



ABV- Indian Institute of Information Technology & Management, Gwalior

Semester I (2023-24)

Major

Course Title: Engineering Mathematics-I

Course Code: ES102

MM: 60 – 65

Duration: 180 minutes

Note:

1. All parts of a question should be answered consecutively.
2. The question paper has six questions in two pages.
3. Questions no. 1 b) and 5 b) are open questions, and the marks will be purely based on the justification and Mathematical explanations.
4. Question No. 3 c), d), 4 a): Marks are only for the proper justification.
5. Question no. 3 b): you must show that the example is a commutative ring with identity but not a field

6

1. a) Solve the following system using LU-Decomposition
 $2x_1 + 3x_2 + x_3 = 9; x_1 + 2x_2 + 3x_3 = 6; 3x_1 + x_2 + 2x_3 = 8$ (5)
 b) How can we construct symmetric and skew symmetric matrix from any random real square matrix (2-5)
 c) State Cayley Hamilton Theorem (CHT) (1)
 d) verify CHT for the following matrix using CHT and also find its inverse using CHT

$$\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$$
 (6)
2. a) Transform the following quadratic form to canonical form
 $3x_1^2 - 2x_2^2 - x_3^2 - 4x_1x_2 + 12x_2x_3 + 8x_1x_3$ (7)
 b) Fill in the blanks and prove: The _____ vectors of a _____ matrix forms a unitary system. (3)

on b/w 2 vector spaces.
3. a) Define Isomorphism ~~between two~~ linear transformation. (2)
 b) Give an example of a finite commutative ring with identity but not a field and prove your claim. (2.5)
 c) Is the vector $(0, 4, -4, 2)$ in the subspace of R^4 spanned by the vectors $(1, 1, 1, 1), (1, 1, 1, -1), (1, 1, -1, 1), (1, -1, 1, 1)$? (2.5)
 d) Find out the vector in R^4 which is not in the subspace spanned by the vectors $(1, 1, 1, 1), (1, 1, 1, -1), (1, 1, -1, 1), (1, -1, 1, 1)$ (2.5)
4. a) Which of the following are linear transformations from R^2 to R^2 (2)
 i) $T(x, y) = (x + 11, y + 23)$, ii) $T(x, y) = (x + 11y, 0)$, iii) $T(x, y) = (x^2, y)$

- b) Let F be a subfield of the complex numbers and let T be the function from F^3 to F^3 defined by $T(x, y, z) = (x - y + 2z, 2x + y, -x - 2y + 2z)$
- i) check whether T is a Linear Transformation
- ii) If (a, b, c) is a vector in F^3 , what are the conditions on a, b, c that the vector be in the range of T ?
- iii) what are the conditions on a, b, c that the vector (a, b, c) be in the nullspace of T ? (2+1.5+1.5)
5. a) Find the spanning set for the null space of the matrix
- $$\begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix} \quad (3)$$
- b) Define Vector space, Construct your own example of a Vector space and discuss any one subspace of the identified vector space. (4.5-6.5)
6. a) Find $\lim_{(x,y) \rightarrow (1,2)} (x^2y^3) - x^3y^2 + 3x + 2y$ (2)
- b) Find out $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ for the following function using limiting conditions (limit definition of derivative) $f(x, y) = 17x^2 + 21xy - 4y^2 + 13x - 23y + 51$ (1.5+1.5)
- c) Find out the absolute maximum and absolute minimum for the following,
 $f(x, y) = x^2 - 2xy + 4y^2 - 4x - 2y + 24$, where $0 \leq x \leq 5$ and $0 \leq y \leq 3$ (7)

End of Question Paper
 Best of Luck