



Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech (CSE/IT)/SEM-4/M-401/2010

2010

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The generators of the cyclic group $(\mathbb{Z}, +)$ are

- | | |
|------------|--------------|
| a) $1, -1$ | b) $0, 1$ |
| c) $0, -1$ | d) $2, -2$. |

ii) The mapping $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x|$, $x \in \mathbb{R}$ is

- | | |
|--------------|-------------------|
| a) Injective | b) Surjective |
| c) Bijective | d) None of these. |

iii) Let S be a finite set of n distinct elements. The number of bijective mapping from S to S is

- | | |
|----------|-------------------|
| a) n^2 | b) $n!$ |
| c) 2^n | d) None of these. |



iv) If three Boolean variables x , y and z are defined on Boolean Algebra B , then which one of the following is a fundamental product ?

- a) $xy'z$ b) xy
c) $xy(x+y)$ d) none of these.

v) If G is binary tree on n vertices, the G has edges

- a) $n(n-1)$ b) $n-1$
c) n d) $\frac{n(n-1)}{2}$.

vi) Solution of the recurrence relation $S_n = 2S_{n-1}$ with $S_0 = 1$ is $S_n =$

- a) 2^n b) 2^{n-1}
c) 2^{n+1} d) none of these.

vii) A complete graph is

- a) regular b) connected simple
c) circuit d) planar graph.

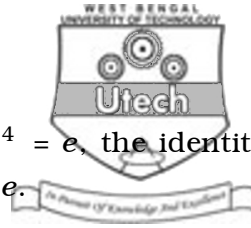
viii) On the set $A = \{1, 2, 3\}$, the relation

$R = \{(2, 1), (1, 2), (3, 3)\}$. Then R is

- a) symmetric b) reflexive
c) transitive d) not a relation at all.

ix) In the additive group Z_6 the order of the element $[4]$ is

- a) 0 b) 2
c) 3 d) 6.



4. Let G be a group, if $a, b \in G$ such that $a^4 = e$, the identity element of G and $ab = ba^2$, prove that $a = e$.
5. Prove that every cyclic group is an Abelian group.
6. Show that the mapping $F : (Z, \bullet) \rightarrow (R, \bullet)$ defined by $f(x) = x^2 \forall x \in Z$ is a monomorphism but not isomorphism.
7. If in a ring R with unity, $(xy)^2 = x^2 y^2 \forall x, y \in R$, then show that R is a commutative.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Examine whether the following two graphs are isomorphic.

Dia.

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- b) Draw the dual of the graph.

Dia.

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- c) Determine the adjacency matrix of the following di-graph :

Dia.

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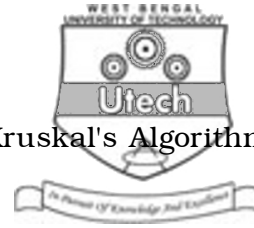
9. a) Construct a simple logic circuit which would satisfy the truth table.

x	y	f
1	1	1
0	1	1
1	0	0
0	0	1

5

- b) Prove that a graph G has a spanning tree if and only if G is connected.

5



- c) Find the minimal spanning tree by Kruskal's Algorithm from the following graph G :

Dia.

5

10. a) Consider the lattice $L = \{ 1, 2, 3, 4, 6, 12 \}$, the divisors of 12 ordered by divisibility. Find the lower and upper bound of L . Is L a complemented lattice ?

5

- b) For any Boolean Algebra, show that.

$$(xy' + xz') + x' = (x' + y + z)(x' + y + z')(x' + y' + z').$$

5

- c) Using generating function solve the recurrence relation,

$$a_n - 7a_{n-1} + 10a_{n-2} = 0, \text{ for } n > 1 \text{ and } a_0 = 3,$$

$$a_1 = 3.$$

5



11. a) Prove that the number of vertices in a binary tree is always odd. 5
- b) Find the truth table of the Boolean function
- $$f = z'xy + xy' + y. \quad 5$$
- c) Prove that a complete graph with n vertices consist of $\frac{n(n-1)}{2}$ number of edges. 5
12. a) Prove that the identity elements and the inverse of an element in a group is unique. 5
- b) Prove that in a group $(G, *)$, $(a * b)^{-1} = b^{-1} * a^{-1}$. 5
- c) Prove that the set of matrices
- $$H = \left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in R, x \neq 0 \right\} \text{ forms a normal}$$
- subgroup of $GL(2, R)$, the group of all real non-singular 2×2 matrices under multiplication. 5
13. a) Using Ford-Fulkerson's algorithm, find the maximum flow in the following network :

Dia.

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b) Using Floyd's algorithm, find the shortest path between

i) w_2 and w_6

ii) w_1 and w_6 .

Dia.

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