Final Assessment Test – November/December 2023

Course: BMAT201L - Complex Variables and Linear Algebra Slot: B2+TB2+TBB2

Class NBR(s): 2024 / 2025 / 2026 / 2029 / 2033 / 2035 Max. Marks: 100

Time: Three Hours

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Verify that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and determine its conjugate. Also find $f(Z)$.

2. If $w = \phi + i\psi$ represents the complex potential for an electric field and $\phi = x^2 - y^2 + \frac{x}{x^2+y^2}$, determine ψ.

3. Fins the image of $|Z - 2i| = 2$ under the transformation $w = \frac{1}{z}$.

4. Find the bilinear transformation which maps the points $2, i, -2$ in the Z- Plane in to the points $1, i, -1$ in to the w-Plane.

5. Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in a Laurent's series if
   (i) $|Z| > 3$
   (ii) $2 < |Z| < 3$

6. Evaluate $\int_c \frac{z^2-2z}{(z+1)^2(z^2+4)} dz$ using residue theorem. Where "c" is the circle $|Z| = 3$.

7. a) Verify that the vectors $(1,2,1)$ $(2,1,0)$ $(1,-2,2)$ form a basis for $R^3(R)$ or not. [5]

   b) Find the rank and nullify for the matrix [5]

$$\begin{bmatrix} 1 & -1 & -2 & 1 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 4 & 0 & 0 & 1 & -2 \\ 2 & 3 & 8 & -2 & -1 \end{bmatrix}.$$

8. Let $T: R^3 \to R^3$ be a linear transformation defined by

$$T(x,y,z) = (x + 3y - 2z, 2x + 3y, y - z)$$

   Verify whether T is invertible or not? If so find T^{-1}.

9. Obtain the matrix representation of a linear transformation $T: R^3 \to R^3$ defined by $T(x,y,z) = (4z,\ 3x + 5y - 2z,\ x + y + 4z)$ relative to the basis $B = \{(1,1,1)(1,1,0)(1,0,0)\}$.

10. Apply Gram-Schmidt orthogonalization process to find an orthogonal basis for the subspace spanned by $\{(1,2,0,3)(4,0,5,8)(8,1,5,6)\}$.

11. Compute the Eigen values and corresponding Eigen vectors of $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$.

12. Solve the following system of equations by Gauss Jordan method

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40.$$

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