Wake Model Description

for the

Optimization Only Case Study

IEA Task 37 on System Engineering in Wind Energy

This is an explanatory enclosure to accompany `iea37-wflocs-announcement.pdf`.

For the Optimization Only Case Study, we will use the enclosed Python file `iea37-aepcalc.py` to evaluate your reported optimal turbine locations in `.yaml` format. If you desire to implement the AEP calculations in a language other than Python, the algorithm's description and wake model equations are provided below. Please insure your implementation computes the same AEP value given in each of the example layouts (`iea37-ex##.yaml`) also enclosed.

AEP Algorithm

1. Read the following input from `.yaml` files:
   - Turbine (x,y) locations.
   - Turbine attributes (cut-in\cut-out\rated wind speed\rated power).
   - Number of wind directional bins, θ_i ($i = 16$ for these Case Studies).
   - Wind frequency at each binned direction, $f(\theta)$.
   - Wind speed at each binned direction, $V_\infty(\theta)$ (invariant for these Case Studies).
2. Calculate the power produced from each turbine at each direction:
   (a) For each binned direction θ, rotate frame of reference so "North" aligns with $-x$ axis:
      i. $\theta = 270° - \theta$
      ii. If $\theta \leq 0°$
         Then $\theta = \theta + 360°$
   (b) Rotate turbine locations (x,y) to match new wind frame of reference (x_w,y_w):
      - $x_w = x \cdot cos(\text{-}\theta) - y \cdot sin(\text{-}\theta)$
      - $y_w = x \cdot sin(\text{-}\theta) + y \cdot cos(\text{-}\theta)$
   (c) Iterating through each turbine in the field:
      - Apply the B. Gaussian Eq. (2) between each turbine pair for wake deficit ($\frac{\Delta U}{U_\infty}$).
      - Use Eq. (4) to calculate total wake loss $(\frac{\Delta U}{U_\infty})_{cmbnd}$ at each turbine.
      - Use wake loss and freestream speed to calculate effective wind speed (V_e):
      $$V_e = V_\infty \cdot \left[1 - \left(\frac{\Delta U}{U_\infty}\right)_{cmbnd}\right]$$
      - Use V_e and the IEA37 3.35MW power curve to calculate each turbine's power:
      $$P_{turb}(V_e) = \begin{cases} 0 & V_e < V_{cut\text{-}in} \\ P_{rated} \cdot \left(\frac{V_e - V_{cut\text{-}in}}{V_{rated} - V_{cut\text{-}in}}\right)^3 & V_{cut\text{-}in} \leq V_e \leq V_{rated} \\ P_{rated} & V_{rated} < V_e < V_{cut\text{-}out} \\ 0 & V_{cut\text{-}out} \leq V_e \end{cases} \tag{1}$$
3. Use calculated turbine power $\left(P_{turb}\right)$ from every wind direction (θ) to compute AEP:
   - For each direction bin θ, sum power from all n turbines, multiply by wind freq. $f(\theta)$:
   $$P_{farm}(\theta) = \sum\nolimits_0^n P_{turb}(\theta) \cdot f(\theta)$$
   - Multiply by hours in a year for AEP at each bin, sum all i bins for total AEP.
   $$AEP = \left(\sum\nolimits_0^i P_{farm}(\theta_i)\right) \cdot 8760\frac{\text{hrs}}{\text{yr}}$$

Wake Model Equations

The wake model for the Optimization Only Case Study is a simplified version of Bastankhah's Gaussian wake model [1]. The governing equations for the velocity deficit in a waked region are:

$$\frac{\Delta U}{U_\infty} = \left(1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/D^2}}\right)\exp\left(-0.5\left(\frac{y_i - y_g}{\sigma_y}\right)^2\right) \tag{2}$$

$$\sigma_y = k_y \cdot (x_i - x_g) + \frac{D}{\sqrt{8}} \tag{3}$$

Where:

Variable	Value	Definition
$\frac{\Delta U}{U_\infty}$	Eq. (2)	Wake velocity deficit
C_T	$\frac{8}{9}$	Thrust coefficient
$x_i - x_g$	-	Dist. from hub generating wake (x_g) to hub of interest (x_i), along freestream
$y_i - y_g$	-	Dist. from hub generating wake (y_g) to hub of interest (y_i), $\perp$ to freestream
σ_y	Eq. (3)	Standard deviation of the wake deficit
k_y	0.0324555	Variable based on a turbulence intensity of 0.075 [1, 2]
D	130 m	Turbine diameter [3]

Note that if the hub of interest is upstream from the hub generating the wake ($[x_i - x_g] \leq 0$), it feels no wake effects ($\frac{\Delta U}{U_\infty} = 0$). In other words:

$$\left(\frac{\Delta U}{U_\infty}\right) = \begin{cases} 0, & (x_i - x_g) \leq 0 \\ Eq.\ (2), & (x_i - x_g) > 0 \end{cases}$$

Partial wake is not considered. Hub coordinates are used for all location calculations. For turbines placed in multiple wakes, the compound velocity deficit is calculated using the square root of the sum of the squares, depicted in Eq. (4):

$$\left(\frac{\Delta U}{U_\infty}\right)_{cmbnd} = \sqrt{\left(\frac{\Delta U}{U_\infty}\right)_1^2 + \left(\frac{\Delta U}{U_\infty}\right)_2^2 + \left(\frac{\Delta U}{U_\infty}\right)_3^2 + \dots} \tag{4}$$

References

[1] Thomas, J. J. and Ning, A., "A method for reducing multi-modality in the wind farm layout optimization problem," *Journal of Physics: Conference Series*, The Science of Making Torque from Wind, Milano, Italy, June 2018.

[2] Niayifar, A. and Porté-Agel, F., "Analytical Modeling of Wind Farms: A New Approach for Power Prediction," *Energies*, September 2016.

[3] Bortolotti, P., Dykes, K., Merz, K., Sethuraman, L., and Zahle, F., "IEA Wind Task 37 on System Engineering in Wind Energy, WP2 - Reference Wind Turbines," Tech. rep., National Renewable Energy Laboratory (NREL), Golden, CO., May 2018.