

Course Name & Code: Discrete Mathematics and Graph theory-MAT1014

Slot: A2

Exam Duration: 90 minutes

Maximum Marks: 50

Answer All the Questions (5 × 10 = 50)

1. a. Show that in a group $(G, *)$, if for any a, b in G , $(a * b)^2 = a^2 * b^2$, then $(G, *)$ must be abelian. (5)
- b. Let a and b be non-identity elements of different orders in a group of order 155. Prove that the only subgroup of G that contains both a and b is G itself. (5)
2. a. Devise a single error correcting group code $(2, 5)$ with parity bits $x_3 = x_1$, $x_4 = x_2$ and $x_5 = x_1 + x_2$. Find its parity check matrix, generator matrix, group code and decoding table. Correct the single error in the received word 10001 using decoding table. (6)
- b. If $x \leq y$ and $z \leq w$, then prove that $x \wedge z \leq y \wedge w$ and $x \vee z \leq y \vee w$. (4)
3. a. Prove that (S_{30}, D) , S_{30} the set of divisors of 30, is a Poset. Draw its Hasse diagram. Find LUB and GLB of every pair of elements and hence show that (S_{30}, D) is a lattice. (4)
- b. In any Boolean Algebra, show that (6)
- (i) $a = b \Leftrightarrow ab' + a'b = 0$
- (ii) $(a + b)(a' + c) = ac + a'b = ac + a'b + bc(a + a')$
4. a. Let $S = \{1, 2, 3\}$. Prove that $(P(S), \cup, \cap, ')$, $P(S)$ the power set of S , is a Boolean algebra. (5)
- List all sub-Boolean algebra of the Boolean algebra. (5)
- b. Expand $f(x, y, z) = x * y + y * z'$ into its sum of products canonical form (5)
5. a. Show that in a lattice with two or more elements, no element is its own complement. (4)
- b. Reduce the Boolean function (6)
- $f(a, b, c, d) = a'b'c'd' + a'b'cd' + ab'c'd' + abcd$ by using Karnaugh map.



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