Sub Code: BAST 102 **ROLL NO……………..……………..**

ODD SEMESTER EXAMINATION, 2024 – 25

1ST Year (1ST Sem) B.Tech.

MATHEMATICS I

Duration: 3:00 hrs **Max Marks: 100**

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	Answer any two parts of the following. (10x2= 20) a) (i) Verify Rolle's theorem for the function $f(x) = sinx + cosx - 1$ in $[0, \frac{\pi}{2}]$. (5 marks) (ii) If $z = x.\log(x+r) - r$, where $x = r\cos\theta$, $y = r\sin\theta$, prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{x+r}$. (5 marks) b) (i) Express $x^4 + 3x^3 - 8x + 20$ in powers of $(x+1)$ using Taylor's theorem. (5 marks) (ii) Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (5 marks) c) If $u = ax^2 + by^2 + c z^2$ where $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$. prove that the stationary values of u satisfy the equation $\frac{l^2}{a-u} + \frac{m^2}{b-u} + \frac{n^2}{c-u} = 0$. (10 marks)
Q 2.	Answer any two parts of the following. (10x2= 20) a) (i) Find the volume if the area of the loop of $y^2 = x^2(x+4)$ is revolved about x –axis. (5 marks) (ii) Evaluate $\iint y\, dxdy$ over the area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. (5 marks) b) Solve $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\, dydx}{\sqrt{x^2+y^2}}$ by Changing the order of integration . (10 marks) c) Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$. (10 marks)
Q 3.	Answer any two parts of the following. (10x2= 20) a) (i) Find the constants m and n such that the surface $\mathrm{mx^2 - 2nyz = (m+4)x}$ will be orthogonal to the surface $\mathrm{4x^2y+z^3 = 4}$ at the point $(1, -1, 2)$. (5 marks) (ii) Prove that $(y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is solenoidal. (5 marks) b) The vector field $\bar{F} = x^2 i + zj + yzk$ is defined over the volume of the cuboid given by $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ enclosing the surface S. Evaluate the surface integral $\iint_S \bar{F}\,.\hat{n}\, dS$. (10 marks) c) Verify Green theorem for $\int_C \{(3x^2 - 8y^2)dx + (4y - 6xy)dy\}$ where C is the boundary of the region bounded by $x = 0$, $y = 0$, $x + y = 1$. (10 marks)
Q 4.	Answer any two parts of the following. (10x2= 20) a) (i) Prove that the intersection of any two vector subspaces of a vector space is a subspace. (5 marks) (ii) Show that the vectors $(1,3,2), (1,-7,-8)$ & $(2,1,-1)$ of $V_3(F)$ are linearly dependent. (5 marks) b) Explain the concept of basis of a vector space. If $S = \{\alpha_1, \alpha_2, \alpha_3, \ldots\ldots\ldots\ldots.. \alpha_n\}\alpha$ is the basis of a vector space $V(F)$, then show that each element of V is uniquely expressed as a linear combination of elements of S. (10 marks) c) Prove that the set $V = \{(a,b): a, b \in R\}$is a vector space with respect to the compositions of addition and scalar multiplication defined as under $(a,b) + (c,d) = (a+c, b+d)$and $k(a,b) = (ka, kb)$. (10 marks)

Q 5.	Answer any two parts of the following. (10x2= 20)
	a) (i) Find the rank of the following matrices A by reducing it into the normal form where $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ (5 marks)
	(ii) Test the consistency of the system of linear equations and hence find the solution $x + 2y - z = 3,\ 3x - y + 2z = 1, 2x - 2y + 3z = 2,\ and\ x - y + z = -1$ (5 marks)
	b) Calculate the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$ (10 marks)
	c) Verify Cayley−Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ Hence find A^{-1}. (10 marks)

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