(Please write your Enrollment Number) Enrollment No. 003

END TERM EXAMINATION
(May, 2019)

Subject Code: BCS 206	Subject: Analysis & Design of Algorithm
Time: 3 Hours	Maximum Marks: 60

Note: Q1 is compulsory. Attempt one question each from the Units I, II, III & IV.

Q1. **(4x5=20)**

(a) Calculate the time complexity of the algorithm given below:

$$\textbf{Function } SQRT(n)\{$$
$$\textbf{if } n = 0 \textbf{ return } 0;$$
$$\textbf{Else return}(2n + SQRT\,(n-1) - 1\,);\}$$

(b) Prove that, $\theta(n-1) + \theta(n) = \theta(n)$ does it follow that, $\theta(n) = \theta(n) - \theta(n-1)$? Justify your answer.

(c) Find the error in the following proof that $O(n) = O(n^2)$.Let $f(n) = n^2, g(n) = n$ and $h(n) = g(n) - f(n)$. It is clear that $h(n) \leq g(n) \leq f(n)$ for all $n \geq 0$. Therefore, $f(n) = max(f(n), h(n))$. Using the maximum rule we conclude,

$$O(g(n)) = O(f(n) + h(n)) = O\left(max(f(n), h(n))\right) = O(f(n))$$

(d) Give an example of a strongly connected directed graph G = (V,E) such that, for every u ∈ V removing u from G leaves a directed graph that is not strongly connected.

(e) What do you mean by Hamiltonian Path Problem and Hamiltonian Cycle Problem?

UNIT I

Q2. **(5,5)**

(a) Rather than separate array A[1:N] into two half size arrays for the purpose of the merge sorting, we might choose to separate it into three arrays of size $(N/3), (N+1)/3)$ and $(N+2)/3)$ to sort each of these recursively, and then to merge the three sorted arrays. Write a more formal description of this algorithm and analyze its execution time also.

(b) The number of additions and subtractions needed to calculate the product of two 2x2 matrices by using Strassen's algorithm seems at first to be 24. Show that this can be reduced to 15 by using auxiliary variables to avoid recalculating terms.

Q3. Solve the following recurrence relations: **(2.5x4=10)**

(a) $T(n) = 2T\left(\frac{n}{2}\right) + 1,\ T(1) = \log n$, Solve using recursion tree only.

(b) $T(n) = \frac{1}{n}T(n-1) + 1\ ,\ with\ T(1) = 0$

(c) $T(n) = T(n-1) + 1/2^n$

(d) $(n) = 4T\left(\frac{n}{2}\right) + n^2 \log n$, Solve using Master Theorem

UNIT II

Q4. **(5,5)**

(a) What is the solution generated by the Job Sequencing with deadlines algorithm when number of jobs are 7 their profits are $(P_1,P_2,\ldots,P_7) = (3,5,20,18,1,6,30)$ and deadlines are $(D_1,D_2,\ldots,D_7) = (1,3,4,3,2,1,2)$.

(b) Write an algorithm to find longest common subsequence (LCS) and then determine LCS of (ABBCCABCA) and (ACBACABABC).

P.T.O

Q5. (5,5)

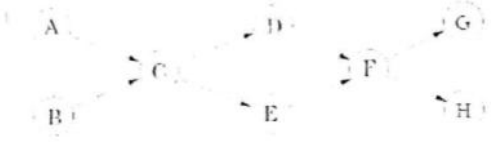
(a) For a given undirected graph **G = (V, E)**, if nodes are $V_1, V_2, \ldots, V_{10}$. Two nodes V_i and V_j are connected if and only if $0 < |I-J| < 3$. Calculate the cost of minimum spanning tree of the graph. How many distinct minimum spanning trees does this graph have? Apply DFS and BFS over the given graph and list the vertices in the order that they are visited?

(b) Consider the chain of four matrices (P,Q,R,S) and its dimensions are (3x4, 4x6, 6x2, 2x7) respectively. Find the optimal parenthesis for given chain of matrices if they are multiplied as in chain.

UNIT III

Q6. (5,5)

(a) Run the topological sorting algorithm on the below given graph. Choice of vertices always picks the one that is alphabetically first. Show the result and how many topological orderings does this graph have?



(b) Consider a directed graph in which the only negative edges are those that leave vertex S; all other edges are positive. Can Dijkstra's algorithm, started at S, fail on such a graph? Prove your answer.

Q7. Write Short notes on any two: **(5x2 =10)**

(a) 8 Queen's problem by using backtracking.

(b) 0/1 Kanpsack problem by using dynamic programming.

(c) Differentiate between dynamic programming, backtracking and branch and bound techniques.

UNIT IV

Q8. (5,5)

(a) Construct the string matching automation for the pattern P = (A B A B) and text is T = (A A A B A B A A B A A B A A B) and show the matching operation.

(b) Consider the CLIQUE problem restricted to graphs in which every vertex has degree at most 3. Call this problem CLIQUE-3. What is wrong with the following proof of NP-completeness for CLIQUE-3?

"We know that the CLIQUE problem in general graphs is NP-complete, so it is enough to present a reduction from CLIQUE-3 to CLIQUE. Given a graph G with vertices of degree <=3, and a parameter g, the reduction leaves the graph and the parameter unchanged: clearly the output of the reduction is a possible input for the CLIQUE problem. Furthermore, the answer to both problems is identical. This proves the correctness of the reduction and, therefore, the NP-completeness of CLIQUE-3".

Q9. (5,5)

(a) How Knuth Morris Pratt (KMP) algorithm works? Using KMP algorithm find whether the pattern P=(00110) is in the text T=(1100011010001010) or not ?

(b) Briefly discuss the strategy used to show a problem to be NP-complete. Explain any five well known NP-complete problems using example.

End-Term Examination

(CBCS) (SUBJECTIVE TYPE)

B. Tech (CSE/IT/CSE AI), 4th Semester

(May, 2022)

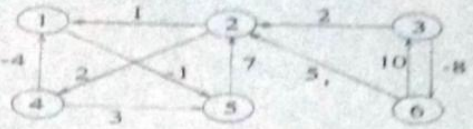
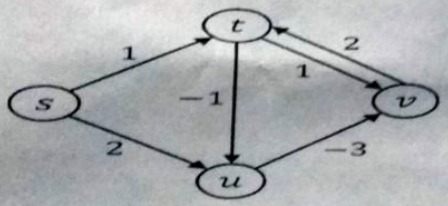
Q.P Code 220517

Subject Code: BCS 204	Subject: Design and Analysis of Algorithms

Time : 3 Hours | Maximum Marks : 60

Note: Q. 1 is compulsory. Attempt one question each from the Units I, II, III & IV.

Q1.		(5*4=20)
	(a) Suppose we are given two n-element sorted arrays A and B that should not be viewed as sets (that is, A and B may contain duplicate entries). Describe an O(n)-time method for computing an array representing the set A ∪ B (with no duplicates).	
	(b)Design a branch and bound algorithm for solving 0/1 Knapsack problem. Explain with an example.	
	(c) Show that the depth first search traversal of a graph can be used to find the strongly connected component of a graph. Explain with an example also	
	(d) You are given an array A[1..n] that is first decreasing and then increasing. More precisely, there is a coordinate $1 \le p \le n$ such that for all $i < p$, $A[i] > A[i+1]$, and for all $i \ge p$, $A[i] < A[i+1]$. Your goal is to find the smallest element in this array. Design O(log n) time algorithm.	
	UNIT-1	
Q2.	An investment company has stock prices for n consecutive days. That is for each day, they have a specific stock price. They want to know on which day they should buy a stock and sell it on some other day so that they make the maximum profit out of it. Hint: Think of it as an array A[0..n] of distinct integers, you need to find two indices i and j such that (j > i) and A[j] − A[i] is maximum. Design a O(n log n) algorithm for this problem.	(10)
Q3.	Suppose you are choosing between the following 3 algorithms: • Algorithm A solves the problem of size n by dividing it into 5 subproblems of size n/2, recursively solving each subproblem, and then combining the solutions in linear time. • Algorithm B solves the problem of size n by recursively solving two subproblems of size n − 1 and then combining the solutions in constant time • Algorithm C solves the problem of size n by dividing it into nine subproblems of size n/3, recursively solving each subproblem, and then combining the solutions in O(n2) time. What are the running times of each algorithm and which would you choose and why?	(10)
	UNIT-2	

Q4.	Run the Floyd Warshall algorithm on the weighted directed graph shown in the figure. Show the matrix D(k) that results after each iteration 	(10)
Q5.	a. We are given a set of n characters along with their frequencies. We are required to represent each character by a unique code word using 0s and 1s such that no code word is a prefix of another. the goal is to find such code words that achieve maximum compression . Write a greedy algorithm and analyse its complexity b. Construct an optimal binary code for four characters a,b, c, d with frequencies 14,3,6,10 respectively. How many bits are needed to encode a string containing 14 a's, 3 b's, 6 c's and 10 d's using this code.? How many bits would be needed if we used 2 bits for each character? Which code is better	(5+5=10)
	UNIT-3	
Q6.	(a)Run the Bellman-Ford algorithm on the following graph to find the shortest path from s to every other node. Show the table that you fill in while running the algorithm.  (b) Now suppose you ran one extra iteration of Bellman-Ford. How could you use that last iteration to detect whether the graph has a negative-weight cycle?	(8+2=10)
Q7.	Suppose that we are given a cable network of n sites connected by communication channels. Unfortunately, the communication channels are not perfect and they may fail with certain probability . Let f(u,v) denote the failure probability for the channel between sites u and v. One of the n sites is the central station and your problem is to find the most reliable paths (paths with lowest failure probability) from the central station to all other sites. Design a greedy algorithm and analyse its complexity.	(10)
	UNIT-4	
Q8.	a. Explain the string matching algorithm using finite state automata. b. Search the pattern 'AGCTA' in the string 'AAGCCTTAGCTAATG' using this algorithm.	(5+5=10)
Q9.	a. Write a short note on NP- Complete problems b. Prove that the clique problem is polynomial-time reducible to the vertex cover problem.	(4+6=10)

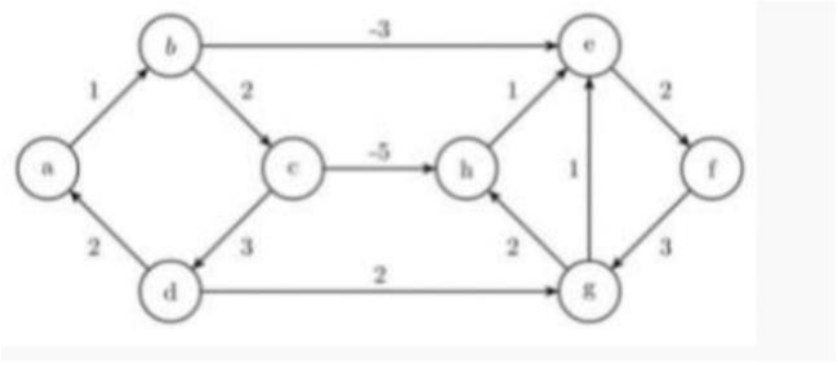
Design and Analysis of Algorithms (End Sem)

(a) T The square of a matrix A is its product with itself, AxA. Show that five multiplications are sufficient to compute the square of a 2 × 2 matrix. Now find what is wrong with the following algorithm for computing the square of an n × n matrix?

"Use a divide-and-conquer approach as in Strassen's algorithm, except that instead of getting 7 sub problems of size n/2, we now get 5 sub problems of size n/2 thanks to first part. Using the same analysis as in Strassen's algorithm, we can conclude that the algorithm runs in time

(b) Working modulo q=11, how many spurious hits does the Rabin-Karp matcher encounter in the text T= 3141592653589793 when looking for the pattern P=26? Justify your answer with suitable steps.

(c) Run the Bellman-Ford algorithm on the given graph and write the output of the algorithm by considering node 'a' as the source node. Justify your answer with suitable steps.



(a) What is the solution generated by the Job Sequencing with deadlines algorithm when number of jobs are 7 their profits are (P1,P2,...,P7) = (3,5,20,18,1,6,30) and deadlines are (D1,D2,....,D7) = (1,3,4,3,2,1,2).

(b) Using KMP algorithm for string matching find the pattern P=(0010) availability in the text T=(1100011010001010). Show every steps of the computation as per KMP.