



Course Code: ES102

MM: 40 – 50.5

Duration: 120 minutes

Note:

1. All parts of a question should be answered consecutively.
2. The question paper has six questions.
3. Questions no. 1 and 2 a), 6 b) are open questions, and the marks will be purely based on the justification and Mathematical explanations.
4. Question no. 5 a): you must show that the example is a commutative ring with identity but not a field

1. Construct your own example of a system of Linear equations (Non-Homogeneous), where it is
  - a) Inconsistent
  - b) Consistent with a unique solution
  - c) Consistent with Infinite solutions(6-12)
2. a) Define a rank of a Matrix (Mathematically) (1.5-4.5)  
b) Determine the non-singular matrices  $P$  and  $Q$  such that  $PAQ$  is in the normal form for  $A$  and find the rank of  $A$ .  
$$A = \begin{pmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{pmatrix}$$
(5)
3. Find the values of  $a$  and  $b$  for which the system has i) no solution, ii) unique solution, iii) infinitely many solutions for the following system,  
$$2x_1 + 3x_2 + 5x_3 = 9, 7x_1 + 3x_2 - 2x_3 = 8, 2x_1 + 3x_2 + ax_3 = b$$
(5)
4. Solve the following system using the Gauss-Jordan Elimination method (7.5)  
$$10x_1 - 7x_2 + 3x_3 + 5x_4 = 6; -6x_1 + 8x_2 - x_3 - 4x_4 = 5;$$
$$3x_1 + x_2 + 4x_3 + 11x_4 = 2; 5x_1 - 9x_2 - 2x_3 + 4x_4 = 7$$
5. a) Define a Field and give an example of a commutative ring with identity but not a field. (4.5)  
b) Justify your answer: Can we have a field of 1147 elements? (1.5)  
c) Find the multiplicative inverse of  $31 \bmod 929$  (2)
6. a) Define a Vector Space and its subspace. (2.5)  
b) Construct your own example of a Vector space and discuss any one subspace of the identified vector space. (4.5-6)