*Name* : ..................................................................

*Roll No.* : ..................................................................

*Invigilator's Signature* : .............................................

**CS/B.Tech (ICE) (O)/SEM-5/IC-504/2012-13**

# 2012

# ADVANCED CONTROL SYSTEM

*Time Allotted* : 3 Hours *Full Marks* : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP – A

## ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) A set of variable for a system is

      a) not unique in general

      b) always unique

      c) never unique.

   ii) For an *n*th order control system, the number(s) of variable is(are)

      a) 1

      b) $n$

      c) $n/2$.

iii) Dynamic equation is a set of equations which is formed by putting together

a) state equation and input equation

b) input equation and output equation

c) output equation and state equation.

iv) State variable approach converts an nth order system into

a) $n$-number second order differential equations

b) two differential equations

c) $n$-number $1^{st}$ order differential equations.

v) The transfer function of a linear system represented by the vector matrix differential equations $\dot{x} = Ax + Bu$ and $Y = Cx + Du$ is given by

a) $C(sI - A)^{-1} B$

b) $C(sI - A)^{-1} B + D$

c) $B(sI - A)^{-1} C + D$.

vi) The properties of state transition matrix $\Phi(t)$ is

a) $\Phi(0) = 1$

b) $\Phi^{-1}(t) = \Phi(t)$

c) $[\Phi(t)]^k = \Phi(-kt)$.

vii) A system is said to be completely observable if

a) any of the state variables affects some output

b) any of the state variable affects all the output

c) all the state variables affects all the outputs.

viii) The second order system $\dot{x} = Ax$ has $\dot{x} = A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$.

The value of its damping and natural frequency are

a) 1 and 1

b) 0·5 and 1

c) 0·707 and 2.

ix) To compute the describing function of a non-linear element for sinusoidal input,

a) the function component of the output is required

b) the dead zone and saturation are to be avoided

c) the non-linear system to be assumed linear.

x) If the both eigenvalues of a second order system are real and negative then it is termed as

a) the saddle point

b) the nodal point

c) the focus point.

xi) A non-linear control system is described by the equation $\frac{d^2x}{dt^2} + \sin x = 0$. The type of singularity at $x = \pi$ and $\frac{dx}{dt} = 0$ is

a) centre

b) stable focus

c) saddle point

d) stable node.

xii) The value of a matrix for the system described by the differential equation $\ddot{y} + 2y + 3\dot{y} = 0$ is

a) $\begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$

## GROUP – B
**( Short Answer Type Questions )**

Answer any *three* of the following. $3 \times 5 = 15$

2. A linear 2nd order servo is described by the equation

   $\ddot{y} + 2tw_n\,\dot{y} + w_n^2 y = w_n^2$

   Where $w_n = 1$, $y(0) = 2{\cdot}0$, $\dot{y}(0) = 0$

   Determine the singular points when

   i) $t = 0$,

   ii) $t = 0{\cdot}15$.

   Construct the phasor trajectory in each case.

3. A linear time invariant system is characterised by the homogeneous state equation

   $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

   a) Compute the solution of the homogenous equation assuming the initial state vector

   $$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \qquad 2\frac{1}{2}$$

   b) Consider now that the system has forcing function and is represented by the following non-homogeneous state equation

   $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U,$$

   where $U$ is a unit step input. compute the solution of this equation assuming initial conditions of sub-question (a). $2\frac{1}{2}$

4. Explain the concept of controllability and observability.

5. Determine $x(k)$ of the system given below :

$$\dot{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

where $x_1(0) = 1$, $x_2(0) = 1$ and $u(k) = 2$. 5

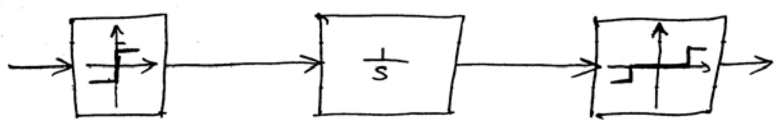
6. Concept of limit cycles in the system analysis of a non-linear control system. 5

## GROUP – C

### ( Long Answer Type Questions )

Answer any *three* of the following. $3 \times 15 = 45$

7. Consider three cascaded elements depicted in the figure given below constituting part of a control system. Two of the elements are non-linear and one is linear (an integrator) :




Why is it not recommendable to obtain the overall frequency response of this system by multiplication of the FTF of the integrator by the individual DF's of each non-linear element ?

Suggest a better method and then use it to obtain the overall DF of the three elements. Plot the DF versus W. 6 + 4 + 5

8. a) The state equations of a system are given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U \text{ and } y = [1\ 0\ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Determine the controllability and observability of the system.

8

b) Consider the system $\dot{x} = Ax + Bu$

where, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Design a linear state variable feedback gain matrix such that the closed-loop poles are located at $(-2 + j4)$, $(-2 - j4)$ and $-10$. 7

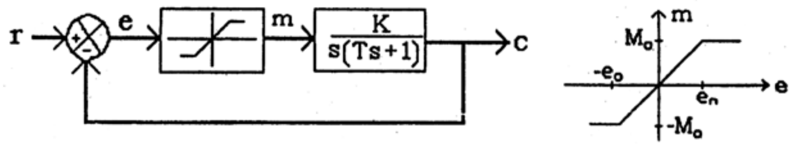
9. a) Define state, state variable, star vector and state space of a system. 4

b) Obtain the state variable model of the system whose transfer function is given by $G(s) = \dfrac{s^2 + 3s + 1}{s^3 + 5s^2 + 7s + 2}$. 7

c) Find the state transition matrix $\Phi(t)$ from the non-homogeneous state equation of a linear control system. 4

10. a) Define phase plane, phase trajectory and phase portrait. 6

b) For the piecewise linear system shown in figure, sketch typical trajectory in the phase-plane where input is $r(t) = 10u(t)$ and $T = 1$, $K = 4$, $e_0 = \pm 0{\cdot}2$, $M_0 = \pm 0{\cdot}2$. 9



11. a) State Lyapunov's direct method of investigating stability of nonlinear system. 4

b) Determine whether or not the following quadratic form is positive definite : 5

$$Q\,(x_1, x_2) = 10\,x_1^2 + 4\,x_2^2 + x_3^2 + 2\,x_1\,x_2 - 2x_2\,x_3 - 4x_1\,x_3.$$

c) A linear system is described by the state equation 6

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x$$

Investigate the stability of this system by using Lyapunov's theorem.