VIT
Vellore Institute of Technology

Winter Semester 2018-19

Continuous Assessment Test – II

Programme Name & Branch: B. Tech.

Course Name & Code: Applications of Differential and Difference Equations (MAT2002)

Slot: D2+TD2

Exam Duration: 90 minutes

Maximum Marks: 50

Answer All the Questions (5 × 10 = 50)

1. A force acting on an undamped system during some interval of time modelled by the differential equation $y''+y=\begin{cases}1-\frac{t^2}{\pi^2}, & 0\le t\le\pi\\ 0, & t\ge\pi\end{cases}$. Find the displacement $y(t)$ with the initial conditions $y(0)=y'(0)=0$, in the interval $0\le t\le\pi$.

2. Solve the following initial value problem $2x^2y''+3xy'-15y=x$, with $y(1)=0, y'(1)=0$.

3. Consider a European forest having one or two varieties of trees. We select some of the oldest trees, those expected to die off in the next few years, then follow the cycle of living trees into dead trees. The dead trees eventually decay and fall from seasonal and biological events. Finally, the fallen trees become humus.

Let variables x, y, z, t be defined by

$x(t)$ = biomass decayed into humus;

$y(t)$ = biomass of dead trees;

$z(t)$ = biomass of living trees;

t = time in decades:

A typical biological model is

$$x'(t) = -x(t) + 2y(t)$$
$$y'(t) = -3y(t) + 5z(t)$$
$$z'(t) = -2z(t).$$

Find the solution of the above system using matrix method when there are no dead trees and no humus at $t = 0$, with initially z_0 units of living tree biomass.

4. Determine the response of a mass spring system under unit impulse time $t=1$, modelled by $y''(t)-6y'(t)+8y(t)=\delta(t-1)$ with the initial conditions $y(0)=0, y'(0)=0$, by Laplace transform.

5. The displacement of an object at any time t is given by the differential equation $s''(t)-4s'(t)+4s(t)=0$. Find a series solution about $t=0$, when $s(0)=1, s'(0)=3$.