End-Term Examination

(CBCS)(SUBJECTIVE TYPE)(Offline)

Course Name:<B.Tech. - IT/MAE/DMAM Semester:<1st>

(Feb-March, 2023)

Subject Code: BAS 101	Subject: Applied Mathematics 1
Time :3 Hours	Maximum Marks :60
Note : Q1 is compulsory. Attempt one question each from the Units I, II, III & IV.	

(2.5*8=20)

Q1

(a) Check if the matrix $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} \end{bmatrix}$ is orthogonal or not.

(b) Prove that eigen value of a Skew-Hermitian matrix are either zero or purely imaginary.

(c) Show that the sequence $\{x_n\}$ where $x_n = \sqrt{n+1} - \sqrt{n}$, is monotonic decreasing and also find the point of convergence.

(d) Calculate the approximate value of $\sqrt{24}$ correct to two decimal places using Taylor's series.

(e) If $y = e^x \sin 2x \sin 3x$, then find y_n.

(f) If $u = \log\frac{x^4+y^4}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$

(g) Find the area lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

(h) Evaluate $\int_0^a \int_0^a \int_0^a (yz + zx + xy)\ dx\, dy\, dz.$

UNIT-I

Q2 (a) Find for what value of α and β the system of linear equations: (5*2=10)

$$x + y + z = 6$$
$$x + 2y + 5z = 10$$
$$2x + 3y + \alpha z = \beta$$

have unique solution, no solution and infinite solutions. Find the solutions wherever it exist.

(b) Check the linear dependence and independence of the vectors [2, -1, 4], [0, 1, 2], [6, -1, 16] and find the relation between them, if possible.

Q3 (a) Find the modal matrix P which transforms the matrix (5*2=10)

$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ to diagonal form.

PTO

(b) Check the linear dependence and independence of the vectors [1, 0, 2], [3, 1, 2], [4, 6, 2]. Also, find the relation between them if it exist.

UNIT-II

Q4 (a) Examine the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^2}$ for convergence and absolute convergence. (5*2=10)

(b) Prove that

$$\tan^{-1}(x+h) = \tan^{-1}x + (h\sin z).\frac{\sin z}{1} - (h\sin z)^2.\frac{\sin 2z}{2} + (h\sin z)^3.\frac{\sin 3z}{3} - \cdots$$

where $z = \cot^{-1}x$.

Q5 (a) Examine the series (5*2=10)

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \cdots$$

for convergence and absolute convergence.

(b) If $y = \tan^{-1}x$, prove that

$$(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0.$$

Also find the values of all derivatives of y when $x = 0$.

UNIT-III

Q6 (a) Trace the curve $xy^2 = 4a^2(2a-x)$ (5*2=10)

(b) Prove that if the perimeter of a triangle is constant, then its area is maximum if the triangle is equilateral.

Q7 (a) Trace the curve $y^2(2a-x) = x^3$. (5*2=10)

(b) Locate the stationary points of the function $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ and determine their nature.

UNIT-IV

Q8 (a) Evaluate the integral (5*2=10)

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$$

by changing the order of integration.

(b) Evaluate $\int_{y=0}^{e} \int_{x=1}^{\log y} \int_{z=0}^{e^x} \log z \; dz\,dx\,dy$.

Q9 (a) Evaluate $\iint \sqrt{a^2-x^2-y^2}\; dx\,dy$ over the semi-circle $x^2+y^2 = ax$ in the positive quadrant by changing the variables into polar coordinates. (5*2=10)

(b) Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\; dx\,dy\,dz$.