

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.TECH (CT-OLD)/SEM-3/M (CT)-301/2011-12**

**2011**

**APPLIED MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) If  $f(z) = \frac{\sin z}{z^3}$  then  $z = 0$  is a pole of order 2.

a) True

b) False.

ii) If  $f(z) = \frac{z+1}{z^4-2z^3}$ , then  $z = 0$  is a pole of order

a) 3

b) 2

c) 1

d) 4.

iii) Residue of  $f(z) = \frac{2+3\sin \pi z}{z(z-1)^2}$  at  $z = 0$  is

a) 1

b) 2

c) 3

d)  $i$ .

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d)  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{isx} ds.$

vii) The residue of  $f(z) = \frac{z+1}{z^2-2z}$  at the pole  $z=0$  is

a)  $-\frac{1}{2}$

b) 0

c)  $\frac{1}{2}$

d)  $\frac{3}{2}$ .

viii) The number of poles of

$$f(z) = \frac{z}{(z-1)(z-2)(z-3)}$$
 inside the circle

$$|z-2| = 2$$
 is

a) 3

b) 1

c) 2

d) 0.

ix) The value of the integral  $\oint_C \frac{dz}{z-4}$  where  $C: |z|=1$

is

a)  $2\pi i$

b)  $-2\pi i$

c) 0

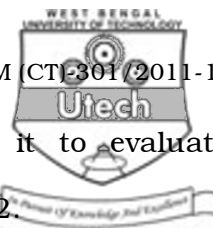
d) none of these.

x) The complete solution of  $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$  is

a)  $z = ax + (1+a)y + c$

b)  $z = ax + (1-a)y + 2$

d)  $z = ax + (1 - a)y + c.$



3. State Cauchy's integral formula and use it to evaluate

$$\int_C \frac{e^{z+1}}{z^2+4} dz \text{ where } C \text{ is the circle } |z-i| = 2.$$

4. Find the value of  $10^{1/2}$  correct up to 4 significant figures using Newton-Raphson method.
5. Using Fourth order Runge-Kutta method, solve

$$\frac{dy}{dx} = x^2 + y^2, \quad y(0) = 1$$

at  $x = 0.2$  using a step length  $h = 0.1$ .

6. Define Fourier cosine transform. Solve the integral equation

$$\int_0^{\infty} f(x) \cos \lambda x dx = e^{-\lambda}.$$

7. Find the complete solution of the following *p.d.e.* :

$$p + q = x.$$

### GROUP – C

#### ( Long Answer Type Questions )

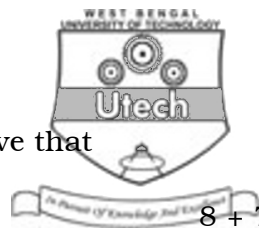
Answer any *three* of the following questions.

$$3 \times 15 = 45$$

8. a) Obtain the Fourier series to represent  $x^2$  in  $-\pi \leq x \leq \pi$ .

Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}.$$



- b) Using Cauchy's Residue theorem, prove that

$$\oint_C \frac{z \cos z}{\left(z - \frac{\pi}{2}\right)^3} dz = -2\pi i.$$

8 + 7

9. a) Use Laurent's series to find the residue of  $\frac{e^{2z}}{(z-1)^2}$  at  $z = 1$ .

- b) Expand  $f(z) = \frac{z-1}{z+1}$  as a Taylor's series about  $z = 1$  and determine the region of convergence. 8 + 7

10. a) Prove that  $\int_0^{+\infty} \frac{dx}{x^2 + 1} = \frac{\pi}{2}$ , by using Cauchy's residue theorem.

- b) Find the Fourier series of the function

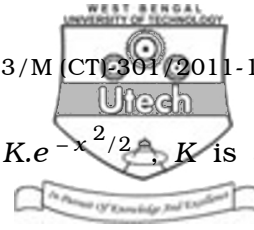
$$f(x) = \begin{cases} \pi + 2x & \text{if } -\pi < x < 0 \\ \pi - 2x & \text{if } 0 \leq x < \pi \end{cases}$$

Hence, deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty = \frac{\pi^2}{8}. \quad 7 + 8$$

11. a) If  $F\{f(x)\} = F(s)$  be the Fourier transform of  $f(x)$ , show that

$$F\{f(x+a)\} = e^{-ias} F(s).$$



b) Find out the Fourier transform of  $K.e^{-x^2/2}$ ,  $K$  is a constant.

c) Show that  $P(\overline{A} / B) = 1 - P(A / B)$ , where  $A$  and  $B$  are any two events. 5 + 6 + 4

12. a) The probability that a teacher will give a surprise test during any class of a particular day is  $\frac{1}{5}$ . If a student is absent, on two days, what is the probability that he will miss at least one test ?

b) If  $f(x) = \frac{1}{4} - Kx$ ,  $0 \leq x \leq 4$   
 $= 0$ , otherwise

is the *p.d.f.* of a random variable  $X$ ,

determine —

i) the value of  $K$  and

ii)  $P(|X - 2| < 0.5)$ .

Also find out the mean of  $X$ .

7 + 8

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