CHENNAI INSTITUTE OF TECHNOLOGY

(An Autonomous Institution, Affiliated to Anna University, Chennai)

CHENNAI - 600 069

B.E. / B.Tech. DEGREE END SEMESTER EXAMINATIONS

NOV / DEC 2024

First Semester

MA4101 – CALCULUS AND LINEAR ALGEBRA

(Common to ALL Branches)

(Regulations 2024)

Time: Three Hours　　　　　　　　　　　　　　　　Maximum Marks

Answer ALL Questions

RBT Level : L1- Remembering, L2 – Understanding, L3 – Applying, L4 – Analyzing, L5 – Evaluating, L6 – Crea

PART – A **(10x2=20 Marks)**

1. Find $\frac{dy}{dx}$, if $x^{2/3} + y^{2/3} = a^{2/3}$.

2. Prove $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ if $f = e^x \sin y$.

3. Evaluate $\int_1^2 \int_3^4 xy\, dy\, dx$.

4. Is the vector $\vec{F} = (e^x \cos y + yz)\vec{i} + (xz - e^x siny)\vec{j} + (xy + z)\vec{k}$ irrotational?

5. Solve $yy' + 36x = 0$.

6. Solve: $y'' + 6y' + 9y = 0$.

7. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

8. Find the sum and product of the Eigenvalues of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

9. Determine whether the sets $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ are bases for R^3.

10. State dimension theorem.

PART –B (5x16=80 Marks) C

11. a) i) Find $\lim\limits_{x\to 0} \frac{(1+x)^{1/x} - e}{x}$. C

ii) Expand $log_e x$ in powers of $(x-1)$ and hence evaluate $log_e 1.1$ correct to 4 decimal places. C

(OR)

b) i) For what values of a and b is $f(x) = \begin{cases} ax + 2b, & x \leq 0 \\ x^2 + 3a - b, & 0 < x \leq 2 \\ 3x - 5, & x > 2 \end{cases}$ continuous at every x?

ii) Find the maximum and minimum values of $3x^4 - 2x^3 - 6x^2 + 6x + 1$ in the interval $(0, 2)$.

12. a) i) Change the order of the integration $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\, dx$ and evaluate the same.

ii) Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\,\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the circular boundary on $z = 0$ plane.

(OR)

b) i) Using polar coordinates, evaluate $I = \int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2)\, dy\, dx$.

ii) Apply Gauss divergence theorem to evaluate

$\iint_S x^3 dydz + x^2 ydzdx + x^2 zdxdy$, where S is the surface of the cube $x = 0, y = 0, z = 0, x = 1, y = 1$ and $z = 1$.

13. a) i) Solve $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$.

ii) Solve $(x^2D^2 + xD + 1)y = \sin(\log x)$.

(OR)

b) i) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

ii) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos[\log(1+x)]$.

14. a) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2xz - 2xy$ to the canonical form.

(OR)

b) Find a singular value decomposition of the matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.

15. a) i) Show that the set of all 2x2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication defined by $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space.

ii) Consider the basis $S = \{v_1, v_2, v_3\}$ for R^3, where $v_1 = (1,1,1)$, $v_2 = (1,1,0)$, $v_3 = (1,0,0)$; let $T: R^3 \rightarrow R^2$ be the linear transformation such that $T(v_1) = (1,0)$, $T(v_2) = (2,-1)$, $T(v_3) = (4,3)$. Find a formula for $T(x_1, x_2, x_3)$; then use this formula to evaluate $T(2,-3,5)$.

(OR)

b) i) Find a basis for the space spanned by the vectors,

$u_1 = (1,-2,0,0,3)$, $u_2 = (2,-5,-3,-2,6)$, $u_3 = (0,5,15,10,0)$, $u_4 = (2,6,18,8,6)$.

ii) Let $u = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ and let $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ be the matrix of T: $R^2 \rightarrow R^2$ with respect to the bases $B = (u, v)$. Evaluate $[T(u)]_B$, $[T(v)]_B$ and also evaluate $T(u)$ and $T(v)$.