Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.TECH(BME(N)/ECE(N)/EE(N)/EEE(N)/EIE(N)/ICE(N)/PWE(N))/SEM-3/M(CS)-301/2011-12

2011

NUMERICAL METHODS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) Which of the following is not a computational error ?

a) Truncation error b) Round-off error

c) Inherent error d) None of these.

ii) Newton-Raphson method fails when

a) $f'(x) = 1$ b) $f'(x) = 0$

c) $f'(x) = -1$ d) $f''(x) = 0$.

iii) Finite difference method is used to solve

a) a system of linear simultaneous equations

b) a system of non-linear simultaneous equations

c) partial differential equations

d) non-linear equations.

iv) Regula-falsi method has a convergence rate of the order of

a) 2 b) 1·62

c) 1 d) none of these.

v) Gauss-Seidel method for solution of a system of linear simultaneous equations converges if

a) $|a_{ii}| \geq \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|$

b) $|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|$

c) $|a_{ii}| / |a_{nn}| = 1$

d) none of these.

vi) Modified Euler's method has a truncation error of the order of

a) h b) h^2

c) h^4 d) h^3.

vii) Divided difference interpolation formula can be used for

a) the tabular values with independent variable unequally spaced

b) inverse interpolation

c) both (a) and (b)

d) none of these.

viii) Truncation error in Simpson's $\frac{1}{3}$ rd rule is given by

a) $\frac{b-a}{180} h^4 f^{iv}(\xi), \quad a \le \xi \le b$

b) $\frac{b-a}{90} h^5 f^{iv}(\xi), \quad a \le \xi \le b$

c) $\frac{b-a}{6} h^4 f'''(\xi), \quad a \le \xi \le b$

d) $\frac{b-a}{90} h^4 f^{iv}(\xi), \quad a \le \xi \le b$.

ix) Which of the following relations is true ?

a) $E = 1 - \Delta, \quad \Delta - \square = \Delta \square$

b) $E = 1 - \Delta, \quad \Delta + \square = \Delta \square$

c) $E = 1 + \Delta, \quad \Delta + \square = \Delta \square$

d) $E = 1 + \Delta, \quad \Delta - \square = \Delta \square$.

x) Trapezoidal method can be used to integrate numerically a function represented in tabular form

a) with odd number of intervals only

b) with even number of intervals only

c) both (a) and (b)

d) none of these.

xi) Condition of convergence for Euler's method

a) $| 1 + hf'(x_i, y_i) | < 1$

b) $| 1 + hf'(x_i, y_i) | \leq 1$

c) $| 1 + hf'(x_i, y_i) | > 1$

d) $| 1 + hf'(x_i, y_i) | \geq 1.$

xii) Milne's corrector formula is

a) $y_{n+1} = y_n + \frac{h}{3}(y'_{n-1} + 4y'_n + 4y'_{n+1})$

b) $y_{n+1} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + 4y'_{n+1})$

c) $y_{n+1} = y_n + \frac{4h}{3}(y'_{n-1} + 4y'_n + 4y'_{n+1})$

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Given the function $y = \frac{1}{x}$, show that the divided difference of n^{th} order

$y[x_0, x_1, x_2, \ldots, x_n] = (-1)^n / (x_0 x_1 x_2 \ldots x_n)$

3. Solve the following system of linear equations by Gauss-Seidel iterative method :

$9x + 2y + 3z = -7$

$x - 6y + 2z = -2$

$x + y + 3z = 5.$

4. Fit a polynomial to the following table of values using Lagrange interpolation formula :

x :	0	1	3	4
y :	– 12	0	6	12

Find the value of y when

a) $x = 2$

b) $x = 3{\cdot}5$.

5. Find the value of $\frac{1}{23}$ using Newton-Raphson method. Result is required to be corrected up to 4 decimal places.

6. Solve the following equation using bisection method :

$$3x + \sin x - e^x = 0$$

Take $x_0 = 1$ and $x_1 = 0$.

Result is required to be corrected up to 2 decimal places.

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Derive the order of convergence for Newton-Raphson method. 5

b) Solve the following initial value problem using Euler's method :

$$\frac{dy}{dx} = x^2 + y \text{ with } y(0) = 1.$$

Compute the first 5 steps of the solution with $h = 0{\cdot}1$.

Compare the results (% relative error) with those obtained from the exact solution

$$y = 3e^x - x^2 - 2x - 2.$$ 10

8. a) Prove by the method of induction :

$$\Delta^m y_r = \nabla^m y_{r+m} .$$ 5

b) Use Newton's formula to find the area of a circle of diameter 98 cm. 5

D (cm) :	80	85	90	95	100
A (cm ^{2})	5026	5674	6362	7088	7854

c) Derive Lagrange interpolation formula. 5

9. a) Derive the expression for total truncation error associated with Simpson's $\frac{1}{3}$ rd method. 8

b) Evaluate the following integral using trapezoidal method :

$$I = \int_0^2 \left(1 / \left(x^2 + 4 \right) \right) dx.$$

Take $h = 0{\cdot}125$. Hence obtain the value of π. 7

10. a) Solve the following system of equations using LU factorization method. 8

$$3x - y + 2z = 12$$

$$x + 2y + 3z = 11$$

$$2x - 2y - z = 2.$$

b) Find the inverse of the following matrix : 7

$$A = \begin{bmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{bmatrix} .$$

11. a) Define Δ, ∇ and E. 5

b) Derive Newton's Backward difference interpolation formula. 5

c) Derive 4th order Runge-Kutta formula for solution of initial value problem of ordinary differential equation. 5