MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BSC301 Mathematics-III (Differential Calculus)

UPID : 003445

Time Allotted : 3 Hours *Full Marks :70*

The Figures in the margin indicate full marks.

Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer any ten of the following : [1 x 10 = 10]

(I) What is the area of the region bounded by x-axis , $y=e^x$, x=0, x=1

(II) What is the general form of clairaut's equation?

(III) If a graph has 5 vertices and 7 edges, then what is the size of its adjacency matrix?

(IV) On which region log(1+x) can be expanded in an infinite series ?

(V) If for any $\vec{A}$, $\vec{\nabla} x \vec{A} = 0$, then $\vec{A}$ will be called as?

(VI) Find the value of $\int_{x=-1}^{1}\int_{y=-2}^{2}\int_{z=-3}^{3} xy^2z^3\,dxdydz$

(VII) If $\int_c ydx + xdy = p$ where c is given by $x = \cos\theta, y = \sin\theta, 0 \le \theta \le \pi/2$, find value of p?

(VIII) Find the value of $\frac{1}{D^2+4}(\sin 2x)$?

(IX) What is the eccentricity of the vertex of a graph having only one vertex?

(X) What is the natue of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots\ldots$

(XI) If $f(x,y) = |x| + |y|$, find the value of $f_x(0,0)$?

(XII) If c is the circle $x^2+y^2=4$, find the value of $\int_c x^2 dx$

Group-B (Short Answer Type Question)

Answer any three of the following [5 x 3 = 15]

2. Test the series $\sum_{n=1}^{\infty}(\frac{n}{n+1})^{n^2}$ [5]

3. If $z = u^2 + v^3$, where u=Sin xy and $v = y^2$, Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ [5]

4. Varify that, $e^{\tan^{-1}x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{6} - ..$ [5]

5. Find $\frac{dy}{dx}$ of the function $(\sin y)^x - (\cos x)^y = 0$ [5]

6. Find the general and singular solution of [5]
$y = 4xp - 16y^3p^2$

Group-C (Long Answer Type Question)

Answer any three of the following [15 x 3 = 45]

7. (a) Test the convergence of the series whose n_{th} term are [3]
$(n^{\frac{1}{n}} - 1)^n$

(b) Examine the convergence of the series [5]
$$\frac{1}{a} - \frac{1}{a+b} + \frac{1}{a+2b} - \frac{1}{a+3b} + \ldots\ldots (a > 0, b > 0)$$

(c) Assuming the validity of expansion, show that [7]
$$sinx = 1 - \frac{(x-\frac{\pi}{2})^2}{2!} + \frac{(x-\frac{\pi}{2})^4}{4!} - \ldots\ldots$$

8. (a) If u=log r and [5]
$r^2 = x^2 + y^2 + z^2$, Prove that $r^2(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) = 1$

(b) Show that [5]
$f(x,y) = 3x^3 + 4x^2y - 3xy^2 - 4y$, neither a maximum nor a minimum at (0,0)

(c) Determine the constant m so that the vector [5]
$\vec{v} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+mz)\hat{k}$ is solenoidal

9. (a) If [5]
$u_n = \frac{3^n}{n+1}$, show that $[u_n]$ is monotonic increasing and bounded above, find its limit.

(b) Expand e^x in power series of (x-1) [5]

(c) Examine the convergence of the series [5]
$\sum u_n$, where $u_n = \frac{(n+1)(n+4)}{n(n+2)(n+5)}$

10. (a) If $u(x,y)=f(x^2+2yz, y^2+2zx)$, prove that [5]
$$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0$$

(b) If [5]
$u = \tan^{-1}(\frac{x^{5/2} + y^{5/2}}{\sqrt{x} - \sqrt{y}})$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$

(c) Show that the function $f(x,y)=4x^2y - y^2 - 8x^4$ has a maximum value at (0,0). [5]

11. (a) The given function [7]
$$f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}, (x,y) \neq (0,0)$$
$= 0, (x,y)=(0,0)$

Find from definition fxy(0,0) and $f_{yx}(0,0)$

(b) If [3]
$A = \pi h^2 \frac{sin\alpha}{1 - sin\alpha}$ find dA, where h and α are independent variables

(c) If [5]
$f(x,y) = \frac{x+y}{1-xy}$ and $g(x,y) = \tan^{-1}x + \tan^{-1}y$ find $Jacobian \frac{\partial(f,g)}{\partial(x,y)}$