

ODD SEMESTER EXAMINATION, 2024 – 25

2nd Year (III Sem) B.Tech. E&CE

Advanced Applied Mathematics/Transformation & Numerical Method

Duration: 3:00 hrs

Max Marks: 100

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	Answer any two parts of the following. <div>(10x2= 20)</div> <div>a) (i) Find the Laplace transform of $f(t) = t \cos at$<div>(5 marks)</div><div>(ii) Find the Laplace transform of $f(t) = t e^{-t} \sin 3t$<div>(5 marks)</div></div><div>b) Using Convolution theorem find the inverse Laplace transform of $f(s) = \frac{1}{(s^2+1)(s^2+9)}$<div>(10 marks)</div></div><div>c) Using Laplace transform to solve the differential equation $\frac{d^2x}{dt^2} - 2 \frac{dx}{dt} + x = e^t$ with $x(0) = 2$ and $x'(0) = -1$<div>(10 marks)</div></div></div>										
Q 2.	Answer any two parts of the following. <div>(10x2= 20)</div> <div>a) (i) Find the Fourier transform of:<div>$f(x) = \begin{cases} 1, & x < 1 \\ 0, & x > 1 \end{cases}$<div>(5 marks)</div><div>(ii) Find the Fourier transform of:<div>$f(x) = \begin{cases} 1 - x^2, & x \leq 1 \\ 0, & x > 1 \end{cases}$<div>(5 marks)</div></div></div><div>b) Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$. Hence derive Fourier sine transform of $g(x) = \frac{x}{1+x^2}$<div>(10 marks)</div></div><div>c) Using finite Fourier transform, solve $\frac{\partial v}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0$, $u(4, t) = 0$ and $u(x, 0) = 2x$ where $0 < x < 4$, $t > 0$.<div>(10 marks)</div></div></div></div>										
Q 3.	Answer any two parts of the following. <div>(10x2= 20)</div> <div>a) (i) Solve the equation $f(x) = x^3 - x - 1 = 0$ using the Newton-Raphson method. Perform three iterations starting with the initial guess $x_0 = 1.5$<div>(5 marks)</div><div>(ii) Compute the value of $f(x)$ for $x = 2.5$ from the following table:<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$f(x)$</td><td>1</td><td>8</td><td>27</td><td>64</td></tr></table><div>Using Lagrange's interpolation formula.<div>(5 marks)</div></div></div></div>	x	1	2	3	4	$f(x)$	1	8	27	64
x	1	2	3	4							
$f(x)$	1	8	27	64							

b) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places. (10 marks)

c) Using Newton's forward difference interpolation formula, find the polynomial for the given data: (10 marks)

x	0	1	2	3
$f(x)$	1	2	5	12

Q 4. Answer any two parts of the following. (10x2= 20)

a) (i) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using simpson's $1/3^{rd}$ rule. (5 marks)

(ii) Evaluate $\int_0^2 \frac{dx}{1+x}$ by using Trapezodial rule. (5 marks)

b) Use Runge-Kutta method to find $y(1.2)$ in step size $h=0.1$ given that $\frac{dy}{dx} = x^2 + y^2$ with $y(1)=1.5$. (10 marks)

c) Given $\frac{dy}{dx} = \frac{1}{x+y}$ with $y(0) = 2$, $y(0.2) = 2.0933$, $y(0.4) = 2.1755$, $y(0.6) = 2.2493$, find $y(0.8)$ by Milne's predictor corrector method. (10 marks)

Q 5. Answer any two parts of the following. (10x2= 20)

a) (i) By the method of least squares, find a straight line that best fits the following data points.

x	0	1	2	3	4
y	1.0	2.9	4.8	6.7	8.6

(5 marks)

(ii) The regression equations calculated from a given set of observations for two random variables are $x = -0.4y + 6.4$ and $y = -0.6x + 4.6$ Calculate \bar{x}, \bar{y} and r . (5 marks)

b) Calculate coefficient of correlation from the following data:

x	1	3	5	7	8	10
y	8	12	15	17	18	20

(10 marks)

c) Fit a second degree parabola to the following data:

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

(10 marks)