

DEPARTMENT OF MATHEMATICS
NATIONAL INSTITUTE OF TECHNOLOGY KURUKSHETRA

End – Semester Examination (May 2024)

MAIC 102: Integral Calculus and Difference Equations

B.Tech 2nd Semester Max. Marks: 50 Duration: 180 Minutes Date: May 08, 2024

(Answer all the questions. Calculator is not allowed)

1. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials [5]
2. Evaluate the double integral $\int_0^{\frac{\pi}{4}} \int_{2y}^{\frac{\pi}{2}} \frac{\cos(x)}{x} dx dy$. [5]
3. Using Beta function, find the exact value of $\int_0^1 \sqrt[11]{1 - \sqrt[3]{x}} dx$. [5]
4. Find the directional derivative of $\text{div}(\text{grad}(\phi))$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$, where $\phi = 2x^3y^2z^4$. [6]
5. Use the Divergence theorem to evaluate $\iint_S (x dy dz + y dz dx + z dx dy)$ where S is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octant. [5]
6. Solve the difference equation $y_{n+2} - 2y_{n+1} + y_n = n^2 2^n$ using complementary function and particular integral method. [6]
7. If $0, 1, 1, 2, 3, 5, 8, 13, \dots$ represents Fibonacci sequence, find the n^{th} term of the sequence using Z-transform. [5]
8. Find the Fourier series expansion for $f(x) = x + \frac{x^2}{4}$, $-\pi \leq x \leq \pi$. [8]
9. Expand $f(x) = e^x$ in half-range cosine series over $(0, 1)$. [5]

_____ * * * _____