Indian Institute of Information Technology and Management, Gwalior

Mathematics-II (IMAS-1201)

End-Term Examination (Session 2022-23)

Maximum Time: 3 Hours | Max marks: 50

Attempt all the questions

1. (a) Find the Solution of the given PDE

$$z(z^2 + xy)(px - qy) = x^4$$

(2.5

(b) Find the integral surface of the linear PDE

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

which contains the straight line

$$x + y = 0, z = 1$$

(2.5)

2. (a) Find a complete integral of

$$z^2(p^2z^2 + q^2) = 1$$

(5

(b) Change the given PDE into canonical form and then find the solution

$$3\frac{\partial^2 z}{\partial x^2} + 10\frac{\partial^2 z}{\partial xy} + 3\frac{\partial^2 z}{\partial y^2} = 0$$

(5

3. (a) Evaluate

$$\int_0^{1+2i} \bar{z}^2 dz$$

1. along line $y = 2x$
2. along the real axis to 1 and then vertically to 1+2i

(5)

(b) Show that $v(x, y) = \sin x \sinh y$, (where v is imaginary part) is harmonic. Find the harmonic conjugate of v and then find an analytic function. **(5)**

4. (a) Evaluate

1. $\int_c \frac{e^z}{z^2+\pi^2}$, where c is a circle $|z| = 4$.
2. $\int_c \frac{\sin^2 z}{(z-\pi/6)^3}$, where c is a circle $|z| = 1$.

(8)

(b) 1. Find the constants a,b,c where

$$f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + cy^2)$$

is analytic express f(z) in terms of z.

2. Show that the function "Ln z" is not continuous on the negative real axis including the point $x = 0$.

(7)

5. (a) Using Laplace transform, find the solution of the given ODE

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$$

with $x(t) = 2$, $\frac{dx}{dt}(t) = -1$ at $t = 0$. **(5)**

(b) Find the Fourier series expansion of

$$f(x) = x - x^2, \qquad -\pi \leq x \leq \pi,$$

and prove that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots = \frac{\pi^2}{12}.$$

(5)

Best wishes