VIT

Vellore Institute of Technology

DEPARTMENT OF MATHEMATICS

SCHOOL OF ADVANCED SCIENCES

FALL SEMESTER 2022-2023

CONTINUOUS ASSESSMENT TEST - I

Programme Name	:	B.Tech
Course Code	:	BMAT101L
Course Name	:	Calculus
Slot	:	B1+TB1
Date of the Examination	:	07.11.2022
Duration	:	90 minutes

Max. Marks:50

Answer all the questions

S.No	Questions	Marks
1	a) Identify the region where the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ is increasing or decreasing and determine its extreme values on $x \in [-2, 3]$.	6M
	b) Verify Rolle's theorem and find an appropriate constant c of the function $f(x) = \sin(x)e^{-x}, x \in [0, \pi]$.	4M
2	Determine by integration the area bounded by three lines $y = 4 - x$, $y = 3x$ and $3y = x$.	10M
3	Determine the volume of the solid generated by rotating the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y–axis.	10M
4	a) Examine the continuity of $f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0) \end{cases}$ at $(0,0)$.	6M
	b) Let $u = \frac{x}{z} + \frac{y}{z}$, where $x = \cos^2 t, y = \sin^2 t, z = \frac{1}{t}$. Find $\frac{du}{dt}$	4M
5	a) Find the value of μ in terms of a and b such that $f(x,t) = \sin(ax)e^{-bt}$ satisfies the heat equation $f_t = \mu f_{xx}$.	6M
	b) Let $u = \sqrt{x^2 + y^2}$ and $v = \sin(x^2 + y^2)$.Find $J(\frac{u,v}{x,y})$ and determine its relation.	4M