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Question Paper Code : 41363

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2024.

Third/Fourth Semester

Electronics and Communication Engineering

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MA 3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to : Biomedical Engineering/Electronics and Telecommunication
Engineering/Medical Electronics)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Use of statistical tables are permitted

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If an experiment has the three possible and mutually exclusive outcomes A, B, and C, check in the following case whether the assignment of probabilities is permissible:

$P(A) = 0.57$, $P(B) = 0.24$, and $P(C) = 0.19$.

2. It has been claimed that in 60% of all solar-heat installations the utility bill is reduced by at least one-third. Accordingly, what is the probability that the utility bill will be reduced by at least one-third in four of five installations?

3. The joint probability density function of a bivariate random variable X and Y is

$$f_{XY}(x, y) = \begin{cases} k(x + y), & 0 < x < 2, 0 < y < 2 \\ 0, & \text{elsewhere} \end{cases}. \text{ Determine the value of } k.$$

4. If there is no linear correlation between two random variables X and Y, what can you say about the regression lines?

5. A random process $Y(t)$ consists of the sum of the random process $X(t)$ and a statistically independent noise process $N(t)$. Obtain the cross correlation function of $X(t)$ and $Y(t)$.

6. If $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, find whether it is regular.

7. Is it possible for a vector u in a vector space to have two different negatives? Justify.
8. Determine whether the set vectors of the form $(a, b, 1)$ is a subspace of R^3 .
9. Prove that the identity $\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2$ holds for any vectors u and v in an inner product space.
10. Verify that the set of vectors $\{(1,0,-1), (2,0,2), (0,5,0)\}$ is orthogonal with respect to the Euclidean inner product.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Suppose traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed so that with probability 0.8 a driver will find the second light to have the same color as the first. Assuming the first light is equally likely to be red or green, what is the probability $P[G_2]$ that the second light is green? Also, what is $P[W]$, the probability that you wait for at least one light? Lastly, what is $P[G_1|R_2]$ the conditional probability of a green first light given a red second light? (8)
 - (ii) The number of messages that arrive at a switchboard per hour is a Poisson random variable with a mean of six. What is the probability for each of the following events:
 - (1) Exactly two messages arrive within one hour.
 - (2) No message arrives in one hour.
 - (3) At least three messages arrive within one hour. (8)
- Or
- (b) (i) Assume that the length of the phone calls made at a particular telephone booth is exponentially distributed with a mean of 3 minutes. If you arrive at the telephone booth just as Chris was about to make a call, find the following:
 - (1) The probability that you will wait more than five minutes before Chris is done with the call.
 - (2) The probability that Chris call will last between two and six minutes. (8)
 - (ii) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modeled by a normal random variable X with mean of 5 and a standard deviation of 4.
 - (1) What is the probability that a randomly selected parcel weighs between one and ten pounds?
 - (2) What is the probability that a randomly selected parcel weighs more than nine pounds? (8)

12. (a) A fair coin is tossed three times. Let X be a random variable that takes the value zero if the first toss is tail and the value one if the first toss is head. Also, let Y be a random variable that defines the total number of heads in the three tosses.
- (i) Determine the joint probability mass function of X and Y . (6)
 - (ii) Find the marginal probability distributions of X and Y . (6)
 - (iii) Are X and Y independent? (4)

Or

- (b) (i) The joint probability density function of the random variables X and Y is defined as follows: $f_{XY}(x, y) = \begin{cases} 25e^{-5y}, & 0 \leq x \leq 0.2, y \geq 0 \\ 0 & \text{elsewhere} \end{cases}$.

(1) Find $f_X(x)$ and $f_Y(y)$.

(2) What is the covariance of X and Y ? (8)

- (ii) The following marks have been obtained by a class of students in two subjects:

X 25 28 35 32 31 36 29 38 34 32

Y 43 46 49 41 36 32 31 30 33 39

Find the linear regression. (8)

13. (a) (i) A random process is defined by $X(t) = K \cos wt$ where w is a constant and K is uniformly distributed between 0 and 2. Determine $E(X(t))$, the autocorrelation function $X(t)$ and the auto covariance of $X(t)$. (8)
- (ii) Three persons A, B, and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B or to A. Show that the process is a Markovian. Find the transition probability matrix and classify the states. (8)

Or

- (b) A random process has sample functions of the form $X(t) = A \cos(wt + \theta)$ where w is a constant, A is a random variable that has magnitude of +1 and -1 with equal probability, and θ is a random variable that is uniformly distributed between 0 and 2π .

Assume that the random variables A and θ are independent.

- (i) Is $X(t)$ a wide sense stationary process? (8)
- (ii) Is $X(t)$ a mean-ergodic process? (8)

14. (a) Determine whether the set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$, where a and b are real, with standard matrix addition and scalar multiplication is a vector space or not. If not, list all axioms that fail to hold.

Or

- (b) Find the basis and the dimension of the solution space of homogeneous system $2x_1 + 2x_2 - x_3 + x_5 = 0$; $-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$; $x_1 + x_2 - 2x_3 - x_5 = 0$; $x_3 + x_4 + x_5 = 0$.
15. (a) (i) Find the rank and nullity of the Matrix

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}. \quad (8)$$

- (ii) Let $T: R^2 \rightarrow R^3$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}. \text{ Find the matrix for the transformation } T$$

with respect to the bases $B = \{u_1, u_2\}$ for R^2 and $B' = \{v_1, v_2, v_3\}$ for

$$R^3, \text{ where } u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}. \quad (8)$$

Or

- (b) Apply Gram-Schmidt process to transform the basis vectors $u_1 = (1, 1, 1)$, $u_2 = (1, 1, 0)$, $u_3 = (1, 0, 0)$ of R^3 with the inner product $\langle u, v \rangle = u_1v_1 + 2u_2v_2 + 3u_3v_3$ into orthonormal basis.