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Final Assessment Test – April 2019

Course: MAT2001 - Statistics for Engineers
Class NBR(s): 0505 / 0508 / 6199
Slot: D1+TD1
Time: Three Hours
Max. Marks: 100

General Instructions :

I. Students are allowed to use the scientific calculators but programming calculators must not be permitted.
II. Statistical tables must be made available to the students during the examinations as per need.
III. Graph papers if needed should be provided during examinations.

Answer any FIVE Questions
(5 X 20 =100 Marks)

1. a) i) For a group of 200 candidates, the mean and standard deviation were found to be 40 and 15 respectively. Later on it was discovered that the scores 43 and 35 were misread as 34 and 53 respectively. Find the corrected mean and standard deviation corresponding to the corrected figures. **[5]**

ii) An incomplete frequency distribution is given below as follows: **[5]**

Variable	Frequency
10-20	12
20-30	30
30-40	?
40-50	65
50-60	?
60-70	25
70-80	18
Total	229

Given the median value is 46. Determine the missing frequencies using the median formula.

b) The expenditure of 100 families is given below as follows: **[10]**

Expenditure	No. of families
0-10	14
10-20	?
20-30	27
30-40	?
40-50	15

If mode value of the distribution is 24. Calculate the missing frequencies.

2. a) i) If the probability density of X is given by **[5]**

$$f(x) = 2(1-x) \quad ; \text{for } 0 < x < 1$$
$$= 0 \quad ; \text{elsewhere}$$

Find $E(x^r) = \frac{2}{(r+1)(r+2)}$ and compute the value of $E[(2X+1)^2]$.

ii) If the joint probability density of random variable X and Y is given by **[5]**

$$f(x, y) = \frac{2}{7}(x + 2y) \quad ; \text{for } 0 < x < 1, 1 < y < 2$$
$$= 0 \quad ; \text{elsewhere}$$

Then, find the expected value of $g(XY) = \frac{X}{Y^3}$.

b) Find the moment–generating function of the random variable 'X' whose probability density is given by **[10]**

$$f(x) = e^{-x} \quad ; \text{for } x > 0$$
$$= 0 \quad ; \text{elsewhere}$$

and hence find its mean and variance.

3. a) If the joint probability distribution of random variable X and Y is given by [10]

		X			Total
		-1	0	1	
Y	-1	1/6	1/3	1/6	2/3
	0	0	0	0	0
	1	1/6	0	1/6	1/3
Total		1/3	1/3	1/3	1

Compute the covariance and comment on your findings.

b) In partially destroyed laboratory record of an analysis of correlation data, the following results only are legible: [10]

Variance of X i.e V(X) = 9 and

The regression equations are : $8X - 10Y + 66 = 0$ and $40X - 18Y = 214$

If both the lines of regression pass through the point $(\bar{X}, \bar{Y})$

Then find the followings

(i) The mean values of X and Y

(ii) The correlation coefficient between X and Y and

(iii) The standard deviation of Y.

4. a) i) According to electrical circuit theory, the voltage drop across a resistor is related to the current flowing through the resistor by the equation V = IR, where R is the resistance level measured in ohms, I is the current in amperes, and V is the voltage in volts. In alternating current systems, the direction and magnitude of the current change in a cyclic pattern. Consequently, measurements of the voltage will have a distribution that is symmetric about the mean. Suppose that the measured voltage in a certain electrical circuit has a normal distribution with mean 120 and standard deviation 2 and five measurements of the voltage are taken. Determine the probability that two of the measurements lie outside the range 118-122. [5]

(ii) A quality control engineer is interested in the fraction of defectives present in a large lot of manufactured items. Due to cost constraints and the fact that the testing procedure can damage the item being tested, it is not possible to test every item in the lot. The engineer selects 100 items at random with replacement and discovers no defects. Determine the probability that more than 2% of the items in the entire lot are defective. [5]

b) If a single bit (0 or 1) is transmitted over a noisy communications channel, it has probability p of being incorrectly transmitted. To improve the reliability of the transmission, the bit is transmitted n times, where n is odd. A decoder at the receiving end, called a majority decoder, decides that the correct message is that carried by a majority of the received bits. Under a simple noise model, each bit is independently subjected to being corrupted with the same probability p. The number of bits that is in error, X, is thus a binomial random variable with n trials and probability p of success on each trial; (here a success is an error). Find the probability that the message is correctly received in 5 trials and with $p = 0.1$. [10]

5. a) Suppose that on the basis of following data (observed frequencies), given below, we want to test/decide at the 0.05 level of significance whether the number of errors the compositor makes in setting a galley of type is a random variable having a Poisson distribution. [10]

No. of errors	0	1	2	3	4	5	6	7	8	9
Observed frequencies(f_i)	18	53	103	107	82	46	18	10	2	1

b) An oil company claims that less than 20% of all car owners have not tried its gasoline. Test this claim at the 0.01 level of significance, if a random check reveals that 22 of 200 car owners have not tried the oil company's gasoline. Write proper implication on your findings. [10]

6. a) Use the data shown in the following table to test at the 0.01 level of significance whether a person's ability in mathematics is independent of his or her interest in statistics. [10]

Interest in statistics	Ability in Mathematics			
		low	Avg.	High
	low	63	42	15
	Avg.	58	61	31
	High	14	47	29

Write appropriate comment on your findings.

b) The data given below are the coded information's on the yield of a chemical process using 5 batches of raw material selected randomly. [10]

B1	B2	B3	B4	B5
9.7	10.4	15.9	8.6	9.7
5.6	9.6	14.4	11.1	12.8
8.4	7.3	8.3	10.7	8.7
7.9	6.8	12.8	7.6	13.4
8.2	8.8	7.9	6.4	8.3
7.7	9.2	11.6	5.9	11.7
8.1	7.6	9.8	8.1	10.7

Using the knowledge of design of experiment can be claim that the batch variance component is significantly greater than zero? If yes then obtain its estimate.

7. a) A particular flashlight requires four batteries to operate. The batteries in this flashlight are configured in series. Suppose that the lifetimes of individual batteries are independent and have Weibul distributions with parameter $\alpha = 2$ and $\beta = 3$, where lifetimes are measured in hours. Determine the probability that the flashlight is still functional and reliable after 2 hours of continuous use. [10]

b) Find the system reliability function of the followings systems [10]

(i) The series system of n independent components.

(ii) The parallel system of n independent components.

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