Reg.No. [redacted]

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Velammal College of Engineering and Technology

Viraganoor, Madurai – 625 009

(Autonomous)

B.E./B.Tech End Semester Examinations April 2025

Fourth Semester **Regulation 2021**

Time : 3 Hours **Max. Marks 100**

21MA205 - Stochastic Process and its Applications

(Common to CSE and IT)

Answer ALL Questions

(Statistical Tables are permitted)

PART-A (10 x 2 = 20 Marks)

1. The random variable X has the p.m.f $P(X=x)=\begin{cases}\frac{c}{x}, & x=1,2,3\\ 0, & otherwise\end{cases}$.

   Find the value of 'c'

2. The mean and standard deviation of the binomial distribution 20 and 4 respectively. Calculate the value of the parameter 'n'.

3. The joint p.d.f of (X,Y) is $f(x,y)=\begin{cases}\frac{1}{4}, & 0<x,y<2\\ 0, & otherwise\end{cases}$. Find P[X+Y≤ 1].

4. Write any two properties of regression coefficients.

5. Define wide-sense stationary (WSS) process.

6. Check whether the Poisson Process is stationary or not.

7. In the usual notation of a (M/M/1):(∞/FIFO) queue system find P(N>2) if λ=12/hr and μ=30/hr.

8. For (M/M/C):(∞/FIFO)model, Summarize the formula for
   (i) Average number of customers in the queue
   (ii)Average waiting time in the system

9. Write down the characteristics of Jackson network.

10. Explain series queues.

Part – B (4 x 16 = 64 Marks)

11. a) (i) In a continuous distribution the probability density function is given by $f(x) = kx(2-x), 0 < x < 2$. Find k, mean and variance, distribution function of X.

(ii) Find the moment generating function of Poisson distribution and hence find its mean and variance.

(8 + 8 Marks)

OR

b) (i) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8. What is the probability that he will finally pass the test on fourth trial? Also find the probability that he will finally pass the test in less than 4 trials?

(ii) The savings bank account of a customer showed an average balance of Rs.150 and standard deviation Rs.50. Assuming that the account balances are normally distributed

(i) What percentage of account is over Rs.200?

(ii) What percentage of account is between Rs.120 and Rs.170?

(iii) What percentage of account is less than Rs.75?

(8 + 8 Marks)

12. a) (i) If X and Y are two random variable having joint p.d.f

$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y); \; 0 < x < 2; \, 2 < y < 4 \\ 0; \; \textit{otherwise} \end{cases}$$

Find (i) $P(X<1 \cap Y<3)$, (ii) $P(X<1/Y<3)$.

(ii) The joint pdf of the random variable (X,Y) is $f(x,y) = x + y, \; 0 \le x \le 1, \; 0 \le y \le 1$, find cov(X,Y).

(8 + 8 Marks)

OR

b) (i) (i)Two random variable X and Y have the joint density

$$f(x,y) = \begin{Bmatrix} 2-x-y & \text{if } 0 < x < 1, \, 0 < y < 1 \\ 0 & \textit{otherwise} \end{Bmatrix}$$

Compute Cov(X,Y) and Correlation coefficient between X and Y.

x(t)

13. a) (i) Show that the process □X□t□□□whose probability distribution under certain conditions is given

$$\text{by } P\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, \; n = 1,2,3,\ldots \\ \dfrac{at}{1+at}, \; n = 0 \end{cases}$$

is evolutionary.

x(t) = Acosλt + Bsinλt

(ii) Show that the process X□t□□□Acos□t □□Bsin□t (where A & B are random variables) is WSS, if (i) E(A) = E(B) = E(AB) = 0,

(ii) $E(A^2) = E(B^2)$.

(8 + 8 Marks)

OR

b) (i) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find (1) the probability that he takes a train on the third day and (2) the probability that he drives to work in the long run.

(ii) The transition probability matrix of the Markov chain $\{X(n)\}$ with n=1,2,3,... having three states 1,2,3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $P^{(0)} = (0.7 \;\; 0.2 \;\; 0.1)$. Find $P(x_2 = 3)$ and $P(x_3 = 2, x_2 = 3, x_1 = 3, x_0 = 2)$. (8 + 8 Marks)

14. a) Customers arrive at a one-man barber shop according to a Poisson process with a mean inter arrival time of 12 min. Customers spend an average of 10min in the barber's chair.

(a) What is the expected number of customers in the barber shop and in the queue?

(b) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait.

© How much time can a customer expect to spend in the barber's shop?

(d) What is the average time customer spends in the queue?

© What is the probability that the waiting time in the system is greater than 30 min?

(f) Calculate the percentage of customers who have to wait prior to getting into the barber's chair.

(g) What is the probability that more than 3 customers are in the system?

OR

b) **Derive** Pollaczek-Khinchine formula for the average number of customers in the M/G/1 queuing system.

Part – C (1 x 16=16 Marks)

15. a) A supermarket owner is experimenting with a new store design and has remodeled one of his stores as follows. Instead of the usual checkout counter design, the store has been remodeled to include a check out "lounge". As customers complete their shopping, they enter the lounge with their carts. If all checkers are busy the customers receive a number. They then park their carts and take a seat. When a checker is free, the next number is called and the customer having that particular number enters the available checkout counter. The store has been enlarged so that for particular purposes, there is no limit on either the number of shoppers that can be in the shopping section or the number that can wait in the lounge, even during the peak hours, customers arrive according to a Poisson process at a mean rate of 40/hr and it takes a customer on the average, ¾ hr to fill his shopping cart. The filling time are approximately exponentially distributed with a mean of 4 minutes, irrespective of the particular checkout counter. The management wishes to know the following.

(i) Minimum number of checkout counters required in operation during peak periods.

(ii) If it is decided to add one more counter than the minimum number of counters required, then what is the average waiting time in the lounge?

(iii) How many people on the average will be in the lounge?

(iv) How many people on the average will be in the entire supermarket?

OR

b) In a library, there are 2 sections, one for English books and the other section for Tamil books. There is only one salesman in each section. Customers from outside arrive at the English book section at a Poisson rate of 5 per hour and at the Tamil book section at a Poisson rate of 4 per hour. The service rates of English book section and Tamil book section are 9 and 11 per hour respectively. A customer after service at English book section is equally likely to go to the Tamil book section or to leave the library. However, a customer upon completion of service at Tamil book section will go the English book section with probability 1/3 and will leave the library otherwise. Find the following.

(i) Joint steady-state probability that there are 2 customers in the English book section and 2 in the Tamil book section,

(ii) average number of customers in the library,

(iii) average waiting time of a customer in the library.