Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS / B.TECH(ME / PE / AUE)OLD / SEM-3 / M-303 / 2012-13

2012

MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be supplied by the Institute on demand.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The value of m for which $2x - 2x^2 + my^2$ may be harmonic is

   a) 0 b) 1

   c) 2 d) 3.

   ii) The residue of a function can be evaluated, only if the pole is an isolated singularity.

   a) False b) True.

iii) A convex set may have finite number of points.

a) True b) False.

iv) The residue of $\frac{z^2}{z^2 + 3^2}$ at $z = i3$ is

a) $-i \,.\, 3/2$ b) $i \,.3/2$

c) 3 d) none of these.

v) Consider the differential equation

$(x - 2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \frac{1}{x}y = 0$. Then $x = 0$ is

a) an ordinary point

b) a singular point but not a regular singular point

c) a regular singular point

d) none of these.

vi) $J_{-1/2}(x) =$

a) $\sqrt{\frac{2}{\pi x}} \cos x$

b) $\sqrt{\frac{2}{\pi x}} \sin x$

c) $-\sqrt{\frac{2}{\pi x}} \left(\sin x + \frac{\cos x}{x} \right)$

d) none of these.

vii) The value of $\int_{-1}^{1} P_n(x)\,dx$, where $P_n(x)$ is Legendre polynomial of degree n is

a) 1, when $n = 0$ b) 0, when $n = 0$

c) 2, when $n = 0$ d) none of these.

viii) Let $S = \{ z \in C : |z| \leq 1 \}$. Then $z = i$ is

a) an interior point b) an exterior point

c) a boundary point d) none of these.

ix) The function $f(z) = \overline{z}$ is

a) continuous at $z = 0$

b) differentiable at $z = 0$

c) analytic

d) none of these.

x) The function $f(z) = \frac{e^{z^2}}{z^4}$ has

a) an essential singularity at $z = 0$

b) a pole of order 4 at $z = 0$

c) a simple pole at $z = 0$

d) no singularity at $z = 0$.

xi) Charnes Big-M method is used to solve

a) a Transportation Problem

b) an Assignment Problem

c) by graphical solution of LPP

d) by simplex method in LPP.

xii) The order and degree of the *p.d.e.*

$z_x^2 + z_y^2 = xy$ are

a) 2, 2 b) 1, 1

c) 2, 1 d) 1, 2.

xiii) "A transportation problem (TP) is basically an LPP."

a) True b) False.

xiv) The value of m for which the function

$f(x, y) = 3x + 3x^2 y + my^3$ is harmonic is

a) 1 b) 0

c) – 1 d) any value of *IR*.

xv) The equation $U_{xx} - U_{yy} = 0$ is

a) parabolic b) hyperbolic

c) elliptic d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta} = \frac{2\pi}{\sqrt{3}}$.

3. Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along $y = x^2$.

4. Prove that
$(n + 1) P_{n+1}(x) = (2n + 1) x P_n(x) - n P_{n+1}(x)$, $n \geq 2$.

5. Find the general solution of the partial differential equation :
$(mz - ny)\, p + (nx - lz)\, q = ly - mx$.

6. Solve the following by graphical method :

Maximize $X = 3x + 2y$

subject $x + y \geq 1$

$x + y \leq 3$

$x, y \geq 0$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions.

$3 \times 15 = 45$

7. a) Find the Taylor series expansion of a function of complex variable
$f(z) = \frac{1}{(z-1)(z-2)}$ about the point $z = 3$. Find the region of convergence.

b) State Cauchy's Integral formula and use it to evaluate

$$\int_C \frac{\cos(\pi z)}{z^2 - 1}\, dz,$$ around a rectangle with vertices

$2 \pm i,\ -2 \pm i.$ 8 + 7

8. A manufacturer uses wood and labour as main resources for producing tables and chairs. It is seen that 8 units of wood per table and 4 units of wood per chair are needed. Also 2 units of labour per table and 3 units of labour per chair are needed. The profit per table is Rs. 50 and that per chair is Rs. 45. If 1500 units of wood and 1000 units of labour are available, find the underlying L.P.P. of this problem and solve it graphically. 7 + 8

9. a) Solve by simplex method :

Maximize $Z = 4x_1 + 10x_2$

subject to $2x_1 + x_2 \le 10$

$2x_1 + 5x_2 \le 20$

$2x_1 + 3x_2 \le 18,$

$x_1,\ x_2 \ge 0.$

b) Use VAM to find the initial feasible solution of the following T.P. :

	D_1	D_2	D_3	Supply ↓
O_1	5	1	7	10
O_2	6	4	6	80
O_3	3	2	5	15
Demand →	75	20	50	

Test the solution for optimality. 8 + 7

10. a) Solve $\sqrt{p} + \sqrt{q} = 1$.

b) Show that $\int_{-1}^{1} P_m(x) P_n(x)\, dx = 0, \quad m \neq n$.

c) Show that $J_{-n}(x) = (-1)^n . J_n(x), \quad n \in N$ and J_n is Bessel function of first kind. 5 + 6 + 4