example. [4]

(c) What is eigen vector? [6]

Find the eigen values and eigen vectors of matrix $\begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}$.

8. Show that the set of all complex numbers is a vector space over real field w.r.t the usual addition between two complex number and multiplication between a real number and a complex number. Find a basis of this space and hence the dimension. [15]

9. What is diagonallzation of matrix? [15]

Diagonalise the matrix $A = \begin{pmatrix} 6 & 4 & -2 \\ 4 & 12 & -4 \\ -2 & -4 & 13 \end{pmatrix}$. Write the matrix P, such that $P^{-1}AP$ is diagonal matrix. Also show that $P^{-1}AP = D$.

10. (a) Prove that $\int_0^\infty \frac{1}{(1+x^2)^5} dx$ $=35\pi/256$ [7]

(b) Obtain the evolute of the parabola $y^2 = 4ax$. [8]

11. (a) Verify Lagrange's MVT for f(x)= x(x-1)(x-2) in [0, 1/2]. [5]

(b) Expand the following functions in power of x in infinite series stating in each case the condition under which the expression is valid: f(x)= Cos x . [5]

(c) Evaluate $\Gamma(1/2)\ \Gamma(3/2)\ \Gamma(5/2)\ \Gamma(7/2)$. [5]

*** END OF PAPER ***