VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
Winter Semester - 2019 ~ 2020

Continuous Assessment Test – I, Jan – 2020

Course Code : MAT3004 — Slot: A2+TA2
Course Name : Applied Linear Algebra — Date: 19.01.2020
Duration : 90 Minutes — Max. Marks: 50

ANSWER ALL

Q1. For what values of a and b the following system [10]

$$x + 2y + 3z = 6;$$
$$x + 3y + 5z = 9;$$
$$2x + 5y + az = b.$$

has (i) no solution (ii) Unique solution (iii) Infinite number of solutions.

Q2. Solve the system of equations by using LU decomposition method [10]

$$x + 2y - z = -3;$$
$$y - z = 1;$$
$$3x - y + z = 4.$$

Q3. Express the given matrix as a product of elementary matrices

$$\begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}.$$ [10]

Q4. (a) Let $x_1, x_2, \ldots, x_n$ be vectors in a vector space V. Then the set $W = \{a_1x_1 + a_2x_2 + \cdots + a_nx_n : a_i \in R\}$ of all linear combinations of $x_1, x_2, \ldots, x_n$ is a subspace of V. [5]

(b) Express the polynomial $v = t^2 + 4t - 3$ in $P(t)$ as a linear combination of the polynomials $P_1 = t^2 - 2t + 5$, $P_2 = 2t^2 - 3t$ and $P_3 = t + 1$. [5]

Q5. (a) Consider the polynomials $p(x) = 1 + 3x + 2x^2$, $q(x) = 3 + x + 2x^2$, $r(x) = 2x + x^2$ in $\mathcal{P}_2$. Where $\mathcal{P}_2$ is collection of all polynomials of degree less than or equal to 2. Is $\{p(x), q(x), r(x)\}$ linearly independent? [5]

(b) Write a basis for the following vector spaces (i) 3×3 symmetric matrices with real entries over R (ii) 3×3 matrices with sum of all main diagonal entries are zero over R. [5]