Final Assessment Test – November 2019

Course: **ECE2005** - **Probability Theory and Random Processes**

Class NBR(s): **0933 / 0941** | Slot: **C2+TC2**

Time: **Three Hours** | Max. Marks: **100**

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. The joint pdf of a bivariate random variable (X, Y) is given by **[10]**

$$f_{X,Y}(x,y) = \begin{cases} k & , \quad 0 < y \le x < 2 \\ 0 & , \quad otherwise \end{cases}$$

Where k is constant

SEARCH VIT QUESTION PAPERS ON TELEGRAM TO JOIN

(a) Determine the value of k

(b) Find the Marginal pdf's of X and Y

(c) Find $P\left(0 < X < \frac{1}{2}, 0 < Y < \frac{1}{2}\right)$

2. Random variables X and Y have the joint density function **[10]**

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{(x+y)^2}{40}; & -1 < x < 1 \quad and \quad -3 < y < 3 \\ 0 \quad ; & else \end{cases}$$

a) Find all the second-order moments of X and Y.

b) What are the variances of X and Y?

c) What is the correlation coefficient?

3. Let X_1 & X_2 be jointly Gaussian random variables where $\sigma^2_{X_1} = \sigma^2_{X_2} = 1$ and $\rho_{X_1X_2} = -1$. Find a rotation transformation matrix such that new random variables Y_1 & Y_2 are statistically independent. **[10]**

4. The random noise voltage X(t) observed at three time instances has the covariance matrix given by **[10]**

$[C_X] = \begin{bmatrix} 3.0 & 1.8 & 1.1 \\ 1.8 & 3.0 & 1.8 \\ 1.1 & 1.8 & 3.0 \end{bmatrix}$. If this is transformed to a new random variables

$$Y_1 = 4X_1 - X_2 - 2X_3$$
$$Y_2 = 2X_1 + 2X_2 + X_3$$
$$Y_3 = -3X_1 - X_2 + 3X_3$$

Find the covariance matrix of the new random variable. Also find $\rho_{y_1y_2}, \rho_{y_1y_3}, \rho_{y_2y_3}$

5. A number of practical systems have square-law detectors that produce an output $W(t)$ that is the square of its input $Y(t)$. Let the detectors output be defined by $W(t) = Y^2(t) = X^2(t)\,\cos^2(\omega_0 t + \theta)$. Where ω_0 is a constant, $X(t)$ is second-order stationary and θ is a random variable independent of $X(t)$ and uniform on $[0, 2\pi]$. **[10]**

Find a) $E[W(t)]$; b) $R_{WW}(t, t+\tau)$; c) Is $W(t)$ wide-sense stationary?

6. A particular commercial system for controlling a petroleum plant has failures (resulting in plant down time) that occur at the average rate of two per 30 days. Assume that the number of failures is a Poisson process and find the probability that one failure will occur during the first 30 days and no other failures will occur for the next 30 days. **[10]**

7. Assume a random process has a spectrum **[10]**

$$S_{XX}(\omega) = \begin{cases} 4 - \frac{\omega^2}{9} & ; |\omega| \le 6 \\ 0 & ; otherwise \end{cases}$$

Find a). The Average power

b). The RMS Bandwidth

8. A pair of noise processes $n_1(t)$ and $n_2(t)$ are related by **[10]**

$$n_2(t) = n_1(t)\cos(2\Pi f_c t + \theta) - n_1(t)\sin(2\Pi f_c t + \theta)$$

Where f_c is a constant and θ is the value of a random variable defined by

$$p_\theta(\theta) = \frac{1}{2\Pi}, 0 \le \theta \le 2\Pi$$

The spectral density of $n_1(t)$ is as shown in fig.1. Find and plot the corresponding spectral density of $n_2(t)$.

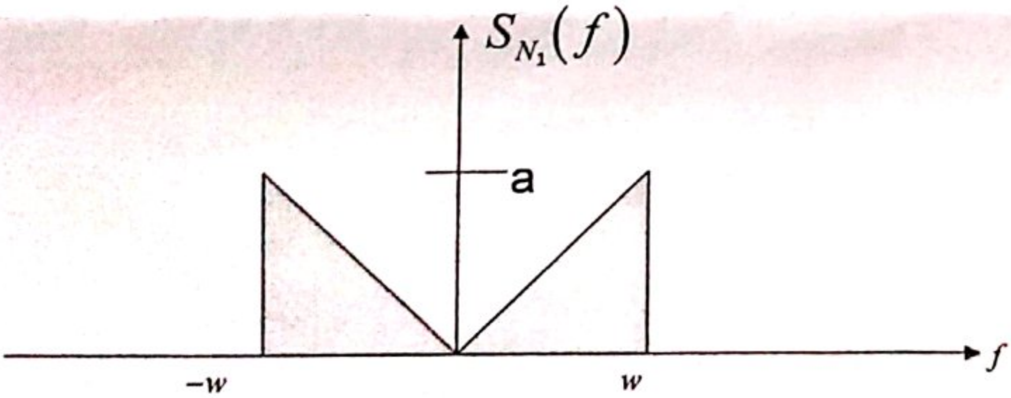

fig.1.

9. A random process X(t) is applied to a network with impulse response $h(t) = u(t)\ e^{-bt}$ where b>0 is a constant. The cross correlation of X(t) with the output Y(t) is known to have the same form $R_{XY}(\tau) = u(\tau)\ e^{-b\tau}$ **[10]**

a). Find the auto correlation of Y(t)

b). What is the average power in Y(t)?

10. The sum of a signal **[10]**

$$x(t) = \begin{cases} W\,t\,e^{-Wt} & ; 0 < t < \frac{2}{W} \\ 0 & ; elsewhere \end{cases}$$

Where $W = 5\times 10^{6}\ rad/sec$ and white noise for which N(t)=$\frac{N_0}{2} = \frac{10^{-8}}{24\pi}$ W/HZ is applied to a matched filter.

a) What is the smallest value of t_0 required for the filter to be causal?

b) For the value of t_0 found in (a), sketch the impulse response of the matched filter

c) Find the maximum output signal-to-noise ratio it provides.

[10]

11. A system's power transfer function is $|H(\omega)|^2 = \frac{16}{[16+\omega^2]}$

a) What is its noise bandwidth?

b) If white noise with power density $6 \times 10^{-3}\ W/Hz$ is applied to the input find the noise power in the system's output.

[6]

12. a) Three networks are cascaded. Available power gains are $G_1 = 8$ (input stage), $G_2 = 6$, and $G_3 = 20$ (output stage). Respective input effective spot noise temperatures are $T_{e1} = 40K, T_{e2} = 100K,$ and $T_{e3} = 280K$. What is the input effective spot noise temperature of the cascade?

[4]

b) Compare the AM transmission with FM transmission from the noise performance perspective.

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