VIT

Vellore Institute of Technology

Continuous Assessment Test – I, FALL 2019-20

Programme Name & Branch: B.Tech (ECE)

Course Code & Name: ECE2005 Probability theory and Random Process

Slot: C1 **Exam Duration:** 90 mins **Maximum Marks:** 50

General instruction(s):

Permit to use Normal probability density and distribution table

Answer all Questions (5X10=50 marks)

1. Consider a pair of random variables X and Y that are uniformly distributed over the unit circle so that (10M)

$$f_{X,Y}(x,y)=\begin{cases}C, x^2+y^2=1\\ 0, otherwise\end{cases}$$

i). Find the C Value.

ii). Find the marginal density functions of X and Y.

iii). Are X and Y statically independent?

2. If $X_1, X_2, X_3, \ldots, X_n$ are Poisson variates with parameter $b = 2$. Use the Central Limit Theorem to estimate $P(120 \le S_n \le 160)$ where $S_n = X_1 + X_2 + X_3 + \cdots + X_n$ and $n=75$. (10M)

3. a) Define new random variables V and W by (5M+5M)

$$V = X + aY$$
$$W = X - aY$$

Where a is a real number and X and Y are random variables. Determine a in terms of moments of X and Y such that V and W are orthogonal.

b) Let $f_{XY}(x,y)=16e^{-4(x+y)}u(x)u(y)$. Find the Joint characteristic function for the random variables X and Y.

4. a). Let X and Y be independent uniform random variables over (0,1). (5M+5M)
Find the probability density function of Z=XY

b). Statistically independent random variables X and Y have moments $m_{10} = 2, m_{20} = 14, m_{02} = 12$ and $m_{11} = -6$. Find μ_{22}