

Maximum Time: 2 Hours

Max marks: 30

Attempt all the questions

1. (a) Find Laplace transform of the following functions

$$f(t) = \frac{\cos \sqrt{t}}{\sqrt{t}}$$

$$f(t) = \sin 2t \delta(t - \pi/4) + \frac{\pi}{2t} \delta(t - \pi/2).$$

(5)

- (b) Find the solution of the following integral equation

$$y(t) = 1 + t + \int_0^t (t - u)y(u)du.$$

(5)

2. (a) Solve the following differential equation

$$y'' + y = t \cos 2t, \quad y(1) = 0, \quad y'(1) = 0.$$

(5)

- (b) Find the Fourier series expansion of

$$f(x) = x^2, \quad -2 \leq x \leq 2,$$

and prove that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}.$$

(5)

- (a) Discuss Gibbs phenomena of the following function

$$f(x) = \begin{cases} 1, & -\pi < x < 0, \\ -1, & 0 \leq x < \pi. \end{cases}$$

or

(5)

Solve the following heat equation

$$u_t(x, t) = u_{xx}(x, t), \quad 0 < x < \infty,$$

$$u(x, 0) = 0, \quad \text{when } x > 0,$$

$$\frac{\partial u}{\partial x} = -\mu, \quad \text{when } x = 0,$$

$$u, \frac{\partial u}{\partial x} \rightarrow 0, \quad \text{as } x \rightarrow \infty, t > 0.$$

(5)

- (b) Find the inverse Fourier transform of $(\sqrt{\pi} \omega e^{-\omega^2/8})(4\sqrt{2}i).$

Best wishes