



**General Instruction :** Statistical tables can be permitted

Answer any **FIVE** Questions

(5 X 20 = 100 Marks)

**SEARCH VIT QUESTION PAPERS ON TELEGRAM TO JOIN**

1. a) The following data represent the length of life in years, measured to the nearest tenth, of 30 similar fuel pumps: [8]

|     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 2.0 | 3.0 | 0.3 | 3.3 | 1.3 | 0.4 |
| 0.2 | 6.0 | 5.5 | 6.5 | 0.2 | 2.3 |
| 1.5 | 4.0 | 5.9 | 1.8 | 4.7 | 0.7 |
| 4.5 | 0.3 | 1.5 | 0.5 | 2.5 | 5.0 |
| 1.0 | 6.0 | 5.6 | 6.0 | 1.2 | 0.2 |

Construct a stem-and-leaf plot using the digit to the left of the decimal point as the stem for each observation. Compute sample mean and sample range.

- b) Calculate the mean deviation about the median for the following data [6]

| Class     | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|-----------|------|-------|-------|-------|-------|-------|
| Frequency | 6    | 7     | 15    | 16    | 4     | 2     |

- c) If  $f(x) = \begin{cases} xe^{-\frac{x^2}{2}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$  [6]

Show that  $f(x)$  is a probability density function. Find its distribution function  $F(x)$ .

2. a) Three cards are drawn without replacement from the 12 face cards (Jacks, Queens and Kings) of an ordinary deck of 52 playing cards. Let  $X$  be the number of kings selected and  $Y$  the number of Jacks. Find [10]

(i) the joint probability distribution of  $X$  and  $Y$ .

(ii)  $P[(x, y) \in A]$ , where  $A = \{(x, y) | x + y \geq 2\}$

- b) The joint probability density function of  $(X, Y)$  is given by [10]

$f(x, y) = 24xy, x > 0, y > 0, x + y \leq 1$  and  $f(x, y) = 0$ , elsewhere, find the conditional mean and variance of  $Y$  given  $X = x$ .

3. a) Suppose that  $X$  and  $Y$  are independent random variables with probability densities and [8]

$$g(x) = \begin{cases} \frac{8}{x^3}, & x > 2 \\ 0, & \text{elsewhere} \end{cases} \quad \text{and} \quad h(y) = \begin{cases} 2y, & 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the expected value of  $Z = XY$  and the covariance of  $X$  and  $Y$ .

- b) Compute the coefficient of correlation between  $X$  and  $Y$  using the following data: [6]

|   |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|
| X | 65 | 67 | 66 | 71 | 67 | 70 | 68 | 69 |
| Y | 67 | 68 | 68 | 70 | 64 | 67 | 72 | 70 |

- c) Find the moment generating function of the Poisson distribution  $P(x, \mu) = \frac{e^{-\mu} \mu^x}{x!}$ . [6]



4. a) Obtain the equation of the line, of regression of  $X$  on  $Y$  for the following data. Also estimate the value of  $X$  for  $Y = 18$ . [8]

|     |    |    |    |    |    |    |    |    |
|-----|----|----|----|----|----|----|----|----|
| $X$ | 22 | 26 | 29 | 30 | 31 | 31 | 34 | 35 |
| $Y$ | 20 | 20 | 21 | 29 | 27 | 24 | 27 | 31 |

- b) From the following data, obtain the multiple correlation coefficients  $R_{1.23}, R_{2.13}, R_{3.12}$ . [12]

|       |   |   |    |    |
|-------|---|---|----|----|
| $X_1$ | 2 | 5 | 7  | 11 |
| $X_2$ | 3 | 6 | 10 | 12 |
| $X_3$ | 1 | 3 | 6  | 10 |

5. a) A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%. [10]

- (i) The inspector of the retailer randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20?  
(ii) Suppose the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be 3 shipments containing at least one defective device?

- b) The daily consumption of milk in excess of 20000 litres in a town is approximately exponentially distributed with parameter  $1/3000$ . The town has a daily stock of 35000 litres. What is the probability that of 2 days selected at random, the stock is insufficient for both days. [10]

6. a) In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? [10]

- b) The following data represent the biological values of protein from cow's milk and buffalo's milk at a certain level. [10]

|         |      |      |      |      |      |      |
|---------|------|------|------|------|------|------|
| Cow     | 1.82 | 2.02 | 1.88 | 1.61 | 1.81 | 1.54 |
| Buffalo | 2.00 | 1.83 | 1.86 | 2.03 | 2.19 | 1.88 |

Examine if the average values of protein in the two samples significantly differ.

7. a) The following data give the number of air-craft accidents that occurred during the various days of a week. [10]

|        |     |      |     |     |     |     |
|--------|-----|------|-----|-----|-----|-----|
| Day    | Mon | Tues | Wed | Thu | Fri | Sat |
| Number | 15  | 19   | 13  | 12  | 16  | 15  |

Test whether the accidents are uniformly distributed over the week.

- b) Two hundred identical units are reliability tested for 50 hours. One unit fails just before completing 12 hours of operation, two units fail just before completing 20 hours of operation and two units fail just before completing 50 hours of operation. [10]

- (i) What is the reliability estimate of these units for a mission of 50 hours?  
(ii) What is the reliability estimate for the end of each failure period including previous failures, when the failed units are replaced with good ones?  
(iii) What is the reliability estimate for each failure period if the failed units are not replaced?

