Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech(New)/CSE/IT/SEM-4/M-401/2013

2013

MATHEMATICS-III

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The number of generations of an infinite cyclic group is

a) 1 b) 2

c) infinite d) none of these.

ii) The standard deviation of a sample mean for SRSWR is

a) σ^2/n b) $\sigma/\sqrt{n}$

c) σ/n d) n.

iii) If t is a statistic such as $E(t^2) = 5$ and $E(t) = 2$, then the standard error of t is

a) 0 b) 1

c) 2 d) none of these.

iv) If the exponential distribution is given by the probability density function $f(x) = e^{-x}$, $0 < x < \infty$, then the mean of the distribution is

a) 1 b) 3

c) $\frac{1}{3}$ d) none of these.

v) The probability of an event A is $\frac{1}{3}$, that of $A + B$ is $\frac{1}{2}$ and that of AB is $\frac{1}{4}$. Then the probability of B is

a) $\frac{1}{12}$ b) $\frac{5}{12}$

c) $\frac{1}{6}$ d) none of these.

vi) Which one of the following sets forms a group under usual multiplication of complex numbers ?

a) $\{1, i\}$ b) $\{1, \omega, \omega^2\}$

c) $\{1, \omega^2\}$ d) $\{1, \omega\}$.

vii) The distribution for which the mean and variance are equal is

a) Poisson b) normal

c) binomial d) exponential.

viii) In a Binomial (n, p) distribution, if its mean and variance are 2 and 4/3 respectivity, then the values of n and p are

a) $8, \frac{1}{4}$ b) $6, \frac{1}{3}$

c) $4, \frac{1}{2}$ d) none of these.

ix) If G is a connected planar graph with n vertices, e edges and f faces, then $n - e + f = 2$. This statement is

a) True b) False.

x) The mean of Binomial variate is

a) np b) $np\ (1 - p)$

c) $\sqrt{np}$ d) none of these.

xi) Kuratowski's graph is a

a) planar graph b) regular graph

c) tree d) none of these.

xii) The order of the dihedral group D_4 is

a) 4 b) 6

c) 8 d) 64.

xiii) Every finite integral domain is a field. This statement is

a) True b) False.

xiv) If A and B are two subgroups of a group G, then which of the following is always a subgroup of G ?

a) $A \cup B$ b) $G - A$

c) $G - B$ d) $A \cap B$.

xv) The symmetric group S_3 has

a) 6 elements b) 8 elements

c) 9 elements d) none of these.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Let (Q, +) be the additive group of rational numbers and (Q^+, .) be the multiplicative group of positive rational numbers. Are these two groups isomorphic ? Justify your answer.

3. Prove Baye's theorem for repeated trials.

4. Examine whether faction $|x|$ in (– 1, 1) and zero elsewhere is a density function.

5. Show that a connected graph is Eulerian if and only if each of its vertices is of even degree.

6. Show that a field does not contain any zero divisor.

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Prove that a planar graph with n vertices, e number of edges and k number of components determines f number of regions, where $f = e - n + k + 1$. 8

   b) Let $\bar{X}$ be the sample mean of samples of size n drawn at random from a population which is normally distributed with mean μ and variance σ^2. Find the standard error of the statistic $\bar{X}$. 7

8. a) Find the mathematical expectation of the number of points obtained in a single throw of an unbiased die. 5

   b) Define Poisson distribution and find its mean and variance. 5

   c) Let f be a ring homomorphism from the ring Z of integers into itself such that $f(1) = 1$. Determine the homomorphism f. 5

9. a) Show that any simple connected planar graph with n vertices ($n \geq 3$) has at most ($3n - 6$) edges. 7

   b) Prove that every nontrivial subgroup of the additive group Z of integers is cyctic. 4

   c) Let R and S be two rings and $f : R \rightarrow S$ be a ring homomorphism. Show that kernel of f is a subring of R. 4

10. a) Determine the mean and variance of exponential distribution. 6

    b) Show that every cyclic group is commutative. 4

    c) Let H be a normal subgroup of a group G and G/H be the set of all cosets of H in G. Show that G/H forms a group under the composition

    $$(aH) \cdot (bH) = (ab)H \text{ for all } a, b \in G.$$ 5

11. a) The probability density function of a random variable X is assumed to be of the form $f(x) = cx^{\alpha}$, $0 \le x \le 1$ for some number and constant c. If $\{ X_1, X_2, \ldots, X_n \}$ is a random sample of size n, find the maximum likelihood estimate of α. 5

b) Let S' be the set defined by $S' = \{ z \in C : |z| = 1 \}$, where C is the set of all complex numbers. Show that S' forms a commutitive group under usual multiplication of complex numbers. 5

c) Let R be the additive group of real numbers and C^* be the multiplicative group of nonzero complex numbers. If $f = R \rightarrow C^*$ is a group homomorphism defined by $f(x) = e^{2\pi ix}$ for all $x \in R$, find the kernel of f. 5