

Fall Semester 2022-2023
Continuous Assessment Test - I
Model Paper 2

Course Code: BMAT101L

Course Title: Calculus

Max. Time: 50 minutes

Max. Marks: 50

Slot:

Answer all the questions

1. **Easy** 10 marks **CO1** **BT1**

Consider $f(t) = t^5 - 5t$ for all real t . Using Rolle's theorem, find the appropriate stationary points of f in $(-3, 3)$. Then use the first derivative test to find the local maxima and local minima of $f(t)$. What are the points of inflection of f ?

2. **Medium** 10 marks **CO1** **BT1**

- (a) Find the points of intersection of the curves $y = x^2$ and $y = x^3$. Use this information to find the area of the region \mathcal{R} , enclosed by these two curves.
- (b) If the region \mathcal{R} is revolved about the y -axis, compute the volume of the solid so generated.

3. **Easy** 10 marks **CO2** **BT2**

- (a) Examine the continuity of

$$f(x) = \begin{cases} \frac{x^3+y^3}{x+y}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0). \end{cases}$$

- (b) If $f(x, y, z) = xy - 2yz + 3zx$, where $x = t^2$, $y = t + 1$, $z = t - 1$. Find the total derivative of f with respect to t in terms of t .

4. **Tough** 10 marks **CO2** **BT2**

Consider $p = x^2 + 4y^2 + 9z^2$, $q = 4xy + 12yz + 6xz$, $r = x + 2y + 3z$. Compute the Jacobian $J \left(\frac{p, q, r}{x, y, z} \right)$ at any point (x, y, z) . Also examine whether p , q and r are functionally related. If so, find the relation among them.

5. **Medium** 10 marks **CO2** **BT1**

Let $f(x, y) = \sin 2x \cos 3y$. Then find all the partial derivatives of f up to third order at the origin, and then obtain a cubic approximation of f near the origin.