Practical no. 1

Topic : limits & Continuity

1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

2) $\lim_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3) $\lim_{x \to \pi/6} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

4) $\lim_{x \to 0} \dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

5) Examine the Continuity of following function

i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, \quad 0 < x \leq \pi/2$
$\quad = \dfrac{\cos x}{\pi - 2x}, \quad \pi/2 < x < \pi$ } at $x = \pi/2$

ii) $f(x) = \dfrac{x^2-9}{x-3}, \quad 0 < x < 3$
$\quad = x+3, \quad 3 \leq x < 6$
$\quad = \dfrac{x^2-9}{x+3}, \quad 6 < x \leq 9$ } at $x = 3$ & $x = 6$

6) Find the value of K so that function f(x) is Continuous at the indicated point

(i) $f(x) = \dfrac{1-\cos 4x}{x^2}, \quad x < 0$
$\quad = K, \quad x = 0$ } at $x = 0$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$
$\quad = K$ } at $x = 0$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}, \quad x \neq \pi/3$
$\quad , x = \pi/3$ } at $x = \pi/3$

7) Discuss the Continuity of the following function which of these function have Removable discontinuity Redefine function so as to Remove the discontinuity?

(i) $f(x) = \dfrac{1-\cos 3x}{x \tan x}, \quad x \neq 0$
$\quad x = 0$ } at $x = 0$

(ii) $f(x) = \dfrac{(e^{3x}-1)\sin x^{\circ}}{x^2}, \quad x \neq 0$
$\quad = \pi/60, \quad x = 0$ } at $x = 0$

8) If $f(x) = \dfrac{(e^{x^2} - \cos x)}{x^2}$

For $x \neq 0$ is continuous at $x = 0$ f (0)

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Q) If $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$ is continuous at $x = \pi/2$ Find $f(\pi/2)$

Solution:-

1) $\lim_{x \to a} \dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$

$= \lim_{x \to a} \dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \times \dfrac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \lim_{x \to a} \dfrac{(a+2x-3x)\sqrt{3a+x} + 2\sqrt{x}}{(3a+x-4x)\sqrt{a+2x} + \sqrt{3x}}$

$= \lim_{x \to a} \dfrac{(a-x)\ \sqrt{3a+x} + 2\sqrt{x}}{(3a-3x)\ \sqrt{a+2x} + \sqrt{3x}}$

$= \lim_{x \to a} \dfrac{(a-x)\sqrt{3a+x} + 2\sqrt{x}}{3(a-x)\sqrt{a+2x} + \sqrt{3x}}$

$= \lim_{x \to a} \dfrac{1}{3} \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \dfrac{\sqrt{3a+a} + 2\sqrt{a}}{3\sqrt{a+2a} + \sqrt{3a}}$

$= \dfrac{\sqrt{4a} + 2\sqrt{a}}{3\sqrt{3a} + \sqrt{3a}}$

$= \dfrac{2\sqrt{a} + 2\sqrt{a}}{3 \times 2\sqrt{3a}} = \dfrac{2\sqrt{a}}{3\sqrt{3a}} = \dfrac{2}{3\sqrt{3}}$

2) $\lim_{y \to 0} \dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}}$

$= \lim_{y \to 0} \dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \dfrac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}}$

$= \lim_{y \to 0} \dfrac{(a+y-a)}{(y\sqrt{a+y})(\sqrt{a+y} + \sqrt{a})}$

$= \lim_{y \to 0} \dfrac{y}{y\sqrt{a+y} \cdot (\sqrt{a+y} + \sqrt{a})}$

$= \lim_{y \to 0} \dfrac{1}{\sqrt{a}\,(\sqrt{a} + \sqrt{a})}$

$= \lim_{y \to 0} \dfrac{1}{\sqrt{a}(2\sqrt{a})}$

$= \dfrac{1}{2a}$

3) $\lim_{x \to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

$x - \pi/6 = h \quad x = h + \pi/6 \quad h \to 0$

$\lim_{h \to 0} \dfrac{\cos(h + \pi/6) - \sqrt{3}(h + \pi/6)}{\pi - 6(h + \pi/6)}$

$\lim_{h \to 0} \dfrac{(\cos h \cdot \cos \pi/6 - \sin h \cdot \sin \pi/6) - \sqrt{3}(\sin h \cos \pi/6 + \cos h \sin \pi/6)}{-6h}$

$$\lim_{h\to 0} \frac{\left(\cos\frac{\sqrt{3}h}{2} - \sin\frac{h}{2}\right) - \left(\sin\frac{3h}{2} + \cos\frac{\sqrt{3}}{2}\right)}{-6h}$$

$$\lim_{h\to 0} \frac{-\frac{\sin 4h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin 2h}{6h}$$

$$= \frac{1}{3} \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{3}$$

4) $\lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

$$= \lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2}}{\sqrt{x^2+5} + \sqrt{x^2}}$$

$$= \lim_{x\to\infty} \frac{8(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2+3)(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$= \lim_{x\to\infty} \frac{8(\sqrt{x^2+3} + \sqrt{x^2+1})}{2(\sqrt{x^2+3} + \sqrt{x^2-3})}$$

$$= \lim_{x\to\infty} \frac{4\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1-3/x^2)}}$$

$$= 4\lim_{x\to\infty} \frac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(\ldots)}}$$

$= 4$

$\sqrt{1-\cos(\pi/2)}$

1) $f(\pi/2) = 6$

$\therefore f$ at $x = \pi/2$ define

2) $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2^+} \frac{\cos x}{\pi - 2x}$

put $x - \pi/2 = h$

$\therefore x = \pi/2 + h$

as $x \to \pi/2$, $h \to 0^+$

$$\lim_{h\to 0^+} \frac{\cos(\pi/2+h)}{\pi - 2(\pi/2+h)} = \lim_{h\to 0^+} \frac{-\sin h}{\pi - \pi - 2h}$$

$$= \lim_{h\to 0^+} \frac{-\sin h}{-2h} = \frac{1}{2}$$

$$\lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^-} \frac{\sin 2x}{\sqrt{1+\cos 2x}}$$

$$= \lim_{x\to\pi/2^-} \frac{2\sin x\cos x}{\sqrt{2\sin^2 x}} = \lim_{x\to\pi/2^-} \frac{2\sin x\cdot\cos x}{\sqrt{2}\,\sin x}$$

$$= \frac{2}{\sqrt{2}} \lim_{x\to\pi/2}$$

$\therefore LHL \neq RHL$

f is not continuous at $x = \pi/2$

5) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$; $3 \le x \le 6$ at $x = 3, 6$; $x = 6$

$= x+3$

$= \frac{x^2-9}{x+3}$

at $x=3$ $= \frac{x^2-9}{\ldots} = 0$

f at x = define.

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\Rightarrow \lim f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)}$

$= LHL = RHL$

f is continous at $x=3$

For $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9}$

$\lim_{x\to 6^+} \frac{(x-3)(x+3)}{(x+3)} = \lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

$LHL \neq RHL$

Function is not continous.

6 / Q)

$f(x) = \frac{1-\cos 4x}{x^2}$ $\left.\begin{matrix} x<0 \\ x=0 \end{matrix}\right\}$ at $x=0$

Sol^n:

f is continous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = K$

$$\lim_{h\to 0} \frac{\sqrt{3}(1-\sqrt{3}\tan h) - \sqrt{3} - \tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)} = \frac{4}{3}\lim_{x\to 0}\left(\frac{\tan h}{h}\right)\left(\frac{1}{1-\sqrt{3}\tan h}\right)$$

$$= \frac{4}{3}\frac{1}{1-\sqrt{3}(0)} = \frac{4}{3}$$

7/(ii) $f(x) = \frac{1-\cos 3x}{x\tan x}$ $x \neq 0$, $x = 0$ } at $x=0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$= \frac{2\sin^2 3/2x}{x\tan x}$$

$$= \frac{2\sin^2 \frac{3x}{2} x^{2 2}}{x^2} \Big/ x\cdot\frac{\tan x}{x^2} \times x^2$$

$$= 2\lim_{x\to 0} (3/2)^2/1$$

$$= \cancel{2}\times\frac{9}{4} = 9/2$$

$\lim_{x\to 0} f(x) = 9/2 \quad 9 = f(0)$

~~∴ f is not cts at x=0~~

Redifine function

$f(x) = \frac{1-\cos 3x}{x\tan x}$ $x\neq 0$

f has Removable discontinuity at $x=0$

7/ii $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$ $x\neq 0$, $x=0$ } at $x=0$

at $x=0$

$$\lim_{x\to 0} (e^{3x}-1)\ \sin\left(\frac{\pi x}{180}\right)/x^2$$

$$3\lim_{x\to 0}\frac{e^{3x}-1}{3x}\ \lim_{x\to 0}\frac{\sin \pi x/180}{x}$$

$$3\log e\ \pi/180 = \frac{\pi}{60} = f(0)$$

8) $f(x) = \frac{e^{x^2}-\cos x}{x^2}$ $x=0$

is continous at $x=0$

f is continous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$$= \frac{e^{x^2}-\cos x}{x^2} = f(0)$$

$$\frac{e^{x^2}-\cos x - 1 + 1}{x^2}$$

$$(e^{x^2}-1) + (1-\cos x)/x^2$$

$$\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0}\frac{2\sin^2 x/2}{x^2}$$

$$= \log e + 2\left(\frac{\sin x/2}{x}\right)^2$$

Multiply with 2 on Numerator & Denominator

$$= 1 + 2\times\frac{1}{4} = \frac{3}{2} = f(0)$$

9) $f(x) = \dfrac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(x)$ is continous at $x = \pi/2$

$$\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$= \frac{2-1+\sin x}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}},$$

$$= \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2\times 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

$$f(\pi/2) = \frac{1}{4\sqrt{2}}$$

6/
(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$, $x \neq 0$
$= K$ $x = 0$ } at $x = 0$

$f(x) = (\sec^2 x)^{\cot^2 x}$
using

$\sec^2 - \tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1+\tan^2 x$

& $\cot^2 x = 1/\tan^2 x$

$\lim (\sec^2 x)^{\cot x}$

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PRACTICAL NO:- 2

Topic: Derivative

Q1 Show that following function defined from R to R are differentiable
1) cot x 2) cosec x 3) sec x

Q2 If $f(x) = 4x+1$, $x \le 2$
$= x^2+5$, $x > 0$ at $x=0$ then find f is differentiable or not

Q3 If $f(x) = 4x+7$, $x \le 2$
$= 3x^2 - 4x + 7$, $x \ge 2$, $x=2$ then find is differentiable or not?

Solutions

Q1 cot x

$f(x) = \cot x$

$= \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$

$= \lim_{x\to a} \frac{1/\tan x - 1/\tan a}{x-a}$

$= \lim_{x\to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$

put $x-a=h$, $x=a+h$ as $x\to a$, $h\to 0$

$Df(a) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$= \lim \tan a - \tan(a+h)$

formula:

$\tan A - \tan B = \tan(A-B)(1+\tan A \cdot \tan B)$

$= \lim_{h\to 0} \frac{\tan(a-a-h)(1+\tan a \tan(a+h))}{h \times \tan(a+h)\tan a}$

$= \lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$

$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$

$= \frac{-\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$Df(a) = -\cos^2 a$

$\therefore$ f is differentiable $\forall a \in R$

2) cosec x

$f(x) = \operatorname{cosec} x$

$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$

$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$

$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$

put $x-a=h$, $x=a+h$ as $x\to a$, $h\to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$Df(a) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

Formula,

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} -\frac{\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2\cos\left(\frac{2a}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

3) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \cancel{\lim_{x \to 0}} \frac{\sec(x) + \sec(a)}{x-a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x-a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$ put $x - a = h$, $x = a+h$

as $x \to a$ $h \to 0$

Q3 Sol^n = RHD:
$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3 \cdot 3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x(x+6)-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9$

$\cancel{LDH} = Df(3^+) = 9$
$LDf = Df(3^-)$
$= \lim_{x \to 3^-} \left(\frac{f(x)-f(3)}{x-3}\right)$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3} = \lim_{x \to 3^-} \frac{4x-12}{x-3} = \lim_{x \to 3^-} \frac{4(x-3)}{(x-3)}$

$Df(3^-) = 4$
RHD ≠ LHD
f is not differentiable at x=3

$f(x) = 8 \times 2 - 5 = 16 - 5 = 11$
Q.4) RHD: $Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$

$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3 \times 2 + 2 = 8$
$Df(2^+) = 8$

LHD: $Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x-5-11}{x-2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$ $Df(2^-) = 8$

LHD = RHD f is differentiable at x=3

TOPIC: Application of Derivation.

1) Find the interval in which function increase or decreasing

(i) $f(x) = x^3 - 5x + 1$

(ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

(iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upward

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$

Solution:-

Q.1) i) $f(x) = x^3 - 5x + 1$

$f'(x) = 3x^2 - 5$

$\because f$ is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \quad \sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

And it is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \quad \sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - \cancel{4} > 0$

$\therefore 2(x - 2) > 0$

$\therefore x - 2 > 0$

$x - 2 > 0$

$x \in (2, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x - 2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3(x + 2) - 5(x + 2) > 0$

$+ \quad -2 \quad - \quad 5/3 \quad +$

$x \in (-\infty, -2) \cup (5/3, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$

$+ \quad -2 \quad - \quad 5/3 \quad +$

$x \in (-2, 5/3)$

4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

$+ \quad -3 \quad - \quad 3 \quad +$

$x \in (-\infty, -3) \cup (3, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$

$+ \quad -3 \quad - \quad 3 \quad +$

$\therefore x \in (-3, 3)$

5) $f(x) = 2x^3 \dots$

$f'(x) = 6x^2 - 18x - 24$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$x \in (-\infty, -1) \cup (4, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$x \in (-1, 4)$

Q2) $y = 3x^2 - 2x^3$

$f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward iff $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\therefore f''(x) > 0$

5A

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$

$+ \quad - \quad +$

$1 \quad 2$

$x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6$

$\therefore f''(x) \not> 0$

$\therefore$ There Exists no interval $(-1/6, \infty)$

PRACTICAL NO – 4

Topic :- Application of derivative & Newton's Method.

Q1) Find maximum & minimum value of following

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ $[-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

Q2) Find the root of the following eq^n by newtons [Takes 4 integration only] Correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ [Takes $x_0 = 0$]

ii) $f(x) = x^3 - 4x - 9$ is $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Q1 $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$\therefore 2x = 32/x^3$

$\therefore x^4 = 32/2$

$\therefore x^4 = 16$

$\therefore x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

Function reaches minimum value at $x = 2$, & $x = -2$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0 \therefore f$ has minimum value at $x = 1$

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0 \therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore f$ has maximum value 5 at $x = -1$ & has the minimum value 1 at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

Consider $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$3x = 0$ or $x - 2 = 0$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0 \therefore f$ has maximum value at $x = 0$

Q.1 $f(x) = x^3 - 3x^2 - 55x + 9.5$ ($x_0 = 0$ 91am)

$f'(x) = 3x^2 - 6x - 55$

By newton's method.

$\therefore x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712)$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.1829 - 1.0272 - 55$

$= -55.9393$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712$

The root of the eqn is 0.1712

1.4 $f(x) = x^3 - 4x - 9$

$= 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

Let x_0

$\therefore$ By newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.792)^2 - 4$

$= 22.5096$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 0.596/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9 = -0.0901$

$f'(3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ [1, 2]

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - (1.8)(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

Lets $x_0 = 2$ be initial approximation by newton method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 2 - 2.2 / f / 8.2$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4606 - 5.6772 - 10$

$= -8.2164$

$x_2 = 1.6892$

$f(x_2) = (1.6893)^3 - 1.8(1.6892)^2 - 10(1.6892) + 17$

$= 4.8677 - 4.9853 - 16.892 + 17$

$x_3 = x_2 - f(x_2) / f'(x_2)$

$= 1.6892 + 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3) / f'(x_3)$

$= 1.6618 + \frac{0.0004}{7.6977}$

$=$

$= 1.668.$

PRACTICAL NO : 5

Topic : Integration

Q.1 Solve the following Integration

i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x}+1)\,dx$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int t^7 \sin(2t^4)\,dt$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$

vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right)dx$

ix) $\int e^{\cos^2 x} \sin 2x\,dx$

x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

1) $\int \frac{1}{x^2+2x-3} dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}} dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$

Substitute
put $x+1=t$
$dx = \frac{1}{t} \times dt$ where $t=1$ $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}} dt$

Using
$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln(1x + \sqrt{x^2-a^2})$

$= \ln(1t + \sqrt{t^2-4})$
$t = x+1$
$= \ln(1x+1+\sqrt{(x+1)^2-4})$
$= \ln(1x+1+\sqrt{x^2+2x-3})$
$= \ln(1x+1+\sqrt{x^2+2x-3}) + c$

2) $\int (4e^{3x}+1) dx$

$= \int 4e^{3x} dx + \int 1 dx$
$= 4\int e^{3x} dx + \int 1 dx$ # $\int e^{ax} dx = \frac{1}{a} \times e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x} dx$
$= \int 2x^2 - 3\sin(x) + 5x^{1/2} dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2 dx - \int 3\sin(x) dx + \int 5x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4) $\int \frac{x^3+3x+4}{\sqrt{x}} dx$

$\int \frac{x^3+3x+4}{x^{1/2}} dx$

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} dx$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}} dx$

$x^{5/2}$ H

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5) $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2t^4$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4 t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du = \frac{t^4 \times \sin(2t^4)}{8}\,du$

Substitute t4 with u/2

$= \int \frac{u/2 \times \sin}{8}\,du$

$= \int \frac{u \times \sin(u)}{2} / 8\,du$

$= \int \frac{u \times \sin(u)}{16}\,du$

$= \frac{1}{16} \int u \times \sin(u)\,du$

$\int u\,dv = uv - \int v\,du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1\,du \quad v = -\cos(u)$

$= \frac{1}{16} (u \times (-\cos(u)) - \int -\cos(u)\,du$

$\int \cos x\,dx = \sin x$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

Relation the distribution $u = 2t^4$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

vi) $\int \sqrt{x}(x^2 - 1)\,dx$

$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= I_1 \cdot \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$

iii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \int \frac{\cos x}{\sin(x)^{3/2}}\,dx$

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(ix) put $t = \sin(x)$

$$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos x}\,dt$$

$$= \frac{1}{\sin x^{3/2}}\,dt$$

$$= I = \int \frac{1}{t^{2/3}}\,dt = \frac{-1}{(2/3-1)\,t^{2/3-1}}$$

$$= \frac{-1}{-1/3\,t^{2/3-1}} = \frac{1}{1/3\,t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$$

$$= 3\sqrt[3]{t}$$

Return distribution $t = \sin(x)$

$$= 3\sqrt[3]{\sin(x)} + c$$

(x) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

put $x^3-3x^2+1 = dt$

$$I = \int \frac{x^2-2x}{} \times \frac{1}{3x^2-3\times 2x}\,dt$$

$$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dt$$

$$= \int \frac{1}{x^3-3x^2+1} + 1/3\,dt$$

$$= \int \frac{1}{3(x^3-3x^2+1)}\,dt = \int \frac{1}{3t}\,dt$$

$$= 1/3 \int 1/t\,dt \qquad \int 1/x\,dx = \ln|x| \times 1$$

$$= 1/3 \times \ln|t| + c$$

$$= 1/3 \ln(1x^3-3x^2+1) + c.$$

Practical no. 6

Find length of the following curve

1) $x = t\sin t,\ y = 1-\cos t \quad t \in [0, 2\pi]$
2) $y = \sqrt{4-x^2} \quad x \in [-2, 2]$
3) $y = x^{3/2}$ in $[0, 4]$
4) $x = 3\sin t,\ y = 3\cos t \quad t \in (0, 2\pi)$
5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Using Simpson's Rule Solve the following

1) $\int_0^2 e^{x^2}\,dx$ with $n=4$

2) $\int_0^4 x^2\,dx$ with $n=4$

3) $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ with $n=6$

Solution:-

1) $y = \sqrt{4-x^2}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$

$= \frac{-x}{\sqrt{4-x^2}}$

$I = \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}$

$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\,dx$

$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\,dx$

$= 2\left[\sin^{-1}(x/2)\right]_{-2}^{2}$

$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$

$y(1f) = \int g(x)(1/f)\,dx + c$

$= \int \sin\frac{x}{2}\ \ x^3\,dx + c$

(ii) $L = \int_0^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = t - \sin t \therefore \frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t \therefore \frac{dy}{dt} = 0 - (-\sin t) = \sin t$

$L = \int_0^{2\pi} \sqrt{(1-\cos t + \sin t)^2 + (\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt = \int_0^{2\pi} \sqrt{2(1-\cos t)} \int_0^{2\pi} \sqrt{2 \times 2\sin^2 t/2}$

$= \int_0^{} \sqrt{4 \sin^2 t/2}$

$= \int_0^{2\pi} 2\left|\sin \frac{t}{2}\right| dt \therefore \sin^2 \frac{t}{2} = \frac{1-\cos t}{2}$

$= \int_0^{2\pi} 2\sin \frac{t}{2}\, dt$

$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$

$= 4 + 4$

$= 8$

$= \cancel{\frac{1}{2}\int_0^4 \sqrt{4 + 9x}\, dx}$

$= \cancel{\frac{1}{2}\left[\frac{(4 + 9x)^{1/2+1}}{1/2 + 1}\right] dx}$

$=$

(ii) $y = \sqrt{4 - x^2} \quad x \in [-2, 2]$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$

$y = \sqrt{4 - x^2} \therefore \frac{dy}{dx} = 2\int_0^2 1 + \left(\frac{-x}{\sqrt{4 - x^2}}\right)^2 dx$

$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4 - x^2}}\, dx$

$= 4\int_0^2 \frac{1}{\sqrt{4 - x^2}}\, dx$

$= 4(\sin^{-1}(x/2))_0^2$

$= 2\pi$

(iii) $y = x^{3/2} \text{ in } [0, 4]$

$f'(x) = \frac{3}{2} x^{1/2}$

$[f'(x)]^2 = \frac{9}{4}x$

$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$$= \int_0^4 \sqrt{1 + \frac{9u}{4}}\, du$$

$$= \int_0^4 \sqrt{\frac{4+9u}{4}}\, du$$

$$= \frac{1}{2} \int_0^4 \sqrt{4+9u}\, du$$

$$= \frac{1}{2} \left[\frac{(4+9u)^{1/2+1}}{1/2+1} \right]_8^4 du$$

$$= \frac{1}{27} \left[(4+9u)^{3/2} \right]_0^4$$

$$= \frac{1}{27} \left[(4+0)^{3/2} - (4+36)^{3/2} \right]$$

$$= \frac{1}{27} (4)^{3/2} - (40)^{3/2}]$$

iv) $x = 3\sin t \qquad y = \cos t$

$$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\, dt$$

$$= \int_0^{2\pi} 3\, dt = 3\int_0^{2\pi} dt = 3[x]_0^{2\pi}$$

$$= 3[2\pi - 0]$$

$$= 6\pi$$

ii) $\int_0^4 x^2\, dx$

$$L = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2 dx = \frac{1}{3}[16 + 4(10) + 8]$$

$$= 64/3$$

$$\int_0^4 x^2 dx = 21.333$$

$$= \frac{1}{2} \int_0^4 \sqrt{4+9u}\, du$$

$$= \frac{1}{2} \left[\frac{(4+9u)^{1/2+1}}{1/2+1} \right]_8^4 du$$

$$= \frac{1}{27} \left[(4+9x)^{3/2} \right]_6^4$$

$$= \frac{1}{27} \left[(4+0)^{3/2} - (4+36)^{3/2} \right]$$

$$= \frac{1}{27} (4)^{3/2} - (40)^{3/2}]$$

iv) $x = 3\sin t \quad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\, dt$$

$$= \int_0^{2\pi} 3\,dt = 3\int_0^{2\pi} dt = 3(t]^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

5.) $x = \frac{1}{6}y^3 + \frac{1}{2y} \quad y = [1,2)$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4-1}{2y^2}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$$

(18) $\int_0^4 x^2\,dx$

$h = \frac{4-0}{4} = 1$

$= \begin{array}{cccccc} x & 0 & 1 & 2 & 3 & 4 \\ y & 0 & 1 & 4 & 9 & 16 \end{array}$

$= \int_0^4 x^2\,dx = \frac{1}{3}\left[16 + 4(10) + 8\right]$

$= 64/3$

$= \int_0^4 x^2\,dx = 21.5333$

$= 1/2 \int_0^4 \sqrt{4 + 9x}\,dx$

$= \frac{1}{2}\left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right] dx$

$= \frac{1}{27}\left(4 + 9x\right)^{3/2}\Big]_0^8$

$= \frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$

$= \frac{1}{27}\left[(4)^{3/2} - (40)^{3/2}\right]$

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PRACTICAL NO:- 7

Topic :- Differential Equation.

1) $x \frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x} \quad Q(x) = \frac{e^x}{x}$

$I.F = e^{\int P(x)dx}$

$= e^{\int 1/x\, dx}$

$= e^{\log x}$

$IF = x$

$y(I.F) = \int Q(x)(I.F)dx + c$

$= \int \frac{e^x}{x} x\, dx + c$

$= \int e^x dx + c$

$= xy = e^x + c$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x}$ (÷ by e^x)

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$$ (divide by x on Both sides)

$p(x) = 3/x \quad q(x) = \sin x / x^3$

$= e^{\int p(x)\,dx}$

$= e^{\int 3/x\,dx}$

$= e^{\ln x^3}$

$IF = x^3$

$y(IF) = \int q(x)\,(IF)\,dx + c$

$= \int \frac{\sin x}{x^3}\, x^3\,dx + c$

$= \int \sin x\,dx + c$

$x^3 y = \cos x + c$

(5) $e^{2x}\frac{dy}{dx} + 2e^{2x} y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{x} = 2e^{-2x}$

$If = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$y\,[IF] = \int Q(x)\,(I.F)\,dx + c$

$= \int 2x\,e^{-2x}\,dx + c\,dx$

(6) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\sec^2 x \tan y\, dx = -\sec^2 y \tan x\, dy$

$\sec^2 x \tan y\, dx$

$$\frac{\sec^2 x\, dx}{\tan x} = \frac{-\sec^2 y\, dy}{\tan y}$$

$$\int \frac{\sec^2 x\, dx}{\tan x} = -\int \frac{\sec^2 y\, dy}{\tan y}$$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x - \tan y| = c$

$\tan x + \tan y = e^c$

(7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Differentiation Both Sides

$x-y+1 = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{1-dy}{dx} = \frac{dv}{dx}$$

$$\frac{1-dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

PRACTICAL NO:- 8

(1) $\frac{dy}{dx} = y+e^{x}-2$ $y(0)=2$, $h=0.5$, find $y(2)$

(2) $\frac{dy}{dx} = 1+y^2$, $y(0)=0$, $h=0.2$ find $y(1)$

(3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0)=1$, $h=0.2$ find $y(1)$

(4) $\frac{dy}{dx} = 3x^2+1$, $y(1)=2$, find $y(2)$
for $h=0.5$, $h=0.25$

(5) $\frac{dy}{dx} = \sqrt{xy}+2$ $y(1)=1$ find $y(1.2)$ with
$h=0.2$

ANSWERS

(1) $\frac{dy}{dx} = y+e^{x}-2$

$f(x,y) = y+e^{x}-2$, $y_0=2$, $x_0=0$, $h=0.5$

n	x_n	y_n	$f(x_n,y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.487	3.57435
2	1	3.5743	4.2925	5.3615

$$\boxed{y_{n+1} = y_n + h\, f(x_n, y_n)}$$

n	x_n	y_n	$f(x_n,y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

$\therefore$ By Euler's formula

$y(2) = 9.2831$

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(2) $\frac{dy}{dx} = 1+y^2$

$f(x,y) = 1+y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

using Euler's iteration formula,

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$
0	0	0	1
1	0.2	0.2	1.04
2	0.4	0.408	1.1665
3	0.6	0.643	1.4113
4	0.8	0.9236	1.8530
5	1	1.2442	

$\therefore$ By Euler's formula

$y(1) = 1.2442$

(9) $\frac{dy}{dx} = 3x^2+1$, $y_0 = 2$, $x_0 = 1$, $n = 0$

for $h = 0.5$

Using Eulers iteration formula

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n\ y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

By Eulers formula

for $h = 0.25$

n	x_n	y_n	$f(x_n\ y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.73594	10.1815	8.9048
4	2	8.9048		

∴ By Eulers formula

$y(2) = 8.9048$

$$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$$

At $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By applying limits

$$= \frac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$$

$$= \frac{-64+3+1-1}{4+8}$$

$$= \frac{-61}{9}$$

$$\lim_{(x,y)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

At $(2,0)$ Denominator $\neq 0$

$\therefore$ By applying limits.

$$= \frac{(0+1)((2)^2+0-4(2))}{2+0}$$

$$= \frac{1(4+0-8)}{2}$$

$$= \frac{-4}{2}$$

$$= -2$$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

At (1,1,1), Denominator = 0

$\therefore \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{(x-yz)(x+yz)}{x^2(x-yz)}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x^2}$

On applying limits

$= \dfrac{1+1(1)}{(1)^2}$

$= 2$

Q2 (i) $f(x,y) = xye^{x^2+y^2}$

$\therefore f_x = \dfrac{\partial}{\partial x}(f(x,y))$

$= \dfrac{\partial}{\partial x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \dfrac{\partial}{\partial y}(f(x,y))$

$= \dfrac{\partial}{\partial y}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

$f_x = \dfrac{\partial}{\partial x}(f(x,y))$

$= \dfrac{\partial}{\partial x}(e^x \cos y)$

$\therefore f = e^x \cos y$

$f_y = \dfrac{\partial}{\partial y}(f(x,y))$

$= \dfrac{\partial}{\partial y}(e^x \cos y)$

$\therefore f = -e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \dfrac{\partial}{\partial x}(f(x,y))$

$= \dfrac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_x = 3x^2y^2 - 6xy$

$f_y = \dfrac{\partial}{\partial y}(f(x,y))$

aa $\frac{d}{dy}(x^3 y^2 \ldots)$

$\frac{d}{dy}(\ldots y + y \ldots 1)$

$= \frac{d}{dx}(x^3y^2 - 3x^2y + \ldots)$

$f_y = 2x^3y - 3x^2 + 3y^2$

$f_y = 2x^3y - 3x^2 + 3y^2$

Q3(i) $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{d}{dx}\left(\frac{2x}{1+y^2}\right)$

$$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x\frac{d}{dx}(1+y^2)}{(1+y^2)}$$

$$= \frac{2+2y^2-0}{(1+y^2)^2}$$

$$= \frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y^2)}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1}$$

$$= 2$$

$f_y = \frac{d}{dy}\left(\frac{2x}{1+y^2}\right)$

$= \frac{x^4(-2x^4 - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2)}{x^6}$

$f_{yy} = \frac{d}{dy}\left(\frac{2y-x}{x^2}\right)$

$= \frac{2-0}{x^2} = \frac{2}{x^2}$

$f_{xy} = \frac{d}{dy}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$ — (1)

$f_{xy} = \left(\frac{-x^2 - 4xy + 2x^2}{x^4}\right)$ — (2)

$= \frac{-x^2 - 4xy + 2x^2}{x^4}$

$f_{yx} = \frac{d}{dx}\left(\frac{2y-x}{x^2}\right)$ — (3)

$f_{yx} = \frac{d}{dx}\left(\frac{2y-x}{x^2}\right)$

$= \frac{x^2\frac{d}{dx}(2y-x) - (2y-x)(x^2)}{(x^2)^2}$

$= \cancel{\frac{-x^2 - 4xy + 2x^2}{x^4}}$ — (4)

From (3) & (4)

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}$ (1)

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{d}{dx}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\cos(xy) + e^{x+y})$

$\cancel{f_{xy}} = -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ — (2)

$f_{yy} = \frac{d}{dy}\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ — (1)

$f_{yx} = \frac{d}{dy}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — (3)

~~From~~ (1) & (2)

$\cancel{f_{xy}} \neq f_{yx}$

Q5 (i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow$ $f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\quad$ $f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad$ $= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\quad$ f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \cancel{\frac{2}{\sqrt{2}}}$

$= \frac{x+y}{\sqrt{2}}$

(ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

$f(\frac{\pi}{2}, 0) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$

$f_x = 0 - 1 + y\cos x$ $\quad$ $f_y = 0 - 0 + \sin x$

f_x at $(\frac{\pi}{2}, 0) = -1 + 0$
$= -1$ $\quad$ f_y at $(\frac{\pi}{2}, 0) = \sin\frac{\pi}{2}$
$= 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)(x - \frac{\pi}{2}) + 1(y-0)$

$= 1 - \cancel{\frac{\pi}{2}} - x + \cancel{\frac{\pi}{2}} + y$

$= 1 - x + y$

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\quad$ $f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\quad$ f_y at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

15. PRACTICAL NO :- 10

1) $f(x,y) = x+2y-3$ $\quad a=(1,-1)$, $u=3i-j$

→ Here

$u = 3i - j$ is not a unit vector.

$\bar{u} = 3\bar{i} - \bar{j}$

$|u| = \sqrt{10}$

Unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3\bar{i} - \bar{j})$

$= \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

Now,

$f(a+hu) = f\left((1,-1)+h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$

$= 1+2(-1)-3 + \left(\frac{3h}{\sqrt{10}}, \frac{-h}{\sqrt{10}}\right) + \left(1+\frac{3h}{\sqrt{10}}, -1-\frac{h}{\sqrt{10}}\right)$

$= -4 + \frac{3h}{\sqrt{10}} - 4 \quad 1 + \frac{3h}{\sqrt{10}} + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$= 1-2-3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$

$= -4 + \frac{h}{\sqrt{10}}$

$\therefore D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{-4+\frac{h}{\sqrt{10}} - (-4)}{h}$

$= \lim_{h\to 0} \frac{h}{h\sqrt{10}}$

$= \frac{1}{\sqrt{10}}$

2) $f(x,y) = y^2 - 4x + 1$ $\quad (3,4)$, $u = i+5j$

→ Here,

$u = i+5j$ is not a unit vector.

$\bar{u} = \bar{i} + 5\bar{j}$

$|\bar{u}| = \sqrt{26}$

$\therefore$ unit vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{26}}(\bar{i}+5\bar{j})$

$= \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

Now,

$f(a+hu) = f\left((3,4)+h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$

$= f\left(3+\frac{h}{\sqrt{26}}, 4+\frac{5h}{\sqrt{26}}\right)$

$= \left(4+\frac{5h}{\sqrt{26}}\right)^2 - 4\left(3+\frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$= \lim \frac{25h^2}{} + \frac{36h}{}$

SS.

$$= \lim_{h \to 0} \left(\frac{25h}{26} + \frac{36}{\sqrt{26}} \right)$$

$$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$$

$$= \frac{36}{\sqrt{26}}$$

iii) $f(x,y) = 2x + 3y$ $\qquad a = (1,2)$, $u = 3i + 4j$

Here,

$u = 3i + 4j$ is not unit vector

$\bar{u} = 3\bar{i} + 4\bar{j}$

$|u| = \sqrt{25} = 5$

$\therefore$ Unit vector along $u = \frac{|u|}{|u|} = \frac{1}{5}(3\bar{i} + 4\bar{j})$

$$= \frac{1}{5}(3,4)$$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

Now, $f(a + hu) = f\left((1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)\right)$

$$= f\left(1 + \frac{3h}{5}, \frac{2 + 4h}{5}\right)$$

$$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= 8 + \frac{18h}{5}$$

$D_u f(a) = \lim_{h \to 0} \frac{f(a + hu) - f(a)}{h}$

$$= \lim_{h \to 0} \frac{8 + \frac{18h}{5} - 8}{h}$$

$$= \lim_{h \to 0} \frac{18h}{5h}$$

ii) $f(x,y) = x^y + y^x$ $\qquad x = (1,1)$

$f_x = y(x^{y-1}) + y^x \log y$

$f_y = x(y^{x-1}) + x^y \log x$

$\nabla f(x,y) = (f_x, f_y)$

$$= (yx^{y-1} + y^x \log y, \; xy^{x-1} + x^y \log x)$$

$\nabla f(x,y)$ at $(1,1)$

$$= (1(1)^0 + 1^1 \log(1), \; 1(1)^{(1-1)} + 1^1 \log 1)$$

$$= (1,1)$$

iii) $f(x,y) = (\tan^{-1} x) - y^2$

$f_x = y^2 \left(\frac{1}{1+x^2}\right) = \frac{y^2}{1+x^2}$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$$

$\nabla f(x,y)$ at $(1,-1)$

$$= \left(\frac{(-1)^2}{1+(-1)^2}, 2(-1) \tan^{-1}(1)\right)$$

$$= \left(\frac{1}{2}, \frac{-2\pi}{4}\right)$$

$$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$$

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iii) $F(x,y,z) = xyz - e^{x+y+z}$

$F_x = yz - e^{x+y+z}$

$F_y = xz - e^{x+y+z}$

$F_z = xy - e^{x+y+z}$

$\nabla F(x,y,z) = (F_x, F_y, F_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$\nabla F(x,y,z)$ at $(1,-1,0)$

$= (-1(0) - e^{1-1+0}, 1(0) - e^{1-1+0}, 1(-1) - e^{1-1+0})$

$= (0-1, 0-1, -1, -1)$

$= (-1, -1, -2)$

Q3

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$F(x,y) = x^2 \cos y + e^{xy} - 2$

$F_x = 2x \cos y + y e^{xy}$

$F_y = -x^2 \sin y + x e^{xy}$

$(x_0, y_0) = (1,0)$

F_x at $(1,0) = 2(1) \cos 0 + 0$

$= 2$

F_y at $(1,0) = -(1)^2 \sin(0) + 1(e)^{1(0)}$

$= 1$

$F_x(x-x_0) + F_y(y-y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$\therefore 2x - 2 + y = 0$

$\therefore 2x + y - 2 = 0$ Equivalent of Tangent

Now,

For equation of Normal:

$bx + ay + d = 0$

$x + 2y + d = 0$

$(1) + 2(0) + d = 0$ at $(1,0)$

$1 + a = 0$

$a = -1$

$\therefore x + 2y - 1 = 0 \longrightarrow$ Equation of Normal

ii) $x^2 y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$F(x,y) = x^2 + y^2 - 2x + 3y + 2$

$F_x = 2x + 0 + 0 + 0$

$= 2x - 2$

$F_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

F_x at $(2,-2) = 2(2) - 2$

$= 2$

F_y at $(2,-2) = 2(-2) + 3$

$= -1$

$\therefore$ Equation of tangent

$F_x(x-x_0) + F_y(y-y_0) = 0$

$2(x-2) + (-1)(y+2) = 0$

$2x - 4 - y - 2 = 0$

$2x - y - 6 = 0 \longrightarrow$ Equation of Tangent

For Equation of Normal:

$bx + ay + d = 0$

$-x + 2y + d = 0$

$-(2) + 2(-2) + d = 0$ at $(2,-2)$

$-2 - 4 + d = 0$

$d = +6$

$\therefore -x + 2y + 6 = 0 \longrightarrow$ Equation of Normal.

#5 at $(2,10)$

Q4. i) $x^2-2yz+3y+xz=7$

$f(x,y,z)=x^2-2yz+3y+xz-7$

$f_x=2x-0+0+z-0$

$=2x+z$

$f_y=-2z+3+0-0$

$=-2z+3$

$f_z=0-2y+0+x-0$

$=2y+x$

$\therefore f_x$ at $(2,1,0)=2(2)+0$

$=4$

f_y at $(2,1,0)=-2(0)+3$

$=3$

f_z at $(2,10)=-2(1)+2$

$=0$

Equation of Tangent,

$f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$

$4(x-2)+3(y-1)+0(z-0)=0$

$4x-8+3y-3=0$

$\therefore 4x+3y-11=0$

Equation of Normal,

$$\frac{x-x_0}{f_x}=\frac{y-y_0}{f_y}=\frac{z-z_0}{f_z}$$

$$\frac{x-2}{4}=\frac{y-1}{3}=\frac{z-0}{0}$$

ii) $3xyz-x-y+z=-4$ at $(1,-1,2)$

$f(x,y,z)=3xyz-x-y+z+4$

$f_x=3yz-1-0+0+0$

$=3yz-1$

$f_y=3xz-0-1+0+0$

$=3xz-1$

$f_x=3xy-0-0+1-0$

$=3xy+1.$

f_x at $(1,-1,2)=3(-1)(2)-1$

$=-7$

f_y at $(1,-1,2)=3(1)(2)-1$

$=5$

f_z at $(1,-1,2)=3(1)+1$

$=-2$

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Now,

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12 - 9$

$= 3 > 0$

Here, $r > 0$ & $rt - s^2 > 0$

$\therefore f$ has minimum at $(0, 2)$

$f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

(7) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy - 0$

$= 8x^3 + 6xy$

$f_y = 0 + 3x^2 - 2$

$= 3x^2 - 2y$.

Now,

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 6xy) = 0$

$4x^2 + 6xy = 0$ — (1)

multiply in (1) by 3 & (2) by (4) & subtract

(2) from (1)

$12x^2 + 18y = 0$

$-12x^2 - 18y = 0$

$24y = 0$

$\boxed{y = 0}$ — (3)

Substituting (3) & (2)

$3x^2 - 2(0) = 0$

$3x^2 = 0$