VIT

Vellore Institute of Technology

Final Assessment Test – November 2019

Course: MAT3003 - Complex Variables and Partial Differential Equations

Class NBR(s): 0505 / 0506 / 0507 / 0508 / 0530 / 6742 Slot: A2+TA2+TAA2+V3

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

Answer any FIVE Questions
(5 X 20 = 100 Marks)

1. a) Find the constants a, b, c such that the function *f(z)*, where $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + cy^2)$ is analytic. Express *f(z)* in terms of z. [10]

   b) Show that the function $u(r,\theta) = r^2\cos 2\theta$ is harmonic. Find its conjugate harmonic function and the corresponding analytic function *f(z)*. [10]

2. a) Find the image of the region bounded by the lines $x - y < 2$ and $x + y > 2$ under the mapping $w = \frac{1}{z}$. [10]

   b) Find the bilinear transformation which maps the points z=1, i, 2+i in the z-plane onto the points w=i, 1, ∞. [10]

3. Let $f(z) = \dfrac{1}{(z+1)(z+2)^2}$. [20]

   Find (i) Taylor's series expansion of *f(z)* and

   (ii) Laurent's series expansion in the valid region (i) $|z-1| < 2$, (ii) $2 < |z-1| < 3$ and $|z-1| > 3$.

4. a) Evaluate the integral $\oint_c \dfrac{dz}{z^2+4}$ where (i) C: $|z-2i| = 1$ and (ii) C: $|z| = 4$. [10]

   b) Evaluate the integral by Contour Integration $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{x^2+2x+2}\,dx$. [10]

5. a) Obtain the partial differential equation by eliminating the arbitrary constants *a* and *b* from the relation $z = x^2 + ax + \dfrac{e^{ay}}{a} + b$. [5]

   b) Solve the partial differential equation $pq - q\cos x = \cos y$ [5]

   c) Find the general solution of the partial differential equation $2xzp + 2yzq = z^2 - x^2 - y^2$ [10]

6. a) Find all possible solutions for the given partial differential equation $\dfrac{\partial^2 y}{\partial t^2} = a^2 \dfrac{\partial^2 y}{\partial x^2}$, using Variable separable method. [10]

   b) Solve $\left(D^3 - 4D^2D' + 4DD'^2\right)z = 2\sin(3x+2y)$ [10]

7. a) Find the Fourier cosine transform of *f(x)* given by $f(x) = \begin{cases} 1 & for\ |x| < a \\ 0 & for\ |x| > a > 0 \end{cases}$ and hence evaluate $\displaystyle\int_0^{\infty} \frac{\sin x}{x}\,dx$. [10]

   b) Find the Fourier cosine transform of $f(x) = e^{-ax}, a > 0$ and hence deduce its inverse. [10]

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