Final Assessment Test – April 2019

Course: MAT2002 - Applications of Differential and Difference Equations

Class NBR(s): 0413 / 0415 / 0417 / 0421 / 0422 / 0424 / 0427 / 0429 / 0430 / 0432 / 0435 / 0437 / 4050 / 5962 / 5967

Slot: D1+TD1

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Time: Three Hours

Max. Marks: 100

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) Find the Fourier series expansion of the periodic function [10]

$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$ and hence find $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots$

b) Express $f(x) = x$ in half-range cosine series of periodicity $2l$ in the range $0 < x < l$ and hence obtain the sum of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$ [10]

2. a) Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to the canonical form through an orthogonal transformation and identify its nature. [10]

b) Use Cayley- Hamilton theorem to simplify [10]

$A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^4 - 5A^3 - 2A + I$, if $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$.

3. a) Solve by the method of undetermined coefficients $y'' + 3y' - 28y = e^{-7t} + 7t - 1$. [10]

b) Solve: $x^2y'' - 4xy' + 6y = \sin(\log x)$ by method of variation of parameters. [10]

4. a) Using the Laplace transform solve the initial value problem $y'' + 2y' - 15y = 6\delta(t-9)$, $y(0) = -5$ and $y'(0) = 7$. [10]

b) The differential equation $2y'' + 5y' - 3y = 0$ represents damped harmonic oscillation of a particle. Initially at time $t = 0$, the particle at a distance of -4 units from the origin and speed away from the origin is 9 units. Converting into system of first order differential equations find $x(t)$ by matrix method. [10]

5. a) Find a series solution for the Bessel's differential equation $x^2D^2y + xDy + (x^2 - 4)y = 0$ where $D = \frac{d}{dx}$. [12]

b) Find the characteristic values and characteristic functions of Strum –Liouville problem $(xy')' + \frac{\lambda}{x}y = 0$ with $y(0) = 0$ and $y(2) = 0$ on the interval $1 < x < 2$. [8]

6. a) Find the $Z -$ transform of $f(n) = \frac{2n+3}{(n+1)(n+2)}$. [5]

b) Find $Z^{-1}\left\{\frac{4z^3}{(z-1)(2z-1)^2}\right\}$. [5]

c) Using convolution theorem, find sum of first n natural numbers. [10]

7. a) Solve : $y_{n+2} - 5y_{n+1} + 6y_n = n^2 + n + 1$; [7]

b) A solid is constructed so that every face is a triangle. Find the number of faces of such a solid having n vertices. [6]

c) Solve : $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$, given that $y_0 = y_1 = 0$ by using the Z transform. [7]

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