Final Assessment Test – April 2019

Vellore Institute of Technology

Course: MAT2002 - Applications of Differential and Difference Equations

Class NBR(s): 0517 / 0521 / 0526 / 0529 / 0532 / 0534 / 0536 / 0537 / 0538 / 0868 / 0872 / 3333 / 5955 / 6000

Slot: E2+TE2

Time: Three Hours

Max. Marks: 100

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) Expand $f(x)=\begin{cases} \frac{1}{4}-x, & \text{if } 0<x<\frac{1}{2} \\ x-\frac{3}{4}, & \text{if } \frac{1}{2}<x<1 \end{cases}$ **[10]**

As the Fourier series of sine terms.

b) The turning moment T is given for a series of values of the crank angle $\theta^0 = 75^0$ **[10]**

θ^0	0	30	60	90	120	150	180
T	0	5224	8097	7850	5499	2626	0

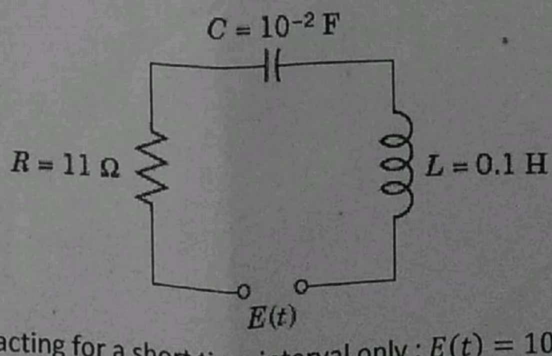
Obtain the first four terms in a series of sines to represent T and calculate T for $\theta^0 = 75^0$.

2. a) Show that $A=\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ is similar to a diagonal matrix, and find the transforming matrix and diagonal matrix. Also identify it's nature. **[12]**

b) If $A=\begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$, verify Cayley-Hamilton theorem. Hence find inverse of A. **[8]**

3. a) Solve $\frac{d^2y}{dx^2}+a^2y=\sec ax$ by the method of variation of parameters. **[10]**

b) Find the response (the current) of the RLC-circuit in the Figure **[10]**



where $E(t)$ is sinusoidal, acting for a short time interval only : $E(t) = 100 \sin 400\,t$ if $0 < t < 2\pi$ and $E(t) = 0$ if $t > 2\pi$ and current and charge are initially zero.

4. a) Solve $ty'' + 2y' + ty = \cos t$, given that $y(0) = 1$ by Laplace transform method. **[10]**

b) Find the general solution of system of non-homogeneous linear system of differential equation $y_1' = -3y_1 + y_2 - 6e^{-2t}$, $y_2' = y_1 - 3y_2 + 2e^{-2t}$ by matrix method. **[10]**

5. a) Obtain Frobenius series solution for the differential equation $xy'' + y' - y = 0$ about $x = 0$. [10]

b) For the Strum-Liouville problem $y'' + \lambda y = 0,\ y(0) = 0, y(l) = 0$, find the eigen functions and verify if they are orthogonal. [10]

6. a) Evaluate $Z^{-1}\left[\dfrac{z^2}{(z-a)(z-b)}\right]$ using convolution theorem, for $a = 2,\ b = 1/2$. [10]

b) Find the Z-transform of [10]
(i) $e^t \sin 2t$ (ii) $n^2 e^{an}$

7. a) Solve $u_{n+2} + u_{n+1} - 6u_n = 8(-2)^n$ by the method of undetermined coefficients. [10]

b) Find the response of the system $y_{n+2} - 5y_{n+1} + 6y_n = u_n; y_0 = 0, y_1 = 1$ and $u_n = 1$ for $n = 0, 1, 2, 3, \ldots$ by the Z transform. [10]

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