VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Winter Semester 2018-19

Continuous Assessment Test – II

Programme Name & Branch: B. Tech.

Course Name & Code: Applied Linear algebra & MAT-3004
Slot: C2+TC2+TCC2

Exam Duration: 90 minutes **Maximum Marks:** 50

Answer All the Questions (5 × 10 = 50)

1. Let V, W be the subspaces of the vector space $P_3(\mathbb{R})$ spanned by $v_1(x)=3-x+4x^2+x^3$ $v_2(x)=5+5x^2+x^3$, $v_3(x)=5-5x+10x^2+3x^3$ and $w_1(x)=9-3x+3x^2+2x^3$ $w_2(x)=5-x+2x^2+x^3$, $w_3(x)=6+4x^2+x^3$ respectively. Find the dimensions and bases for $V+W$ and $V\cap W$. (10M)

2. (a) Find the equation of a circle that passes through the three points $(2,-2),(3,5)$ and $(-4,6)$ in the plane $\mathbb{R}^2$. (4M)

   (b) Let P_3 denote the vector space of all polynomials of degree 3 or less with real coefficients. Consider the ordered basis $B=\{1+x,1+x^2,x-x^2+2x^3,1-x-x^2\}$. Write the coordinate vector for the polynomial $f(x)=-3+2x^3$ in terms of the basis B. (6M)

3. Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be defined as $T(x,y,z)=(3x,x-y,2x+y+z)$. Prove that T is invertible and find T^{-1}. Also prove that $(T^2-I)(T^2-3I)=\bar{0}$ (10M)

4. Find the matrix representations $[T]_\alpha$ and $[T]_\beta$ of each of the following linear transformations T on $\mathbb{R}^3$ with respect to the standard basis $\alpha=\{\overline{e_1},\overline{e_2},\overline{e_3}\}$ and $\beta=\{\overline{e_3},\overline{e_2},\overline{e_1}\}$;

   (a) $T(x,y,z)=(2x-3y+4z,5x-y+2z,4x+7y)$.

   (b) $T(x,y,z)=(2y+z,x-4y,3x)$.

   Also find the matrix representation $[T]_\alpha^\beta$ of each of the linear transformations T. (10M)

5. Let α be the standard basis for $\mathbb{R}^3$, and let $S,T:\mathbb{R}^3\to\mathbb{R}^3$ be two linear transformations given by

   $S(\overline{e_1})=(2,2,1)$, $S(\overline{e_2})=(0,1,2)$, $S(\overline{e_3})=(-1,2,1)$ and $T(\overline{e_1})=(1,0,1)$ $T(\overline{e_2})=(0,1,1)$. $T(\overline{e_3})=(1,1,2)$ Compute $[S+T]_\alpha$, $[2T-S]_\alpha$ and $[T\circ S]_\alpha$. (10M)