

2] Easy

1. Insertion
2. Selection
3. Bubble

Advanced

1. merge
2. Quick

1. Insertion Worst - $O(n^2)$ Best - $O(n)$
 2. Selection Worst - $O(n^2)$ Best - $O(n)$
 3. Bubble Worst - $O(n^2)$ Best - $O(n^2)$

* Advanced Algo:-

1. Merge Sort:-

a
b
c

"Merge two sorted arrays":

Q:- Two arrays a_1 & a_2

Input:- $a_1 = [2, 4, 5, 6]$ // Sorted

$a_2 = [1, 3, 4, 8]$ // Sorted

→ Merge these two sorted arrays

O/P:- $[1, 2, 3, 4, 4, 5, 6, 8]$

Approach-1

length of $a_1 = m$
length of $a_2 = n$

1. Create an empty array of size $m+n$ // $O(1)$

→ $[2, 4, 5, 6, 1, 3, 4, 8]$ $O(m+n)$

2. Copy elements from a_1 & a_2 into new array.

3. Apply any known sorting algo. to sort the new array. $\Rightarrow O((m+n)^2)$

$O(1) + O(m+n) + O((m+n)^2)$

$\Rightarrow O((m+n)^2)$

Approach-2

$a_1 = [2, 4, 5, 6]$ // m
 $a_2 = [1, 3, 4, 8]$ // n

Create an empty array of size $m+n$

res: $[]$

if $(a_1[p_1] < a_2[p_2])$ if it is true
 $res[k] = a_1[p_1]$
 $k++$

else // $a_1[p_1] > a_2[p_2]$
 $res[k] = a_2[p_2]$

if $(a_1[p_1] < a_2[p_2])$ {

$res[k] = a_1[p_1]$

$k++$

} else {

$res[k] = a_2[p_2]$

$k++$

}

Sorted

p_1

p_2

Sorted
 $a_1 = [2, 4, 5, 6]$ // m
 $a_2 = [1, 3, 4, 8]$ // n

res: []

Sorted
 $a_1 = [2, 4, 5, 6]$ // m
 $a_2 = [1, 3, 4, 8]$ // n
 $4 < 4$

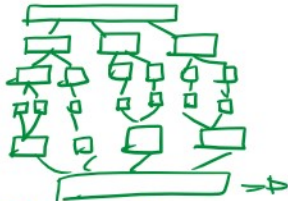
res: []

$a_1 = [2, 4, 5, 6]$ // Sorted
 $a_2 = [1, 3, 4, 8, 9, 10]$ // Sorted

[1, 2, 3, 4, 4, 5, 6, 8, 9, 10]

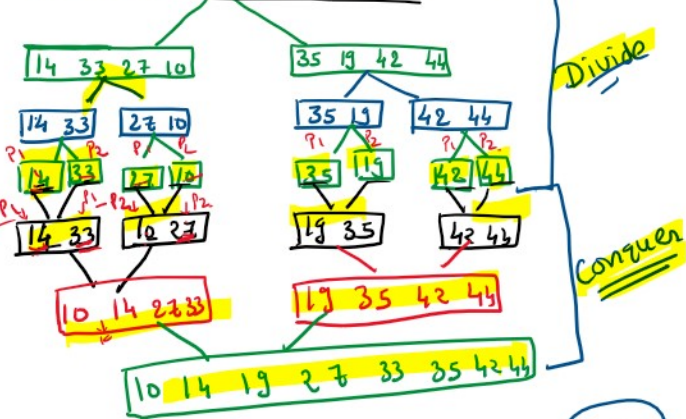
④ Merge Sort:-

1) Divide and Conquer.



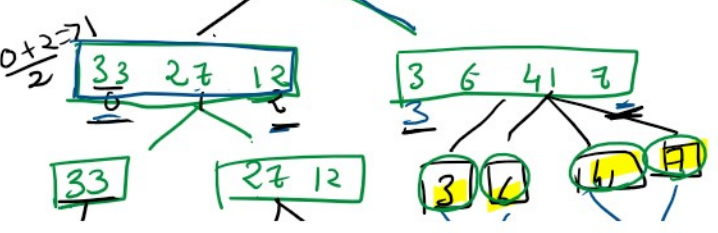
① Divide & conquer

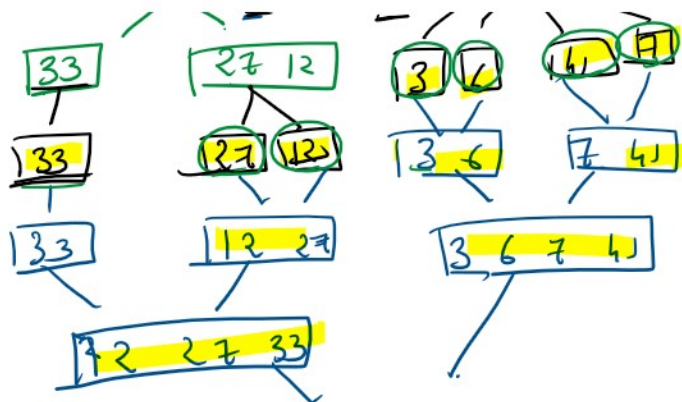
14 33 27 10 35 19 42 44



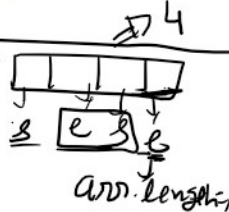
33 27 12 3 6 41 7

$$\frac{0+6}{3} = 2$$





3 6 7 12 27 33 41

function mergeSort (arr, s, e) ^{arr} 
 {
 [if (start > end)] Base Case
 return;
 var mid = parseInt((start+end)/2);
 mergeSort (arr, start, mid);
 mergeSort (arr, mid+1, end);

```
function MergeSort(arr, start, end){
  if(start >= end)
    return;

  //divide process
  let mid = parseInt((start+end)/2);
  MergeSort(arr, start, mid);
  MergeSort(arr, mid+1, end);
  // elements are not divided further

  //conquer process
}
```

```
function merge(arr, start, mid, end){
}
```

arr, s, mid arr, mid+1, e

MergeTwoSortedArrays (arr, start, mid, end)

8 elems

MS (arr, 0, 7)



$$\frac{0+7}{2} \Rightarrow 3.5 \Rightarrow 3$$

MS (arr, 0, 3)

MS (4, 7)

(arr, mid+1, e)

arr, s, mid

MS (arr, 0, 1)

MS (arr, 2, 3)

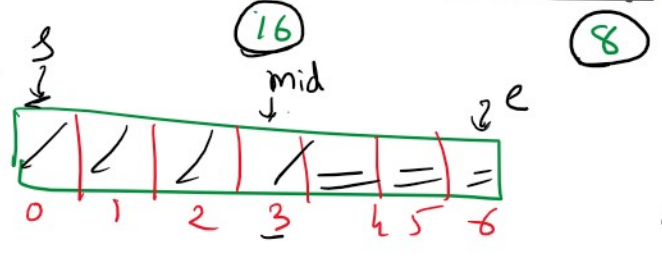
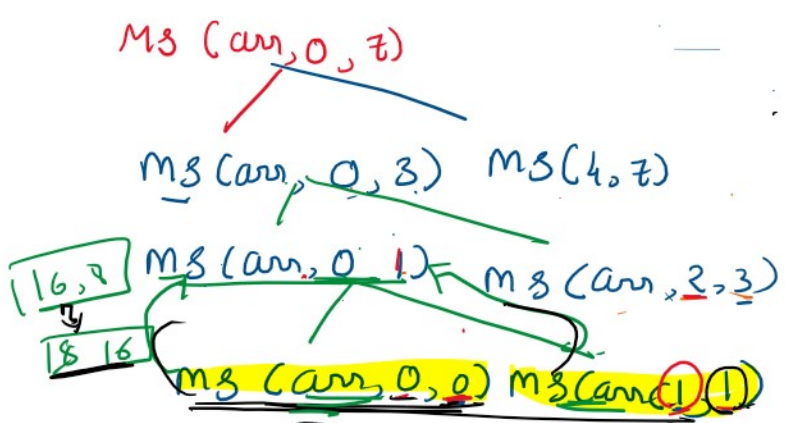
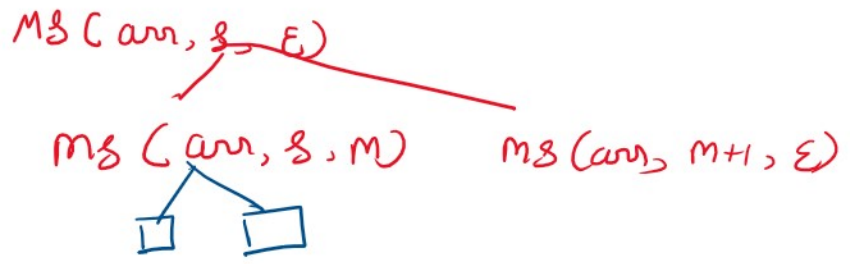
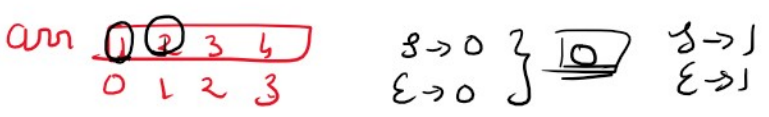
$$\frac{0+1}{2} \Rightarrow 0.5 \Rightarrow 0$$

MS (arr, 0, 0) MS (arr, 1, 1)

$$\frac{1+1}{2} \Rightarrow 1$$

$$0+0 \Rightarrow 0$$

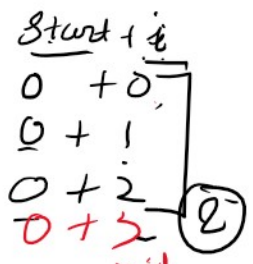
$\frac{1+1}{2}$
 $\text{ms(arr, 0, 0) ms(arr, 1, 1)}$
 $\frac{0+0}{2} \Rightarrow 0$

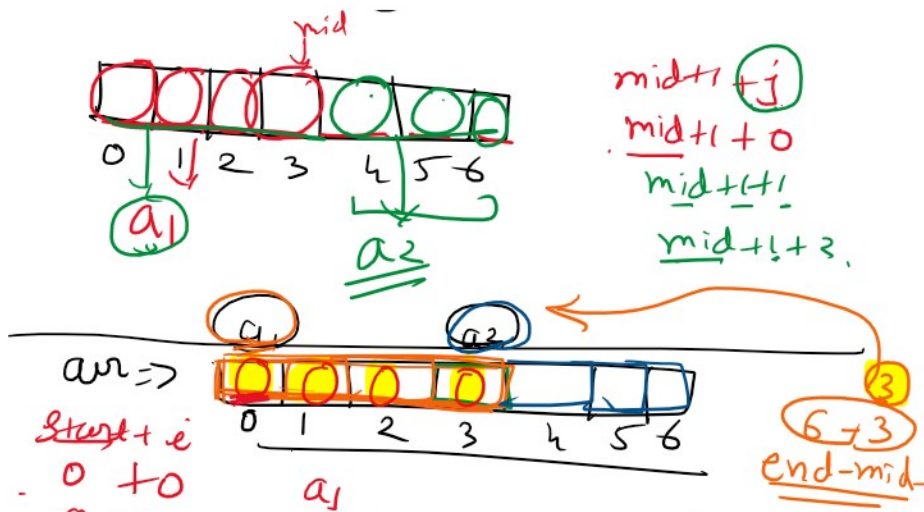


$\frac{0+6}{2} \Rightarrow 3$

$a_1 = \text{mid} - s + 1 = 3 - 0 + 1 \Rightarrow 4$
 $a_2 = e - \text{mid} = 6 - 3 = 3 \Rightarrow a_2$

$m \Rightarrow 4$





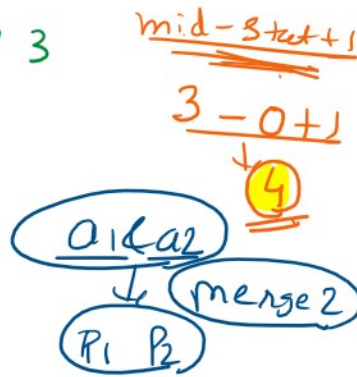
```
function merge(arr, start, mid, end) {
    let m1 = mid - start + 1;
    let m2 = end - mid;

    let a1 = new Array(m1);
    let a2 = new Array(m2);

    for (i = 0; i < m1.length; i++) {
        a1[i] = arr[start + i];
    }

    for (i = 0; i < m2.length; i++) {
        a2[i] = arr[mid + 1 + i];
    }

    merge2(a1, a2, start, mid, end);
}
```



- ① MS (first half) (arr, s, m)
- ② MS (second half) (arr, m+1, e)
- ③ Merge $\frac{n}{2} \times \frac{n}{2}$

R.R

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + O(n)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n) \quad \text{--- (1)}$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \frac{n}{2} \quad \text{--- (2)}$$

② → ①

$$T(n) = 2 \left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2} \right] + n$$

$$T(n) = 2 \left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2} \right] + n \Rightarrow 2n$$

$$T(n) = 2 \left[2T\left(\frac{n}{2}\right) + \frac{n}{2} \right] + n \rightarrow (1)$$

$$T\left(\frac{n}{2}\right) = 2 + \left(\frac{n}{2}\right) + \frac{n}{2}$$

$$T(n) = 2 \cdot 3T\left(\frac{n}{2}\right) + 3n \rightarrow (3)$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$$

$$T(1)$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log n = k \log_2 2$$

$$k = \log n$$

$$2^{\log_2 n} T(1) + n \log n$$

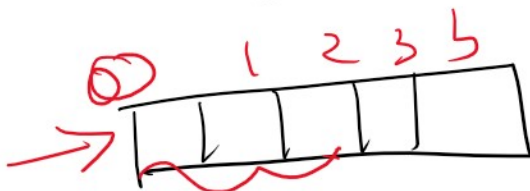
$$n \times 1 + n \log n$$

$$\Rightarrow n \log n$$

$$S.C \Rightarrow O(n)$$

$$\text{Time} \Rightarrow O(n \log n) \rightarrow \begin{matrix} B \\ W \\ A \end{matrix}$$

$$S.C \Rightarrow O(n)$$



Siddhanta

