VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

DEPARTMENT OF MATHEMATICS

SCHOOL OF ADVANCED SCIENCES

Fall Semester 2022-23

Continuous Assessment Test –II (October-2022)

Slot: A2+TA2+TAA2

Course Code: BMAT201L **Course Title:** Complex Variables and Linear Algebra

Max. Time: 90 minutes **Max. Marks:** 50

Answer *all* the Questions

1. Find the Laurent's series expansion of $f(z) = \frac{z}{(z^2+1)(z^2+4)}$ in the region

   $(a)\ 1 < |z| < 2$ $(b)\ |z| > 2$ **(10 M)**

2. (a) For the function $f(z) = \frac{1-e^{2z}}{z^4}$, find the poles and residues at each of the poles.

   (b) Using Cauchy's integral formula,

   evaluate $\int_c \frac{z \sec z}{1-z^2}\, dz$ where C is the ellipse $4x^2 + 9y^2 = 9$. **(5+5 M)**

3. Using contour integration evaluate $\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta}\, d\theta$. **(10 M)**

4. For a matrix $A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 3 \end{pmatrix}$. Find the eigenvalues of A and hence verify eigenvalues of A^{-1}. **(10 M)**

5. Find the values of a and b for which the following system of equations has (i) no solution and (ii) unique solution:

   $x + 2y + 2z = 10;$

   $2x - 2y + 3z = 1;$

   $4x - 3y + az = b.$

   Solve the system by Gauss-Elimination method for $a = 5$ and $b = 4$. **(10 M)**

Department of Mathematics

School of Advanced Sciences

Fall Semester 2022-2023

Continuous Assessment Test - II (October, 2022)

Course Code: BMAT201L **Course Title:** Complex Variables & Linear Algebra

Max. Time: 90 minutes **Max. Marks:** 50

Slot: A1+TA1+TAA1

Answer all the questions.

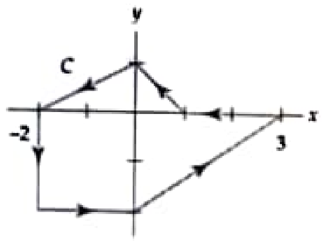
Write the answers in detail.

1. Find the Laurent's series of $f(z) = \dfrac{1}{(z-3)(z+1)}$ valid in the regions:

   (a) $\mathscr{A} : 0 < |z-3| < 1,$ (5 marks)

   (b) $\mathscr{B} : 1 < |z-2| < 3.$ (5 marks)

2 (a). Employing Cauchy's integral formula, find $\oint_C f(z)\, dz$, where $f(z) = \dfrac{z-1}{z(z+i)}$ and C is as shown below: (5 marks)

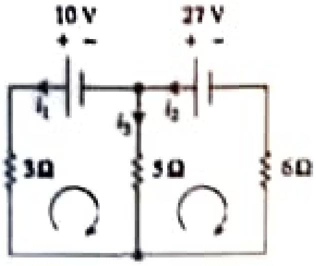


2 (b). Using the residue theorem, find $\oint_\gamma \dfrac{1+4i}{(z-2)(z+2i)^2}\, dz$, where γ is the rectangle with sides $x = \pm 1,\ y = \pm\pi$. (5 marks)

3. Compute $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x}{x^2 - 10x + 9}\, dx$, through contour integration.

4. Find the characteristic polynomial of $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$. Using Caley-Hamilton's theorem, express A^4 in terms of a quadratic matric polynomial in A and find A^{-1}. What are the eigenvalues of A and A^2 and A^{-1}?

5 (a). The currents i_1, i_2 and i_3 in the network:



are described by the system of linear equations:

$$i_1 - i_2 + i_3 = 0,$$
$$3i_1 - 5i_3 = 10,$$
$$6i_2 + 5i_3 = 27.$$

Use *Gauss' elimination* method to solve the system for i_1, i_2 and i_3. (5 marks)

5 (b). Find the values of k such that the system

$$-3x_1 + 2x_2 - 2x_3 = 0,$$
$$x_1 - 2x_2 + 2x_3 = 0,$$
$$-2x_1 + 4x_2 + kx_3 = 0$$

has infinitely many nonzero solutions, and hence find all the corresponding nonzero solution vectors of the system. (5 marks)

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Vellore – 632014, Tamil Nadu, India

DEPARTMENT OF MATHEMATICS

SCHOOL OF ADVANCED SCIENCES

FALL SEMESTER 2022-2023

CONTINUOUS ASSESSMENT TEST – II

Programme Name & Branch	:	B.Tech.
Course Code	:	BMAT201L
Course Name	:	Complex Variables and Linear Algebra
Slot	:	B2+TB2+TBB2
Date of the Examination	:	11/10/2022
Duration	:	**90 minutes**

Max. Marks : 50

General instruction(s): Open Book / Notebook Examination. Answer ALL the Questions

Q. No	Question	Marks	Course Outcome (CO)	Bloom's Taxonomy (BL)
1.	Find Laurent series expansions of $f(z) = \frac{1}{z(1-2z)}$ about (i) the origin and (ii) about $z = \frac{1}{2}$. Using the expansions write the residue value at each singular points.	10	CO2	L1
2.	Evaluate the integral using contour integration $\int_0^{2\pi} \frac{d\theta}{10-6\cos\theta}$.	10	CO3	L5
3.	(i) Identify and classify all the singular points of $f(z) = \frac{1}{z^6+1}$ (ii) Evaluate $\oint \sec z \, dz$ where the contour C is the circle $\lvert z\rvert = 1$	10	CO3	L4
4.	For a matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (i) Find the Eigen values and Eigen vectors of A (ii) Using the Eigen values of A, write (a) the trace of A (b) the Determinant of A. (c) the Eigen values of A^{-1} (iii) Using Cayley-Hamilton theorem, find the inverse of A.	10	CO5	L1
5.	Use Gauss Jordan to the system $x_1 + x_2 + x_3 = 5$; $2x_1 + 3x_2 + 5x_3 = 8$; $4x_1 + 5x_3 = 2$ and find the solution vector X?	10	CO5	L3

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DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
FALL SEMESTER 2022-2023

CONTINUOUS ASSESSMENT TEST – II

Programme Name & Branch : BTech
Course Code : BMAT201L
Course Name : Complex Variables and Linear Algebra
Slot : C2+TC2+TCC2
Date of the Examination : 12.10.22
Duration : 90 minutes **Max. Marks : 50**

Q. No	Question	Marks
1.	Find the Laurent's series expansion of $f(z) = \frac{1}{(z^2+1)(z^2+2)}$ in the region a) $1 < \lvert z\rvert < \sqrt{2}$ b) $\lvert z\rvert > \sqrt{2}$	10
2.	a) Find the residue of $\frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles. b) Evaluate $\int_C \frac{dz}{z^2+4}$, where C is $\lvert z-i\rvert = 2$ in the positive orientation.	10
3.	Using contour integration, evaluate the real integral $\int_0^\infty \frac{dx}{(x^2+4)^3}$.	10
4.	Evaluate $A^8 - A^7 + 5A^6 - A^5 + A^4 - A^3 + 6A^2 + A - 2I$ if $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$.	10
5.	Solve $Ax = b$, where $A = \begin{bmatrix} 4 & 1 & 1 & -2 \\ -4 & 0 & -1 & 4 \\ -12 & -1 & 4 & 5 \\ 0 & 0 & 14 & -7 \end{bmatrix}$ and $b = \begin{bmatrix} -7 \\ 8 \\ 0 \\ -49 \end{bmatrix}$ using Gauss elimination method.	10

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Fall Semester 2022-23

School of Advanced Sciences

Department of Mathematics

Continuous Assessment Test –II

Course Code & Name: BMAT201L: Complex Variables and Linear Algebra

Slot: C1+TC1+TCC1

Exam duration: 90 minutes **Max. Marks:** 50

General Instructions:

i. Students are permitted to bring one text book / hand written note book only.

ii. Answer all the following questions.

Q.No.	Question	Marks
1.	Find the Laurent's series expansion of $f(z) = \frac{z+4}{(z+3)z^2}$ in the annulus $1 < \lvert z+1 \rvert < 2$.	10
2.	(a) Find the singularities of $f(z) = \frac{\sin z}{z^2 - z}$ and classify them. (b) Evaluate $\oint_C \frac{z\,e^{2z}}{(z+1)(z+2)}\,dz$ using Cauchy's integral formula, where C is the circle $\lvert z \rvert = \pi$.	4+6
3.	Evaluate the integral $\int_0^\infty \frac{x \sin x}{(x^2+1)(x^2+4)}\,dx$ using the contour integration.	10
4.	(a) Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ -1 & 1 & -1 \end{bmatrix}$. What are the eigenvalues of A^{-1} and $B = A^3 + 2A^2 + I$? (b) Find e^{At} using Cayley Hamilton theorem, for $A = \begin{bmatrix} -1 & 1 \\ 4 & -1 \end{bmatrix}$.	5+5
5.	Express the system $2x + y + z = 4; x + y - 2z = 3; -x - 2y + z = 1$ in matrix form as $AX = B$. Solve the system by Gauss elimination method. Also find a non-zero vector X such that $AX = \lambda X$ for some λ.	10

$\frac{3}{2} + \frac{3}{2}(-5)$

$3 + \frac{3}{2}(2)$