*Name :* .....

*Roll No. :* .....

*Invigilator's Signature :* .....

**CS/B.TECH/ICE(N)/SEM-5/IC-502/2012-13**

# 2012
# CONTROL SYSTEM

*Time Allotted :* 3 Hours *Full Marks :* 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP – A

### ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) A set of variables for a system is

      a) Not unique in general

      b) Always unique

      c) Never unique

      d) May be unique.

   ii) State variable approach converts an *n*th order system into

      a) *n*-number second order differential equation

      b) two differential equations

      c) two *n*-order differential equations

      d) *n*-number of 1st order differential equations.

iii) The transfer function of a linear system represented by the vector-matrix differential equations $\dot{X} = Ax + Bu$ and $Y = Cx + Du$ is given by

a) $C(sI - A)^{-1}B$

b) $C(sI - A)^{-1}B + D$

c) $B(sI - A)^{-1}C + D$

d) $B(sI - A)^{-1}D + C$.

iv) A system is said to be completely observable if

a) any of the state variables affects some output

b) any of the state variables affects all the outputs

c) all the state variables affects all the outputs.

d) all the state variables affects some output.

v) The second order system $\dot{x} = Ax$ has $A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$. The value of its damping and natural frequency are

a) 1 and 1 b) 0·5 and 1

c) 0·707 and 2 d) 1·41 and 1.

vi) The properties of the state transition matrix $\Phi(t)$ is

a) $\Phi(0) = 1$ b) $\Phi(t)^{-1} = \Phi(t)$

c) $[\Phi(t)]^k = \Phi(-kt)$ d) $[\Phi(-t)]^k = \Phi(kt)$.

vii) The transfer function of a zero order hold circuit is

a) $\frac{1-e^{-Ts}}{s}$ b) $\frac{1+e^{-Ts}}{s}$

c) $\frac{1+e^{+Ts}}{s}$ d) $\frac{s}{1-e^{-Ts}}$.

viii) The system matrix $A$ for the system described by the differential equation $\ddot{y}+2\dot{y}+3y=0$ is

a) $\begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ -3 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

ix) $\dot{x} = f(x)$ is called

a) an autonomous system

b) an overdamped system

c) an underdamped system

d) a critically damped system.

x) If the $z$-transform of a function is $\frac{z\sin\ \omega T}{z^2-2z\ \cos\omega T+1}$. Its Laplace transform will be

a) $\frac{s}{s^2+\omega^2}$ b) $\frac{\omega}{s^2+\omega^2}$

c) $\sin\ \omega t$ d) $\tan\ \omega t$.

xi) The eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix}$ are

a) 1, 2, 3 b) –1, 2, – 3

c) – 1, – 2, – 3 d) 1,– 2, 3.

xii) The variable $z$ in $z$-transform theory is equal to ( $s$ is the Laplace operator and $T$ is the sampling period )

a) $z = Ts$ b) $z = e^{Ts}$

c) $z = e^{-Ts}$ d) $z = T^2 s$.

## GROUP – B

### ( Short Answer Type Questions )

Answer any *three* of the following. $3 \times 5 = 15$

2. A linear time-invariant system is described by the state model $\dot{x} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

obtain the state transition matrix.

3. State and explain the theorems of controllability and observability in control systems.

4. Determine $x$ ( $k$ ) of the system given below.

$$\dot{x}\ (k+1) = \begin{bmatrix} 0 & 2 \\ -3 & -5 \end{bmatrix} x\ (k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

where $x_1(0) = 1$, $x_2(0) = 1$ and $u$ ( $k$ ) = 2.

5. Determine the expression for the describing function of a hysteresis type non-linearity.

6. Determine whether or not the following quadratic form is positive definite :

$$Q(x_1, x_2) = 10x_1^2 + 4x_2^2 + x_3^2 + 2x_1x_2 - 2x_2x_3 - 4x_1x_3$$

## GROUP – C

### ( Long Answer Type Questions )

Answer any *three* of the following. $3 \times 15 = 45$

7. a) For an electrical R-L-C series circuit as shown in the Fig.-1, find out the state space model in the physical form. Also draw the relative state diagram.

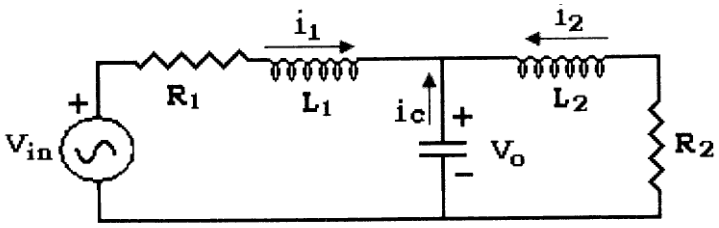



Figure-1

b) Find the state transition matrix $\Phi(t)$ from the homogeneous state equation of a linear control system.

c) Obtain a state space representation of the system whose transfer function is given by

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 3s + 1}{S^3 + 5s^2 + 7s + 2}$$

where $Y$ is the output and $U$ is the input of the system.

6 + 3 + 6

8. a) The state equations of a system are given by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U \text{ and}$$

$$Y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Determine the controllability and observability of the system.

b) Consider the system, $\dot{x} = Ax + Bu$

Where, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Design a linear state variable feedback gain matrix such that the close-loop poles are located at $( -2 + j4 )$, $( -2 - j4 )$ and $-10$. 7 + 8

9. a) Draw the phase trajectory of the system shown in Fig.-2 when it is subjected to a step input $r(t) = R$.

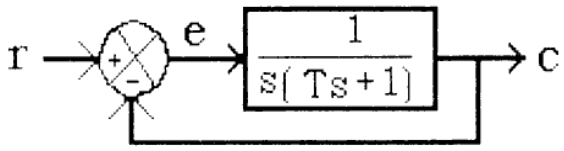



Figure-2

b) State Lyapunov's second method for investigating the stability of a nonlinear system.

c) Consider the following system :

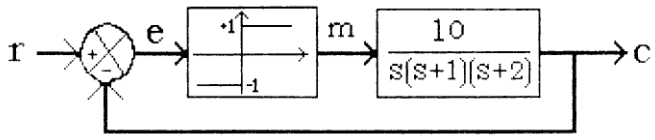
$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

The only equilibrium state is the origin, $x = 0$, Determine the stability of this system. 7 + 3 + 5

10. a) A nonlinear element $N$ has an input $x$ and an output $y$. Determine the describing function $N(x)$ for the non-linear element.

b) Explain how Nyquist stability criterion can be extended to determine possibility of limit cycle in nonlinear control system analysis.

c) For the system shown in Fig-3, determine the amplitude and frequency of the limit cycle using describing function analysis.




4 + 6 + 5

Figure-3

11. a) Compute the $z$-transform of the sinusoidal function

$x(t) = 0$ for $t < 0$

$= \sin \omega t$ for $t \geq 0$.

b) Solve the difference equation

$$x(k+2) + 3x(k+1) + 2x(k) = u(k)$$

The initial condition are $x(0) = 0$, $x(1) = 1$

c) In continuous time, a system is given by the transfer function $G(s) = \dfrac{K}{s+a}$

Find the $z$-transform $G(z)$. 4 + 6 + 5

12. Write short notes on any *three* of the following : 3 × 5

(i) Nonlinear relay

(ii) Asymptotic stability

(iii) Zero order hold

(iv) Stability analysis by phase plane method

(v) Limit cycle.