VIT

Vellore Institute of Technology

School of Advanced Sciences
Department of Mathematics
Winter Semester 2019~20
Continuous Assessment Test –II

Programme Name & Branch: B. Tech/M. Tech (SE) **Slot:** C1+TC1+TCC1
Course Code & Name: MAT3004 – Applied Linear Algebra **Date:** 03-03-2020
Maximum Marks: 50 **Exam Duration:** 90 mins
Exam Type: Closed book

Answer any Five questions 5 x 10 = 50 Marks

1. Let $V = \{(a,b,c,d) \in \mathbb{R}^4 \mid b + c + d = 0\}$ and $W = \{(a,b,c,d) \in R^4 \mid a + b = 0, c = 4d\}$ be two subspaces of R^4. Find the bases for $V + W$ and $V \cap W$. [10]

2. (a) Determine the nullity and rank of the following matrix: [5]

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 4 & 2 & 0 & 0 & 3 \\ 1 & 1 & 1 & -2 & 1 \\ 2 & 2 & 0 & 0 & 2 \\ 1 & 1 & 2 & -4 & 1 \end{bmatrix}$$

(b) Find the left inverse of the matrix $A = \begin{bmatrix} 3 & 4 \\ -1 & 0 \\ 1 & 2 \end{bmatrix}$ [5]

3. (a) A jet fighter's position on an aircraft carrier's runway was timed during landing: [5]

Time t, sec	0	2	4
Position x, m	0	1	2

where x is the distance from the end of the carrier. Fit an appropriate interpolating polynomial of the form $x = a_0 + a_1 t + a_2 t^2$. Hence estimate the velocity of jet fighter at the instant $t = 3$.

(b) Let the transformation $T: R^2 \to R^1$ be defined by $T((a,b)) = a^2 + b^2$. Is T a linear transformation? [5]

4. (a) Calculate a basis and $\dim(\ker(T))$ for the linear transformation $T: R^3 \to R^2$ defined by $T((a,b,c)) = (a + 2b + c, -a + 3b + c)$. [5]

(b) The transformation $T: R^3 \to R^2$ given by $T((x,y,z)) = (xz, yz)$. Verify that the dimension theorem $(\dim(\ker(T)) + \dim(range(T)) = \dim(R^3))$ is satisfied. If it is not satisfied justify. [5]

5. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by $T((1,1)) = (0,1)$ and $T((-1,1)) = (2,3)$. Let $\alpha = \{e_1, e_2\}$ be the standard basis of $\mathbb{R}^2$ and $\beta = \{v_1, v_2\}$, where $v_1 = (1,0), v_2 = (4,3)$ be another ordered basis for $\mathbb{R}^2$. Determine the associated matrices of T with respect to α and β. [10]

6. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator given by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 5y + 3z \\ 2x + 3y + z \\ 3x + 4y + z \end{pmatrix}$. Is T invertible? Check if T is invertible and if so then determine the inverse transformation T^{-1}. [10]