School of Electronics Engineering
Winter Semester (2019-20)
CAT 1

ot: B2 | **Duration**: 90 min

ourse Code: ECE2005 | **Max Marks**: 50

rse Name: Probability Theory and Random Process

nstructions: Answer **all** questions. Each question carries **ten** marks

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A random variable X has a probability density

$$f_X(x) = \begin{cases} \left(\frac{3}{32}\right)(-x^2+8x-12)\,; & 2 \le x \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

Find the following moments: (a) m_0 (b) m_1 (c) m_2 and (d) μ_2

2. Given the function

$$f_{XY}(x,y) = \begin{cases} b(x+y)^2; & -2 < x < 2 \text{ and } -3 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the constant such that this is a valid joint density function.

(b) Determine the marginal density functions $f_X(x)$ and $f_Y(y)$

3. The Joint Characteristic function of two random variables and X and Y is given by

$$\Phi_{X,Y}(\omega_1,\omega_2) = e^{j(\mu_1\omega_1+\mu_2\omega_2)}e^{-(\omega_1^2\sigma_1^2+2\omega_1\omega_2\sigma_1\sigma_2+\omega_2^2\sigma_2^2)/2}$$

Compute

(a) Mean values of X and Y

(b) Variances of X and Y

(c) Correlation Coefficient ρ_{XY}

Please turn over

4. If $W = X + Y$ and $Z = X - Y$, where X and Y are independent random variab having density functions $f_X(x) = \frac{1}{2}\delta(x-1) + \frac{1}{3}\delta(x-2) + \frac{1}{6}\delta(x-3)$ and $f_Y(y)$ $\frac{1}{4}\delta(y-1) + \frac{3}{4}\delta(y-3)$ respectively, then find the density functions of W and Z.

5. Zero mean Gaussian random variables X_1 and X_2 having covariance matrix $[C] = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ are transformed to new variables

$$Y_1 = X_1 + 4X_2$$

$$Y_2 = 3X_1 + 5X_2$$

Find the covariance matrix and joint density function of Y_1 and Y_2.

End of exam

Reg No