VIT

Final Assessment Test – November 2019

Course: MAT3004 - Applied Linear Algebra

Class NBR(s): 0344 / 0510 / 0512 / 0513 / 2730

Slot: A1+TA1+TAA1+V1

Time: Three Hours

Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

SEARCH VIT QUESTION PAPERS ON TELEGRAM TO JOIN

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1\. a) Determine if the following system is consistent: $y - 4z = 8$; $2x - 3y + 2z = 1$; $5x - 8y + 7z = 1$. [10]

b) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$, using Gauss-Jordan elimination process. [10]

2\. a) Determine whether the polynomials $p_1 = 1 - x$, $p_2 = 5 + 3x - 2x^2$, $p_3 = 1 + 3x - x^2$ are linearly dependent or independent in P_2. [10]

b) Determine whether the vectors $v_1 = (1,1,2)$, $v_2 = (1,0,1)$, $v_3 = (2,1,3)$ span the vector space R^3. [10]

c) Show that the matrices $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $M_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, $M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ form a basis for the vector space M_{22} of 2×2matrices. [10]

3\. a) Let $U = \{(x_1, x_2, x_3, x_4, x_5): 2x_1 - x_2 - x_3 = 0 = x_4 - 3x_5\}$, $V = \{(x_1, x_2, x_3, x_4, x_5): x_3 + x_4 = 0\}$ be two subspaces of R^5. Find the dimensions of $U, V, V + W$ and $V \cap W$. [10]

U, V

b) (i) With $= \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$, let $u = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$. Determine if u is in null space of A. [5]

$Ax = 0$

(ii) Could a 6×9 matrix have a two-dimensional null space. [5]

4\. a) Let $T: P_2(\mathbb{R}) \to M_{2\times2}(\mathbb{R})$ be defined by $T(p) = \begin{bmatrix} p(0) & p(1) \\ p'(0) & p'(1) \end{bmatrix}$. Find the matrix associated to T with respect to the standard bases $\beta = \{1, x, x^2\}$ and $\gamma = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$ of $P_2(\mathbb{R})$ and $M_{2\times2}(\mathbb{R})$ respectively. [10]

b) Let $T: R^4 \to R^3$ be the linear transformation defined by $T(x, y, z, u) = (x + 2y, x - 3z + u, 2y + 3z + 4u)$. Let α and β be the standard bases for R^4 and R^3 respectively. Find $[T]_\alpha^\beta$ [10]

5\. a) Apply the Gram-Schmidt process to transform the basis vectors $v_1 = (1,1,1)$, $v_2 = (0,1,1)$, $v_3 = (0,0,1)$ into an orthogonal basis $\{u_1, u_2, u_3\}$ and then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{w_1, w_2, w_3\}$. [1(

b) Prove that if $x_1, x_2, \dots, x_k$ are nonzero mutually orthogonal vectors in an inner product space V, then they are linearly independent. [

c) Let U be a subspace of an inner product space V, and let $x \in V$. Then, prove that the orthogonal projection $Proj_U(x)$ of x satisfies $\|x - Proj_U(x)\| \le \|x - y\|$ for all $y \in U$.

$-2/3 \; x^2/3$ $\frac{4}{6}$ $\frac{3-2}{3}$ $\frac{1}{3}$

6. a) Find all least square solutions x in $\mathbb{R}^3$ of $Ax = b$ and then determine the orthogonal projection b_c of b into the column space $C(A)$, where [10]

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & -1 \\ -1 & 1 & 2 \\ 3 & -5 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$x = y + z$

$x = y$

z

b) Find the point on the plane $x - y - z = 0$ that is closest to $(1, 2, 0)$. [10]

7. a) Decode 9,45,37,13,53,41,27,60,48,15,42,27 using $A \leftrightarrow 1, B \leftrightarrow 2, C \leftrightarrow 3, \ldots, Z \leftrightarrow 26$, space $\leftrightarrow 27$ and $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. [10]

b) Find a polynomial whose graph passes through the points $(1,3), (2,-2), (3,-5), (4,0)$. [10]

- QR
- LU
- least
- Gauss
- Poly