ABV- Indian Institute of Information Technology & Management Gwalior
Department of Computer Science and Engineering
Data Structures

Assignment # 1 Due date: Monday, 1-May-2023

Queue

1. Suppose we have a queue that initially contains 7 elements: 2, 4, 6, 8, 10, 12, and 14. If we perform three dequeue operations followed by two enqueue operations with the elements 16 and 18, what will be the element at the front of the queue?

2. Suppose we have a stack S containing n elements and a queue Q that is initially empty. Describe how we can use Q to scan S to see if it contains a certain element x, with the additional constraint that our algorithm must return the elements back to S in their original order. We may not use an array or linked list—only S and Q and a constant number of reference variables.

3. Suppose we have an n×n two-dimensional array A that we want to use to store integers, but we don't want to spend the $O(n^2)$ work to initialize it to all 0's, because we already know that we are only going to use up to n of these cells in our algorithm, which itself runs in O(n) time (not counting the time to initialize A). Show how to use an array-based stack S storing (i, j, k) integer triples to allow us to use the array A without initializing it and still implement our algorithm in O(n) time, even though the initial values in the cells of A might be total garbage.

4. When a share of common stock of some company is sold, the capital gain (or, sometimes, loss) is the difference between the share's selling price and the price originally paid to buy it. This rule is easy to understand for a single share, but if we sell multiple shares of stock bought over a long period of time, then we must identify the shares actually being sold. A standard accounting principle for identifying which shares of a stock were sold in such a case is to use a FIFO protocol—the shares sold are the ones that have been held the longest (indeed, this is the default method built into several personal finance software packages). For example, suppose we buy 100 shares at $20 each on day 1, 20 shares at $24 on day 2, 200 shares at $36 on day 3, and then sell 150 shares on day4 at $30 each. Then applying the FIFO protocol means that of the 150 shares sold, 100 were bought on day 1, 20 were bought on day 2, and 30were bought on day 3. The capital gain in this case would therefore be $100 \times 10 + 20 \times 6 + 30 \times (-6)$, or $940. Write a program that takes as input a sequence of transactions of the form "buy x share(s) at $y each"or "sell x share(s) at $\$y$ each," assuming that the transactions occur on consecutive days and the values x and y are integers. Given this input sequence, the output should be the total capital gain (or loss) for the entire sequence, using the FIFO protocol to identify shares.

Hash Tables

5. Demonstrate what happens when we insert the keys 5; 28; 19; 15; 20; 33; 12; 17; 10into a hash table with collisions resolved by chaining. Let the table have 9 slots,and let the hash function be $h(k) = k \ mod \ m$.

6. Professor Marley hypothesizes that he can obtain substantial performance gains by modifying the chaining scheme to keep each list in sorted order. How does the professor's modification affect the running time for successful searches, unsuccessful searches, insertions, and deletions?

7. Consider a hash table of size 10, implemented using linear probing for collision resolution. The hash function used maps keys to indices in the range [0, 9]. Initially, the hash table is empty. Perform a series of $put(k, v)$ operations with the following keys: 7, 17, 27, 37, 47, 57, 67, 77, 87, 97. Assume that the hash function generates collisions for all keys, resulting in linear probing. Calculate the final state of the hash table after inserting all the elements and determine the total number of probes required for successful insertion.

Binary Search trees

8. Suppose that we have numbers between 1 and 1000 in a binary search tree, and wewant to search for the number 363. Which of the following sequences could not bethe sequence of nodes examined?

   1. 2, 252, 401, 398, 330, 344, 397, 363.
   2. 924, 220, 911, 244, 898, 258, 362, 363.
   3. 925, 202, 911, 240, 912, 245, 363.
   4. 2, 399, 387, 219, 266, 382, 381, 278, 363.
   5. 935, 278, 347, 621, 299, 392, 358, 363.

9. Let M be an n$\times$m integer matrix in which the entries of each row are sorted in increasing order (from left to right) and the entries in each column are in increasingorder (from top to bottom). Give an efficient algorithm to find the position of aninteger x in M, or to determine that x is not there. How many comparisons of x with matrix entries does your algorithm use in worst case?

10. A company database consists of 10,000 sorted names, 40% of whom are known as good customers and who together account for 60% of the accesses to the database.There are two data structure options to consider for representing the database:

    - Put all the names in a single array and use binary search.
    - Put the good customers in one array and the rest of them in a second array.
    - Only if we do not find the query name on a binary search of the first array do we do a binary search of the second array.

    Demonstrate which option gives better expected performance. Does this changeif linear search on an unsorted array is used instead of binary search for both options?

Height Balanced Trees

11. Consider an empty AVL tree. Insert the following integers into the AVL tree in the given order: 5, 10, 15, 20, 25, 30, 35, 40. After each insertion, provide the resulting AVL tree structure and the height of the tree.

12. In a magical forest, there was a group of wise owls who were the guardians of the trees. Each owl represented a unique integer key, and they resided in an AVL tree to maintain balance and order within the forest. One day, a mischievous squirrel named Sam decided to play a prank on the owls. Sam started by inserting the integers 5, 10, 15, 20, 25, and 30 into the AVL tree in a random order. The owls were initially content and enjoyed the tranquility of the forest. However, Sam wasn't satisfied with just that. He knew that the owls had the power to restore balance to the forest. So, he decided to challenge the owls by performing a series of operations on the AVL tree.

    Challenge:

    1. Sam first inserted the integer 12 into the AVL tree. How did the AVL tree structure change after this insertion, and what was the resulting height of the tree?
    2. Feeling mischievous, Sam then deleted the owl with key 20 from the AVL tree. How did the AVL tree structure change, and what was the height of the tree after this deletion?
    3. Realizing that the owls were trying to restore balance to the forest, Sam inserted the integer 17 into the AVL tree. Describe the resulting AVL tree structure and its height.

    Finally, Sam attempted to remove the owl with key 10 from the AVL tree. However, the wise owls managed to keep the forest in balance and refused to let the deletion affect the tree. Describe the AVL tree structure and its height after this failed deletion. The owls were determined to maintain harmony in the forest and restore the AVL tree's balance after each operation. *Can you help them by providing the answers to the challenges posed by Sam?*

13. Once upon a time, in the enchanting land of Bharatvan, there stood a mesmerizing garden filled with vibrant, towering trees. Each tree in the garden represented a unique integer key, and they were organized in an AVL tree to maintain balance and harmony. Among the inhabitants of Bharatvan was a mischievous sprite named Leela. Leela was known for her playful nature and her love for exploring every nook and cranny of the garden. One sunny afternoon, as Leela fluttered through the garden, she noticed a peculiar tree with the key values: 10, 20, 30, 40, 50, and 60 forming an AVL tree structure.

    Curiosity sparked within Leela, and she decided to test the resilience of the AVL tree. She decided to perform a daring act by removing a key from the tree, hoping to witness the tree adapt and maintain its balance.

    Question: If Leela removes the key with a value of 40 from the AVL tree, what would be the resulting AVL tree structure? Describe the new arrangement of the tree, including any necessary rotations, and specify the new heights of the affected nodes.

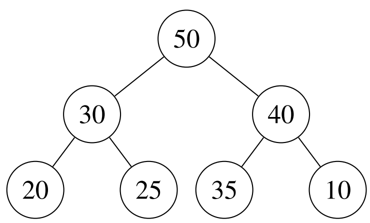
Heaps

14. What is the effect of calling MAX-HEAPIFY(A, i) when the element $A[i]$ is larger than its children?

15. Illustrate the operation of MAX-HEAPIFY$(A, 3)$ on the array $A = \langle 27; 17; 3; 16; 13; 10; 1; 5; 7; 12; 4; 8; 9; 0 \rangle$.

16. In a mystical kingdom called Ratnadesh, there existed a legendary treasure known as the Ratna Kund. It was believed that this sacred pool possessed incredible powers, as it held precious gemstones with unique numerical values. The Ratna Kund was organized in a structure reminiscent of a binary tree, with the most valuable gemstone always placed at the top. A courageous warrior named Rajveer embarked on a quest to explore the mysteries of the Ratna Kund. Armed with his mighty sword and an unwavering spirit, Rajveer was determined to unlock the hidden secrets within the gemstone pool.

    Question: As Rajveer delved deeper into his quest, he encountered a mesmerizing sight - the Ratna Kund. The current state of the pool was as follows:



    Inspired by the enchanting beauty of the Ratna Kund, Rajveer decided to perform a series of operations. First, he immersed a new gemstone with a value of 45 into the pool. Describe how the Ratna Kund would be modified after this insertion, ensuring that it maintains its structure with the most valuable gemstone at the top.

    Filled with a sense of accomplishment, Rajveer was eager to continue his exploration. His next task involved removing the most valuable gemstone from the Ratna Kund. If Rajveer were to extract the topmost gemstone, what would be the resulting structure of the pool? Depict the modified Ratna Kund and indicate how the remaining gemstones would be rearranged to maintain the pool's properties.

    Embark on this numerical adventure alongside Rajveer and unravel the secrets of the Ratna Kund. Use your understanding of gemstone insertion and deletion to guide you through this captivating journey in the mystical kingdom of Ratnadesh!