Sub Code: AHT003 **ROLL NO……………..…………..**

ODD SEMESTER EXAMINATION, 2024 – 25

1^{st}Year (1^{st} - Sem) B.Tech.

INTRODUCTION TO ENGINEERING MATHEMATICS

Duration: 3:00 hrs **Max Marks: 100**

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	Answer any two parts of the following. (10x2= 20) a) (i) Show that the function $f(x) = \begin{cases} x \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$ is continuous but not differentiable at a point x = 0. (5 marks) (ii) State Lagrange's mean value theorem, find the value of *c* by Lagrange's mean value theorem where $f(x) = 2x^2 + 3x + 4$ in [1, 2]. (5 marks) b) State Taylor's theorem in two variables. Expand $e^x \sin y$ in power of x and y, x = 0, y = 0 as far as terms of third degree. (10 marks) c) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, Show that (i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$. (ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$. (10 marks)
Q 2.	Answer any two parts of the following. (10x2= 20) a) (i) Evaluate $\iint x\,y\,dx\,dy$, *over R where R is quadrant of the cricle* $x^2 + y^2 = a^2$ where $x \geq 0, y \geq 0$. (5 marks) (ii) Evaluate $\int_0^\infty e^{-x^2}\,dx$ *with the help of Gamma function.* (5 marks) b) Trace the curve $x^3 + y^3 = a^2x$. (10 marks) c) Evaluate by change of order of integration $\int_0^1 \int_{x^2}^{2-x} x\,y\,dy\,dx$. (10 marks)
Q 3.	Answer any two parts of the following. (10x2= 20) a) (i) If x + y + z = u, y + z = uv, z = uvw, show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$. (5 marks) (ii) Find area enclosed between the parabola $y = x^2$ and the straight-line y = x. (5 marks) b) Evaluate $\iiint (x^2 + y^2 + z^2)$ dx dy dz, over R where R denotes the region bounded by x = 0, y = 0, z = 0, and x + y + z = a (a > 0). (10 marks) c) A triangular thin plate with vertices (0, 0), (2, 0) and (2, 4) has density $\rho = 1 + x + y$, then find (i) The mass of the plane (ii) Centre of gravity. (10 marks)
Q 4.	Answer any two parts of the following. (10x2= 20) a) (i) If $\vec{f} = x^2yi - 2xzj + 2yzk$, find div f, curl f, and curl(curl f). (5 marks) (ii) If $\vec{f}$= 2zi – xj + yk, Evaluate $\iiint \vec{f}\,dV$, *over V where V is region bounded by the surface* $x = 0$, y = 0, x = 2, y = 4 and z =

	x^2, z = 2. (5 marks) b) State Green's theorem for the plane. Verify Green's theorem in the plane for $\int (xy + y^2)\, dx + x^2 dy$, over C, where C is the region bounded by y = x and y = x^2. (10 marks) c) State Gauss Divergence theorem, Verify Gauss divergence theorem for $\vec{f}$ = 4xzi – y^2j + yzk, take over the cube bounded by the planes x = 0, x = 1, y = 0, y = 1 and z = 0, z = 1. (10 marks)
Q 5.	Answer any two parts of the following. (10x2= 20) a) (i) Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. (5 marks) (ii) Find the eigen value of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (5 marks) b) Find for what values of λ and μ the system of linear equations: x + y + z = 16 x + 2y + 5z = 10 2x + 3y + λz = μ Has (i) A unique solution (ii) No solution (iii) Infinite solution. Also find the solution for λ = 2 and μ = 8. (10 marks) c) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A (Cayley- Hamilton Theorem) and hence obtain A^{-1}. (10 marks)

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