9. a. Evaluate the triple integral $\iiint xyz\, dx\, dy\, dz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$. **(CO4 K3) 8M**

b. Evaluate the following integral by changing into polar coordinates

$$\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dx\,dy$$ **(CO4 K3) 7M**

* * *

Reg. No: | | | | | | | | | | |

VELAGAPUDI RAMAKRISHNA

SIDDHARTHA ENGINEERING COLLEGE

(AUTONOMOUS)

I/IV B.Tech. DEGREE EXAMINATION, FEBRUARY - 2024

First Semester

23BS1101 LINEAR ALGEBRA AND CALCULUS

(All Branches)

Time: 3 hours ***Max. Marks: 70***

Part-A is compulsory

Answer One Question from each Unit of Part - B

Answer to any single question or its part shall be written at one place only

PART-A

5 x 2 = 10M

1. a. Find the rank of the matrix $A=\begin{bmatrix}1&2&1\\-1&0&2\\2&1&-3\end{bmatrix}$. **(CO1 K2)**

b. If $A=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}$ then write the eigenvalues of adj (A) and A^{-1}. **(CO2 K2)**

c. if $x=\frac{u^2}{v}, y=\frac{v^2}{u}$ *find* $\frac{\partial(u,v)}{\partial(x,y)}$ **(CO3 K2)**

d. State Lagranges mean value theorem. **(CO3 K1)**

e. Evaluate $\int_0^2\int_0^x y\,dy\,dx$. **(CO4 K2)**

PART-B

4 x 15 = 60M

UNIT-I

2. a. Apply Gauss-Jordan method to find the inverse of the matrix

$$A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}.$$ **(CO1 K3) 7M**

b. Reduce the matrix $A=\begin{bmatrix}1 & 3 & -1 & 2\\ 0 & 11 & -5 & 3\\ 2 & -5 & 3 & 1\\ 4 & 1 & 1 & 5\end{bmatrix}$ into Normal form and hence find its rank. **(CO1 K3) 8M**

(or)

3. Solve, by Gauss-Seidal iteration method, the equations

$20x+y-2z=17, 3x+20y-z=-18, 2x-3y+20z=25$

(CO1 K3) 15M

UNIT-II

4. a. Verify Cayley-Hamilton theroem for the matrix $A=\begin{bmatrix}3 & 1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{bmatrix}$ hence find A^{-1} and A^4. **(CO2 K3) 8M**

b. Find the Eigenvalues and corresponding Eigenvectors of

$$A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$$ **(CO2 K3) 7M**

(or)

5. Reduce the quadratic form $3x^2+2y^2+3z^2-2xy-2yz$ into canonical form by an Orthogonal transformation and hence find rank, index and signature of the quadratic form. **(CO2 K3) 15M**

UNIT-III

6. a. If f(x)=log(1+x), x > 0, using Maclaurin's theorem, prove that

$$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3(1+\theta x)^3} \text{ for } 0<\theta<1$$ **(CO3 K3) 8M**

b. Verify Rolle's theorem for $f(x)=(x+2)^3(x-3)^4$ in (-2, 3) **(CO3 K2) 7M**

(or)

7. a. Find the dimensions of the rectangular box open at the top of maximum capacity whose surface area is 108 sq.inches. **(CO3 K3) 8M**

b. Expand $e^x \cos y$ in terms of x and y. **(CO3 K2) 7M**

UNIT-IV

8. a. Change the order of integration and evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$ **(CO4 K3) 8M**

b. Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dxdydz$. **(CO4 K3) 7M**

(or)