VIT

Vellore Institute of Technology

Continuous Assessment Test –II

Programme Name & Branch: B.Tech all **Exam Duration: 90 mins**

Slot: A1+TA1+TAA1

Semester: Winter 2019-2020 **Maximum Marks: 50**

Course Code: MAT3004 **Course Title: Applied Linear Algebra**

Exam Type: Closed book

Answer any five questions ($5 \times 10 = 50$ Marks)

1. Let V and W be the subspaces of the vector space $\mathbb{R}^4$ spanned by $v_1 = (3,-1,4,1)$, $v_2 = (5,0,5,1)$, $v_3 = (5,-5,10,3)$ and $w_1 = (9,-3,3,2)$, $w_2 = (5,-1,2,1)$, $w_3 = (6,0,4,1)$, respectively. Find the bases and dimensions for $V + W$ and $V \cap W$, and hence prove that $dim(V+W) = dim(V) + dim(W) - dim(V \cap W)$. [10]

2. (a) Let A and B be two $n \times n$ matrices. Show that if $AB = 0$, then the column space of B is a subspace of the null space of A. [5]

   (b) Find the right inverse of the matrix $\begin{bmatrix} 6 & 4 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, if exists. [5]

3. (a) Find a cubic parametric curve that passes through the points $(0,0), (2,2), (0,3)$ and $(2,4)$. [5]

   (b) If $T: \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation defined as $T(x,y,z) = (x+2y, y+4z, x+3y+4z)$, find the Kernel of T. [5]

4. (a) Let V and W be two vector spaces over $\mathbb{R}$. Prove that if V and W are isomorphic, then $dim\, V = dim\, W$. [5]

   (b) Find the transition matrix from the standard ordered basis α to another basis β for $\mathbb{R}^3$, where $\beta = \{(1,1,0), (1,1,1), (0,1,1)\}$. [5]

5. Let $T: P_2(\mathbb{R}) \to P_1(\mathbb{R})$ be a transformation defined as: $T(f(x)) = f'(x)\ \forall f(x) \in P_2(\mathbb{R})$, and $S: P_1(\mathbb{R}) \to P_2(\mathbb{R})$ be transformation defined as: $S(g(x)) = xg(x)\ \forall g(x) \in P_1(\mathbb{R})$. Prove that S and T are linear transformations and also find the matrix representation of ToS w.r.t. standard bases $\alpha = \{1, x, x^2\}$ of $P_2(\mathbb{R})$ and $\beta = \{1, x\}$ of $P_1(\mathbb{R})$, i.e., find $[ToS]_\beta$. [10]

6. Find the general formula for $T: \mathbb{R}^3 \to \mathbb{R}^3$, if $T(1,0,1) = (1,2,0)$, $T(1,-2,1) = (0,1,0)$ and $T(0,0,1) = (0,2,-1)$. Also find T(2,-3,1). [10]

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$B = A^T(AA^T)^{-1}$