CHENNAI INSTITUTE OF TECHNOLOGY

(An Autonomous Institution, Affiliated to Anna University, Chennai)

CHENNAI - 600 069

B.E. / B.Tech. DEGREE END SEMESTER EXAMINATIONS

APR / MAY 2025

Second Semester

MA4201 – PROBABILITY AND STATISTICS

(Common to ALL Branches)

(Regulations 2024)

(Normal, t, F, and Chi-square tables are permitted)

Time: Three Hours | Maximum Marks:

Answer ALL Questions

RBT Level : L1- Remembering, L2 – Understanding, L3 – Applying, L4 – Analyzing, L5 – Evaluating, L6 – Creat

PART – A **(10x2=20 Marks)**

1. If an experiment has the three possible and mutually exclusive outcomes *A, B,* and *C*, check whether the following assignment of probabilities P(A) = 0.35, P(B) = 0.52, and P(C) = 0.26 is permissible.

2. Can the following be treated as Bernoulli trials? Justify.
'Drivers stopped at a roadblock will be checked for failure to wear a seatbelt'.

3. Prove that the covariance of X and Y is zero if X and Y are independent.

4. The joint probability mass function of two random variables X and Y is $P(X,Y)=\begin{cases} k(2x+y), & x=1,2; y=1,2 \\ 0, & otherwise \end{cases}$ where k is a constant. Find the value of k.

5. 'The sum of two independent Poisson processes is a Poisson process'. Prove or disprove.

6. Consider the random process X(t) = cos(t+ϕ), where ϕ is uniformly distributed in the interval $-\pi/2$ to $\pi/2$. Check whether the process is stationary or not.

7. Define Type-I and Type-II errors.

8. Write any two conditions for the validity of Chi-square test.

9. What are the basic principles of experimental design?

10. Why a 2X2 Latin square is not possible?

PART – B (5x16=80 Marks)

11. a) i) A student buys 1000 integrated circuits (ICs) from supplier A, 2000 ICs from supplier B, and 3000 ICs from supplier C. He tested the ICs and found that the conditional probability of an IC being defective depends on the supplier from whom it was bought. Specifically, given that an IC came from supplier A, the probability that it is defective is 0.05; given that an IC came from supplier B, the probability that it is defective is 0.10; and given that an IC came from supplier C, the probability that it is defective is 0.10. If the ICs from the three suppliers are mixed together and one is selected at random, what is the probability that it is defective? Given that a randomly selected IC is defective, what is the probability that it came from supplier A?

ii) The discrete random variable K has the following probability mass function,

$$P_K(k) = \begin{cases} b, & k = 0 \\ 2b, & k = 1 \\ 3b, & k = 2 \\ 0, & otherwise \end{cases}$$

1) What is the value of b?

2) Determine the values of $P[K \leq 2]$ and $P[0 < K < 2]$.

3) Determine the cumulative distribution function of K.

(OR)

b) i) Assume that the length of phone calls made at a particular telephone booth is exponentially distributed with a mean of three minutes. If you arrive at the telephone booth just as Chris was about to make a call, calculate the following.

1) The probability that you will wait more than 5 minutes before Chris is done with the call.

2) The probability that Chris' call will last between 2 minutes and 6 minutes.

ii) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modeled by an N (5; 16) normal random variable, X.

1) What is the probability that a randomly selected parcel weighs between 1 and 10 pounds?

2) What is the probability that a randomly selected parcel weighs more than 9 pounds?

12. a) A fair coin is tossed three times. Let X be a random variable that takes the value 0 if the first toss is a tail and the value 1 if the first toss is a head. Also, let Y be a random variable that defines the total number of heads in the three tosses.

1) Determine the joint probability mass function of X and Y.

2) Are X and Y independent?

3) Find the marginal probability mass functions of X and Y.

4) Find the conditional probability of Y given X.

(OR)

X	9	8	7	6	5	4	3	2	1
Y	15	16	14	13	11	12	10	8	9

ii) If X and Y are independent random variables with joint probability density function f(x, y), find the probability density function of U=X+Y, V=X. CO2

13\. a) Three are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are inter changed. Let the state a_i of the system be the number of red marbles in A after i changes. What is the probability that there are 2 red marbles in A after 3 steps? In the long run, what is the probability that there are 2 red marbles in urn A? CO3

(OR)

b) i) Suppose that a customer arrives at a bank according to Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 min. CO

1) Exactly 4 customers arrive.

2) More than 4 customers arrive.

ii) Prove that the random process $X(t) = A\cos(\omega t + \theta)$ where ω and θ are constants, A is a random variable with zero mean and variance one and θ is uniformly distributed on the interval (0, π). Assume that the random variable A and theta are independent. Is X(t) is mean-ergodic process? CO

14\. a) The following random samples are measurements of the heat-producing capacity of specimens of coal from two mines, CO

Mine 1	8260	8130	8350	8070	8340	
Mine 2	7950	7890	7900	8140	7920	7840

Use the 0.05 level of significance to test whether it is reasonable to assume that the variances of the two populations sampled are equal.

(OR)

b) To determine whether there really is a relationship between an employee's performance in the company's training program and his or her ultimate success in the job, the company takes a sample of 400 cases from its very extensive files and obtains the results shown in the following table, C

		Performance in training program		
		Below average	Average	Above average
Success in job	Poor	23	60	29
	Average	28	79	60
	Very good	9	49	63

Apply Chi-square test to test the null hypothesis that the performance in the training program and success in the job are independent. Use the 0.01 level of significance.

15. a) A manufacturer of paper used for making grocery bags is interested in improving the tensile strength of the product. Product engineering thinks that tensile strength is a function of the hardwood concentration in the pulp and that the range of hardwood concentrations of practical interest is between 5 and 20%. A team of engineers responsible for the study decides to investigate four levels of hardwood concentration: 5%, 10%, 15%, and 20%. They decide to make up six test specimens at each concentration level, using a pilot plant. All 24 specimens are tested on a laboratory tensile tester, in random order. The data from this experiment are shown in the following table, CC

Hardwood concentration (%)	Observations					
	1	2	3	4	5	6
5	7	8	15	11	9	10
10	12	17	13	18	19	15
15	14	18	19	17	16	18
20	19	25	22	23	18	20

Use an appropriate analysis of variance to test the hypothesis that different hardwood concentrations do not affect the mean tensile strength of the paper with $\alpha = 0.01$.

(OR)

b) An experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following "cleanness" readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines, C

	Engine 1	Engine 2	Engine 3
Detergent A	45	43	51
Detergent B	47	46	52
Detergent C	48	50	55
Detergent D	42	37	49

Looking at the detergents as treatments and the engines as blocks, obtain the appropriate analysis of variance table and test at the 0.01 level of significance whether there are differences in the detergents or in the engines.