http://www.makaut.com

Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/B.TECH (CT-NEW)/SEM-4/M(CT)-401/2012

2012
MATHEMATICS - III

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The period of $\sin 2x$ is

a) 1 b) π
c) 2 d) 2π.

ii) If $F\{f(x)\} = F(s)$ represents the Fourier transform of $f(x)$, then $F\{f(x-a)\}$ (a being a constant) equals

a) $e^{isa} F(s)$ b) $F(s/a)$
c) $e^{-isa} F(s)$ d) $\frac{1}{a^2} F(as)$.

iii) The value of α such that $3y - 5x^2 + \alpha y^2$ is a harmonic function is

a) 5 b) 0
c) – 5 d) 3.

iv) The value of the integral $\int_C \frac{z^2 - z + 1}{z - 1} dz$ where $C : |Z| = 1$, is

a) πi b) $-\pi i$

c) $-2\pi i$ d) $2\pi i$.

v) The residue of the function $f(z) = z^2/(z-1)^2(z+2)$ at the pole $z = -2$ is

a) $\frac{9}{4}$ b) $\frac{2}{9}$

c) $\frac{4}{9}$ d) $\frac{9}{2}$.

vi) Four coins are tossed simultaneously. The probability of getting 2 heads is

a) $\frac{3}{8}$ b) $\frac{1}{8}$

c) $\frac{3}{4}$ d) $\frac{1}{4}$.

vii) The random variable X has the following p.d.f. :

$$f(x) = \begin{cases} k, -2 < x < 2 \\ 0, \text{otherwise} \end{cases}$$

The value of the constant k is

a) $\frac{1}{8}$ b) $\frac{1}{12}$

c) $\frac{1}{2}$ d) $\frac{1}{4}$.

viii) The variance of a Poisson distribution with parameter λ is

a) λ^2 b) $\frac{1}{\lambda}$

c) λ d) $\frac{1}{\lambda^2}$.

ix) A solution $u(x, y)$ of the PDE $u_{xx} - u = 0$ is

a) $A(y)\, e^{x} + B(y)\, e^{-x}$

b) $A(x)\, e^{y} + B(x)\, e^{-y}$

c) $A(x)\, e^{y} + B(y)\, e^{x}$

d) none of these.

x) The value of $J_{-\frac{1}{2}}(x)$ is

a) $\sqrt{\frac{2}{\pi x}}\sin x$ b) $\sqrt{\frac{2}{\pi x}}\cos x$

c) $-\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x}\right)$ d) $-\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x}\right)$.

xi) The value of Legendre's polynomial $P_1(x)$ is

a) 0 b) 1

c) x d) x^2.

xii) If $\alpha \neq \beta$, then the value of $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx$ is

a) $\frac{1}{2}\left\{J_{n+1}(\alpha)\right\}^2$ b) 0

c) $\frac{1}{2}\left\{J_n(\alpha)\right\}^2$ d) none of these.

xiii) If $P_n(x)$ is the Legendre's polynomial of degree n, then $\int_{-1}^{1} P_n(x)\, dx$ in

a) 1, when $n = 0$ b) 0, when $n = 0$

c) 2, when $n = 0$ d) none of these.

xiv) If $f(z) = u(x, y) - i\, v(x, y)$ is analytic, then $f'(z)$ equals

a) $\frac{\partial u}{\partial x} - i \frac{\partial u}{\partial y}$ b) $\frac{\partial u}{\partial x} + i \frac{\partial u}{\partial y}$

c) $\frac{\partial v}{\partial x} - i \frac{\partial v}{\partial y}$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

3. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$. 3 + 2

4. Show that the polar form of Cauchy – Riemann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}.$$

Deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. 3 + 2

WEST BENGAL UNIVERSITY OF TECHNOLOGY
Utech

5. Evaluate $\int_c \frac{e^z}{(z^2+\pi^2)^2}\, dz$, where C is $|z| = 4$.

6. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution.

3 + 2

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions.

3 × 15 = 45

7. a) If $f(x) = |\cos x|$, expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$.

b) Using Parseval's identities, prove that

$$\int_0^{\infty} \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$$

c) Find Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^{\infty} \frac{x \sin mx}{1+x^2}\, dx = \frac{\pi e^{-m}}{2}$, $m > 0$.

8. a) Find the analytic function, whose real part is $\sin(2x)/\{\cosh(2y) - \cos(2x)\}$. 3 + 2

b) Show that under the transformation $w = \frac{z-i}{z+i}$, real axis in the z plane is mapped into the circle $|w| = 1$. Which portion of the z plane corresponds to the interior of the circle. 3 + 2

c) Evaluate $\int_0^{2+i} (\bar{z})^2\, dz$ along (i) the line $y = x/2$, (ii) the real axis to 2 and then vertically to $2+i$. 2 + 3

9. a) A has one share in a lottery in which there is 1 prize and 2 blanks; B has three shares in a lottery in which there are 3 prizes and 6 blanks ; compare the probability of A's success to that of B's success.

b) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts ?

10. a) Solve the following equation by the method of separation of variables $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, 0) = \sin \frac{n \pi x}{l}$.

b) Show that the solution of the heat equation

$K \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $-\infty < x < \infty, t > 0$ subject to the condition

$u(x, t) = 0$ at $x = \pm \infty$, $\frac{\partial u}{\partial x} = 0$ at $x = \pm \infty$ and

$u(x, 0) = f(x)$, $-\infty < x < \infty$ can be written in the form

$$u(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-Ks^2 t - isx} ds$$

where $F(s)$ is the Fourier transform of $f(x)$. 8 + 7

11. a) Obtain the series rotation of the equation

$$x(1 - x) \frac{d^2 y}{d x^2} - (1 + 3x) \frac{dy}{dx} - y = 0.$$

b) Show that $J_n(x) = \frac{1}{\pi}\int_o^{\pi} \cos(n\,\theta - x \sin\theta)\, d\,\theta$, n being an integer. 8 + 7

12. a) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomial.

b) Show that

$$\int_{-1}^{1} x^2 P_{n-1}(x)\, P_{n+1}(x)\, \mathrm{d}x = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

8 + 7