MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BS-M101/BSM101 Mathematics -IA

UPID : 001004

Time Allotted : 3 Hours

Full Marks :70

The Figures in the margin indicate full marks.

Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer *any ten* of the following : [1 x 10 = 10]

(I) The radius of curvature of the parabola $y^2 = 4x$ at its vertex is

(II) If f(x) satisfies the conditions of Rolle's theorem in [a,b], then we get a point on the curve in which the tangent is parallel to....................................

(III) If A is an idempotent matrix, then I-A is..........................

(IV) If $T(x,y,z) = (x,y,0)$ for all $(x,y,z) \in R^3$ is a Linear transformation, then Kernel T is

(a) $(0,0,0)$ (b) x-axis (c) y-axis (d) z-axis

(V) If 4 is an eigen value of the matrix A then the eigen value of the matrix A+kI is...............................

(VI) A function of x and y possessing continuous partial derivatives of the first and second orders is called a harmonic function if it satisfies

(a) Homogeneous equation (b) Laplace equation (c) Lagrange's equation (d) none of these

(VII) If Rolle's theorem is applied for the function $f(x)=x(x^2-1)$ in [0,1], then c=

(VIII) The value of $\begin{vmatrix} 100 & 101 & 102 \\ 105 & 106 & 107 \\ 110 & 111 & 112 \end{vmatrix}$ is

(a) 2 (b) 0 (c) 405 (d) −1

(IX) The eigen value of $A = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$ is

(a) 2,4 (b) 0,4 (c) 0,2 (d) 0,0

(X) The eigen values of the matrix $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ are

(a) $-1,-2$ (b) $1,2$ (c) $3,1$ (d) $-3,-1$

(XI) $\Gamma(m)\Gamma(1-m) =$ (a) $\frac{2\pi}{\sin\pi}$ (b) $\frac{3\pi}{\sin m\pi}$ (c) $\frac{\pi}{\sin m\pi}$ (d) none of these

(XII) If $\lim_{x\to 0}\frac{ae^x - b}{x} = 2$, then

(a) $a=2; b=2$ (b) $a=1; b=1$ (c) $a=0; b=1$ (d) none of these

Group-B (Short Answer Type Question)

Answer *any three* of the following : [5 x 3 = 15]

2. Show that intersection of two subspaces of a vector space V, is a subspace. [5]

3. Determine k so that the set S is linearly dependent in R^3 [5]
   S={ (1,2,1), (k,3,1), (2,k,0)}

4. Define a basis set of a vector space V^3. Show that the set of vectors {(1,-2,3), (2,3,-1), (-1,3,2)} forms a basis of the vector space V^3 over the field of real numbers. [5]

5. Define linearly dependence and independence of vectors. Prove that a set of vectors containing null vector is linearly dependent. [5]

6. Show that W ={ (x,y,z) $\in R^3$ / x+y+z=0} is a subspace of R^3. Find also a basis of W. [5]

Group-C (Long Answer Type Question)

Answer *any three* of the following : [15 x 3 = 45]

7. (a) Prove that $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\log 2$ [5]

(b) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$ about the major axis [5]

(c) Show that $\Gamma(n+1)=n\Gamma(n),\ n>0$ [5]

8. (a) Use mean value theorem, show that $0<\frac{1}{x}\log\frac{e^x-1}{x}<1$, for $x>0$. [5]

(b) Show that if $0<\theta<1$, $a^x = 1 + x\log a + \frac{x^2}{2!}(\log a)^2 + \dots + \frac{x^{n-1}}{(n-1)!}(\log a)^{n-1} + \frac{x^n}{n!}a^{\theta x}(\log a)^n$ [5]

(c) Evaluate $\lim_{x\to 0}\frac{\tan x - x}{x-\sin x}$ [5]

9. (a) Show that $\begin{vmatrix} 1-a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2-b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1-a^2-b^2)^3$ [5]

(b) Expand by Laplace's method,to show that $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af-be-cd)^2$ [5]

(c) Given, $x-4y-2z=1$; $2x-7y-5z=2k$; $4x-ay-10z=2k-1$ [5]
Find for what values of k and a, the system has (i)unique solution (ii) no solution
(iii) many solutions.

10. (a) Show that an orthogonal set of non-null vectors in an inner product space is independent. [5]

(b) Show that A={(5,0,0), (0,3,0), (0,0,1)} is an orthogonal set of R^3. Express r = (2,1,4) as a linear combination of the vectors of A. [5]

(c) Use Gram-Schmidt process to convert the basis {(1,2,-2), (2,0,1),(1,1,0)} of R^3 into an orthogonal basis and then to an orthonormal basis. [5]

11. (a) Determine the eigen vectors of $A=\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ and then diagonalise with the help of basis eigen vectors. [5]

(b) Use Gram Schmidt process to obtain an orthonormal basis of the subspace of the Euclidean space R^4 with standard inner product generated by the linearly independent set $\{(1,1,0,1),(1,1,0,0),(0,1,0,1)\}$ [5]

(c) Find a basis of a real vector space R^3 containing the vectors (1,1,2) and (3,5,2) [5]

*** END OF PAPER ***