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Paper Code : PCC- CS401/PCC-CS401/PCCCS 401/PCCCS401 Discrete Mathematics

UPID : 004407

Time Allotted : 3 Hours *Full Marks :70*

The Figures in the margin indicate full marks.
Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer *any ten* of the following : [1 x 10 = 10]

(I) What is Pendent vertex and Isolated vertex. Give examples.

(II) Define Cantor's Power Set Theorem.

(III) How many ways can you arrange 3 different books on a shelf?

(IV) Give examples of one proposition and one non-proposition statements.

(V) What is the number of elements of order 10 in Z_{10} ?

(VI) Define path for a graph.

(VII) State fundamental theorem of arithmetic.

(VIII) How many ways can 4 people sit in a row if 2 specific people must sit together?

(IX) What is the inverse of p -> ~q ?

(X) What is the domain of the relation R = {(1, 2), (2, 3), (5, 4)}?

(XI) For which m and n does the graph $K_{m,n}$ contain a Euler path and Euler circuit? Explain.

(XII) If A = {1, 2, 3} and B = {2, 3, 4}, then determine the value of A Δ B.

Group-B (Short Answer Type Question)

Answer *any three* of the following : [5 x 3 = 15]

2. Find the number of ways to form a committee of 4 members from a group of 10 people, such that at least 1 member is a woman, given that there are 4 women and 6 men in the group. [5]

3. Let f(x) = x + 2 and $g(x) = x^3$. Then find out f(g(x)) & g(f(x)) . [5]

4. Are the statements, "it will not rain or snow" and "it will not rain and it will not snow" logically equivalent? Justify with truth table. [5]

5. Show that the set G = {1,2,3,4,5,6} form a cyclic group under the operation multiplication modulo 7. Find all generators of this group. [5]

6. Draw and explain the graph coloring problem taking four different colors. What is the chromatic number of a planar graph? [5]

Group-C (Long Answer Type Question)

Answer *any three* of the following : [15 x 3 = 45]

7. (a) Prove that the sum of the degrees of the vertices of any finite graph is even. [5]

(b) Prove that a complete graph with n vertices contains n(n – 1)/2 edges. [5]

(c) Show that every simple finite graph has two vertices of the same degree. [5]

8. (a) Define injective, surjective and bijective mapping with proper examples. [8]

(b) If P = {4, 5, 6, 7, 8} and Q = {6, 7, 8, 10}, is P – Q = Q – P and P U Q = Q U P ? Justify. [4]

(c) If P = {4, 5, 6, 7, 8} and Q = {6, 7, 8, 10}, find P Δ Q. [3]

9. (a) A University offers three elective courses: Machine Learning (ML), Data Science (DS), and Artificial Intelligence (AI). Among 200 students:85 students have enrolled in ML,75 students have enrolled in DS,60 students have enrolled in AI,30 students have enrolled in both ML and DS,25 students have enrolled in both DS and AI,20 students have enrolled in both ML and AI,10 students have enrolled in all three courses.
Using the Principle of Inclusion-Exclusion (PIE), find the number of students who have enrolled in at least one of these courses. [6]

(b) Find the number of students who have enrolled in exactly one course. [7]

(c) Determine the number of students who have not enrolled in any of these courses. [2]

10. (a) Formalize the following sentences: [6]

i). If Paola is happy and paints a picture then Renzo isn't happy.

ii). If Paola is happy then she paints a picture.

iii). Paola is happy only if she paints a picture.

(b) If I work out hard then I am sore. If I am sore, I take an aspirin. I did not take an aspirin. So, I did not work out hard. [5]

Write the consequences in logical language.

(c) What is necessity & sufficiency condition? [4]

11. (a) Show that every group of prime order is cyclic. [5]

(b) If G is a group and H is a subgroup of index 2 in G, prove that H is a normal subgroup of G. [5]

(c) If G= is a cyclic group of order 40, find all distinct elements of the cyclic subgroup [5]

(i)$<a^8>$ (ii) $<a^{10}>$.

*** END OF PAPER ***