



VIT

Vellore Institute of Technology

DEPARTMENT OF MATHEMATICS

SCHOOL OF ADVANCED SCIENCES

Fall Semester, 2018 - 2019

Continuous Assessment Test - II, Sep - 2018

Course Code : MAT 2002

Course Name : Applications of Differential and Difference Equations

Class NBR(s) : VL2018191004478

Duration : 90 Minutes

Slot : B2+TB2

Date : 24-09-2018

Max. Marks : 50

ANSWER ALL QUESTIONS

(50 marks)

1. (a) Solve by the method of variation of parameters:

$$x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = x^4 \sec^2 x. \quad [6]$$

(b) Solve: $(3x + 2)^2 \frac{d^2 y}{dx^2} + 5(3x + 2) \frac{dy}{dx} - 3y = x^2 + x + 1. \quad [6]$

2. Solve by using Laplace transforms $\frac{d^2 y}{dt^2} + 9y = f(t), y(0) = 0, y'(0) = 4,$

where $f(t) = \begin{cases} 8 \sin t, & 0 < t < \pi, \\ 0, & t \geq \pi \end{cases} \quad [12]$

3. Solve the initial value problem by matrix method:

$$\frac{dx}{dt} = \begin{bmatrix} 6/7 & -15/14 \\ -5/7 & 37/14 \end{bmatrix} x + \begin{bmatrix} e^{2t} \\ e^{-t} \end{bmatrix}, \quad x(0) = \begin{bmatrix} 4 \\ -1 \end{bmatrix}. \quad [13]$$

4. Determine all the eigen values and corresponding eigen functions of the boundary value problem:

$$4(e^{-x} y')' + (1 + \lambda)e^{-x} y = 0, \quad y(0) = y(1) = 0. \quad [13]$$

Handwritten solutions for Question 4:

Let $y = v e^{-x}$. Then $y' = v' e^{-x} - v e^{-x}$.

Substituting into the equation:

$$4(e^{-x} (v' e^{-x} - v e^{-x}))' + (1 + \lambda)e^{-x} v e^{-x} = 0$$

$$4(v'' e^{-2x} - 2v' e^{-2x} + v e^{-2x}) + (1 + \lambda)v e^{-2x} = 0$$

$$4v'' - 8v' + (4 + 1 + \lambda)v = 0$$

$$4v'' - 8v' + (5 + \lambda)v = 0$$

Let $\lambda = -5 + \mu^2$. Then the equation becomes:

$$4v'' - 8v' + \mu^2 v = 0$$

The characteristic equation is:

$$4r^2 - 8r + \mu^2 = 0$$

$$r = \frac{8 \pm \sqrt{64 - 16\mu^2}}{8} = \frac{8 \pm 4\sqrt{4 - \mu^2}}{8} = \frac{4 \pm 2\sqrt{4 - \mu^2}}{4} = 1 \pm \sqrt{4 - \mu^2}$$

For $\mu = 0$, $\lambda = -5$, the equation is $4v'' - 8v' = 0$. The solution is $v = C_1 + C_2 x$. Then $y = C_1 e^{-x} + C_2 x e^{-x}$. Applying boundary conditions $y(0) = 0$ and $y(1) = 0$, we get $C_1 = 0$ and $C_2 = 0$. So $\lambda = -5$ is not an eigen value.

For $\mu \neq 0$, the solution is $v = e^{(1 + \sqrt{4 - \mu^2})x} + e^{(1 - \sqrt{4 - \mu^2})x}$. Then $y = e^{-x} (e^{(1 + \sqrt{4 - \mu^2})x} + e^{(1 - \sqrt{4 - \mu^2})x}) = e^{(1 + \sqrt{4 - \mu^2})x} + e^{(1 - \sqrt{4 - \mu^2})x}$. Applying boundary conditions $y(0) = 0$ and $y(1) = 0$, we get $1 + e^{(1 + \sqrt{4 - \mu^2})} + 1 + e^{(1 - \sqrt{4 - \mu^2})} = 0$ and $e^{(1 + \sqrt{4 - \mu^2})} + e^{(1 - \sqrt{4 - \mu^2})} = 0$. This implies $e^{(1 + \sqrt{4 - \mu^2})} = -e^{(1 - \sqrt{4 - \mu^2})}$. Taking the logarithm, we get $1 + \sqrt{4 - \mu^2} = 1 - \sqrt{4 - \mu^2} + i\pi$. This implies $\sqrt{4 - \mu^2} = i\pi/2$. Squaring both sides, we get $4 - \mu^2 = -\pi^2/4$. This implies $\mu^2 = 4 + \pi^2/4$. So $\mu = \pm \sqrt{4 + \pi^2/4}$. Then $\lambda = -5 + \mu^2 = -5 + 4 + \pi^2/4 = -1 + \pi^2/4$. So the eigen values are $\lambda = -1 + \pi^2/4$ and the corresponding eigen functions are $y = e^{-x} (e^{(1 + \sqrt{4 - \mu^2})x} + e^{(1 - \sqrt{4 - \mu^2})x})$.