Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS / B.TECH(CHE / FT)OLD / SEM-3 / M-315 / 2012-13

2012

MATHEMATICS - III

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) If x is a uniform distribution defined in the interval $(4, 7)$, the variance is

a) 4 b) 3

c) $\frac{3}{4}$ d) 1.

ii) If $x = c$ is a point of discontinuiy then the sum of the fourier series $f(x)$ at $x = c$ for given $f(x) =$

a) $\frac{f(x+c)+f(x-c)}{2}$ b) $f(c)$

c) $\frac{f(x-c)+f(x+c)}{2}$ d) $\frac{f(c+0)+f(c-0)}{2}$.

iii) The value of $\sum_{i=1}^{n} f_i \left(x_i - \bar{x} \right)$ is zero.

a) True b) False.

iv) let $P_n(x)$ be Legendre polynomial of n-th order.

Then $\int_{-1}^{1} \left[p_n(x) \right]^2 \, dx =$

a) $\frac{1}{n+2}$ b) $\frac{2}{2n+1}$

c) $\frac{1}{n+1}$ d) $\frac{1}{n-1}$.

v) The value of K such that

$f(k) = K_x (1 - x)$, $0 < x < 1$

$= 0$, elsewhere

is a probability density function, is

a) 6 b) 2

c) 5 d) 0.

vi) For the equation $\frac{\partial^2 z}{\partial x^2} = 2xy \sqrt{\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}} = 5$, the order and the degree are

a) 2, 1 b) 2, 2

c) 2, 0 d) 2, $\frac{1}{2}$.

vii) An example of linear partial differential equation is,

a) $\frac{\partial^2 p}{\partial x \, \partial y} + \left(\frac{\partial p}{\partial x} \right)^2 = 0$

b) $\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 = xy$

c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

d) $\sqrt{\frac{\partial z}{\partial x}} + \sqrt{\frac{\partial z}{\partial y}} = x - y$.

viii) The complete solution of $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = 0$ is

a) $z = f_1(y + 2x) + f_2(y - 3x)$

b) $z = f_1(y - 2x) + f_2(y - 3x)$

c) $z = f_1(y - 2x) + f_2(y + 3x)$

d) $z = f_1(y + 2x) + f_2(y + 3x)$.

ix) If $f(x)$ is an odd function then for the Fourier expansion of $f(x)$, given by

$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, the correct statement is

a) $a_0 = 0, a_n \neq 0$ b) $a_0 \neq 0, a_n = 0$

c) $a_0 = 0, a_n = 0$ d) $a_0 \neq 0, a_n \neq 0$.

x) If $f(x)$ is a periodic function of period T, then the correct statement is

a) $f(x + T) = f(x) + T$

b) $f(x + t) = f(x) - T$

c) $f(x + T) = f(x)$

d) $f(x + T) = f(x) - f(T)$.

xi) The graph of the periodic function

$f(x) = -k, -a \leq x < 0$

$= k, 0 < x \leq a$

and $f(x + 2a) = f(x)$ for all x is a

a) square waveform b) saw-toothed waveform

c) triangular waveform d) half-wave rectifier.

xii) For the random variable X, the expression for $Var(aX + b)$ is

a) $a\,Var(X)$ b) $a\,Var(X) + b$

c) $Var(X) + ab$ d) $a^2 Var(X)$

xiii) The mean of a random variable having Poisson distribution with parameter 4 is

a) 1 b) 2

c) 3 d) 4.

xiv) Let (x, y) be a bivariate data assumed by a bi-variate (X, Y), then correlation coefficient of x & y is

a) $\frac{1}{\sigma_x \sigma_y}$ b) $\frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$

c) $\frac{\sigma_x \sigma_y}{\text{cov}(x, y)}$ d) $\sigma_x \sigma_y$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

3 × 5 = 15

2. Obtain the Fourier expansion of $x \sin x$ in $-\pi \leq x \leq \pi$.

3. Eliminate the arbitrary functions from the relation $z = f_1 (x + ay)$ and form the corresponding partial differential equation.

4. Solve the following partial differential equation

$$(mz - ny) \frac{\partial z}{\partial x} + (nx - lz) \frac{\partial z}{\partial y} = (ly - mx)$$

5. Solve in series the equation $\frac{d^2 y}{dx^2} + (x - 1) \frac{dy}{dx} + y = 0$ in the power of $(x - 2)$.

6. A sample of 506 persons showed the income distribution given below :

Income (Rs.)	***No. of persons***
150 — 300	232
300 — 450	128
450 — 600	60
600 — 750	40
750 — 900	28
900 — 1100	12
1100 — 1150	6

Find out the standard deviation.

7. If the probability of the horse *A* winning the race is $\frac{1}{5}$ and the probability of the horse *B* winning the same race is $\frac{1}{6}$, what is probability that one of the horses will win the race ?

GROUP – C

(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

8. a) Show that when n is a pentive integer, $J_{-n}(x) = (-1)^n J_n(x)$.

Solve : $\left(D^2 - 7DD'^2\right) z = e^{x+2y}$ $\left(D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y}\right)$

b) Find the particular solution of the partial differential equation

$(y - z)\frac{\partial z}{\partial x} + (z - x)\frac{\partial z}{\partial y} = x - y$ which passes through the curve $xy = 4,\ z = 0$.

c) Solve by the method of separation of variables :

$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(x, 0) = 4e^{-x}$. 5 + 5 + 5

9. a) Find the Fourier expansion of x^2 on $[-\pi, \pi]$. Hence prove that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi}{6}.$$

b) Find the half range sine series of the function $f(x) = x,\ 0 \le x \le 2$.

c) Find the Fourier series corresponding to the function

$$f(x) = -x - \pi,\ -1 < x < 0$$

$$= \pi - x,\ 0 < x < \pi$$

What is the value of the series at $x = 0$.

10. a) If X has Binomial distribution with parameters n and p then prove that its mean is np and variance is $np\,(1-p)$.

b) If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and s.d. Rs. 70 and Rs. 5 respectively, find the expected number of workers whose weekly wages are

i) between Rs. 66 and Rs. 72

ii) less than Rs. 66

iii) more than Rs. 72

$\left[\text{Given that } \frac{1}{\sqrt{2\pi}}\int_0^z e^{\frac{t^2}{2}}\,dt = 0{\cdot}1554 \text{ \& } 0{\cdot}2881 \text{ according as } z = 0{\cdot}4 \text{ \& } z = 0{\cdot}8\right]$

11. a) Let the lines of regression of two variables x and y be given by $y = 32 - x$ and $x = 13 - 0{\cdot}25\,y$. Obtain the values of the mean and the correlation coefficients.

b) Find the regression equation from the following data :

$$\sum_{i=1}^{n} X = 24 \quad \sum_{i=1}^{n} Y = 44 \quad \sum_{i=1}^{n} XY = 306$$

$$\sqrt{\frac{2}{\pi x}}\sin x, \quad \sum_{i=1}^{n} Y^2 = 574, \; n = 4$$

Find the estimated values of X when $Y = 6$.

c) The mean of a normal distribution is 50 and 5% of the values are greater then 60. Find the standard deviation of the distribution (given that the area under the standard normal curve between $Z = 0$ and $Z = 1{\cdot}64$ is $0{\cdot}45$). 5 + 5 + 5

12. a) If $P_n(x)$ is a Legendre polynomial of degree n and α is such that $P_n(\alpha) = 0$ then prove that

$P_{n-1}(\alpha)$ and $P_{n+1}(\alpha)$ are of opposite signs.

b) Solve $xy'' + 8y' + 5y = 0$ in series.

c) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.