Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH (AUE)/SEM-4/AUE-401/2010

2010

ENGINEERING ANALYSIS & NUMERICAL METHODS

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following :

$10 \times 1 = 10$

i) Newton-Raphson method used for finding the real roots of a numerical equation is

a) analytical method

b) graphical method

c) iterative method

d) none of these.

ii) If the interval of differencing is unity and $f(x) = ax^2$, find which one of the following choices is wrong.

a) $\Delta f(x) = a(2x+1)$

b) $\Delta^3 f(x) = 2$

c) $\Delta^4 f(x) = 0$

d) None of these.

iii) The accuracy attainable with Newton-Raphson method depends upon the value of the derivative $f'(x)$. It is

a) True b) False.

iv) In evaluating $\int_a^b f(x)\,dx$, the error in Trapezoidal rule is of the order

a) h^3 b) h^4

c) h^5 d) none of these.

v) The rate of convergence of Newton-Raphson method is

a) 1 b) 2

c) 3 d) none of these.

vi) The shift operator E is equal to

a) $1 + \Delta$ b) $(1+\Delta)^{-1}$

c) $1 - \Delta$ d) $1 - \Delta^2$.

vii) Which of the following is not true ?

a) $\Delta = E - 1$ b) $\Delta . \nabla = \Delta - \nabla$

c) $\frac{\Delta}{\nabla} = \Delta + \nabla$ d) $\Delta = 1 - E^{-1}$.

viii) One of the iterative methods by which we can find the solution of simultaneous linear equation is

a) Gauss Elimination method

b) Gauss-Seidel method

c) Matrix Inversion method

d) none of these.

ix) For the differential equation $\frac{dy}{dx} = 1 - y$; $y(0) = 0$, the value of $y(0 \cdot 2)$ is

a) $0 \cdot 1$ b) $0 \cdot 01$

c) $0 \cdot 2$ d) none of these.

x) The error in 4th order Runge-Kutta method is of order

a) h^3 b) h^4

c) h^5 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* from the following. $3 \times 5 = 15$

2. Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using Simpson's $\frac{1}{3}$ rule taking $h = \frac{1}{6}$.

   Hence compute an approximate value of π.

3. Find a real root of the equation $x^3 - 2x - 5 = 0$ by Newton-Raphson method, correct up to three decimal places.

4. Solve the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 1$, $x \in [0, 1]$, by Taylor's series method to obtain y for $x = 0 \cdot 1$.

5. Compute $y(0 \cdot 3)$ by Euler's method

   $$\frac{dy}{dx} = y^2 - x^2 \quad \text{with} \quad y(0) = 1$$

   and correct up to three decimal places.

6. Using Stirling's formula, find $f(2 \cdot 9)$ from the following table :

***x* :**	1	2	3	4	5
***f (x)* :**	1	– 1	1	– 1	1

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Using the Trapezoidal rule evaluate

$$\int_1^2 \int_1^2 \frac{dx\,dy}{x+y}$$

with the length of subinterval $h = 0{\cdot}25$ (along x-axis) and $k = 0{\cdot}25$ (along y-axis). Compare the exact solution.

b) Find a real root of the equation $x^2 - y^2 = 4$ and $x^2 + y^2 = 16$ by Newton-Raphson method correct up to two decimal places. Take $x_0 = y_0 = 2\sqrt{2}$. 7 + 8

8. a) Find all eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

by Jacobi method.

b) Determine the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

by Power method. 8 + 7

9. a) Using Gauss-Jacobi method, find the solutions of the equations

$$8x - 3y + 2z = 20$$

$$6x + 3y + 12z = 35$$

$$4x + 11y - z = 33$$

correct up to three decimal places.

b) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 2 & -3 \\ -3 & 2 & 2 \\ 2 & -3 & 2 \end{bmatrix}$$

by Gauss-Jordon method.

c) Show that $\Delta \log f(x) = \log \left[1 + \frac{\Delta f(x)}{f(x)} \right]$

6 + 6 + 3

10. a) Find $f'(1 \cdot 1)$ and $f''(1 \cdot 1)$ from the following table :

x **:**	1·0	1·2	1·4	1·6	1·8	2·0
f(x) **:**	0·0	0·1280	0·5540	1·2960	2·4320	4·0000

b) Solve the following system of equations of Gauss-Seidel method :

$$x + 5y - z = 10$$

$$4x + 2y + z = 14$$

$$x + y + 8z = 20$$

correct up to three significant figures. 7 + 8

11. a) Compute $y\,(\,0{\cdot}1\,)$, $y\,(\,0{\cdot}2\,)$, $y\,(\,0{\cdot}3\,)$ from the following differential equation :

$$\frac{dy}{dx} = x + y \; ; \; y\,(\,0\,) = 1 \quad \text{takeing} \quad h = 0{\cdot}1.$$

b) Given the initial value problem

$$\frac{dy}{dx} = 1 + y \; ; \; y\,(\,0\,) = 0$$

Find $y\,(\,0{\cdot}6\,)$ by Runge-Kutta 4th order method taking $h = 0{\cdot}2$. 8 + 7