## Wake Model Description

for the

Optimization Only Case Study IEA Task 37 on System Engineering in Wind Energy

## Wake Model

The wake model for this study is a simplified version of Bastankhah's Gaussian wake model [1]. The governing equations for the velocity deficit in a waked region are:

$$\frac{\Delta U}{U_{\infty}} = \left(1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/D^2}}\right) \exp\left(-0.5\left(\frac{y - \delta}{\sigma_y}\right)^2\right) \tag{1}$$

$$\sigma_y = k_y \cdot x + \frac{D}{\sqrt{8}} \tag{2}$$

Where:

Variable	Value	Definition
$\frac{\Delta U}{U_{\infty}}$	-	Wake velocity deficit
$C_T$	$\frac{8}{9}$	Thrust coefficient
$y - \delta$	_	Dist. from hub of interest to the wake center in cross-stream direction
D	130 m	Turbine diameter
$\sigma_y$	Eq. (2)	Standard deviation of the wake deficit
$k_y$	0.0324555	Variable based on a turbulence intensity of 0.075 [1, 2]
x	-	Downstream dist. from hub generating wake to hub of interest

Partial wake is not considered. Hub coordinates are used for all location calculations. For turbines placed in multiple wakes, the compound velocity deficit is calculated using the square root of the sum of the squares, depicted in Eq. (3):

$$\left(\frac{\Delta U}{U_{\infty}}\right)_{cmbnd} = \sqrt{\left(\frac{\Delta U}{U_{\infty}}\right)_{1}^{2} + \left(\frac{\Delta U}{U_{\infty}}\right)_{2}^{2} + \left(\frac{\Delta U}{U_{\infty}}\right)_{3}^{2} + \dots}$$
(3)

## References

- [1] Thomas, J. J. and Ning, A., "A method for reducing multi-modality in the wind farm layout optimization problem," *Journal of Physics: Conference Series*, The Science of Making Torque from Wind, Milano, Italy, June 2018.
- [2] Niayifar, A. and Porté-Agel, F., "Analytical Modeling of Wind Farms: A New Approach for Power Prediction," *Energies*, September 2016.