Final Assessment Test – November 2019

VIT
Vellore Institute of Technology

Course: MAT3004 - Applied Linear Algebra
Class NBR(s): 0345 / 0346 / 0347 / 0514 / 0515 / 0516 / 0532 / 6564 / 7261
Slot: A2+TA2+TAA2+V3
Time: Three Hours
Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS EXAM MALPRACTICE

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) Solve the following system of linear equations using an LU factorization method $x_1+x_2+x_3=1, x_1+4x_2+5x_3=3$ and $x_1+4x_2+7x_3=5$. [10]

   b) Determine all values of the make the following system consistent. $x+y-z=b_1, 2y+z=b_2, y-z=b_3$. [10]

2. a) Let U, W be subspaces of a vector space V. [10]

   (i) Suppose that Z is a subspace of V contained in both U and W. Show that Z is also contained in $U \cap W$.

   (ii) Suppose that Z is a subspace of V containing both U and W. Show that Z also contains $U+W$.

   b) In the 3-space $\mathbb{R}^3$ let (x_1, x_2, x_3) satisfy the equation $x_1 - x_2 - x_3 = 0$ Prove that W is a subspace of $\mathbb{R}^3$. Find a basis for the subspace W. [10]

3. a) Let V, W be the subspaces of the vector space $P_3(\mathbb{R})$ spanned by [10]

   $\{v_1(x)=3-x+4x^2+x^3, v_2(x)=5+5x^2+x^3, v_3(x)=5-5x+10x^2+3x^3\}$ and
   $\{w_1(x)=9-3x+3x^2+2x^3, w_2(x)=5-x+2x^2+x^3, w_3(x)=6+4x^2+x^3\}$ respectively.
   Find the dimensions and bases for $V+W$ and $V \cap W$.

   b) Find the equation of a circle that passes through the three points (2, -2), (3, 5), and (-4, 6). [10]

4. a) Prove that two vector spaces V, W are isomorphic if and only if $\dim V = \dim W$. [10]

   b) Let α be the standard basis for $\mathbb{R}^3$, and let $S, T: \mathbb{R}^3 \to \mathbb{R}^3$ be two linear transformations given by $S(\overline{e_1})=(2,2,1), S(\overline{e_2})=(0,1,2), S(\overline{e_3})=(-1,2,1)$ and $T(\overline{e_1})=(1,0,1), T(\overline{e_2})=(0,1,1)$, $T(\overline{e_3})=(1,1,2)$. Compute $[S+T]_\alpha$, $[2T-S]_\alpha$ and $[T\circ S]_\alpha$ [10]

5. a) Let $\beta=\{\overline{v_1}, \overline{v_2}, \overline{v_3}\}$ be a basis for the 3-space $\mathbb{R}^3$ where $\overline{v_1}=(1,1,0)$ $\overline{v_2}=(1,0,1)$ and $\overline{v_3}=(0,1,1)$. [5]

   Let T be the linear transformation $\mathbb{R}^3$ on given by the matrix $[T]_\beta = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \\ -1 & 1 & 1 \end{bmatrix}$.

   Let $\alpha=\{e_1, e_2, e_3\}$ be the standard basis. Find the basis-change matrix $[id]_\alpha^\beta$ and $[T]_\alpha$.

   b) In $\mathbb{R}^2$, equipped with an inner product $\langle \overline{X}, \overline{Y}\rangle = x_1y_1 + x_2y_2$, find the angle between $\overline{X}=(1,1), \overline{Y}=(1,0)$. [5]

   c) Find an orthogonal basis for the subspace W of the Euclidean space $\mathbb{R}^3$ given by $x+2y-z=0$. [10]

$\frac{1}{2}$ -2 $\frac{1}{2}$ $+1$

6. (a) Find the QR –factorization of $A=\begin{bmatrix}1 & -1 & -1\\ 1 & 0 & 0\\ 1 & -1 & 0\\ 0 & 1 & -1\end{bmatrix}$. error **[14]**

(b) Find a point on the plane $x-y+z=0$ that is closest to $P=(1,2,0)$ using orthogonal projection. **[6]**

7. a) Find all least squares solutions $\overline{X}$ in $\mathbb{R}^3$ of $A\overline{X}=\overline{b}$. Where $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 2\\ -1 & 1 & -1\\ -1 & 2 & 0\end{bmatrix}$, $\overline{b}=\begin{bmatrix}3\\ -3\\ 0\\ -3\end{bmatrix}$. **[10]**

b) Decode the cipher text "19, 45, 26, 13, 36, 41" using A→0, B→1, Z→25 and $A=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 1 & 1 & 1\end{bmatrix}$. **[5]**

c) Encode the message "GOOD LUCK" using the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 1 & 1 & 1\end{bmatrix}$. **[5]**

⇔⇔⇔

$(AX=b)\ A^T$

$$\frac{A^T A (X)}{A^T A} = \frac{A^T b}{A^T A}$$

$$(\overline{X}) = (A^T A)^{-1} (A^T b)$$

$$A^T A = (\quad)\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

$A^T b$,

$A^T A$