



VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

School of Advanced Sciences

Department of Mathematics

CAT-I, August 2019

MAT 3003-Complex variables and Partial Differential Equations

Max. Time : 90 minutes.

Max. Marks: 50

Answer ALL questions (5 X 10 = 50 marks)

1. In a two-dimensional fluid flow, if the velocity potential is given by $\phi = x^4 - 6x^2y^2 + y^4$, then

- Find the stream function ψ and write the corresponding complex potential.
- Write the expression for velocity and hence find speed.
- Verify that the family of level curves $\phi = c_1$ and $\psi = c_2$ intersect orthogonally. (10M)

2. a) It is given that a function $f(z)$ and its conjugate $\overline{f(z)}$ are both analytic. Determine the function $f(z)$. (4M+6M)

b) Show that the function $u(x, \theta) = r^2 \cos 2\theta$ is harmonic. Find its conjugate harmonic function and the corresponding analytic function $f(z)$ in terms of z .

3. a) Find the Bilinear transformation which maps $z = 1, 0, -1$ onto $w = \infty, -1, 0$ respectively. (7M)

b) Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$. (6M)

c) Find the image of the semi-infinite strip $x > 0, 0 < y < 2$ under the transformation $w = iz + 1$. (7M)
Also draw the graph of the region.

4. a) Obtain the Taylor's series expansion of $\cosh z$ about $z_0 = \pi i$. Also find the radius of convergence of the series. (4M+6M)

b) Find the Laurent's series for the function $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ in the following regions:

i) $|z| > 3$

ii) $2 < |z| < 3$

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