VIT UNIVERSITY
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VELLORE ■ CHENNAI
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SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS
Continuous Assessment Test –I, August 2016

Course Code: MAT3003 **Slot: A1+TA1+TAA1**

Course Name: Complex Variables and Partial Differential Equations **Date of Exam: 21.8.2016**

Max. Marks : 50 **Answer ALL the Questions** **Duration: 90 Minutes.**

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1. (a) Show that an analytic function with constant modulus is constant. (4)

   (b) Show that the families of curves $r^n = \alpha secn\theta$ & $r^n = \beta cosecn\theta$,

   intersect orthogonally, where α & β are arbitrary constants. (3)

   (c) Is the function $f(z) = \cosh z$ is analytic or not? Justify your answer. (3)

2. (a) In a two dimensional fluid flow, find if $x^2 - y^2 - 3x - 2y + 2xy$ can represent The Stream function. If so, find the corresponding velocity potential and also the Complex potential. (7)

   (b) Find the equation of the orthogonal trajectories of the family of curves given by

   $2x - x^3 + 3xy^2 = a.$ (3)

3. Find the bilinear transformation that maps the points $z_1 = 1, z_2 = i, z_3 = -1$

   into the points $w_1 = i,\ w_2 = 0, and\ w_3 = -i.$ Hence find

   (i) The image of $|z| < 1$. (ii) Find the invariant points of this transformation. (10)

4. (a) Find the Laurent's series expansion of $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ in the region

   (i) $0 < |z-1| < 4$ (ii) $|z-1| > 4.$ (7)

   (b) Expand $f(z) = sin^3 z$ in a Taylor's series about $z = 0.$ (3)

5. (a). Discuss the transformation $w = e^z.$ (7)

   (b) Find the residues at each pole of the function $f(z) = \frac{z^2}{(z-1)^2\,(z+2)}$ (3)