



Name :

Roll No. :

Invigilator's Signature :

CS/B.TECH(NEW)/BME/ECE/EE/EIE/PWE/ICE/EEE/SEM-3/M-302/2011-12

2011

MATHEMATICS – III

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Short Answer Type Questions)

1. Answer any *ten* of the following : 10 × 2 = 20

a) Evaluate $\oint_C \frac{z}{z^2 - 1} dz$ where $C : |z| = 2$.

b) Find the singularities of $f(z) = \frac{1}{\sin z - \cos z}$.

c) Find the Fourier Sine Transform of

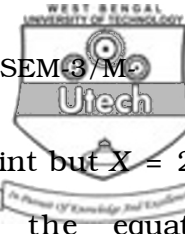
$$f(x) = \frac{1}{x}.$$

d) Find the period of the function $f(x) = 2 |\cos^2 x|$.

e) Prove that every function can be represented as a sum of even and odd function.

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[Turn over



- f) Show that $X = 0$ is a regular singular point but $X = 2$ is not a regular singular point of the equation
- $$x(x-2)^3 y'' + 3(x-2)^3 y' + 4y = 0.$$

- g) Find the general value of $\log(-i)$.

- h) From the discrete distribution find k :

| | | | | | | | | |
|----------|---|---|----|----|----|-------|--------|------------|
| $X :$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $P(X) :$ | 0 | k | 2k | 2k | 3k | k^2 | $2k^2$ | $7k^2 + k$ |

- i) Determine whether the function is even or odd :

$$f(x) = \log(x + \sqrt{x^2 + 1}).$$

- j) If $f(x) = x$, $0 < x < a$,

$$= 0, \quad x > a,$$

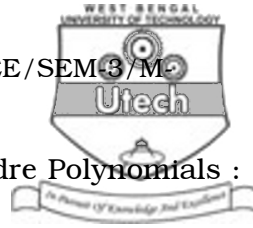
find the Fourier Sine Transform of $f(x)$.

- k) If $f(x) = e^{-x^2/2}$, then show that $F(s) \approx \sqrt{2\pi} e^{-s^2/2}$,

where $F(s)$ is the Fourier transform of $f(x)$.

- l) If $f(x) = e^{-x}$, $x \geq 0$, show that Fourier sine transform of $f(x)$ is $s F'(s)$, where $F(s) = \tan^{-1} s$.

- m) A random variable X has the density function $f(x) = x$, $0 \leq x \leq 1$, $f(x) = \frac{1}{2}$, $1 < x \leq 2$. Find the mean of X .



- n) Express $x^3 + x^2$ in terms of Legendre Polynomials :

$$P_0(x), P_1(x), P_2(x), P_3(x).$$

- o) A random variable X has the following probability density function :

| | | | | | |
|---------------------|---|----|----|---|---|
| $X :$ | 0 | 1 | 2 | 3 | 4 |
| $P(X = x) = f(x) :$ | 0 | 5k | 3k | k | k |

Determine the value of k .

GROUP – B

(Long Answer Type Questions)

Answer any *five* questions from the following :

$$5 \times 10 = 50$$

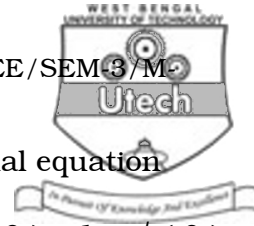
2. i) Let $f(x) = x$, $0 < x < 2$. Find the half range cosine series.

Write Parseval's Identity corresponding to $f(x)$. Hence show that

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots + \frac{1}{n^4} = \frac{\pi^4}{90}.$$

- ii) For a function defined by $f(z) = \sqrt{|xy|}$; show that the Cauchy-Riemann equations are satisfied at $(0, 0)$ but the function is not differentiable at that point.

$$5 + 5$$



3. i) Obtain series solution of the differential equation $\frac{d^2 y}{dx^2} + y = 0$ near $x = 0$ such that $y(0) = 1, y'(0) = 2$.

- ii) Expand the function $f(z) = \frac{1}{(z-1)(z-2)}$ between the annular region of $|z| = 1$ and $|z| = 2$. 5 + 5

4. i) If $U(x, y) = 4xy - x^3 + 3xy^2$, verify that U is harmonic function and obtain its conjugate $V(x, y)$ so that $f = U + iV$ is an analytic function and also find $f(z)$, where $z = x + iy, x, y \in \mathbb{R}$.

- ii) Find the Fourier sine transform of the function

$$f(x) = \begin{cases} 1 & \text{for } 0 < x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$$

and hence evaluate the integral

$$\int_0^{\infty} \frac{1 - \cos p\pi}{p} \sin px \, dp. \quad \text{5 + 5}$$

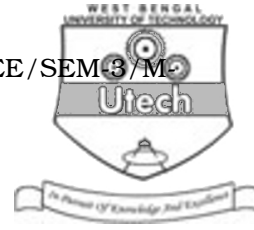
5. i) A random variable X has the following :

$$\text{p.d.f. } f(x) = cx^2, \quad 0 \leq x \leq 1$$

$$= 0, \text{ otherwise}$$

Find (i) C

$$\text{(ii) } P\left(0 \leq x \leq \frac{1}{2}\right).$$



- ii) a) Show that :

$$\int_{-1}^1 P_m(x) P_n(x) dx = 0, m \neq n.$$

- b) Show that $J_{-n}(x) = (-1)^n \cdot J_n(x)$, $n \in N$ and J_n is Bessel function of first kind. 5 + 5

6. i) Use the method of Frobenius to solve the following differential equation :

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0.$$

- ii) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, ($x > 0$, $t > 0$) subject to the conditions

a) $u(0, t) = 0$

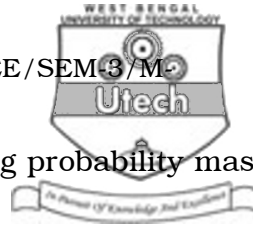
b) $u(x, 0) = 1, 0 < x < 1$

$= 0, x \geq 1$

- c) $u(x, t)$ is bounded. 5 + 5

7. i) Three bags contain respectively 4W, 7B, 9R; 5W, 8B, 7R and 6W, 9B, 5R balls. One ball is chosen at random from each bag. What is the probability that the balls are

- (a) of same colour (b) 2W and 1R ?



- ii) A random variable X has the following probability mass function :

| | | | | | | | |
|------------|-----|------|------|------|------|-------|-------|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $P(X = x)$ | k | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |

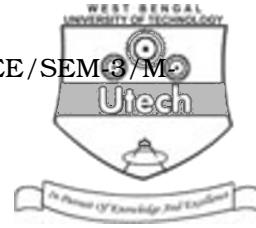
- Find the value of k
 - Find $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 5)$
 - Obtain the distribution function $F(x)$
 - What is the smallest value of x for which $P(X \leq x) > 0.5$?
8. i) The radius of a circle has distribution given by the p.d.f.

$$f(x) = 1, 1 < x < 2$$

$$= 0, \text{ otherwise}$$

Find the mean and variance of the area of the circle.

- If the weekly wage of 10,000 workers in a family follows normal distribution with mean and standard deviation Rs. 70 and Rs. 5 respectively, find the expected number of workers whose weekly wages are
 - between Rs. 66 and Rs. 72
 - less than Rs. 66
 - more than Rs. 72.



9. i) Show that

$f(z) = [x^3(1+i) - y^3(1-i)] / (x^2 + y^2)$,
 $z \neq 0, f(0) = 0$ is continuous and Cauchy's Reiman's
equation are satisfied at the origin yet $f'(0)$ does not
exist.

ii) Find the first four terms of the Taylor's series
expansion of $f(z) = (z+1) / (z-3)(z-4)$ about
 $z = 2$ and also find the region of convergence. 5 + 5

