Faculty of Engineering & Technology

First Semester B.Tech. (Applied Science Humanities)/AI/AI & DS/AI & ML/Robotics & AI (NEP) 2024-25 Examination

BASIC CALCULUS & DIFFERENTIAL EQUATIONS

Time Three Hours

[Maximum Marks: 70

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INSTRUCTIONS TO CANDIDATES

- (1) All questions carry marks as indicated.
- (2) Solve Question No. 1 OR Question No. 2.
- (3) Solve Question No. 3 OR Question No. 4.
- (4) Solve Question No. 5 OR Question No. 6.
- (5) Solve Question No. 7 OR Question No. 8.
- (6) Solve Question No. 9 OR Question No. 10.
- (7) Use of non-programmable calculator is permitted.
- 1. (a) If $y = (x^2 1)^n$ then prove that :

$$(x^2 - 1)y_{m+2} + 2(m+1-n)xy_{m+1} + (m-2n)(m+1)y_m = 0.$$

(b) Evaluate:

(i)
$$\lim_{x\to 0} x \tan\left(\frac{\pi}{2} - x\right)$$

(ii)
$$\lim_{x\to 0} \left(\cot x - \frac{1}{x}\right)$$
.

OR

(a) Given $f(x) = x^3 + 8x^2 + 15x - 24$. Find the value of $f\left(\frac{11}{10}\right)$ by using Taylor's theorem.

(b) Evaluate:

(i)
$$\lim_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$$

(ii)
$$\lim_{x \to x} \left(\frac{1}{x}\right)^{\frac{1}{x}}$$
.

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3 (a) Prove that
$$\frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}^2} + \frac{\partial^2 \mathbf{u}}{\partial \mathbf{y}^2} + \frac{\partial^2 \mathbf{u}}{\partial \mathbf{z}^2} = 0$$

where
$$u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$$

(b) If
$$u = tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$$
 then prove that

$$x^{2} \frac{\partial^{2} u}{\partial x^{2}} + 2xy \frac{\partial^{2} u}{\partial x \partial y} + y^{2} \frac{\partial^{2} u}{\partial y^{2}} = \sin 2u (1 - 4\sin^{2} u)$$

OR

4. (a) If
$$u = \frac{x+y}{1-xy}$$
, $v = \tan^{-1}x + \tan^{-1}y$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.

State whether u & v are functionally related. If so find the relation between them

- (b) Prove that the rectangular solid of maximum volume which can be inscribed in a given sphere is a cube.
- (a) Investigate the value of λ and μ so that the system of equations x + y + z = 6, x + 2y + 3z = 10, $x + 2y + \lambda z = \mu$ have (i) no solution, (ii) unique solution, (iii) an infinite solution.
- (b) Find the eigen values and corresponding eigen vectors for the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & 2 & 0 \end{bmatrix}$

OR

- 6. (a) Investigate the linear dependence of vectors $X_1 = (1, 2, 4), X_2 = (2, -1, 3), X_3 = (0, 1, 2)$ and $X_4 = (-3, 7, 2)$. Find the relation if possible. https://www.rtmnuonline.com 7
 - (b) By using Cayley Hamilton's theorem find the matrix represented by $A^8 5A^7 + 7A^6 3A^5$

$$+ A^4 - 5A^3 + 8A^2 - 2A + I \text{ where } A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

(a) Solve
$$\sqrt[4]{\frac{dy}{dx}} + y = 1 - y$$
.

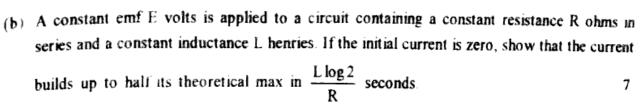
(b) Solve:
$$\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y(\log y)^2}{x^2}.$$

OR

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$$g = (a)$$
 Solve $ye^{xy} dx + (xe^{xy} + 2y)dy = 0$.



9. (a) Solve:
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = -37\sin 3x$$
.

(b) Solve
$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin(e^x)$$
 by method of variation of parameters.

OR

10. (a) Solve:
$$x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + 5y = 10 - \frac{4}{x}$$
.

(b) An emf E sin pt is applied at t = 0 to a circuit containing a capacitance C and inductance L. The current i satisfies the equation $L\frac{di}{dt} + \frac{1}{C}\int idt = E \sin pt$. If $p^2 = \frac{1}{LC}$ and initially the current i and the charge q are zero. Show that the current at time t is $\frac{Et}{2L} \sin pt$, where

$$i = \frac{dq}{dt}$$
.



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