



VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

School of Advanced Sciences

Department of Mathematics

CAT-I, August 2019

MAT 3003-Complex variables and Partial Differential Equations

Max. Marks: 50

Max. Time : 90 minutes.

Answer ALL questions (5 X 10 =50 marks)

1. In a two-dimensional fluid flow, if the velocity potential is given by  $\phi = x^4 - 6x^2y^2 + y^4$ , then
  - i) Find the stream function  $\psi$  and write the corresponding complex potential.
  - ii) Write the expression for velocity and hence find speed.
  - iii) Verify that the family of level curves  $\phi = c_1$  and  $\psi = c_2$  intersect orthogonally. (10M)
2. a) It is given that a function  $f(z)$  and its conjugate  $\overline{f(z)}$  are both analytic. Determine the function  $f(z)$ . (4M+6M)  
 b) Show that the function  $u(r, \theta) = r^2 \cos 2\theta$  is harmonic. Find its conjugate harmonic function and the corresponding analytic function  $f(z)$  in terms of  $z$ .
3. a) Find the Bilinear transformation which maps  $z = 1, 0, -1$  onto  $w = \infty, -1, 0$  respectively. (7M)  
 b) Find the image of  $|z - 2i| = 2$  under the transformation  $w = \frac{1}{z}$ . (6M)  
 c) Find the image of the semi-infinite strip  $x > 0, 0 < y < 2$  under the transformation  $w = iz + 1$ . (7M)  
 Also draw the graph of the region.
4. a) Obtain the Taylor's series expansion of  $\cosh z$  about  $z_0 = \pi i$ . Also find the radius of convergence of the series. (4M+6M)  
 b) Find the Laurent's series for the function  $f(z) = \frac{z^2 - 1}{(z + 2)(z + 3)}$  in the following regions:
  - i)  $|z| > 3$
  - ii)  $2 < |z| < 3$

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