Final Assessment Test – Jan/Feb 2023

VIT
Vellore Institute of Technology

Course: BMAT101L - Calculus
Class NBR(s): 5008/ 5012/ 5015/ 5017/5022/ 5027/
5029/5031/5034/5035/5037/5041/5043/5046/5049/ Slot: B1+TB1
5051/5418/ 5424/5483/5490/6423/6443
Time: Three Hours   Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. a) Use the first derivative test to find the location of all local extrema for. **[5]**
    $f(x) = x^3 - 3x^2 - 9x - 1$. Sketch the graph to confirm your results. Also find the
    region where the function increasing and decreasing.

    b) Find the area of the region bounded by the parabolas $y = 6x - x^2$ and $y = x^2 - 2x$. **[5]**

2. a) Find a point on the curve $y = \sin x + \cos x - 1$, $x \in [0, \pi/ 2]$, where the **[5]**
    tangent is parallel to the x axis.

    b) A solid is formed by rotating the triangle with vertices $(0, 0)$, $(2, 0)$ and **[5]**
    $(1, 1)$ about x-axis. Find the resulting volume.

3. a) If $u = x \log (xy)$ where $x^3 + y^3 + 3xy = 1$ find $\frac{du}{dx}$. **[5]**

    b) Prove that if $u = \frac{x}{y}$, and $v = \frac{x+y}{x-y}$ are functionally dependent and find the **[5]**
    relation between them.

4. Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's theorem up to **[10]**
    third degree terms.

5. Find the minimum value of $x^2 + y^2 + z^2$, given that $xyz = 1$ **[10]**

6. Evaluate the volume of the region enclosed by the surfaces $z = x^2 + 3y^2$ and **[10]**
    $z = 8 - x^2 - y^2$ by using the idea of triple integral.

7. Change the order of integration and evaluate $\int_0^a \int_0^y \frac{dxdy}{\sqrt{(a^2 + x^2)(a - y)(y - x)}}$. **[10]**