Final Assessment Test – November 2019

VIT
Vellore Institute of Technology

Course: MAT1014 - Discrete Mathematics and Graph Theory

Class NBR(s): 0481 / 0482 / 0485 / 0493 / 0546 Slot: A1+TA1+TAA1+V1

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) Obtain the PDNF and PCNF of the statement $(P \wedge Q) \vee (P \wedge R) \vee (Q \wedge R)$. [10]

   b) "If there was a ball game, then traveling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time. Therefore, there was no ball game. "Show that these statements constitute a valid argument". [10]

2. a) Show that the conclusion $(\forall x)(F(x) \rightarrow \neg S(x))$ follows logically from the premises $(\exists x)(F(x) \wedge S(x)) \rightarrow (\forall y)(M(y) \rightarrow W(y))$ and $(\exists y)(M(y) \wedge \neg W(y))$. [10]

   b) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book." [10]

3. a) State and prove Lagrange's theorem. [10]

   b) (i) Show that in a group $(G,*)$, if for any $a, b \in G$, $(a * b)^2 = a^2 * b^2$ then $(G,*)$ must be abelian. [5]

   (ii) Show that if every element in a group is its own inverse, then the group must be abelian. [5]

4. a) Give two partially ordered sets. In each case explain why they are lattices and not lattices? [10]

   b) Let $< L, \leq >$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively. Prove that for any $a, b \in L$ $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$. [10]

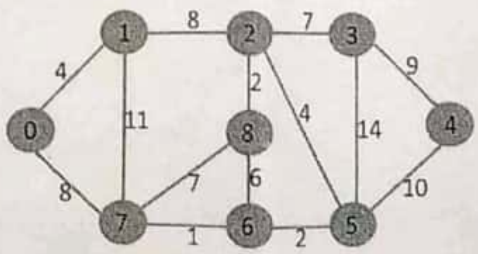
5. a) Find the sum of product expansion of the Boolean function $F(x, y, z) = (x + z)y$. [5]

   b) Use Karnaugh map to minimize the sum of product expansion $xyz + xy\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$. [5]

   c) Use McCluskey algorithm to minimize the sum of product expansion $wxy\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + \bar{w}xyz + \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}z$. [10]

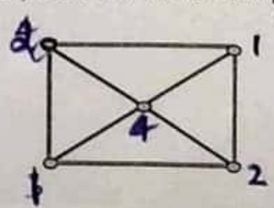
6. a) Prove that a given connected graph is an Euler graph if and only if all vertices of G are of even degree. [10]

   b) Use Dijkstras algorithm to find the shortest path between the vertices 0 and 4 [10]



7. a) Prove a tree with n vertices has exactly n-1 edges. [10]

   b) Find the chromatic polynomial of the following graph [10]



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Final Assessment Test – November 2019

Course: MAT1014 - Discrete Mathematics and Graph Theory
Class NBR(s): 0494 / 0496 / 0547 / 7205 Slot: A2+TA2+TAA2+V3
Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

Answer any **FIVE** Questions

(5 X 20 = 100 Marks)

1. a) Obtain the product of sum canonical form of $(7P \to R) \wedge (Q \leftrightarrow P)$ and hence find its sum of product canonical form. [10]

   b) Show that the following set of premises are inconsistent: [10]

   If the contract is valid, then John is liable for penalty. If John is liable for penalty, he will go bankrupt. If the bank will loan him money, he will not go bankrupt. As a matter of fact, the contract is valid and the bank will loan him money.

2. a) Write the negation of the statements (i) There is an honest politician, (ii) All Americans eat cheese burgers, (iii) $\forall x(x^2 > x)$, (iv) $\exists x(x^2 = 2)$, (v) All mammals are animals. [10]

   b) Verify whether the hypothesis "All hummingbirds are richly coloured", "No large birds live on honey", "Birds that do not live on honey are dull in colour" lead to the conclusion "Hummingbirds are small". [10]

3. a) Determine whether (S,*) where S={1,2,3,6,9,18} and a*b= L.C.M of (a,b) is a (i) semigroup (ii) Monoid (iii) Group. Also test whether it is an abelian group. [10]

   b) Let $e: B^2 \to B^6$ be an (2,6) encoding function defined as $e(00)$=000000, $e(01)$=011101 $e(10)$=001110, $e(11)$=111111. [10]

   (i) Find the minimum distance. (ii) How many errors can e detect? (iii) How many errors can e correct? (iv) Decode the word 101001.

4. a) Let S_n be a set of divisors of 'n'. (i) Draw the Hasse diagram of (S_n, /) for n=30 (ii) Test whether (S_n, /) is a Lattice. If so find all possible sublattices. [10]

   b) In a distributive Lattice show that $(a*b) \oplus (b*c) \oplus (c*a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$. [10]

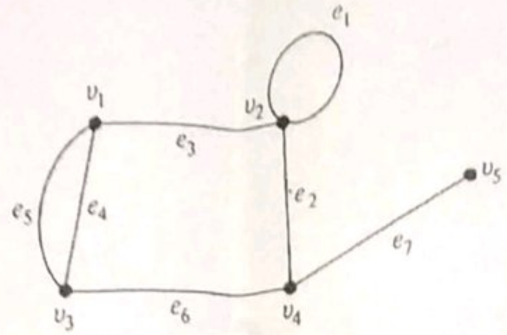
5. a) State and prove De Morgan's laws in a Boolean Algebra. [10]

   b) Simplify the Boolean Expression $xy'z' + xyz' + xyz + xy'z + x'y'z$ (i) using K-map (ii) without using k-map. [10]

$\overline{a+b} = \bar{a} * \bar{b}$

6. a) Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. [10]

b) For the given graph G, find (i) Incidence matrix, (ii) Adjacency matrix (iii) Degree of all vertices [10]



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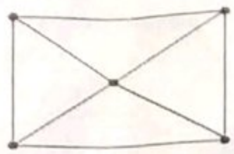
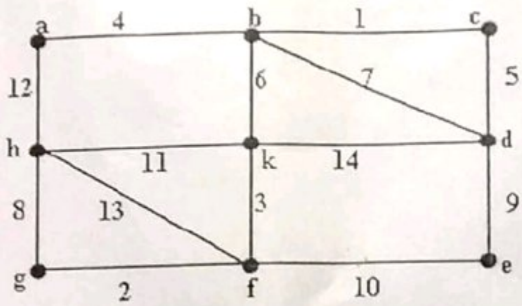
7. a) Find the Chromatic polynomial of the given graph H_1 [10]



H_1

b) (i) Determine the minimum spanning tree from the given graph H_2 using Krushkal's Algorithm. [6]



H_2

(ii) Find the number of edges of a tree with 20 vertices. [4]

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