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DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
Continuous Assessment Test – I, January 2020

Course Code: MAT 3003 Date of Exam: 19-01-2020
Course Name: Complex Variables and Partial Differential Equations
Slot:A1+TA1+TAA1
Max. Marks: 50 Duration: 90 minutes

Answer ALL the questions

1. Find the analytic function $f(z) = u + iv$, given that $u = e^{-x}\cos y + xy$. Also show that the level curves $u(x,y) = \alpha, v(x,y) = \beta$ cut orthogonally. [10]

2. Discuss the transformation $f(z) = z^2$ as the flow of fluid around a corner. Draw the equipotential lines and the stream lines. Also find the complex velocity and speed of the flow. [10]

3. (a) Show that under the mapping $w = 1/z$, all circles and straight lines in the z–plane are transformed to circles and straight lines in the w–plane. [5]

   (b) Under the mapping $w = e^{-z}$, find the image in the w–plane of the rectangle $R: 0 \leq x \leq 1, 0 \leq y \leq \pi/4$ in the z–plane. Is the mapping conformal? [5]

4. Determine the bilinear transformation that maps the points $z = 0, 1, \infty$ into the points $w = -i, 1, i$ respectively. Find the invariant points of this transformation. Find the image of $|z| < 1$ under this transformation. [10]

5. (a) Find the Taylor's series expansion of $f(z) = \dfrac{1}{4-3z}$ about the point $z_0 = 1 + i$. Determine the region of convergence. [4]

   (b) Expand the function $f(z) = \dfrac{1}{(z-1)^2(z-3)}$ in Laurent series valid in the regions: [6]

   (i) $0 < |z-1| < 2$ (ii) $|z| > 3$.

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