Final Assessment Test – November 2019

Course: MAT1014 · Discrete Mathematics and Graph Theory

VIT
Vellore Institute of Technology

Class NBR(s): 0481 / 0482 / 0485 / 0493 / 0546   Slot: A1+TA1+TAA1+V1

Time: Three Hours   Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

Answer any **FIVE** Questions

(5 X 20 = 100 Marks)

1. a) Obtain the PDNF and PCNF of the statement $(P \wedge Q) \vee (P \wedge R) \vee (Q \wedge R)$. [10]

    b) "If there was a ball game, then traveling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time. Therefore, there was no ball game. "Show that these statements constitute a valid argument". [10]

2. a) Show that the conclusion $(\forall x)(F(x) \rightarrow \neg S(x))$ follows logically from the premises $(\exists x)(F(x) \wedge S(x)) \rightarrow (\forall y)(M(y) \rightarrow W(y))$ and $(\exists y)(M(y) \wedge \neg W(y))$. [10]

    b) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book." [10]

3. a) State and prove Lagrange's theorem. [10]

    b) (i) Show that in a group $(G,*)$, if for any $a, b \in G$, $(a*b)^2 = a^2 * b^2$ then $(G,*)$ must be abelian. [5]

    (ii) Show that if every element in a group is its own inverse, then the group must be abelian. [5]

4. a) Give two partially ordered sets. In each case explain why they are lattices and not lattices? [10]

    b) Let $<L, \leq>$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively. Prove that for any $a, b \in L$ $a \leq b \Leftrightarrow a*b = a \Leftrightarrow a \oplus b = b$. [10]

5. a) Find the sum of product expansion of the Boolean function $F(x, y, z) = (x + z)y$. [5]

    b) Use Karnaugh map to minimize the sum of product expansion
    $x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$. [5]

    c) Use McCluskey algorithm to minimize the sum of product expansion
    $wxy\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + \bar{w}xyz + \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}z$. [10]
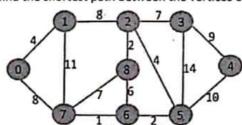
6. a) Prove that a given connected graph is an Euler graph if and only if all vertices of G are of even degree. [10]

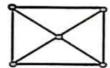   b) Use Dijkstras algorithm to find the shortest path between the vertices 0 and 4 [10]

7. a) Prove a tree with n vertices has exactly n-1 edges. [10]

    b) Find the chromatic polynomial of the following graph [10]

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