VIT®
Vellore Institute of Technology

Final Assessment Test – April 2019

Course: MAT2002 - Applications of Differential and Difference Equations

Class NBR(s): 0443 / 0444 / 0445/ 0448 / 0450 / 0455 /0457 / 0459 / 0462 / 0465 / 0467 /0468 / 5969 / 5980

Slot: D2+TD2

Time: Three Hours

Max. Marks: 100

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Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) Find the Fourier series expansion of $f(x) = \frac{(\pi-x)^2}{2}$ $(0, 2\pi)$ and deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+ \ldots\ldots = \frac{\pi^2}{8}$. [10]

   b) Find the Fourier series expansion of the following periodic function of period 4 : [10]

$$f(x) = \begin{cases} 2+x, & -2 \le x \le 0 \\ 2-x, & 0 < x \le 2 \end{cases}$$

2. a) Find the matrix A, where eigen values are 1, -1, and 2 and the corresponding eigen vectors are $(1\,1\,0)^T$, $(1\,0\,1)^T$ and $(3\,1\,1)^T$. [10]

   b) Reduce the quadratic form $3x^2 - 2y^2 - z^2 - 4xy + 8zx + 12yz$ to canonical form by orthogonal reduction, and identify its nature. [10]

3. a) Using the method of variation of parameters, solve: $y'' - 2y' + y = e^x.\ \log x$. [10]

   b) Solve: $x^2y'' - 3x' + 5y = x^2\sin(\log x)$. [10]

4. a) Solve by the method of Laplace transform, the equation $y'' + 2y' + 5y = e^{-t}\sin t$. [10]

   b) Solve the following system of linear differential equations by matrix method: [10]

$$x'(t) = 2x + y + 2z$$
$$y'(t) = 2x + 2y - 2z$$
$$z'(t) = 3x + y + z,$$

   Where $x(0) = y(0) = z(0) = 1$.

5. a) Solve the equation in Frobenius series: $3x\left(\frac{d^2y}{dx^2}\right) + (1-x)\left(\frac{dy}{dx}\right) - y = 0$. [10]

   b) Find the eigen values and eigen functions of $y'' + \lambda y = 0$ with $y(0) = 0$ and $y(\pi) = 0$. [10]

6. a) Find the Z – transform of : (i) $\frac{1}{(n+1)(n+2)}$ (ii) $n^2 \cdot e^{n\theta}$ [10]

   b) Using convolution theorem, find the inverse Z-transform of $\frac{z^2}{(z-3)^3}$. [10]

7. a) Solve : $u_{n+2} - 4u_{n+1} + 4u_n = n^2 \cdot 2^n$. [10]

   b) Using Z – transform , solve : $y_{n+2} - 5y_{n+1} + 6y_n = 1$, where $y_0 = 0$ and $y_1 = 1$. [10]

Possession of Mobile Phone in the exam hall even in switched off condition is a malpractice