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Paper Code : BCAC201 Discrete Structures

UPID : 200050

Time Allotted : 3 Hours *Full Marks :70*

The Figures in the margin indicate full marks.
Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer any ten of the following : [1 x 10 = 10]

(I) Composition of Mappings is ______________ but not ________________ in general.

(II) Which rule of inference is used in deriving the conclusion: "If it is Sunday, then the Mall will be crowded. It is Sunday. Thus, the Mall is crowded."

(III) The maximum number of diagonals can be drawn in a hexagon is ______.

(IV) What is the identity element in the group G = {2, 4, 6, 8} under ordinary multiplication modulo 10?

(V) The sum of the out-degrees of all the vertices in a digraph is 20. Then the number of edges in the graph is ____.

(VI) If set A and B have 2 and 5 elements respectively, then the number of subsets of set (A X B) is ______.

(VII) If A = {{Φ}, {Φ, {Φ}}}, then power set of A has ______ elements.

(VIII) Let P and Q be propositional symbols such that P is equivalent to Q. Then ________ will be tautology.

(IX) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is ________.

(X) The set of integer modulo *n* forms a field if *n* is _____ number.

(XI) If *F1, F2* and *F3* be three propositions such that both of the following two propositions are Tautologies:
$(F1 \wedge F2) \rightarrow F3 \quad and \quad F1 \wedge (F2 \rightarrow F3)$
Then given the following statements based on the above two propositions, which one is correct?
a: Both F1 and F2 are Tautologies but F3 is Contradiction
b: Both F1 and F2 are Contradictions
c: F1 is Tautology but F2 is Contradiction
d: F1 is Contradiction but F2 is Tautology

(XII) If *T(n)* be the time to *recursively* calculate the factorial of a integer number *n>1*, then *T(n)* must satisfy the recurrence relation _________

Group-B (Short Answer Type Question)

Answer any three of the following : [5 x 3 = 15]

2. $Prove\ that\ A-(B\cup C)=(A-B)\cap(A-C)$ [5]

3. Construct the truth table of the following proposition: [5]
$(P\vee Q)\wedge(\neg P\wedge(\neg P\wedge Q))$

4. *Check the validity of the following argument (H1: 1st Premises, H2: 2nd Premises, C: Conclusion):* [5]
$H1: P \rightarrow (Q \rightarrow R);\ \ H2: P\wedge Q;\ \ C: R$

5. $If\ (R,+,.)\ is\ a\ Ring\ such\ that\ a^2 = a\ \ \forall a \in R,\ prove\ that$ [5]
$(i)\ a+a=0\ \ \forall a\in R$
$(ii)\ a+b=0 \Rightarrow a=b$

6. $Solve\ the\ recurrence\ relation:$ [5]
$a_n + 3a_{n-1} + 3a_{n-2} + a_{n-3} = 0;\ a_0 = 1, a_1 = -2, a_2 = -1$

Group-C (Long Answer Type Question)

Answer any three of the following : [15 x 3 = 45]

7. (a) $Let\ f:\mathbb{R}\rightarrow\mathbb{R}\ and\ g:\mathbb{R}\rightarrow\mathbb{R}\ be\ defined\ by f(x)=x^2+3, g(x)=x+6.\ Then\ find\ f\circ g\ and\ g\circ f.$ [5]

(b) Show that the following relation R defined on Z is symmetric, transitive but not reflexive. R = {a, b}: a, b ∈ Z and ab>0. [5]

(c) Show that the following function g is neither surjective nor injective: $g:\mathbb{R}\rightarrow\mathbb{R}\ defined\ by\ g(x)=x^2, x\in\mathbb{R}$ [5]

8. (a) $Without\ truth\ table,\ prove\ that\ P\wedge(P\vee Q)\equiv P$ [5]

(b) *Prove that the following argument is valid:* [5]

$P \vee Q,\ P \to R,\ Q \to R \vdash R$

(c) *Without using truth table, prove that the following proposition is a Tautology:* [5]

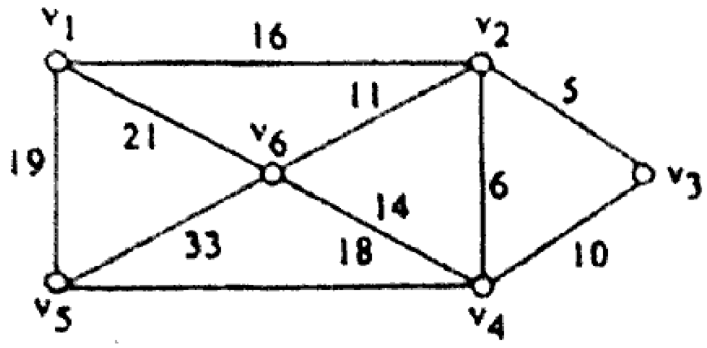
$(P \wedge Q) \to (P \to Q)$

9. (a) *For any three sets A,B,C, show that* $A-(B-C)=(A-B)\cup(A\cap C)$ [5]

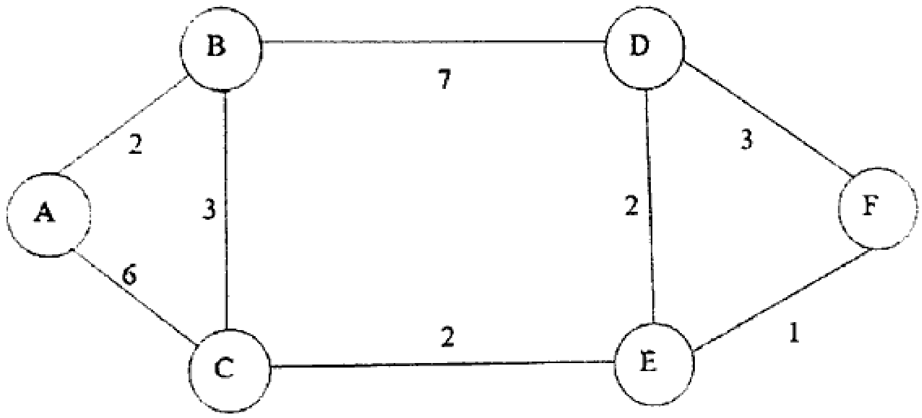
(b) *How many numbers must be selected from the set {1, 2, 3, 4} to guarantee that at least one pair of these numbers add up to 7?* [5]

(c) *If A,B are non − empty sets, then prove that* $(A-B)$ *and* $(A\cap B)$ *are pairwise disjoint.* [5]

10. (a) Find the minimal spanning tree (MST) of the following graph using Kruskal's Algorithm and also calculate the weight of the MST: [5]



(b) Using Dijkstra Algorithm, find the shortest path between A to F in the following graph and also calculate the length of the shortest path: [5]



(c) Given the following distance matrix of an weighted graph, find the minimal spanning tree (MST) using Prim's Algorithm and also determine the weight of the MST (Distance ∞ means no direct edge between the vertices): [5]

Distance	A	B	C	D	E	F	G
A	0	12	∞	∞	14	∞	20
B	12	0	12	10	6	∞	∞
C	∞	12	0	4	∞	∞	∞
D	∞	10	4	0	∞	6	∞
E	14	6	∞	∞	0	6	8
F	∞	∞	∞	6	6	0	4
G	20	∞	∞	∞	8	4	0

11. (a) If G is a simple graph with n vertices and k components, prove that G can have atmost $\frac{(n-k)(n-k+1)}{2}$ number of edges. [9]

(b) Prove that the number of internal vertices in a binary tree is one less than the number of pendant vertices. [3]

(c) If δ(G) and Δ(G) be the min degree and max degree of an(p,q)graph respectively, prove that $\delta(G) \le \frac{2q}{p} \le \Delta(G)$ [3]

*** END OF PAPER ***