VIT
Vellore Institute of Technology

Final Assessment Test – November 2018

Course: MAT2002 - Applications of Differential and Difference Equations
Class NBR(s): 2195 / 2212 / 4478 Slot: B2+TB2
Time: Three Hours Max. Marks: 100

Answer any FIVE Questions
(5 X 20 = 100 Marks)

1. a) A sinusoidal voltage $E\sin wt$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. Find the Fourier series of the following of the $E\sin wt$ resulting periodic function $u(t)=\begin{cases}0 & if\ -L<t<0\\ E\sin wt & if\quad 0<t<L\end{cases}$. [10]

b) The following table gives the variation of periodic current over a period. [10]

t sec	0	T/6	T/3	T/2	2T/3	5T/6	T
A *amp.*	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 *amp* in the variable current and obtain the amplitude of the first harmonic.

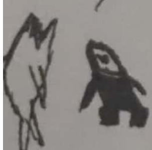
2. a) Reduce the quadratic form $8x^2+7y^2+3z^2-12xy-8yz+4zx$ into canonical form by an orthogonal transformation and give the matrix transformation. Also state the nature, rank, index and signature of it. [10]

b) Verify Caley-Hamilton theorem for the matrix $\begin{bmatrix}1&0&2\\0&1&1\\0&0&2\end{bmatrix}$ and hence find A^{-1}, A^{-2} and A^{3}. [10]

AA^{-1}

3. a) Find the charge and maximum value of the current at any subsequent time t in an RLC-circuit with $R=100$ ohms, $L=0.1$ henry, $C=10-3$ farad, which is connected to a source of voltage $E(t)=155\sin 377t$ (60 cycles/sec), assuming zero charge and current when $t=0$ by the method of undetermined coefficients. [10]

b) The radial displacement u in a rotating disc at a distance r from the axis is given by $r^2\frac{d^2u}{dr^2}+r\frac{du}{dr}-u+kr^3=0$, where k is constant. Solve the equation under the conditions $u=0$ when $r=0$, $u=0$ when $r=a$. [10]



$2=\frac{1}{\sqrt{R^2+\omega^2L^2}}$

4. a) In an electrical circuit with *e.m.f.* $E(t)$, resistance R and inductance L, the current i builds up at the rate given by $L\frac{di}{dt}+Ri=E(t)$. If the switch is connected at $t=0$ and disconnected at $t=a$, find the current i at any instant by Laplace transform method. [10]

b) Suppose that the mass in a mass-spring –damper system with $m=10, c=9$, and $k=2$ is set in motion with initial position $x(0)=0$ and initial velocity $v_0=5\,m/s$. Find the position function $x(t)$ so that transforming the governing model into first order system of equations by Matrix method. [10]

$C_{A+B}=C_AC_B-S_AS_B$
$C_{A-B}=C_AC_B+S_AS_B$
$S_{A+B}=S_AC_B+C_AS_B$
$S_{A-B}=S_AC_B-C_AS_B$
$S_AC_B=S\frac{A+B}{2}+S\frac{A-B}{2}$

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5\. a) Find the eigen values and eigen functions of the Strum-Liouville boundary value problem $-y''(x)=\lambda y(x)$, posed on the interval $[0, L]$ as $y'(0)=0=y'(L)$ and prove that their orthogonality of eigen functions. [10]

b) Obtain Bessel's solutions of first and second kind of order 'n' by the method of Frobenius. What role do the zeros of Bessel functions play in the context of their applications in engineering? [10]

6\. a) Use Z-transforms to find the sum of the squares of all integers from 1 to n: $y_n=\sum_{1}^{n}k^2$. [10]

b) Calculate the convolution $\{y_n\}$ of the sequences $\{u_n\}=\{a^n\}$ and $\{v_n\}=\{b^n\}$ where $a\neq b$ (i) directly (ii) using partial fractions. [10]

7\. a) Obtain the (i) unit impulse response and (ii) unit step response of the system specified by the second order difference equation $y_n-\frac{3}{4}y_{n-1}+\frac{1}{8}y_{n-2}=x_n$ by Z-transforms. Assume that both responses refer to the case of zero initial conditions. [10]

b) Find the transient motion and steady periodic oscillations of a damped mass-spring system with $m=1$, $c=2$, and $k=26$ under the influence of an external force $F(t)=82\cos 4t$ with $x(0)=6$ and $x'(0)=0$. Also, investigate the possibility of practical resonance for this system. [10]

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$2(a_n)=\sum_{n=0}^{\infty}\frac{k_i}{2x}$

$\frac{n(2n+1)(2n+1)}{6}$