WS 2018-19

VIT
Vellore Institute of Technology

Continuous Assessment Test – II

Programme Name & Branch: B. Tech.

Course Name & Code: Applied Linear Algebra – MAT3004

Slot: C1+TC1+TCC1+V2

Class Number(s): VL2018195000777

Exam Duration: 90 minutes

Maximum Marks: 50

Answer All the Questions (5 × 10 = 50)

S. No.	Question	
1.	Let $A = \begin{pmatrix} 1 & -1 & 2 & 0 \\ 2 & -2 & 4 & 0 \\ 3 & -3 & 7 & 0 \end{pmatrix}$	
	a) Find a basis for the column space $C(A)$.	(3)
	b) Find a basis for the null space $N(A)$.	(3)
	c) Write the complete solution to $Ax = b$, where $b = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$	(4)
2.	a) Show that the column space of the matrix A is $\{(x,y,z) \in \mathbb{R}^3 : x+y+z=0\}$, where $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$.	(4)
	b) State the condition for a $m \times n$ matrix to have a left inverse. Find at least two left inverse of the matrix $A = \begin{pmatrix} -3 & -4 \\ 4 & 6 \\ 1 & 1 \end{pmatrix}$ if it exists.	(6)
3.	Let V be a vector space of polynomials of degree ≤ 2 with basis $\{1,\ x-1,\ (x-1)^2\}$ and W be the same vector space with basis $\{1,\ x,\ x^2\}$. Let $T: V \to W$ be a transformation defined by $T(p(x)) = p(x+1)$.	
	a) Show that T is linear.	(2)
	b) Find the kernel of T and nullity of T.	(3)
	c) Write the matrix for T.	(5)
4.	a) Complete the following sentences appropriately for a 3×3 matrix. i) If the column space is a plane, the null space is a ________ ii) If the column space is a line, the null space is a ________ iii) If the column space is all of R^3, the null space is a ________ iv) If the column space is the zero vector, the null space is a ________	(4)
	(b) Find a 7×7 matrix A whose column space equals it null space, or argue briefly it cannot exist.	(6)
5.	Let $T: R^3 \to R^3$ be a linear transformation such that $T(e_1) = (8,6,1)$, $T(e_2) = (2,0,5)$ and $T(e_3) = (1,2,0)$, where $\{e_1, e_2, e_3\}$ is the standard basis for R^3. Find $T(x,y,z)$ and hence find $T(-1,2,6)$,	(10)

$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -3 & -4 \\ 4 & 6 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -15+8 & -2 \\ -9+8 & -12 \end{bmatrix}$