Final Assessment Test – April 2019

VIT® Vellore Institute of Technology

Course: MAT1014 - Discrete Mathematics and Graph Theory

Class NBR(s): 0668 / 0670 / 0671 / 0675 /0678 / 0680 / 0683 / 0684 / 0687 / 0692 / 0711 / 0713 / 0715 / 3334

Slot: A1+TA1+TAA1+V1

Time: Three Hours

Max. Marks: 100

Answer any FIVE Questions
(5 X 20 = 100 Marks)

1. a) Obtain the PCNF and PDNF of S: $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$. Also determine a unique representation for the PCNF and PDNF of S. Is S a tautology? **[10]**

   b) Show that R is a valid conclusion from the premises $P \rightarrow Q$, $Q \rightarrow R$, $\neg(P \wedge R)$, $P \vee R$. **[10]**

2. a) Verify the validity of the following argument. **[10]**

   'Every living thing is a plant or an animal. John's gold fish is alive and it is not a plant. All animals have hearts. Therefore John' s gold fish has a heart.'

   b) Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)\, P(x) \vee (\exists x)\, Q(x)$ using indirect method of proof. **[10]**

3. a) State and prove Lagrange's theorem for finite groups. **[10]**

   b) Let $G = \{1, -1, i, -i\}$. **[10]**

   i. Is G is a group under usual multiplication?

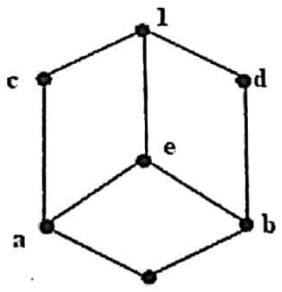
   ii. Is G abelian?

   iii. Is G cyclic?

   iv. What is the order of every element in G?

4. a) Let S be any set, and ρ(S) its power set. Verify if $(\rho(S), \leq)$, is a *partially ordered relation, where* $\leq$ denotes the relation 'is a subset of '. Draw the Hasse diagram of $(\rho(S), \leq)$, when X = {1, 2, 3, 4}. **[10]**

   b) Given the Hasse diagram (L, ≤), verify if it is modular, distributive and complemented. Justify your answer. **[10]**



(L, ≤)

5. a) Verify if the following Boolean polynomials P and Q are equivalent. If so determine their sum of product canonical form. **[10]**

   $P(x, y, z) = (x \oplus y) * (x' \oplus z) * (y \oplus z)$.

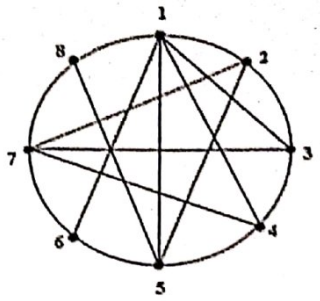
   $Q(x, y, z) = (x \oplus y) * (x' \oplus z)$.

   b) Minimize the following expression using Quine-McCluskey's method **[10]**

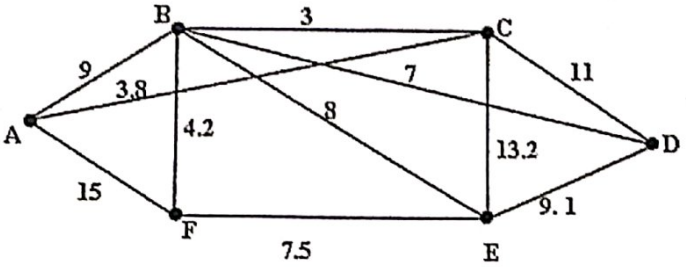
   $wxyz + w\bar{x}yz + wx\bar{y}z + w\bar{x}\bar{y}z + wxyz$.

6. a) State and prove a necessary and sufficient condition for a connected graph G to be Euler. [10]

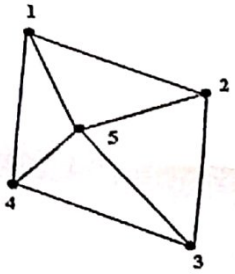
b) State Kuratowski's theorem on graph planarity. Hence verify If the following graph is planar using Kuratowski s theorem. [10]



7. a) Determine a minimum weighted spanning tree for the following graph G using Prim's and Kruskal's algorithm. [10]



b) *Given the following graph G, determine* [10]



i. The chromatic number of G.

ii. A chromatic partition of G.

iii. The chromatic polynomial for G.

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