Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech(OLD)/EEE,ICE,IT,ECE/SEM-3/M(CS)-312/2012-13

2012

NUMERICAL METHODS AND PROGRAMMING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) Which of the following relations is true ?

a) $E = 1 + \Delta$ b) $E = 1 - \Delta$

c) $E = 1/\Delta$ d) None of these.

ii) By evaluating $\int_0^1 \frac{dx}{1+x^2}$ by a numerical integration method, we can obtain an approximate value of

a) $\log_e 2$ b) π

c) e d) $\log_{10} 2$.

iii) If a be the actual value and e be its estimated value, the formula for relative error is

a) $\frac{a}{e}$ b) $\frac{|a-e|}{a}$

c) $\frac{(e-a)}{e}$ d) $\frac{|a-e|}{e}$.

iv) In Trapezoidal rule, the portion of curve is replaced by

a) straight line b) circular path

c) parabolic path d) none of these.

v) The error involved in 4^{th} order R-k method is given by

a) $O(h^2)$ b) $O(h^4)$

c) $O(h^3)$ d) $O(h^5)$.

vi) An $n \times n$ matrix A is said to be diagonally dominant if

a) $|a_{ii}| \leq \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|$

b) $|a_{ii}| < \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|$

c) $|a_{ii}| > \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|$

d) $|a_{ii}| \geq \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|$.

vii) Find the output of the following program :

```
main( )
{
    char a, b ;
    a = 'b' ;
    b = a ;
    printf( "b = %c\n", b ) ;
}
```

a) *a* b) *b*

c) garbage value d) none of these.

viii) Lagrange's interpolation formula is used for

a) equispaced arguments only

b) unequispaced arguments only

c) both equispaced and unequispaced arguments

d) none of these.

ix) If $f(3) = 5$ and $f(5) = 3$, then the linear interpolation function $f(x)$ is

a) $f(x) = 8 - x$ b) $f(x) = 8 + x$

c) $f(x) = x^2$ d) $f(x) = x + x^2 + 8$.

x) If $f(x) = \frac{1}{x}$, the divided difference $[a, b, c]$ is

a) $\frac{1}{a+b+c}$ b) $\frac{1}{abc}$

c) $\frac{1}{a^2+b^2}$ d) $\frac{1}{a+b-c}$.

xi) If $\frac{dy}{dx} = x + y$ and $y(1) = 0$, then $y(1.1)$ according to Euler's method is $[h = 0{\cdot}1]$

a) 0·1 b) 0·3

c) 0·5 d) 0·9.

xii) Which one of the following results is correct ?

a) $\Delta x^n = nx^{n-1}$ b) $\Delta x^{(n)} = nx^{n+1}$

c) $\Delta^n e^x = e^x$ d) $\Delta \cos x = -\sin x$.

xiii) In the method of iteration the function $\phi(x)$ must satisfy

a) $|\phi'(x)| < 1$ b) $|\phi'(x)| > 1$

c) $|\phi'(x)| = 1$ d) $|\phi'(x)| = 2$.

xiv) The inherent error for Simpson's $\frac{1}{3}$ rd rule of integration is as (the notations have their usual meanings)

a) $-\frac{nh^5}{180} f''(x_0)$ b) $-\frac{nh^5}{140} f''(x_0)$

c) $-\frac{nh^3}{12} f''(x_0)$ d) none of these.

xv) $(\Delta - \nabla)\, x^2$ is equal to (the notations have their usual meanings)

a) h^2 b) $-2h^2$

c) $2h^2$ d) none of these.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. From the following table find the values of $f(12)$ by Newton's divided difference interpolation formula :

x :	11	13	14	18	19	21
$f(x)$:	1342	2210	2758	5850	6878	9282

3. Solve the following system by Matrix Inversion Method :

$2x + y + z = 10$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 16.$

4. a) Evaluate the missing terms in the following table :

x :	0	1	2	3	4	5
$f(x)$:	0	—	8	15	—	35

b) What is ternary operator ? Give an example.

5. a) Solve by Taylor's series method $\frac{dy}{dx} = 2x + 3y^2$, given $y = 0$ when $x = 0$ at $x = 0{\cdot}2$.

b) Using Euler's method obtain the solution of $\frac{dy}{dx} = x - y$, with $y(0) = 1$ and $h = 0{\cdot}2$ at $x = 0{\cdot}4$.

6. Find the first approximation of the root lying between 0 and 1 of the equation

$x^3 + 3x - 1 = 0$ by Newton-Raphson formula.

7. Find $y^I(x)$, $y^{II}(x)$, given

x :	0	1	2	3	4
f (x) :	1	1	15	40	85

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

8. a) From the following table, estimate the number of students who obtained marks between 40 and 45 :

Marks :	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of Students :	31	42	51	35	31

b) Using Newton's divided difference formula, evaluate $f(8)$ and $f(15)$, given :

X :	4	5	7	10	11	13
f (X) :	48	100	294	900	1210	2028

7 + 8

9. a) Find the positive real root of $x^3 = 18$ using the bisection method of 4 iterations.

b) Find the root of the equation $x^3 + x^2 + x + 7 = 0$ using Regula Falsi method.

c) A curve passes through the points as given in the following table. Find the area bounded by the curve, x-axis, $x = 1$ and $x = 9$:

X	1	2	3	4	5	6	7	8	9
Y	0·2	0·7	1	1·3	1·5	1·7	1·9	2·1	2·3

5 + 5 + 5

10. a) Write a program in *C* to solve the equation $x^3 - 3x - 5 = 0$ within (1, 2) by Bisection method correct up to 3 places of decimal.

b) Write a program in *C* using recursive function to calculate the sum of all digits of any number. 8 + 7

11. a) Evaluate $\int_0^1 xe^x \, dx$ by using Trapezoidal rule taking $n = 6$.

b) Use Lagrange's interpolation formula to find the value of $f(x)$ for $x = 0$, given the following :

x :	– 1	– 2	2	4
$f(x)$:	– 1	– 9	11	69

c) Prove that Newton-Raphson method has a quadratic convergence. 5 + 5 + 5

12. a) Solve the following system of equations by $L - U$ Factorization Method :

$$x_1 + x_2 - x_3 = 2$$

$$2x_1 + 3x_2 + 5x_3 = -3$$

$$3x_1 + 2x_2 - 3x_3 = 6.$$

b) Solve the following set of equations by Gauss-Seidel method correct to 2 places of decimal :

$$9x - 2y + z = 50$$

$$x + 5y - 3z = 18$$

$$-2x + 2y + 7z = 19.$$

c) Write a *C* program to approximate a real root of the following equation :

$4 * \sin(x) = e^x$ by Bisection method. 5 + 5 + 5

13. a) Write a *C* program to interpolate a given function at a specified argument by Lagrange's interpolation formula.

b) Find the value of $\log 2^{1/3}$ from $\int_0^1 \frac{x^2}{1+x^3}\,dx$ using Simpson's $\frac{1}{3}$ rd rule with $n = 4$.

c) Calculate the approximate value of $\int_0^{\pi/2} \sin x\,dx$ by Composite Trapezoidal Rule by using 11 ordinates. Also compare it with the actual value of the integral.

5 + 5 + 5